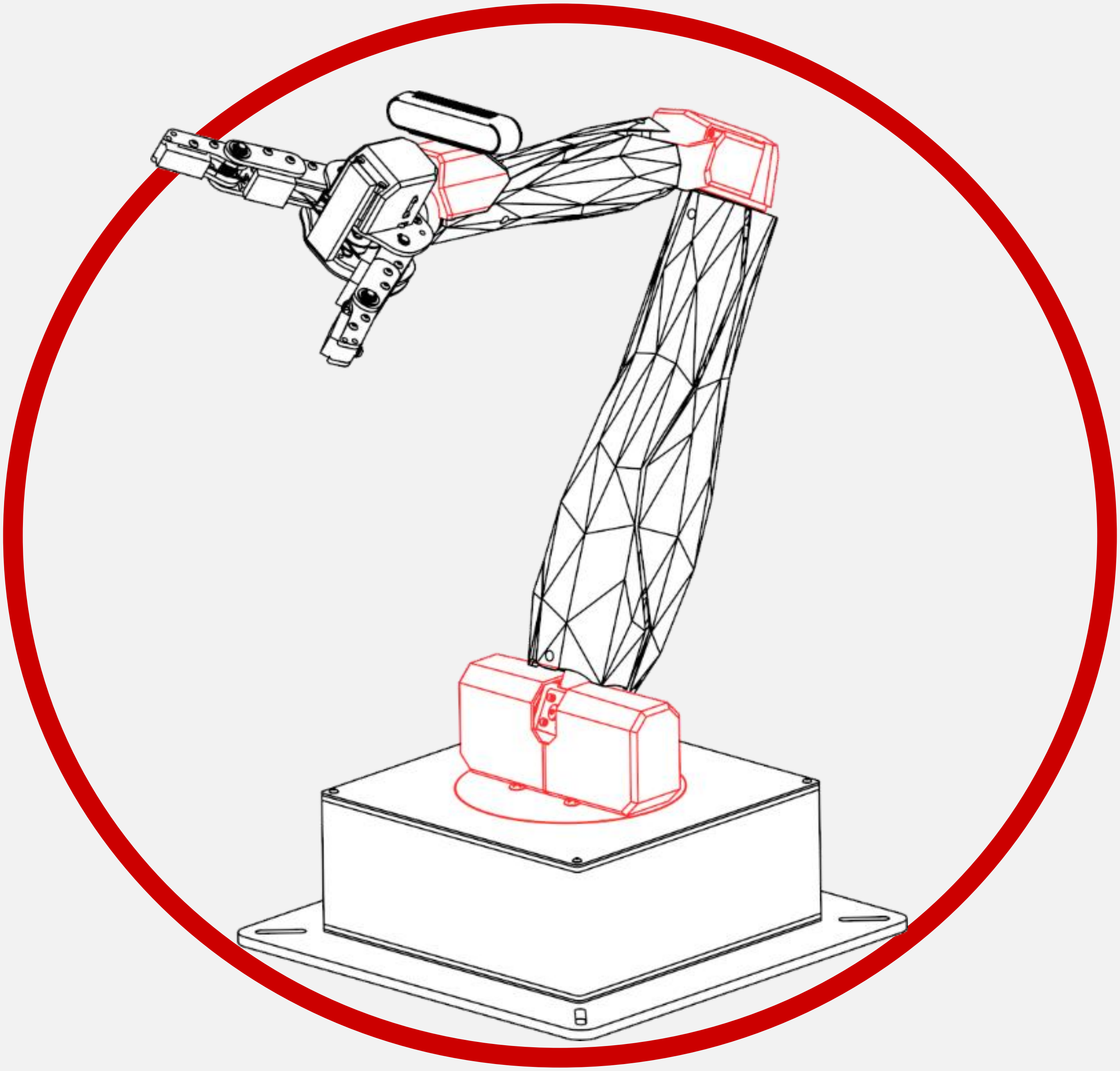

QArm

Singularity Identification

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Caution

This equipment is designed to be used for educational and research purposes and is not intended for use by the public. The user is responsible for ensuring that the equipment will be used by technically qualified personnel only. Users are responsible for certifying any modifications or additions they make to the default configuration.

Singularity Identification

What is Sigularity Identification?

When operating a manipulator in task space, such as commanding the end-effector to traverse a path at a constant speed, you may notice that the manipulator stops and refuses to travel further. This may happen even if the manipulator has not reached its physical joint limits. A joint configuration that prevents the end-effector to move in a certain direction is called a singularity. Its commonly said that this loss of motion causes the manipulator to lose one or more degrees of freedom.

To prevent singularities, you must first be able to first identify them. In this lab you will identify singularities using three different methods: geometrically, mathematically, and visually. The geometric approach identifies certain geometrical configurations, such as joint angles or alignment of the robot axes, which result in limited movement of the end-effector. The mathematical approach examines the manipulator's Jacobian and its inverse to identity singularities. In the visual approach, you will identify loss of end-effector movement along the cartesian axes by applying speed disturbances to the joints.

