计算机视觉

Computer Vision

Lecture 8 Camera Calibration

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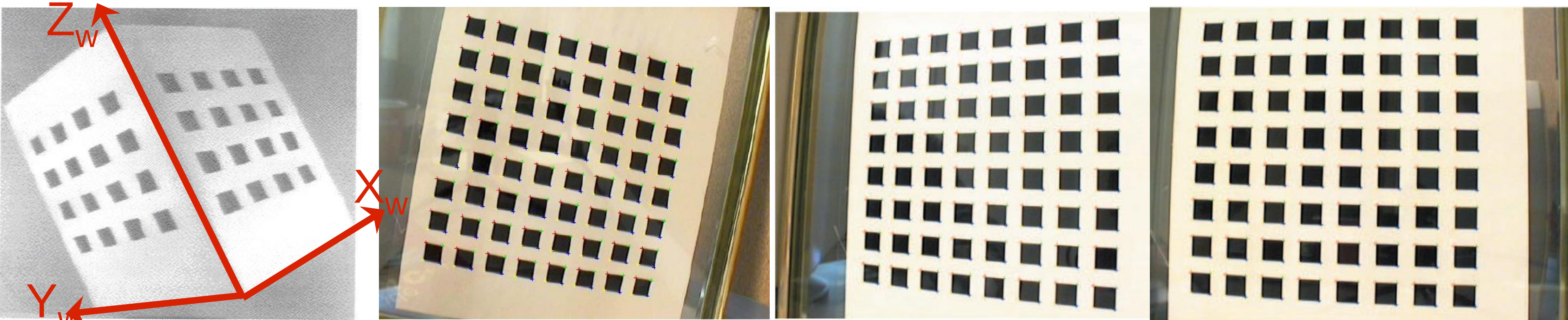

Camera Calibration

- **Calibration: Problem definition**
- **Solution by Nonlinear Least Squares**
- **Multi-plane calibration**
- **Calibration Software**

Camera Calibration

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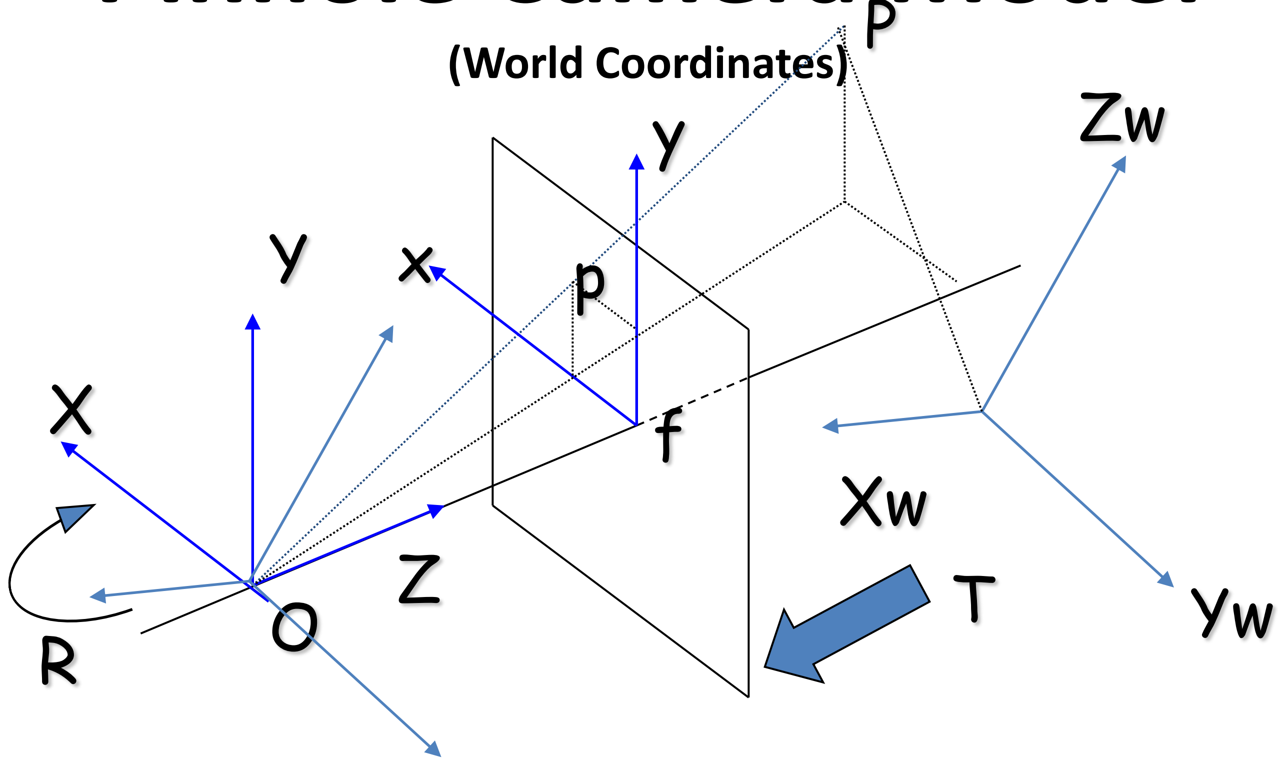
Pinhole Camera Model

(World Coordinates)



$$P = R \cdot T \cdot P_w = M_{\text{ext}} \cdot P_w$$

$$p = M_{\text{int}} P = M_{\text{int}} M_{\text{ext}} \cdot P_w$$

Pinhole Camera Model

- Extrinsic parameters (R, T):

$$P = R \cdot T \cdot P_w = M_{\text{ext}} \cdot P_w$$

- Intrinsic parameter (f):

$$p = M_{\text{int}} P = M_{\text{int}} M_{\text{ext}} \cdot P_w$$

$$p = M \cdot P_w$$

Projective camera

$$P' = M\,P_w = \underbrace{K}_{\text{Internal parameters}}\underbrace{\begin{bmatrix} R & T \end{bmatrix}}_{\text{External parameters}} P_w$$

Internal parameters

External parameters

$$\boxed{\mathcal{M} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta}\boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix}_{3\times 4}}$$

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o \\ 0 & \dfrac{\beta}{\sin\theta} & v_o \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \qquad T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Goal of calibration

Estimate intrinsic and extrinsic parameters from 1 or multiple images

$$P' = M\, P_w \quad = K\begin{bmatrix} R & T \end{bmatrix} P_w$$

$$\mathcal{M} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta}\boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix}_{3\times 4}$$

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o \\ 0 & \dfrac{\beta}{\sin\theta} & v_o \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \qquad T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

