Multiple Prizes and Idea Diversity in Innovation Contests[*]

Ying-Ju Chen[†] Zhuoqiong Chen[‡] Yangguang Huang[§] Bin Liu[¶]

November 2024

Abstract

We analyze data from design proposal contests and find that multiple prizes are widely adopted, especially in building projects requiring diverse and creative ideas. The data also indicates that contests with higher compensation for non-winning participants attract more entrants. We extend the standard incomplete-information contest model by incorporating the contest organizer's valuation of both the total effort and idea diversity. Our model explains the coexistence of different prize structures: when the value of diverse ideas is significant, multiple prizes are optimal; otherwise, a winner-take-all approach is preferred. Furthermore, as the importance of idea diversity increases, the optimal prize distribution should be more equally distributed among participants.

Keywords: innovation contests, multiple prizes, endogenous entry, idea diversity, procurement

JEL codes: D47, D82, L74

[*]We greatly appreciate Mikhail Drugov and Jingfeng Lu for very helpful discussions and comments. We thank participants at HKUST IO Workshop, China Meeting of the Econometric Society, and International Industrial Organization Conference. Chen gratefully acknowledges support from the National Natural Science Foundation of China (No. 71903046) and the "Shenzhen Peacock Plan" start-up research grant. Liu gratefully acknowledges financial support from National Natural Science Foundation of China (No. 72073115).

[†]Hong Kong University of Science and Technology. Email: imchen@ust.hk.

[‡]Harbin Institute of Technology, Shenzhen. Email: chenzq926@gmail.com.

[§]Hong Kong University of Science and Technology. Email: huang_sunny@outlook.com.

[¶]Chinese University of Hong Kong, Shenzhen. Email: binliu@cuhk.edu.cn.

1 Introduction

Since the seminal work by Tullock (1980), contests have been widely used to study competitive environments in which agents exert costly effort to compete for one or multiple prizes. Contests are a popular mechanism for procuring innovative products, such as architectural designs, inventions, and solutions to scientific and engineering problems. In a typical innovation contest, the contest organizer (seeker or procurer, referred to as "she") establishes a prize scheme that specifies prizes to be rewarded to agents (solvers or contestants, referred to as "he/they") based on the ranking of their performance. Agents are invited to submit proposals demonstrating their ideas, which are then evaluated to determine their ranking.

In the literature, many researchers have shown that a winner-take-all prize structure is optimal in many contest environments (Kalra and Shi, 2001; Ales et al., 2017). Moldovanu and Sela (2001) establish the unique optimality of devoting all resources to the first prize in contests with incomplete information when the effort cost function is linear or concave. Liu and Lu (2014) show that given (potentially) heterogeneous prizes, an all-pay auction with winner-take-all maximizes the total effort given a fixed budget. Liu and Lu (2019) introduce endogenous entry into the model of Moldovanu and Sela (2001) and show that winner-take-all is still optimal despite the new tradeoff between encouraging the entry of potential contestants and eliciting effort from entrants. In fact, the majority of studies on contests focus on the case of winner-take-all (Fu and Wu, 2019).

However, in practice, many innovation contests offer multiple prizes. For example, in 1993, Boeing and Lockheed-Martin were each given compensation of USD 2.2 billion to produce a prototype in competition for the contract for the Joint Strike Fighter (Kaplan et al., 2002; Matros and Armanios, 2009). In 2012, the Google Xprize foundation launched a contest for ideas to develop affordable transportation to the moon. The contest offers a USD 20 million grand prize, a USD 5 million second prize, and several USD 1 million "diversity prizes."[1] In 2007, the Shanghai government launched an international proposal contest for the architectural design of the China Pavilion for EXPO 2010 Shanghai. The contest offered a CNY 120 million prize to the winner and a CNY 0.5 million prize to each of the eight designers who were shortlisted in the final round.[2] The majority of innovation contests organized by Topcoder and InnoCentive have given multiple awards (Korpeoglu et al., 2021; Stouras et al., 2022). In addition, we obtain a dataset of 327 contests of design proposals from a public procurement platform in China. We find that approximately 70% of these contests offer multiple prizes. Moreover, contests for building projects tend to offer more prizes and attract more agents to participate than other types of projects.

