Frame Assignments

1 Frame Diagram

The Quanser Arm's frame diagram is attached in Figure 1. This was developed using the Standard Denavit Hartenberg (DH) parameters [1].

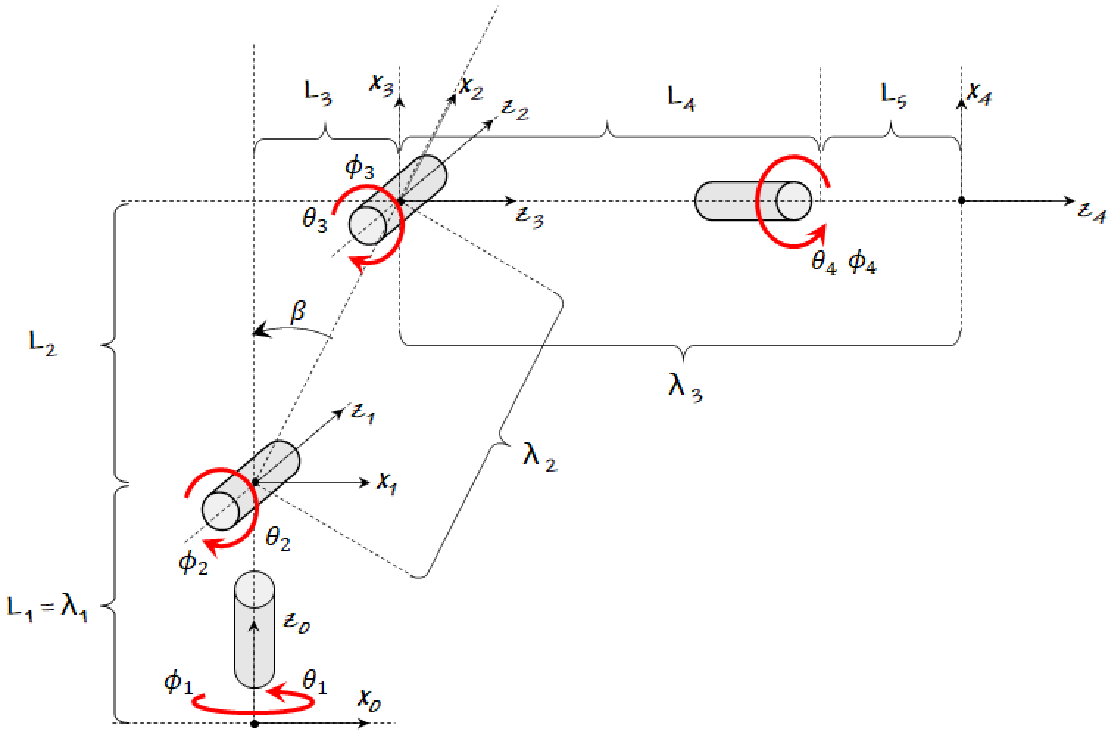

Figure 1. Frame diagram for the Quanser Arm manipulator

Note that the manipulator shown in Figure 1 is currently in its home position. In this state, the joint space vector $\overrightarrow{\theta}$ is,

$$\overrightarrow{\theta} = \begin{bmatrix} 0 \\ \beta - \frac{\pi}{2} \\ -\beta \\ 0 \end{bmatrix} \tag{3}$$

The manipulator's encoders and actuators though, are calibrated at this position. Thus, an actuator command of $[0\ 0\ 0\ 0]^T$ would move the manipulator to the home position, where it's encoders would read a joint position of $[0\ 0\ 0\ 0]^T$ as well. We can represent this alternate joint space as $\overrightarrow{\phi}$. A mapping summarized in Table 1 will allow us to describe the manipulator in $\overrightarrow{\phi}$ space, while we carry out the mathematics in $\overrightarrow{\theta}$ space without having to carry around the offset in equation 3. For example, a command of $\phi_2 = 0$ will imply $\theta_2 = \beta - \frac{\pi}{2}$ which corresponds to joint 2 in the home position. Algebraically, a reference to θ_2 in the mathematics already includes this offset.

New parameter	Original Parameter
λ_1	L_1
λ_2	$\sqrt{{L_2}^2 + {L_3}^2}$
λ_3	$L_4 + L_5$
β	$\tan^{-1}\left({}^{L_3}/_{L_2}\right)$

New parameter	Original Parameter
ϕ_1	θ_1
ϕ_2	$\theta_2 + \frac{\pi}{2} - \beta$
ϕ_3	$\theta_3 + \beta$
ϕ_4	θ_4

Table 1. Linear mapping to simplify the mathematical formulations

2 DH TABLE

The DH table corresponding to the frame diagram in Figure 1 is presented in Table 2.

$\boldsymbol{i}$	$\boldsymbol{a_i}$	$\boldsymbol{\alpha_i}$	$\boldsymbol{d_i}$	$\boldsymbol{\theta_i}$
1	0	$-\pi/2$	λ_1	θ_1
2	λ_2	0	0	θ_2
3	0	$-\pi/2$	0	θ_3
4	0	0	λ_3	θ_4

Table 2. DH Table