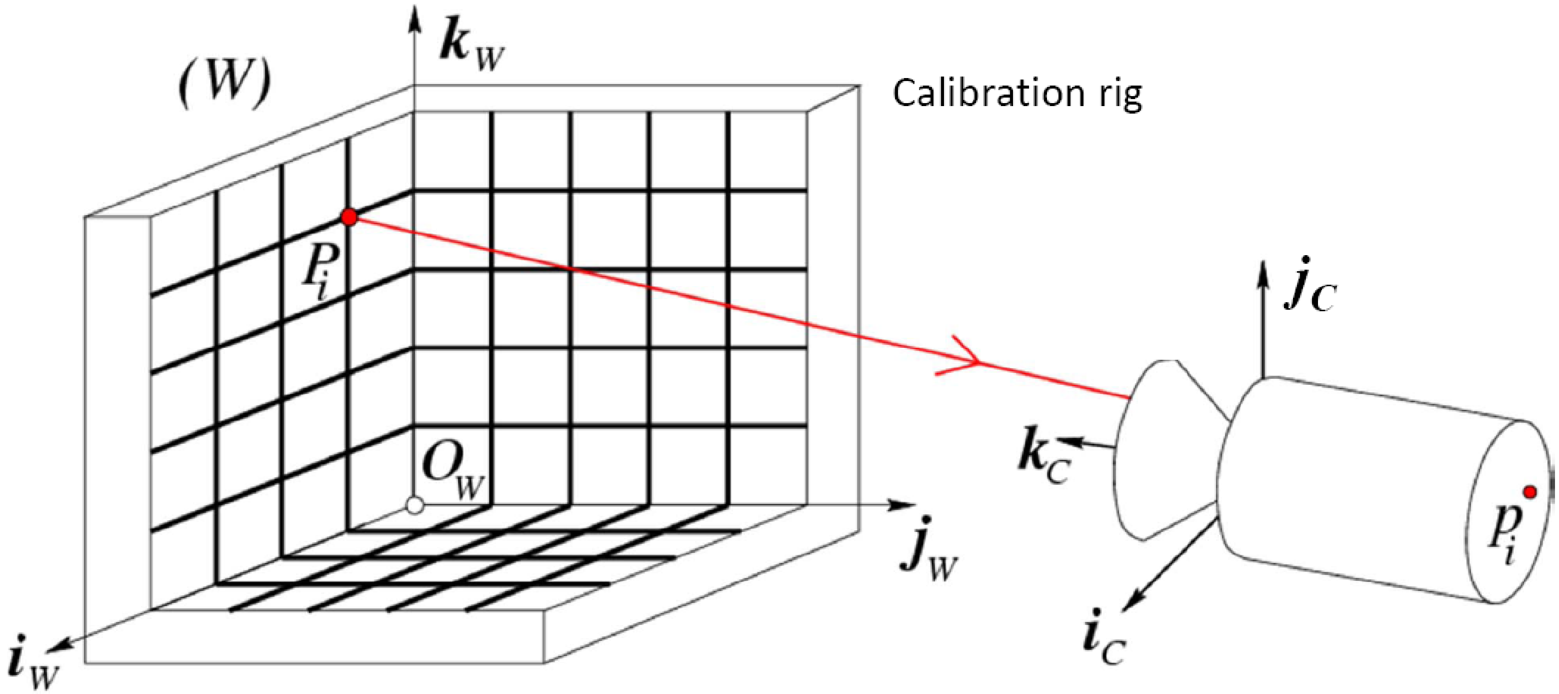
Change notation:
$P = P_w$

$p = P'$

Camera Calibration

- Determine extrinsic and intrinsic parameters of camera
  - Extrinsic
    - 3D location and orientation of camera
  - Intrinsic
    - Focal length
    - The size of the pixels

Camera Calibration



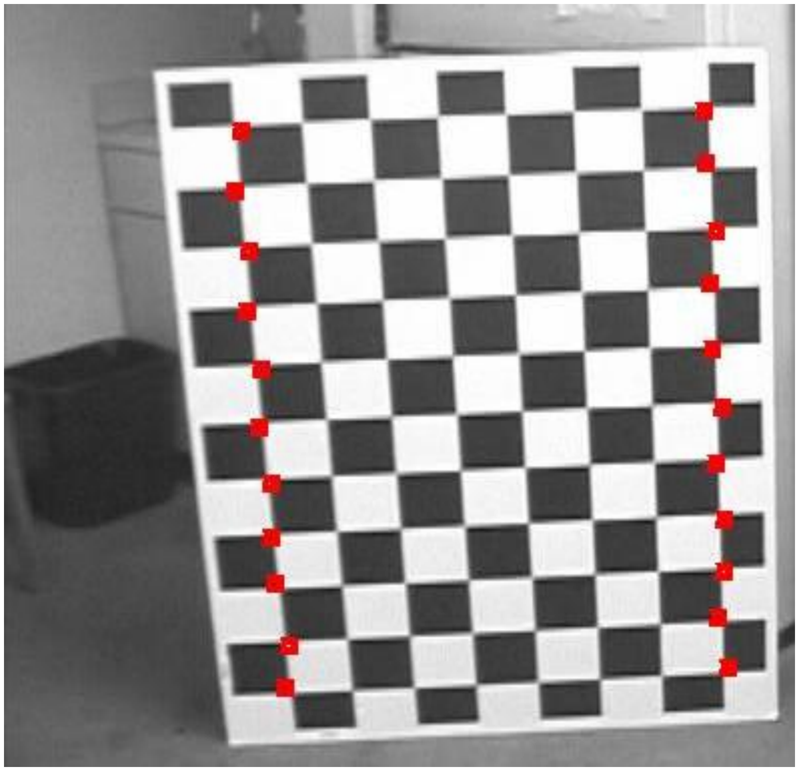
- $P_1 \ldots P_n$ with known positions in $[O_w, i_w, j_w, k_w]$
- $p_1, \ldots p_n$ known positions in the image

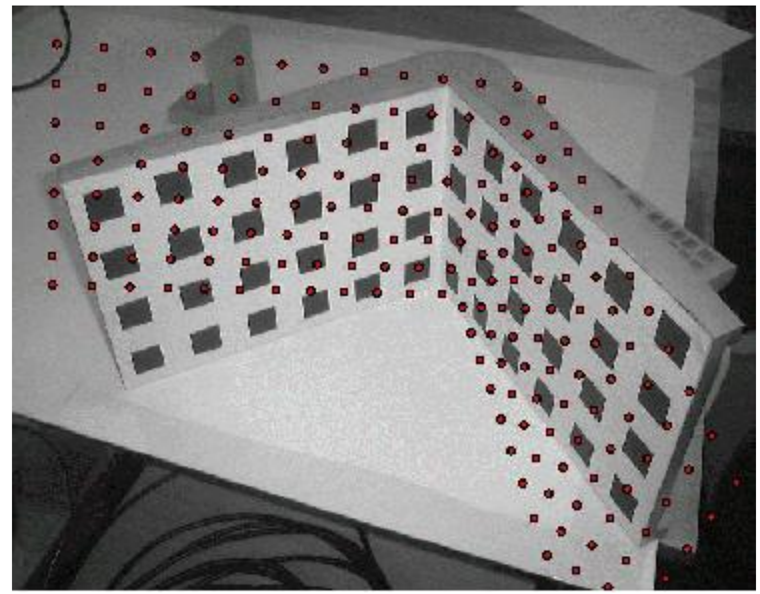
Goal: compute intrinsic and extrinsic parameters

