Concept Review

# Inertial Measurement Units

## Why Use an Inertial Measurement Unit?

Inertial Measurement Units (IMU) are typically a combination of multiple sensors that can provide kinematic information such as angular speed and linear acceleration. IMUs are cost-effective, and relatively easy to integrate into complex systems. They form a core part of modern pose estimation processes not only in aerial vehicles, but also day to day devices such as smartphones, game controllers and VR headsets.

## Inertial Measurement Unit

The IMU is typically placed near the center of gravity of an object you are interested in tracking. It often includes an accelerometer, gyroscope, and magnetometer, but could also include an onboard temperature sensor and barometer to adjust for variances due to weather and altitude.

Review the Frames of Reference concept review to ensure you are comfortable with concepts regarding the Inertial, Horizon and Body frames. In addition to translation, the Body frame can also reorient itself with respect to the inertial frame. This rotation is typically measured as roll, pitch and yaw angles, but can also be represented as a rotation matrix.

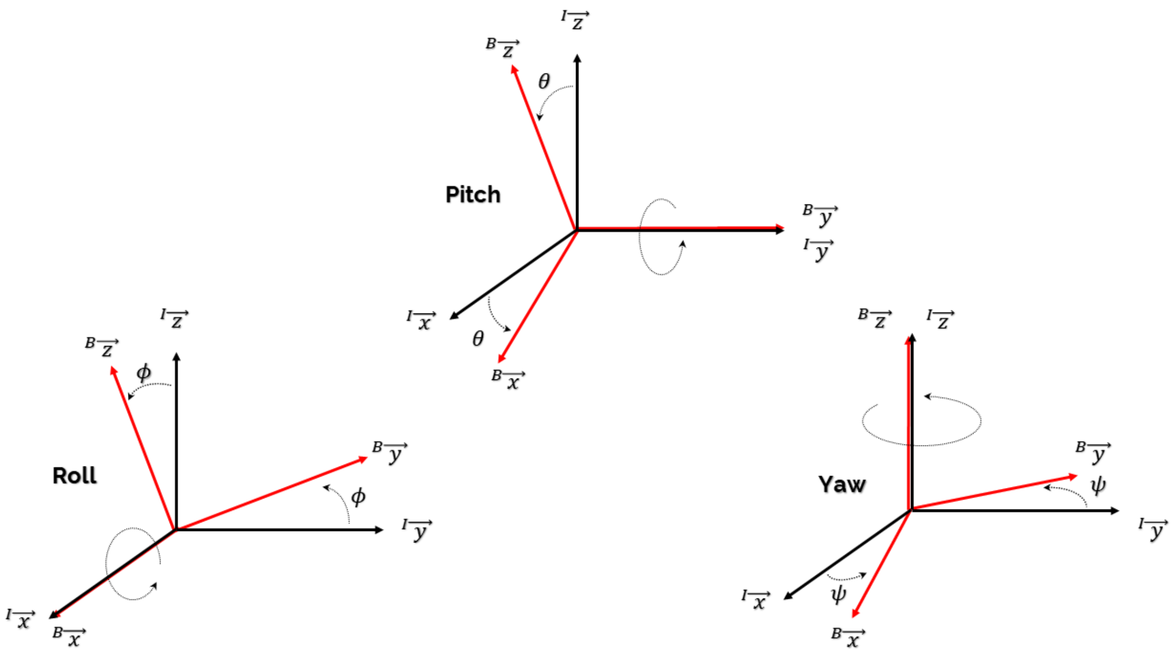


Figure 1: Roll, pitch and yaw angles measured about the x, y and z axis respectively

## Accelerometers

This sensor measures accelerations along 3 cartesian directions. It can thus provide translational accelerations of a body it is attached to (in the local body frame). The sensor will also pick up the acceleration due to gravity (m/s2). When stationary, the measurements of the sensor in its three axis can be used to estimate the roll/pitch attitude of the vehicle knowing that the acceleration due to gravity points straight downwards. However, since any rotation about the vertical axis would not change the components of gravity in any of the sensor's three directions, this sensor cannot be used to estimate the yaw angle.

