

1. For how many integers k with $0 < k < 18$ is

$$\frac{5 \sin(10k^\circ) - 2}{\sin^2(10k^\circ)} \geq 2 ?$$

2. If $\frac{1}{\cos x} - \tan x = 3$, what is the value of $\sin x$

3. Without using a calculator, determine positive integers m and n for which

$$\sin^6 1^\circ + \sin^6 2^\circ + \sin^6 3^\circ + \dots + \sin^6 87^\circ + \sin^6 88^\circ + \sin^6 89^\circ = \frac{m}{n}$$

4. Simplify:

$$\sqrt{(\sin^4 x + 4 \cos^2 x)} + \sqrt{(\cos^4 x + 4 \sin^2 x)}$$

5. If $\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} + \frac{1}{\tan^2 x} + \frac{1}{\cot^2 x} = 7$, find the value of $\sin^2 2x$

6. Given $Q = \sin^6 x + \cos^6 x - m(\sin^4 x + \cos^4 x)$. Find real number m such that the value of the expression Q does not depend on the value of x , and calculate the new value of Q (if m exists).

7. Determine all real numbers $x > 0$ for which

$$\log_4 x - \log_x 16 = \frac{7}{6} - \log_x 8$$

8. Determine all real values of x for which $\log_2 (2^{x-1} + 3^{x+1}) = 2x - \log_2 (3^x)$

9. Determine all real numbers x for which $(\log_{10} x)^{\log_{10} (\log_{10} x)} = 10000$

10. Determine all values of x such that $\log_{2x}(48\sqrt[3]{3}) = \log_{3x}(162\sqrt[3]{2})$
11. Determine all pairs (a, b) of real numbers that satisfy the following system of equations:

$$\sqrt{a} + \sqrt{b} = 8$$

$$\log_{10} a + \log_{10} b = 2$$