Calibration

- Known calibration object, many views
- Compute intrinsics and extrinsics
- (Retain intrinsics, toss extrinsics)

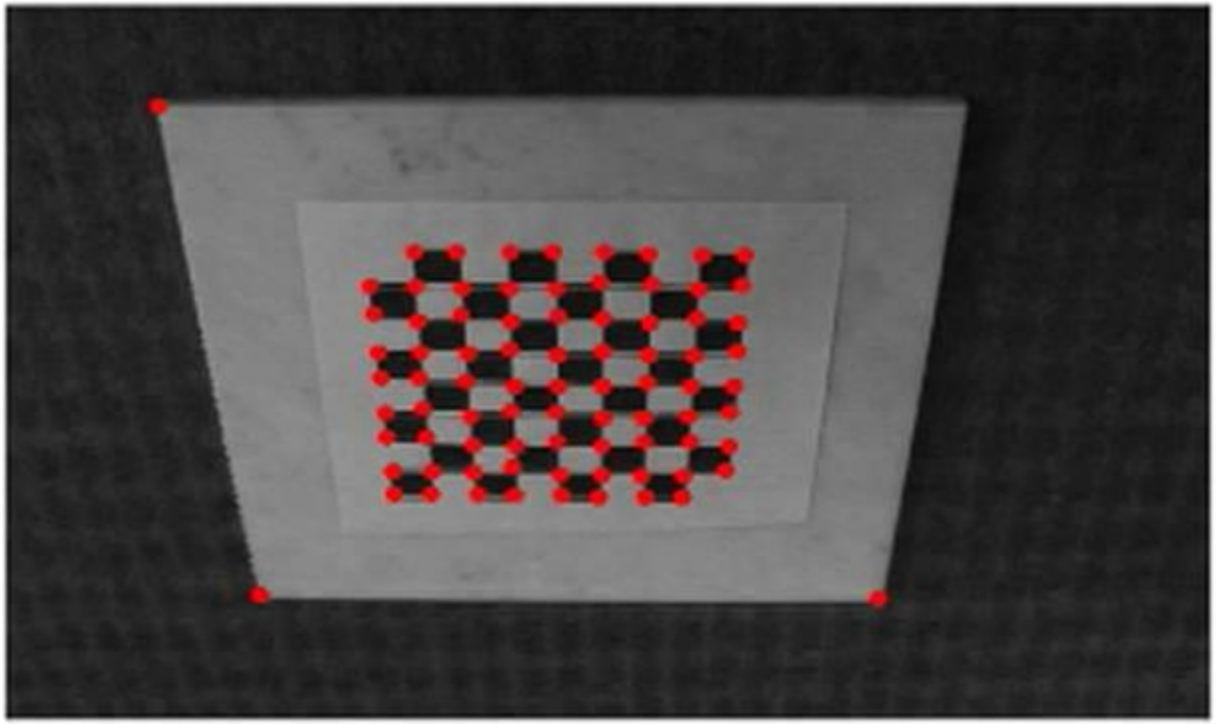
Example Calibration Pattern





Calibration Pattern: Object with features of known size/geometry

Harris Corner Detector



Direct linear calibration - homogeneous

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

$$u_i = \frac{m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}}$$

$$v_i = \frac{m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}}$$

$$u_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}) = m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}$$

$$v_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}) = m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}$$

One pair of equations for each point

$$\begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -u_iX_i & -u_iY_i & -u_iZ_i & -u_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -v_iX_i & -v_iY_i & -v_iZ_i & -v_i \end{bmatrix} \begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \\ m_{03} \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{20} \\ m_{21} \\ m_{22} \\ m_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Direct linear calibration - homogeneous

$$\underbrace{\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\ & & & & & & \vdots & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n & -u_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n & -v_n \end{bmatrix}}_{\substack{\mathbf{A} \\ 2n \times 12}} \underbrace{\begin{bmatrix} m_{00} \\ m_{10} \\ m_{02} \\ m_{03} \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{20} \\ m_{21} \\ m_{22} \\ m_{23} \end{bmatrix}}_{\substack{\mathbf{m} \\ 12}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{\substack{\mathbf{0} \\ 2n}}$$

This is a homogenous set of equations.

When over constrained, defines a least squares problem

– minimize $\|\mathbf{Am}\|$

- Since **m** is only defined up to scale, solve for unit vector $\mathbf{m}^*$
- Solution: $\mathbf{m}^*$ = eigenvector of $\mathbf{A}^T\mathbf{A}$ with *smallest* eigenvalue
- Works with 6 or more points

Extracting camera parameters

$$\frac{\mathcal{M}}{\rho} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha\cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha\cot\theta t_y + u_0 t_z \\ \frac{\beta}{\sin\theta}\boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \frac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix} = K\begin{bmatrix} R & T \end{bmatrix}$$

$$A \qquad \mathbf{b}$$

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o \\ 0 & \frac{\beta}{\sin\theta} & v_o \\ 0 & 0 & 1 \end{bmatrix}$$

Box 1

$$A = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\rho = \frac{\pm 1}{|\mathbf{a}_3|} \qquad u_o = \rho^2(\mathbf{a}_1 \cdot \mathbf{a}_3) \qquad v_o = \rho^2(\mathbf{a}_2 \cdot \mathbf{a}_3)$$

$$\cos\theta = \frac{(\mathbf{a}_1 \times \mathbf{a}_3)\cdot(\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_1 \times \mathbf{a}_3| \cdot |\mathbf{a}_2 \times \mathbf{a}_3|}$$

