

## Recommended Assessment

# State Space Modeling

### Deriving the State Space Representation

1. Using the provided equations of motion  $J_r\ddot{\theta} + m_p l r \ddot{\alpha} = \tau - b_r \dot{\theta}$  and  $J_p \ddot{\alpha} + m_p l r \ddot{\theta} + m_p g l \alpha = -b_p \alpha$ , rearrange and solve to find explicit expressions for the angular accelerations  $\ddot{\theta}$  and  $\ddot{\alpha}$  in terms of the given parameters.

*Hint:* Use MATLAB's symbolic toolbox and solve function to assist with solving the equations simultaneously.

2. Based on the output state  $y(t)$  defined in  $y(t) = [\theta(t) \ \alpha(t)]^T$ , find the state space matrices  $C$  and  $D$  in  $y(t) = Cx(t) + Du(t)$
3. Using the solution from Question 1 and  $x(t) = [\theta(t) \ \alpha(t) \ \dot{\theta}(t) \ \dot{\alpha}(t)]^T$ , derive matrices  $A$  and  $B$  in  $\dot{x}(t) = Ax(t) + Bu(t)$ .
4. Complete the **A**, **B**, **C**, and **D** matrices in the file `qs3_rotpen_ABCD_eqns_down.m`. The provided file `qs3_rotpen_param.m` contains all the defined model parameters. Paste the code from the MATLAB script that defines the matrices  $A$ ,  $B$ ,  $C$  and  $D$ .

### Model Validation

5. Attach a screenshot of the rotary arm and pendulum scopes. Do the responses match closely? If there are discrepancies between the measured and modeled responses, give a possible reason why the responses don't match.