

# Recommended Assessment

## Proportional Control

### Second Order Systems

1. Use the closed-loop equation for the Qube-Servo 3 under unity feedback (equation below) and find the natural frequency ( $\omega_n$ ) and damping ratio ( $\zeta$ ) of the system in terms of  $K$  and  $\tau$ .

$$\frac{\Theta_d(s)}{V_m(s)} = \frac{\frac{K}{\tau}}{s^2 + \frac{1}{\tau}s + \frac{K}{\tau}}$$

2. Using the previous answer, solve for the natural frequency ( $\omega_n$ ) and damping ratio ( $\zeta$ ) using the model parameters  $K$  and  $\tau$  calculated in the modeling labs, or use  $K = 24$  and  $\tau = 0.1$  as defaults if you have not done any of those labs.
3. Calculate the expected peak time and percent overshoot based on the calculated natural frequency ( $\omega_n$ ) and damping ratio ( $\zeta$ ).
4. Attach a screenshot of the response captured during the lab. Write down the peak time and percent overshoot of the response.
5. Do the measured and calculated peak time and percent overshoot match? If they do not match exactly, give one reason why.

### Proportional Position Control

6. Is it possible to get any desired second-order system response (by specifying natural frequency ( $\omega_n$ ) and damping ( $\zeta$ )) by only using proportional feedback gain?  
*Hint:* Compare the Qube's closed loop transfer function to the standard second order system transfer function.

$$\frac{Y(s)}{R(s)} = \frac{K k_p}{\tau s^2 + s + K k_p}$$

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

7. The voltage that can be applied to the Qube – Servo 3 is limited to  $\pm 10V$ . Determine the maximum proportional gain for the square wave reference signal of  $\pm 0.5$  rad as shown in the lab that does not saturate the Qube – Servo 3.

8. Attach scopes from the result of your position controller with a proportional gain of 1.5. What was the percent overshoot and peak time? Was there a steady-state error, if so, what was it?
9. When varying  $k_p$  between 1 and 5, what happened? Same when varying between 0.1 and 1. How did the proportional gain affect the position control response in both cases?

## Proportional Speed Control

10. Based on the proportional speed control figure in the lab procedure, what is the closed-loop transfer function  $E(s)/R(s)$  in that represents the dynamics between a desired speed,  $R(s) = \Omega_d(s)$ , and the error,  $E(s) = R(s) - Y(s) = \Omega_d(s) - \Omega_m(s)$
11. Find the steady-state error of the system,  $e_{ss}$ , for the reference step input  $R(s) = R_0/s$  where  $R_0$  the desired angular rate step amplitude. (*Hint:* Use the Final-Value Theorem described in the concept review). Evaluate it the proportional gain is  $0.5 \text{ V}\cdot\text{s} / \text{rad}$  and the system is given a step amplitude of  $15 \text{ rad/s}$ . Use the  $K$  found in one of the modeling laboratories, e.g. Step Response modeling lab. (or  $K = 24$  if no modeling lab has been done).
12. What was the response of your controller with a proportional gain of 0.5? Add your scopes. What was the steady-state error? Compare it with your calculated value.
13. Show how the error can be decreased by half the current magnitude. Validate your results with the Qube - Servo 3 and show your response.  
*Hint:* Recall how the steady state error was defined and identify which parameters you can adjust.  
*Hint:* Consider the effect of the tachometer vs differentiating the position for lower speeds.