Extracting camera parameters

$$\frac{\mathcal{M}}{\rho} = \left(\underbrace{\begin{matrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T \\ \dfrac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T \\ \boldsymbol{r}_3^T \end{matrix}}_{A} \quad \underbrace{\begin{matrix} \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ t_z \end{matrix}}_{\mathbf{b}} \right) = K\begin{bmatrix} R & T \end{bmatrix}$$

$$A = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\alpha = \rho^2 |\mathbf{a}_1 \times \mathbf{a}_3| \sin\theta$$

$$\beta = \rho^2 |\mathbf{a}_2 \times \mathbf{a}_3| \sin\theta$$

Extracting camera parameters

$$\frac{\mathcal{M}}{\rho} = \left(\begin{array}{c|c} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta}\boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{array}\right) = K\begin{bmatrix} R & T \end{bmatrix}$$

$$\underbrace{}_{\mathrm{A}} \qquad \underbrace{}_{\mathbf{b}}$$

$$A = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Extrinsic

$$\mathbf{r}_1 = \frac{(\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_2 \times \mathbf{a}_3|} \qquad \mathbf{r}_3 = \frac{\pm\,\mathbf{a}_3}{|\mathbf{a}_3|}$$

$$\mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1 \qquad T = \rho\, K^{-1}\mathbf{b}$$

Direct linear calibration (transformation)

- Advantage:
  - Very simple to formulate and solve. Can be done, say, on a problem set
  - These methods are referred to as "algebraic error" minimization.
- Disadvantages:
  - Doesn't directly tell you the camera parameters (more in a bit)
  - Doesn't model radial distortion
  - Hard to impose constraints (e.g., known focal length)
  - Doesn't minimize the right error function

For these reasons, *nonlinear methods* are preferred

- Define error function E between projected 3D points and image positions
  - E is nonlinear function of intrinsics, extrinsics, radial distortion
- Minimize E using nonlinear optimization techniques
  - e.g., variants of Newton's method (e.g., Levenberg Marquart)

Camera Calibration

- **Calibration: Problem definition**
- **Solution by Nonlinear Least Squares**
- **Multi-plane calibration**
- **Calibration Software**

$i = \text{feature}$

$k = \text{image}$

Calibration constraints

- **Step 1: Transform into camera coordinates**

$$\begin{pmatrix} \tilde{X}^C[i,k] \\ \tilde{Y}^C[i,k] \\ \tilde{Z}^C[i,k] \end{pmatrix} = \begin{pmatrix} \cos\phi[k] & \sin\phi[k] & 0 \\ -\sin\phi[k] & \cos\phi[k] & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\varphi[k] & 0 & \sin\varphi[k] \\ 0 & 1 & 0 \\ -\sin\varphi[k] & 0 & \cos\varphi[k] \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\psi[k] & \sin\psi[k] \\ 0 & -\sin\psi[k] & \cos\psi[k] \end{pmatrix} \begin{pmatrix} X^W[i] \\ Y^W[i] \\ Z^W[i] \end{pmatrix} + \begin{pmatrix} T_X[k] \\ T_Y[k] \\ T_Z[k] \end{pmatrix}$$

- **Step 2: Transform into image coordinates**

$$x_{im}[i,k] = -\frac{f}{s_x} \frac{\tilde{X}^c[i,k]}{\tilde{Z}^c[i,k]} + o_x$$

$$y_{im}[i,k] = -\frac{f}{s_y} \frac{\tilde{Y}^c[i,k]}{\tilde{Z}^c[i,k]} + o_y$$

Camera Calibration

$$\sum_{i,k}\left\{\begin{pmatrix} x_{im}[i,k] \\ y_{im}[i,k] \end{pmatrix} - \begin{pmatrix} -\dfrac{f}{s_x}\dfrac{\begin{pmatrix}\cos\phi[k] & \sin\phi[k] & 0\end{pmatrix}\begin{pmatrix}\cos\varphi[k] & 0 & \sin\varphi[k] \\ 0 & 1 & 0 \\ -\sin\varphi[k] & 0 & \cos\varphi[k]\end{pmatrix}\begin{pmatrix}1 & 0 & 0 \\ 0 & \cos\psi[k] & \sin\psi[k] \\ 0 & -\sin\psi[k] & \cos\psi[k]\end{pmatrix}\begin{pmatrix}X^W[i] \\ Y^W[i] \\ Z^W[i]\end{pmatrix} + T_X[k]}{\begin{pmatrix}0 & 0 & 1\end{pmatrix}\begin{pmatrix}\cos\varphi[k] & 0 & \sin\varphi[k] \\ 0 & 1 & 0 \\ -\sin\varphi[k] & 0 & \cos\varphi[k]\end{pmatrix}\begin{pmatrix}1 & 0 & 0 \\ 0 & \cos\psi[k] & \sin\psi[k] \\ 0 & -\sin\psi[k] & \cos\psi[k]\end{pmatrix}\begin{pmatrix}X^W[i] \\ Y^W[i] \\ Z^W[i]\end{pmatrix} + T_z[k]} + o_x \\ -\dfrac{f}{s_y}\dfrac{\begin{pmatrix}-\sin\phi[k] & \cos\phi[k] & 0\end{pmatrix}\begin{pmatrix}\cos\varphi[k] & 0 & \sin\varphi[k] \\ 0 & 1 & 0 \\ -\sin\varphi[k] & 0 & \cos\varphi[k]\end{pmatrix}\begin{pmatrix}1 & 0 & 0 \\ 0 & \cos\psi[k] & \sin\psi[k] \\ 0 & -\sin\psi[k] & \cos\psi[k]\end{pmatrix}\begin{pmatrix}X^W[i] \\ Y^W[i] \\ Z^W[i]\end{pmatrix} + T_Y[k]}{\begin{pmatrix}0 & 0 & 1\end{pmatrix}\begin{pmatrix}\cos\varphi[k] & 0 & \sin\varphi[k] \\ 0 & 1 & 0 \\ -\sin\varphi[k] & 0 & \cos\varphi[k]\end{pmatrix}\begin{pmatrix}1 & 0 & 0 \\ 0 & \cos\psi[k] & \sin\psi[k] \\ 0 & -\sin\psi[k] & \cos\psi[k]\end{pmatrix}\begin{pmatrix}X^W[i] \\ Y^W[i] \\ Z^W[i]\end{pmatrix} + T_z[k]} + o_y \end{pmatrix}\right\}^2 \rightarrow \min$$