## Roll/Pitch angles

There is a constant gravitational force acting on objects, which is inertially pointed downwards. This fact can be exploited to approximate the roll and pitch angle of a vehicle if the accelerometer is attached to the body of the vehicle and rotates with it. If the body frame accelerometer data reads,

$$^{B}\vec{a} = \begin{bmatrix} ^{B}a_x \\ ^{B}a_y \\ ^{B}a_z \end{bmatrix}$$

It can be rotated to the inertial horizon frame (a frame rotated from the inertial frame to align with the current body heading angle) using the rotations,

$$^{I}\vec{a} = ^{I}R_B\, ^{B}\vec{a} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} {}^{B}\vec{a}$$

Consider the case with no roll. Calculating the inertial accelerations and equating them to $[0 \quad 0 \quad g]^T$ yields the equation for calculating the pitch angle.

$$\begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} = \begin{bmatrix} ^{I}a_x \\ ^{I}a_y \\ ^{I}a_z \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} ^{B}a_x \\ ^{B}a_y \\ ^{B}a_z \end{bmatrix} = \begin{bmatrix} ^{B}a_x \cos\theta + ^{B}a_z \sin\theta \\ ^{B}a_y \\ ^{B}a_z \cos\theta - ^{B}a_x \sin\theta \end{bmatrix}$$

$$0 = ^{B}a_x \cos\theta + {}^{B}a_z \sin\theta$$

$$\tan\theta = -\frac{^{B}a_x}{^{B}a_z}$$

$$\theta = \tan^{-1}\left(-\frac{^{B}a_x}{^{B}a_z}\right)$$

Also notice that the third element, $^{I}a_z$ is related to the x and z body frame acceleration components $^{B}a_z$ and $^{B}a_z$, as well as the pitch angle approximation $\theta$, This is illustrated in Figure 2. Geometrically, the pitch angle approximation can be calculated by taking an inverse tangent on the x and z components of the accelerometer data. A similar calculation based on the geometry of the rotation can be used to obtain the roll angle approximation $\phi$, That is,

$$\tan\phi = \frac{^{I}a_y}{^{I}a_z}$$

$$^{I}a_z = {}^{B}a_z \cos\theta - {}^{B}a_x \sin\theta$$

$$\phi = \tan^{-1}\left(\frac{^{I}a_y}{^{I}a_z}\right) = \tan^{-1}\left(\frac{^{B}a_y}{-^{B}a_x \sin\theta + ^{B}a_z \cos\theta}\right)$$

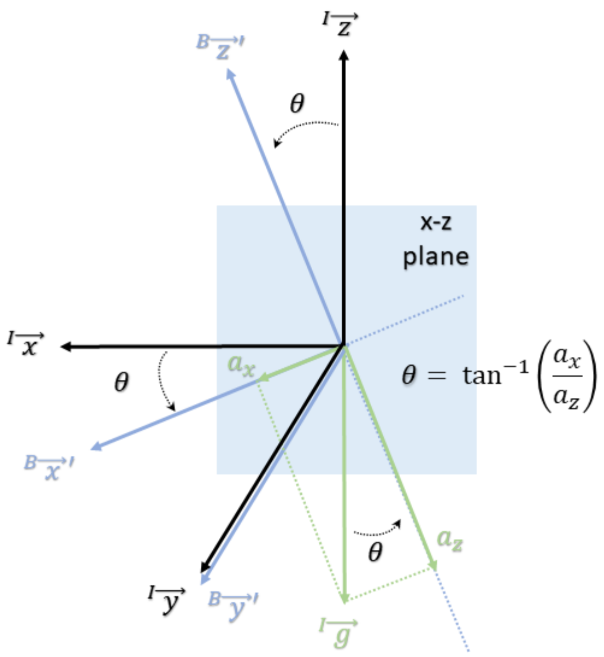


Figure 2: Roll angle approximation from the accelerometer data

Also note that the accelerometer cannot be used to estimate the yaw angle of the vehicle. One way to realize this is that gravity points straight down, and any rotation about this axis does not change the accelerometer measurement. A workaround involves using a magnetometer. See Appendix A for more information. This approximation is inherently noisy and measured at a slow rate (< 500 Hz). It is valid for a relatively stationary vehicle with small roll/pitch attitude (±10 deg). This information is often combined with gyroscopic rates to improve the estimate.

## Gyroscopes

This sensor measures the angular rate of rotation of a body it is attached to. It can be integrated to estimate the angular position, but this will tend to drift due to an accumulated error over time. It is often combined with an accelerometer and magnetometer to provide angular position and rates.

### Angular Rate Estimates

The gyroscope measures angular rates in radians per second directly and therefore can be used to provide angular rate estimates.

$$\begin{bmatrix} \dot{\phi}_e \\ \dot{\theta}_e \\ \dot{\psi}_e \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

A filter can also be introduced to reduce noise. For example, a second-order low-pass filter can be added,

$$\begin{bmatrix} \dot{\phi}_e \\ \dot{\theta}_e \\ \dot{\psi}_e \end{bmatrix} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

Where $\omega_n$ is the natural frequency of the filter in rad/s. As $\omega_n$ goes to infinity, the filter becomes less effective, although lowering the frequency increases the delay. The damping rate $\zeta$ is set to 1.