Differential Kinematic Formulation

The differential kinematic formulation of the Quanser QArm in matrix form as follows:

$$\begin{bmatrix} v_x \\ v_y \\ v_z \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} -\lambda_2 s_1 c_2 + \lambda_3 s_1 s_{23} & -\lambda_2 c_1 s_2 - \lambda_3 c_1 c_{23} & -\lambda_3 c_1 c_{23} & 0 \\ \lambda_2 c_1 c_2 - \lambda_3 c_1 s_{23} & -\lambda_2 s_1 s_2 - \lambda_3 s_1 c_{23} & -\lambda_3 s_1 c_{23} & 0 \\ 0 & -\lambda_2 c_2 + \lambda_3 s_{23} & \lambda_3 s_{23} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} \tag{1}$$

$$V = {}^{\mathbf{0}}\boldsymbol{J}\,\dot{\Theta}$$

where ${}^{\mathbf{0}}\boldsymbol{J}$ is the manipulator's Jacobian expressed in base frame {0}. This mapping relates the differential changes in joint space (i.e. joint velocities) to the differential changes in Cartesian space (i.e. end-effector velocity).

Singularity Analysis

At singular configurations, we lose the ability to map the joint velocities and task space velocities to each other. Different methods can be used to determine the singular configurations a of a manipulator, three of which will be analyzed in the lab procedure.

Jacobian Determinant

A singularity occurs when the determinant of the Jacobian is 0, which means the matrix is noninvertible. The determinant of the Jacobian expressed in Equation 1 is derived as:

$$\begin{aligned} \det {}^{\mathbf{0}}\boldsymbol{J} &= (-\lambda_2 s_1 c_2 + \lambda_3 s_1 s_{23})\big((-\lambda_2 s_1 s_2 - \lambda_3 s_1 c_{23})(\lambda_3 s_{23}) - (-\lambda_2 c_2 + \lambda_3 s_{23})(-\lambda_3 s_1 c_{23})\big) \\ &\quad + (\lambda_2 c_1 c_2 - \lambda_3 c_1 s_{23})\big((-\lambda_2 c_2 + \lambda_3 s_{23})(-\lambda_3 c_1 c_{23}) - (-\lambda_2 c_1 s_2 - \lambda_3 c_1 c_{23})(\lambda_3 s_{23})\big) \\ \det {}^{\mathbf{0}}\boldsymbol{J} &= -\lambda_2 \lambda_3 \big(s_1(-\lambda_2 s_1 c_2 + \lambda_3 s_1 s_{23})(s_2 s_{23} + c_2 c_{23}) - c_1(\lambda_2 c_1 c_2 - \lambda_3 c_1 s_{23})(c_2 c_{23} + s_2 s_{23})\big) \\ \det {}^{\mathbf{0}}\boldsymbol{J} &= \lambda_2 \lambda_3 \big(s_1^2(\lambda_2 c_2 - \lambda_3 s_{23})(s_2 s_{23} + c_2 c_{23}) + c_1^2(\lambda_2 c_2 - \lambda_3 s_{23})(c_2 c_{23} + s_2 s_{23})\big) \end{aligned} \tag{2}$$

$$\det {}^{\mathbf{0}}\boldsymbol{J} = \lambda_2 \lambda_3 (\lambda_2 c_2 - \lambda_3 s_{23})(s_2 s_{23} + c_2 c_{23})$$

Since λ_2 and λ_3 are both non-zero constants, the determinant of the Jacobian is zero if either of the last two terms of the final expression given in Equation 2 are zero. This results in the two singularity conditions as described below.

Singularity Condition 1

Under the first singularity condition:

$$\lambda_2 c_2 - \lambda_3 s_{23} = 0 \tag{3}$$

Geometrically, this occurs when the end-effector is positioned directly above the base.

Singularity Condition 2

Under the second first singularity condition:

$$s_2 s_{23} + c_2 c_{23} = 0 \tag{4}$$

Expanding the above expression yields:

$$s_2 s_2 c_3 + s_2 c_2 s_3 + c_2 c_2 c_3 - c_2 s_2 s_3 = 0 \tag{5}$$

which holds true if:

$$c_3 = 0 \tag{6}$$

Geometrically, this corresponds to the arm either folded into itself or completely stretched out (i.e. $\theta_3 = 90°$ or $\theta_3 = -90°$).