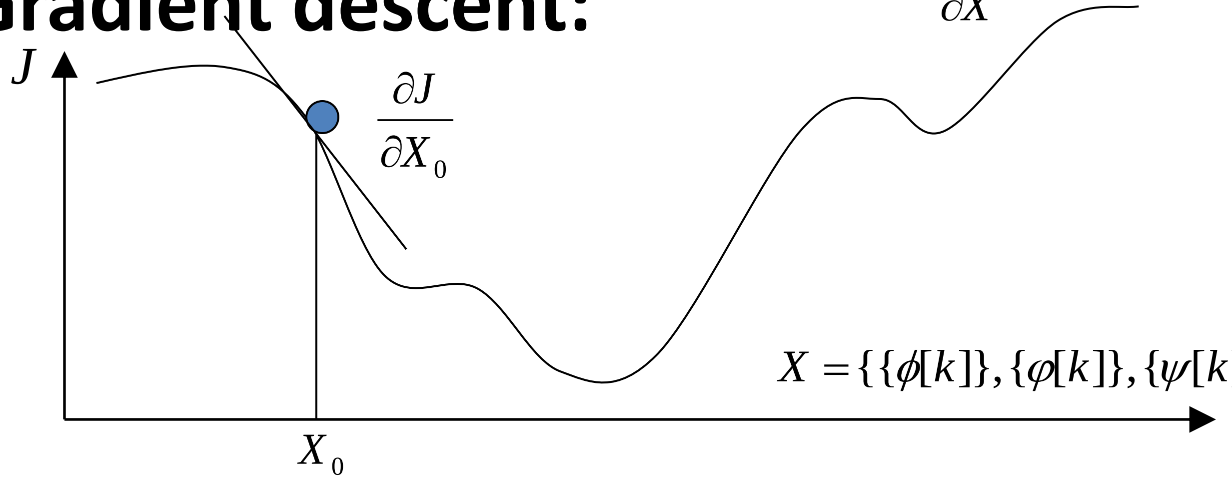
Calibration by nonlinear Least Squares

- **Least Mean Square**

$$J = \sum_{i,k} \left[\begin{pmatrix} x_{im}[i,k] \\ y_{im}[i,k] \end{pmatrix} - g\left(\begin{pmatrix} X^W[i] \\ Y^W[i] \\ Z^W[i] \end{pmatrix}, \phi[k], \varphi[k], \psi[k], T[k], f, s_x, s_y, o_x, o_y \right) \right]^2 \rightarrow \min$$

- **Gradient descent:**

$$X_k \leftarrow X_{k-1} - 0.1 \cdot \frac{\partial J}{\partial X}(X_{k-1})$$

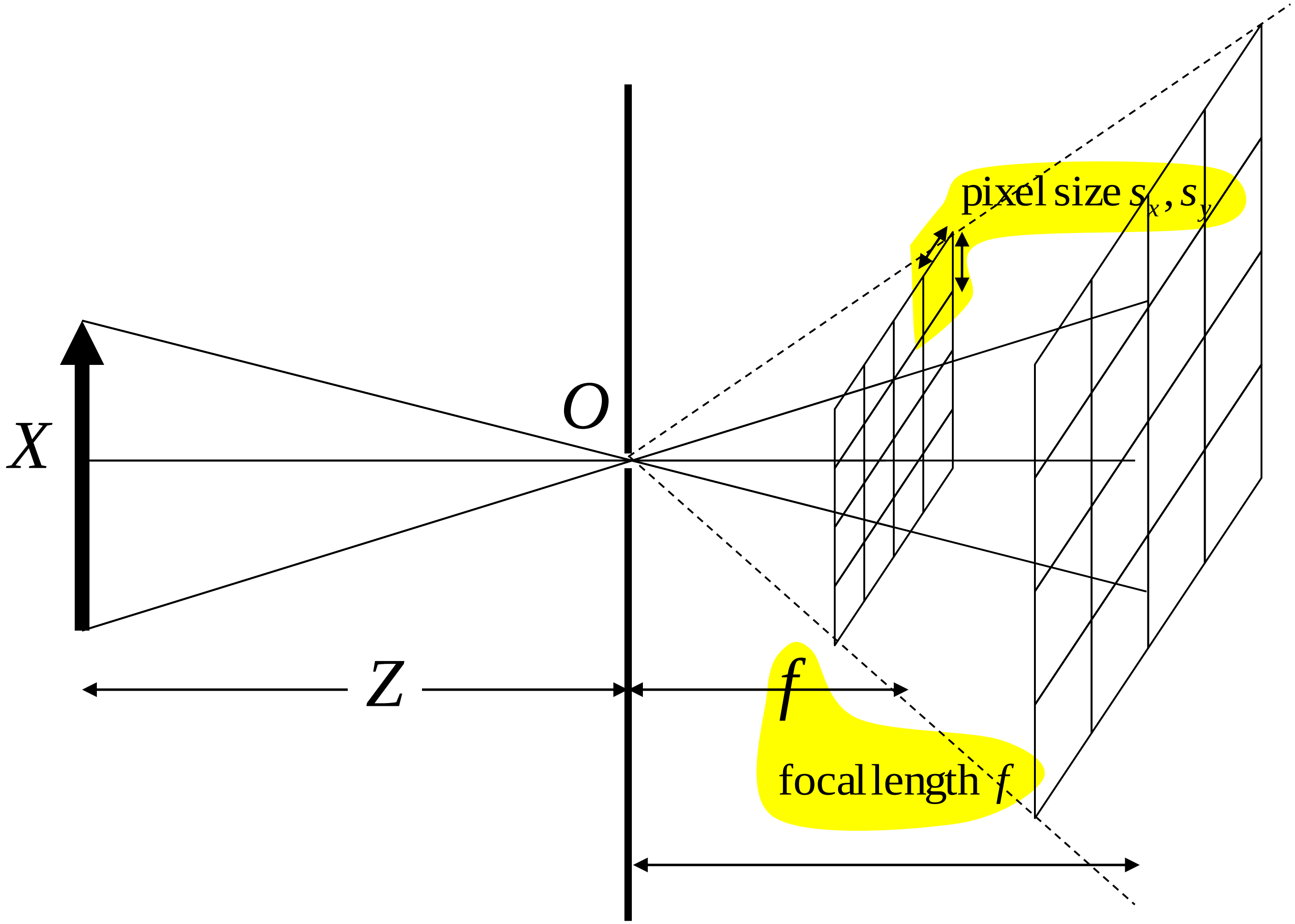


$$X = \{\{\phi[k]\}, \{\varphi[k]\}, \{\psi[k]\}, \{T[k]\}, f, s_x, s_y, o_x, o_y\}$$

The Calibration Problem

- **Given**
  - **Calibration pattern with *N* corners**
  - ***K* views of this calibration pattern**
- **How large would *N* and *K* have to be?**
- **Can we recover all intrinsic parameters?**

Intrinsic Parameters, Degeneracy



Summary Parameters, Revisited

- **Extrinsic**
  - **Rotation** $\phi[k], \varphi[k], \psi[k]$
  - **Translation** $T[k]$
- **Intrinsic**
  - ~~**Focal length**~~ ~~f~~ Focal length, in pixel units $\dfrac{f}{s_x}$
  - ~~**Pixel size**~~ ~~(s_x, s_y)~~ Aspect ratio $\alpha = \dfrac{s_x}{s_y}$
  - **Image center coordinates** (o_x, o_y)

The Calibration Problem

- Given
  - Calibration pattern with *N* corners
  - *K* views of this calibration pattern
- How large would *N* and *K* have to be?
- Can we recover all intrinsic parameters?