### Angular Acceleration Estimates

The angular acceleration estimates are calculated by differentiating the gyroscopic data and filtering them,

$$\begin{bmatrix} \ddot{\phi}_e \\ \ddot{\theta}_e \\ \ddot{\psi}_e \end{bmatrix} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} \begin{bmatrix} d\dot{\phi}/dt \\ d\dot{\theta}/dt \\ d\dot{\psi}/dt \end{bmatrix}$$

## Magnetometers

This sensor measures the direction of the magnetic field in the environment. Combining this information with a known roll/pitch attitude (for example, from an accelerometer) yields the yaw angle of the body the sensor is attached to. The magnetometer is susceptible to interferences from local electromagnetic devices and iron in the surrounding environment and must first be calibrated for these (often termed soft and hard iron calibration).

### Calibration

Magnetometers must be calibrated for tilt, hard-iron distortions and soft-iron defects. These are summarized below.

Tilt correction

When the magnetometer (attached to the body) is perfectly level with the horizontal plane, no tilt correction is required. However, as the body rolls or pitches, the magnetometer readings must be transformed from the body frame ${}^B\vec{m}$ to the horizontal frame ${}^I\vec{m}$. This is done using a rotation matrix ${}^IR_B$ as a sequence of roll and pitch rotations,

$$
{}^{I}\vec{m} = {}^{I}R_B\,{}^{B}\vec{m} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} {}^{B}\vec{m}
$$

Hard-iron distortions

The tilt-corrected magnetometer readings must next be calibrated for hard-iron distortions, which are produced by materials that exhibit a magnetic field of their own, for example, speakers or iron beams. This defect is additive and manifests itself as a constant bias. To calculate what the bias is for subsequent removal, hold the magnetometer with its z axis in the horizontal plane and rotate it in an infinity (sideway figure-8) pattern (see Figure 3) in the vertical plane. While rotating it, gradually rotate yourself about the vertical axis. When the magnetometer readings are plotted in pairs (x vs. y, y vs. z and z vs. x), three roughly circular plots are achieved as shown in Figure 4.

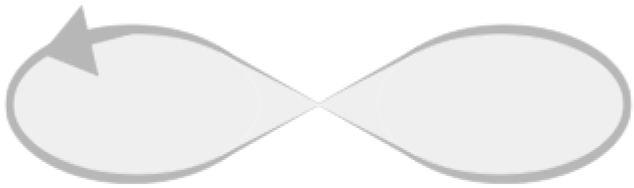

Figure 3: Infinity pattern to rotate the magnetometer around in

The blue circles represent the raw magnetometer data. Notice that the circles are not centred about (0,0), which signifies that a hard-iron calibration is required. Averaging the maximum and minimum X readings will provide the x axis offset. Repeating this in the y and z axis will yield the y and z offsets as well. Subtracting these offsets from the raw magnetometer data will yield the red circles, which are correctly centred about (0,0).

The offsets must be removed from all future magnetometer readings before yaw estimation.

$$
\begin{aligned}
{}^{I}m_{xH} &= {}^{I}m_x - \frac{(max({}^{I}m_x) + min({}^{I}m_x))}{2} \\
{}^{I}m_{yH} &= {}^{I}m_y - \frac{(max({}^{I}m_y) + min({}^{I}m_y))}{2} \\
{}^{I}m_{zH} &= {}^{I}m_z - \frac{(max({}^{I}m_z) + min({}^{I}m_z))}{2}
\end{aligned}
$$

