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1.

(a)

Since $P(A|C) = \dfrac{P(A,C)}{P(C)}$

$P(B|C) = \dfrac{P(B,C)}{P(C)}$

Since $P(A|B,C) = \dfrac{P(A,B,C)}{P(B,C)}$

$= P(B|A,C) = \dfrac{P(B,A,C)}{P(A,C)}$ $\left(\Rightarrow \dfrac{P(A,C)}{P(B,C)} = \dfrac{P(A,B,C)}{P(A,B,C)} = 1\right)$

Thus $P(B,C) = P(A,C)$

Thus $P(A|C) = P(B|C)$

(b)

~~Since~~ $\cancel{P(A,B|C) = \frac{P(A,B,C)}{P(C)} = P(A,B)}$

Since $P(A|C) = \sum_b P(A,B=b|C)$

$= \sum_b P(A,B)$

$= P(A)$

So $P(A|C) = P(A)$

Since $P(B|C) = \sum_a P(A=a, B|C) = \sum_a P(A=a, B)$

$= P(B)$

So $P(B|C) = P(B)$

(c) Since $P(A,B,C) = P(A)P(B)P(C)$

Thus A, B, C are mutually independent

~~Since~~ Thus $P(A,B|C) = \frac{P(A,B,C)}{P(C)} = \frac{P(A)P(B)P(C)}{P(C)}$

$= P(A)P(B)$

$= P(A,B)$

2.

(a) $P(\text{toothache}) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2$

(b) $P(\text{cavity}) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2$

(c) $$P(\text{toothache} \mid \text{cavity}) = \frac{P(\text{toothache}, \text{cavity})}{P(\text{cavity})}$$

$$= \frac{0.108 + 0.012}{0.2}$$

$$= 0.6$$

(d) $$P(\text{toothache} \mid \sim\text{cavity}) = \frac{P(\text{toothache}, \sim\text{cavity})}{P(\sim\text{cavity})}$$

$$= \frac{0.016 + 0.064}{1 - 0.2}$$

$$= 0.1$$

(e)

Since $\alpha P(\text{toothache} \mid \text{cavity}) P(\text{catch} \mid \text{cavity}) P(\text{cavity}) + \alpha P(\text{toothache} \mid \sim\text{cavity}) P(\text{catch} \mid \sim\text{cavity}) P(\sim\text{cavity}) = 1$

Thus $$\alpha\left(0.6 \times \frac{0.108 + 0.072}{0.2} \times 0.2\right) + \alpha\left(0.1 \times \frac{0.016 + 0.144}{1 - 0.2} \times (1 - 0.2)\right) = 1$$

Solve for α, we have $\alpha = 8.07$

(f)

$$P(\sim cavity \mid toothache, catch)$$

$$= \alpha P(toothache \mid \sim cavity) P(catch \mid \sim cavity) P(\sim cavity)$$

$$= 8.07 \times 0.1 \times \frac{0.016 + 0.144}{1 - 0.2} \times (1 - 0.2)$$

$$= 0.13$$

3.

(a)

$$P(m \mid s) = \frac{P(s \mid m) P(m)}{P(s)}$$

$$P(\sim m \mid s) = \frac{P(s \mid \sim m) P(\sim m)}{P(s)}$$

Thus $\alpha P(s \mid m) P(m) + \alpha P(s \mid \sim m) P(\sim m) = 1$

(b)

Thus $\alpha (0.5 \times \frac{1}{50000}) + \alpha (0.05 \times (1 - \frac{1}{50000})) = 1$

Thus $\alpha = 20$

Thus $P(m \mid s) = 0.5 \times \frac{1}{50000} \times 20 = 0.0002$