NO

N	1	3	1	3	4	4	6
K	1	1	3	3	3	4	6

Constraints

- N points
- K images $\Rightarrow$ $2NK$ constraints

- 4 intrinsics (distortion: +2)
- $6K$ extrinsics

$\Rightarrow$ need $2NK \geq 6K+4$

$\Rightarrow$ $(N-3)K \geq 2$

The Calibration Problem

N	1	3	1	3	4	4	6
K	1	1	3	3	3	4	6
	No	No	No	No	Yes	Yes	Yes

need $(N\text{-}3)K \geq 2$

Hint: may not be co-linear

Problem with Least Squares

- Many parameters (=slow)
- Many local minima! (=slower)

Camera Calibration

- Calibration: Problem definition
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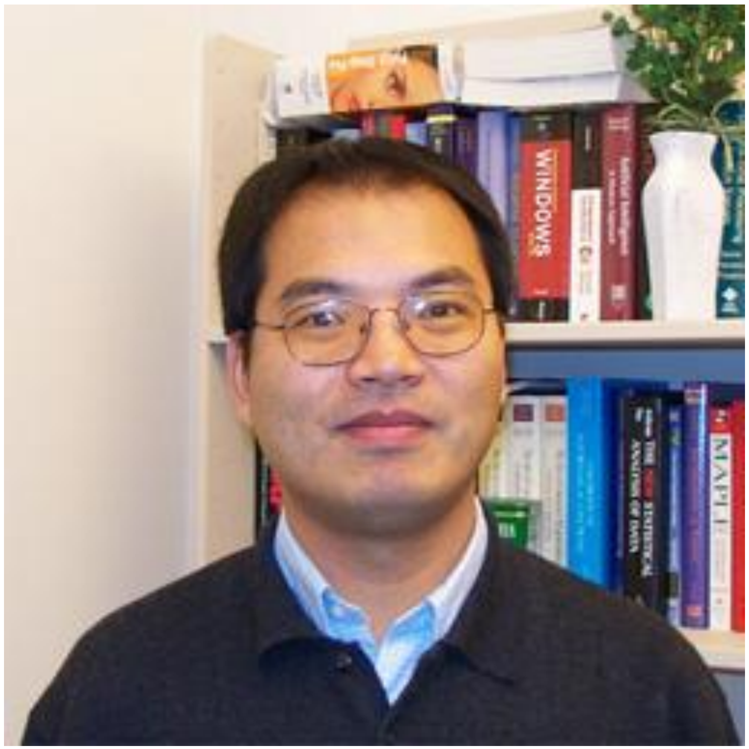
About Zhang Zhengyou



- Senior Researcher at Microsoft Research
- Ph.D. degree in computer science from the [University of Paris XI](), Orsay, France, in 1990. Advisor: [Olivier Faugeras]()

About Zhang Zhengyou

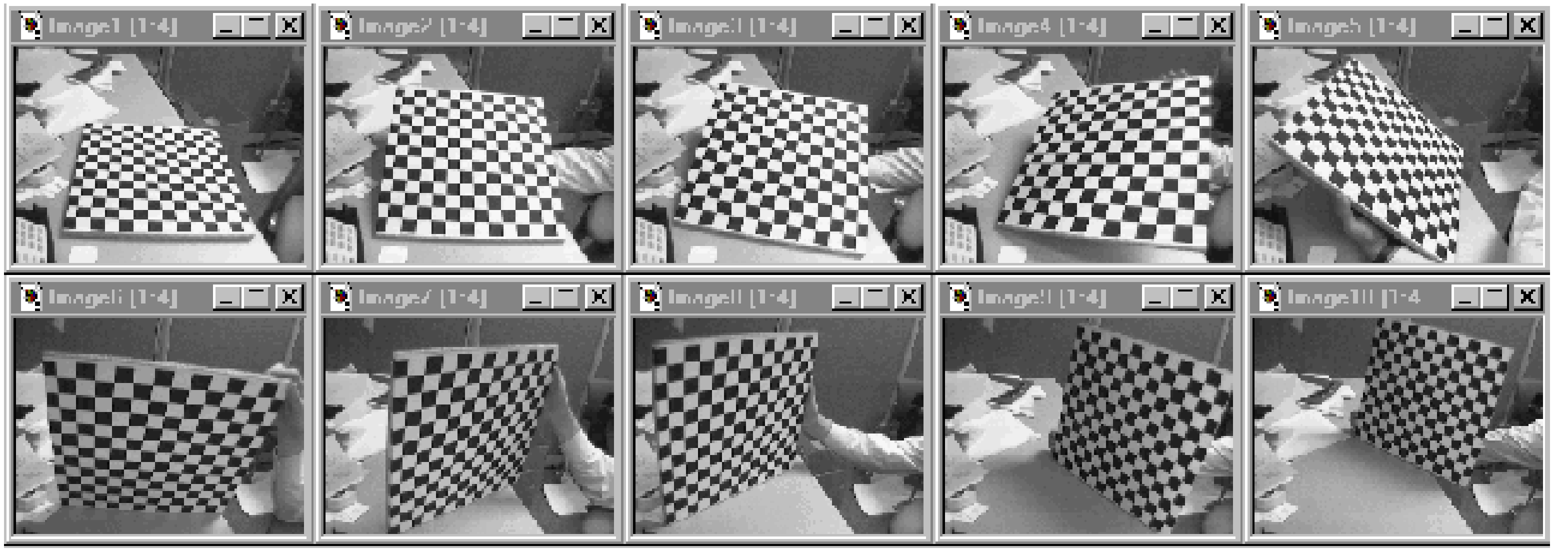
Z. Zhang, “A flexible new technique for camera calibration”, IEEE Transactions on Pattern Analysis and Machine Intelligence, 22(11):1330–1334, 2000

- **2013 ICCV Helmholtz Prize**
- **Cited more than 7,400 times**
- Most widely used camera-calibration algorithm， such as in NASA’s Mars Rover, and used to recalibrate **Kinect** sensors

Multi-plane calibration

- Easy to implement calibration technique
  - It only requires a planar pattern
  - Images at a few orientations
  - Motion need not be known
- Radial lens distortion modeled
- Process consists of:
  - Closed-form solution followed by
  - Nonlinear refinement

Multi-plane calibration



Images courtesy Jean-Yves Bouguet, Intel Corp.