Jacobian and its Inverse

Another method for identifying singularities is determining the rank of the Jacobian. Any joint configuration that causes the Jacobin to lose a rank makes it noninvertible and therefore results in a singularity. A drop in rank indicates a loss in dimensionality. This can happen when a column of the Jacobian approaches all 0s. This happens during the first Singularity Condition. Simplifying the Jacobian's first column yields,

$$\begin{bmatrix} -\lambda_2 s_1 c_2 + \lambda_3 s_1 s_{23} & -\lambda_2 c_1 s_2 - \lambda_3 c_1 c_{23} & -\lambda_3 c_1 c_{23} & 0 \\ \lambda_2 c_1 c_2 - \lambda_3 c_1 s_{23} & -\lambda_2 s_1 s_2 - \lambda_3 s_1 c_{23} & -\lambda_3 s_1 c_{23} & 0 \\ 0 & -\lambda_2 c_2 + \lambda_3 s_{23} & \lambda_3 s_{23} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -s_1(\lambda_2 c_2 - \lambda_3 s_{23}) & -\lambda_2 c_1 s_2 - \lambda_3 c_1 c_{23} & -\lambda_3 c_1 c_{23} & 0 \\ c_1(\lambda_2 c_2 - \lambda_3 s_{23}) & -\lambda_2 s_1 s_2 - \lambda_3 s_1 c_{23} & -\lambda_3 s_1 c_{23} & 0 \\ 0 & -\lambda_2 c_2 + \lambda_3 s_{23} & \lambda_3 s_{23} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Note that with Singularity Condition 1, when $\lambda_2 c_2 - \lambda_3 s_{23} = 0$, the first column becomes all zeros regardless of the value of θ_1. A loss in rank can also occur when at least two columns are linearly dependent (i.e. one column is a multiple of another). This happens during Singularity Condition 2. When the cosine of the third joint angle c_3 is 0, we can simplify certain expressions,

$$if\ c_3 = 0,\ \ then\quad \theta_3 = \pm\frac{\pi}{2}\ , and\ c_{23} = \mp s_2\ ,\ \ s_{23} = \ \pm c_2 \tag{8}$$

$$
\begin{bmatrix} -\lambda_2 s_1 c_2 + \lambda_3 s_1 s_{23} & -\lambda_2 c_1 s_2 - \lambda_3 c_1 c_{23} & -\lambda_3 c_1 c_{23} & 0 \\ \lambda_2 c_1 c_2 - \lambda_3 c_1 s_{23} & -\lambda_2 s_1 s_2 - \lambda_3 s_1 c_{23} & -\lambda_3 s_1 c_{23} & 0 \\ 0 & -\lambda_2 c_2 + \lambda_3 s_{23} & \lambda_3 s_{23} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} -\lambda_2 s_1 c_2 \pm \lambda_3 s_1 c_2 & -\lambda_2 c_1 s_2 \pm \lambda_3 c_1 s_2 & \pm\lambda_3 c_1 s_2 & 0 \\ \lambda_2 c_1 c_2 \mp \lambda_3 c_1 c_2 & -\lambda_2 s_1 s_2 \pm \lambda_3 s_1 s_2 & \pm\lambda_3 s_1 s_2 & 0 \\ 0 & -\lambda_2 c_2 \pm \lambda_3 c_2 & \pm\lambda_3 c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} -s_1 c_2(\lambda_2 \mp \lambda_3) & -(\lambda_2 \mp \lambda_3) c_1 s_2 & \pm\lambda_3 c_1 s_2 & 0 \\ c_1 c_2(\lambda_2 \mp \lambda_3) & -(\lambda_2 \mp \lambda_3) s_1 s_2 & \pm\lambda_3 s_1 s_2 & 0 \\ 0 & -(\lambda_2 \mp \lambda_3) c_2 & \pm\lambda_3 c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Note that here, column 2 is column 3 multiplied by the factor $\mp(\lambda_2 \mp \lambda_3)/\lambda_3$.

Alternatively, one can examine the condition number of the Jacobian. The condition number is a mathematical operation that quantifies how sensitive a function's output is to changes or errors in the input. When the manipulator is in a singular configuration, we lose a task space motion direction. This signifies that you will not find a meaningful joint space velocity solution for a certain task space solution. This is equivalent to saying, a small change in the task space velocity desired will result in large fluctuations in the joint space solution. The condition number here is the largest term in the Jacobian's inverse. A *large* condition number indicates that the Jacobian may not be invertible and therefor our joint configuration represents a singularity.

Background

The QArm content contains 3 labs that focus on velocity manipulation. They focus on tool manipulation, singularity identification and singularity avoidance respectively. This lab focuses on understanding singularities and finding where they are.

Getting started

The goal of this lab is to understand how to avoid singularity positions based on the singularity identification lab before.

Ensure you have completed the following labs

- **Kinematic Manipulation Labs**
- **Tool Manipulation**

Before you begin this lab, ensure that the following criteria are met.

- The QArm has been setup and tested. See the QArm Quick Start Guide for details on this step.
- You are familiar with the basics of Simulink. See the [Simulink Onramp]() for more help with getting started with Simulink.