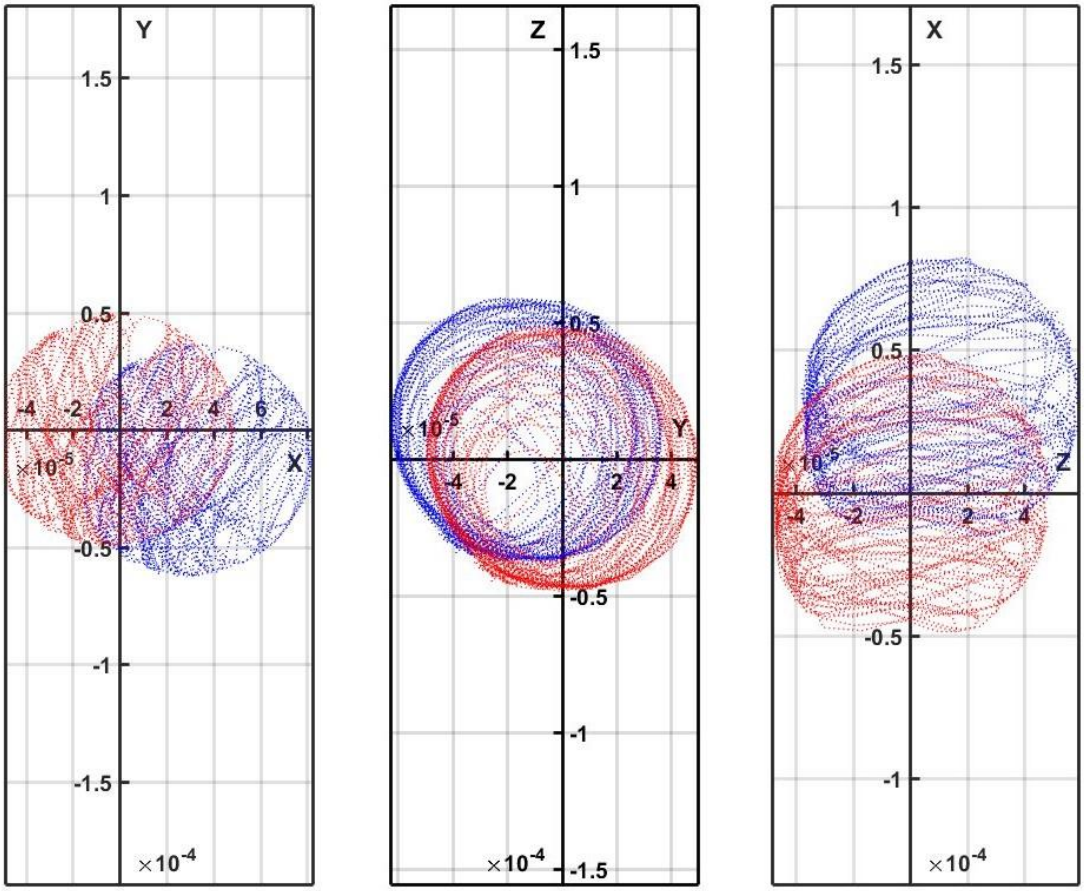


Figure 4: Hard-iron calibration result

## Soft-iron defects

These defects are caused by materials in the environment that distort the magnetic field, but do not produce a magnetic field of their own. These defects are thus not additive, but multiplicative. If the hard-iron calibrated magnetometer data circles do not have the same maximum and minimum magnitudes, the circles will actually be ellipses. Find the maximum magnitude of the three axis, and calculate their 2-norm. Then use the ratio of the norm and maximum magnitude in each axis to scale the corresponding axis data to yield true circles. This is summarized in Equations 18. This effect is usually minor, and a corresponding soft-iron calibration is optional.

$$||m|| = \sqrt{max(|^I m_{xH}|)^2 + \ max(|^I m_{yH}|)^2 + \ max(|^I m_{zH}|)^2}$$

$$^I m_{xHS} = \frac{||m||}{max(|^I m_{xH}|)} \, ^I m_{xH} \qquad ^I m_{yHS} = \frac{||m||}{max(|^I m_{yH}|)} \, ^I m_{yH}$$

$$^I m_{zHS} = \frac{||m||}{max(|^I m_{zH}|)} \, ^I m_{zH}$$

### Yaw angle

The magnetometer readings that are now tilt corrected, and hard/soft iron calibrated can be represented as a vector $\vec{m}$:

$$\vec{m} = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = \begin{bmatrix} {}^I m_{xHS} \\ {}^I m_{yHS} \\ {}^I m_{zHS} \end{bmatrix}$$

The heading angle with respect to the x axis can be calculated using an inverse tangent on the x and y components.

$$\psi = \tan^{-1}\left(\frac{m_y}{m_x}\right)$$

The average heading reading $\psi_0$, obtained during the first few seconds when the drone is stationary, can be subtracted from future heading calculations to get a heading angle with respect to the initial heading position of the body. This yaw angle is inherently noisy, and the sensor provides data at a relatively slow rate (<500 Hz). It is typically combined with gyroscopic angular rates (~1000 Hz) to improve the estimate.

## Approximating Pitch using Angular Velocity

In many cases, such as when a vehicle is accelerating vertically, or when measuring yaw, there is no way to use acceleration to calculate the angular position of the vehicle. In this case, the position is often approximated by integrating the angular velocity measured by the gyroscope. In this way, the angle at time t is given by

$$\theta_t = \theta_0 + \int_0^t \omega dt \tag{3}$$

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