Advantage

- Only requires a plane
- Don’t have to know positions/orientations

Basic Equations

- Use 2D-3D correspondences
- If we restrict our 3D points to be coplanar, then we only have to print out our calibration object, and paste it to something flat!
- Also, it simplifies the mapping between 3D and 2D:

$$s\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_u & \gamma & u_0 \\ & \alpha_v & v_0 \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{21} & r_{31} & t_1 \\ r_{12} & r_{22} & r_{32} & t_2 \\ r_{13} & r_{23} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Doesn't Contribute!

Is zero!

- Since z is zero, We can drop a column from our extrinsic matrix. Our 3x4 projection matrix becomes 3x3…

Basic Equations

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- If we restrict our 3D points to be coplanar, then we only have to print out our calibration object, and paste it to something flat!
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$$s\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_u & \gamma & u_0 \\ & \alpha_v & v_0 \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{21} & t_1 \\ r_{12} & r_{22} & t_2 \\ r_{13} & r_{23} & t_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Since z is zero, We can drop a column from our extrinsics matrix. Our 3x4 projection matrix becomes 3x3…

Identifying a homography between Model plane and Image plane

- We'll call this 3x3 transformation a homography:

$$H = \begin{bmatrix} \alpha_u & \gamma_u & u_0 \\ & \alpha_v & v_0 \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{21} & t_1 \\ r_{12} & r_{22} & t_2 \\ r_{13} & r_{23} & t_3 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} & h_{31} \\ h_{12} & h_{22} & h_{32} \\ h_{13} & h_{23} & h_{33} \end{bmatrix}$$

- There should be one homography per image
- Remember, it should satisfy the following equation:

$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = H \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Finding the Homography

- With a 2D homogenous image point [u,v,w]…

$$uw = \begin{bmatrix} x & y & 1 \end{bmatrix} \bar{h}_1^T = -u \begin{bmatrix} x & y & 1 \end{bmatrix} \bar{h}_3^T$$
$$vw = \begin{bmatrix} x & y & 1 \end{bmatrix} \bar{h}_2^T = -v \begin{bmatrix} x & y & 1 \end{bmatrix} \bar{h}_3^T$$

- Where M_j is a 3-element, homogenous 3D point, and where $\bar{h}_i$ is a row of our homography…

$$\begin{bmatrix} M_j^T & 0 & -uM_j^T \\ 0 & M_j^T & -vM_j^T \\ & \vdots & \end{bmatrix} \begin{bmatrix} \bar{h}_1^T \\ \bar{h}_2^T \\ \bar{h}_3^T \end{bmatrix} = 0$$

M=(X,Y,1)and
n points

- We can find our unknown "h" vector by singular value decomposition

Finding the Intrinsics

- With our homography as follows…

$$H = [h_1 \quad h_2 \quad h_3] = A[r_1 \quad r_2 \quad t]$$

- We already know two properties:
  - **Since r_1 and r_2 orthogonal…** $r_1 \cdot r_2 = 0$
  - **And r_1 and r_2 have equal lengths…** $r_1 \cdot r_1 = r_2 \cdot r_2$
- From our first equation, we can deduce that…

$$r_1 = A^{-1}h_1 \qquad r_2 = A^{-1}h_2$$

- So we know that these relationships must hold:

Orthogonal: $h_1^T A^{-T} A^{-1} h_2 = 0$

Equal length: $h_1^T A^{-T} A^{-1} h_1 = h_2^T A^{-T} A^{-1} h_2$

- To get our intrinsics, we just need to solve for $A^{-T} A^{-1}$

Dealing with radial distortion

$$u_{distorted} = u + (u - u_0) \cdot r \qquad v_{distorted} = v + (v - v_0) \cdot r$$

$$r = [k_1(x^2 + y^2) + k_2(x^2 + y^2)^2]$$

- **We can estimate k_1 and k_2 after having estimated the other parameters, which will give us the ideal pixel coordinates (u, v)**
- **(u', v') is the corresponding real observed image coordinates**

m points in
n images

$$\begin{bmatrix} (u - u_0)r & (u - u_0)r^2 \\ (v - v_0)r & (v - v_0)r^2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} u' - u \\ v' - v \end{bmatrix}$$

$$Dk = d \qquad k = (D^T D)^{-1} D^T d$$

Complete Maximum Likelihood Estimation

- We can minimize those pixel errors by refining our solution with (again) a nonlinear minimization technique, like Levenberg-Marquardt
- Let's minimize the following

The computed projection of point M_j in image I

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left\| m_{ij} - m(A, k_1, k_2, R_i, t_i, M_j) \right\|^2$$

- This requires initial guesses for all the parameters, which we've calculated including the radial distortion (which we can also guess is zero, and let refinement take its course)

Summary

- The recommended calibration procedure:
  - Print a pattern and attach it to a planar surface
  - Take a few images of the model plane under different orientations by moving either the plane or the camera
  - Detect the feature points in the images
  - Estimate the five intrinsic parameters and all the extrinsic parameters using the closed-form solution
  - Estimate the coefficients of the radial distortion
  - Refine all the parameters

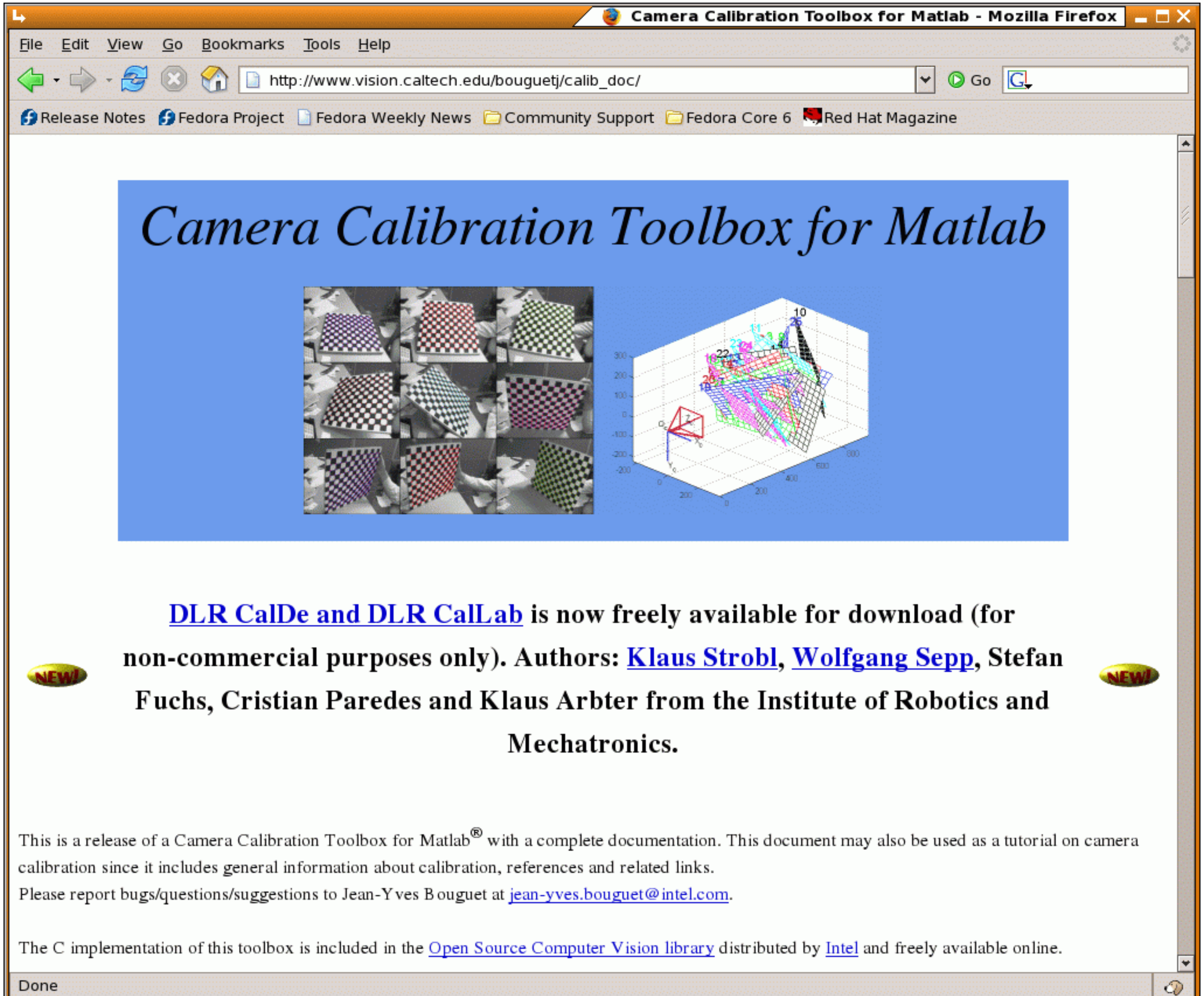
Camera Calibration

- **Calibration: Problem definition**
- **Solution by Nonlinear Least Squares**
- **Multi-plane calibration**
- **Calibration Software**

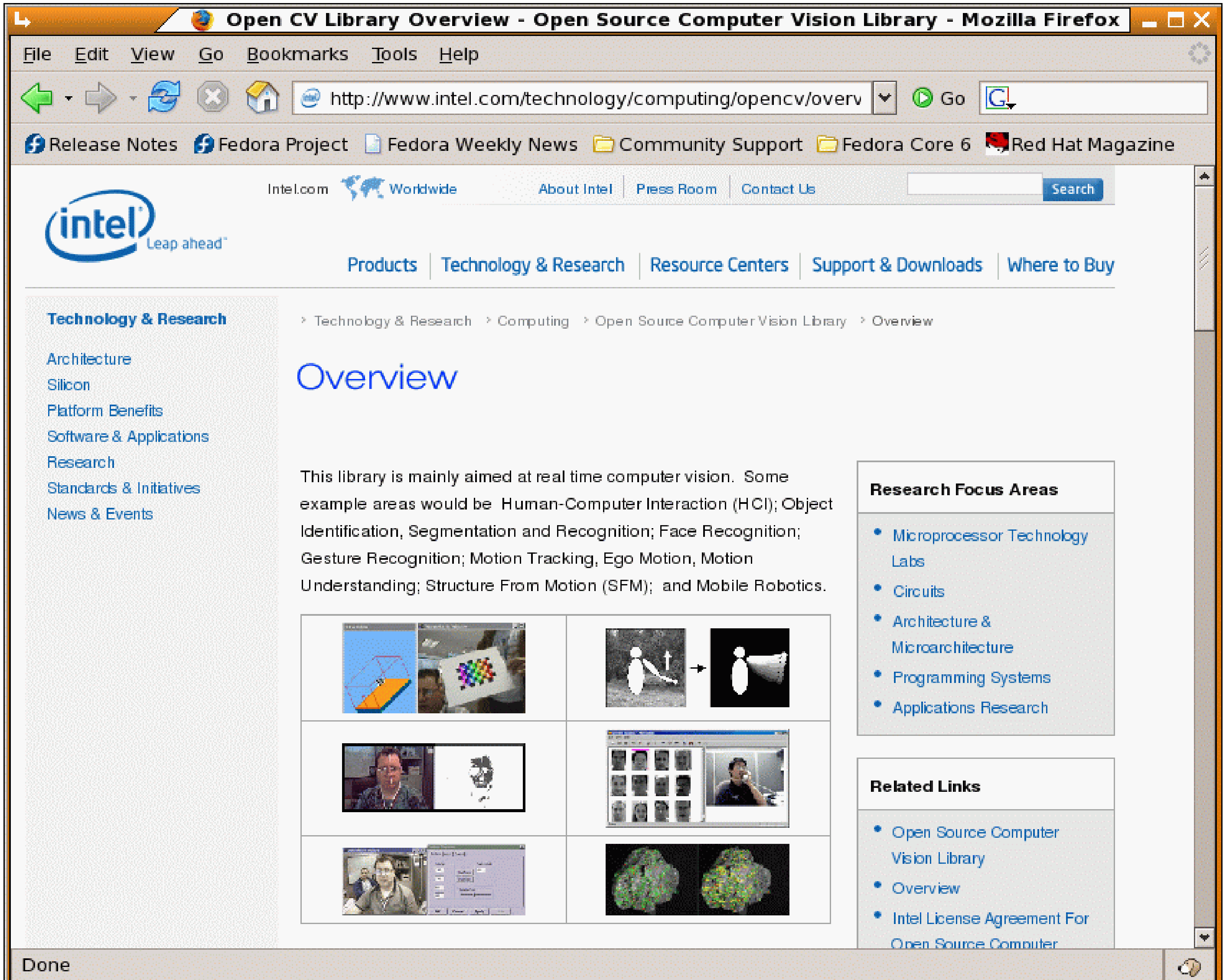
Calibration Software

- Good code available online!
  - Intel's OpenCV library: http://www.intel.com/research/mrl/research/opencv/
  - Matlab version by Jean-Yves Bouget: http://www.vision.caltech.edu/bouguetj/calib_doc/index.html
  - Zhengyou Zhang's web site: http://research.microsoft.com/~zhang/Calib/

Calibration Software: Matlab



Calibration Software: OpenCV



Readings

- **Chapter 1.3**

- **Zhengyou Zhang, A Flexible New Technique for Camera Calibration, Technical Report MSR-TR-98-71**
- **Trucco, Chapter 6.**