Why are multiple prizes popular in innovation contests? Why do some contests use a winner-take-all prize while others adopt multiple prizes? How does rewarding contest losers affect potential participants' entry decisions and their contest performance? In this paper, we answer these ques-

[1] See `lunar.xprize.org`.

[2] The contest attracted 344 designers from throughout the world to submit proposals. See `en.wikipedia.org/wiki/China_pavilion_at_Expo_2010`.

tions by incorporating two important aspects of innovation contests into the well-adopted workhorse model of contests pioneered by Moldovanu and Sela (2001). First, each agent endogenously decides whether to enter the contest at a cost; second, the contest organizer is uncertain about the ideal design of the project (or the best solution to the problem).

In our model, the contest organizer sets up a rank-order prize scheme subject to a fixed budget and invites agents to submit their proposals. Each agent is endowed with an efficiency parameter as his private information and draws an idea from the idea space. Agents spend costly effort in preparing proposals to demonstrate their ideas. The proposals are evaluated by the organizer or a committee of experts, whose ratings or votes determine the ranking of agents.

The nature of innovation contests determines that the contest organizer is unsure about the ideal approach and thus cannot fully specify the evaluation criteria in the contest announcement. For example, in architectural design contests, the organizers are not able to fully describe the ideal design before reviewing the proposals from design firms (Zhu, 2019); in the development of COVID-19 vaccines, there are at least nine different approaches, and many of them may end up in failure (Le et al., 2020). In the InnoCentive open platform, regulatory issues or technical feasibility may vary over time, and thus may not be predefined in the initial evaluation criteria and left out in the judging process.

Given the uncertainty of ideal design, we assume that the proposal from each agent yields two kinds of value to the contest organizer. First, a high-quality proposal requires that the agent spends effort preparing the proposal to demonstrate his idea based on the evaluation rules. Second, the novelty of the idea or the innovative element of the design is valuable to the contest organizer, but the organizer cannot specify how to measure novelty when announcing the contest. Therefore, the contest organizer not only cares about the total effort spent by agents but also derives value from diverse ideas proposed by these agents. A greater number or diversity of ideas makes it more likely to obtain a proposal with a desirable design. Lakhani et al. (2007) provide empirical evidence that attracting solvers with diverse scientific backgrounds greatly improves the success rate of the designated problems. In practice, the organizer sometimes combines the ideas from several proposals to develop the final plan, so each idea can be valuable to the organizer. For example, in the EXPO 2010 China pavilion design contest, the final construction plan used the main idea from the winner in conjunction with elements from three other proposals.[3]

Therefore, we assume that the contest organizer chooses the prize scheme to maximize the summation of total effort from all participating agents and the total value of their ideas. The total value of ideas is a decreasing function of the entry threshold as fewer entrants to the contest results in fewer ideas being submitted. We find that the organizer's benefit from eliciting diverse

[3]See `en.wikipedia.org/wiki/China_pavilion_at_Expo_2010`. According to the Bidding Law of the People's Republic of China (`www.lawinfochina.com/display.aspx?id=27151&lib=law&EncodingName=big5`), paying design compensation to losers allows the procurer to claim the intellectual property of the losing proposals. In most contest announcements, we observe the following clause: "Except for authorship, the other intellectual property rights belong to the procurer. The procurer has the right to refer to and use them in the project implementation process without paying additional fees."

ideas explains her choice between winner-take-all and multiple prizes. With both a positive entry cost and a sufficiently high value of diverse ideas, the contest organizer will offer multiple prizes to attract entry because she can benefit from diverse ideas from many agents. We also find that the cutoff value of ideas is substantially greater than zero, which explains why winner-take-all is still used even in some industries that value diversity. This result provides a rationale for the coexistence of winner-take-all and multiple prizes used in innovation contests. It is also consistent with our findings from the design contest data: multiple prizes are much more common in contests for building projects that require more creativity. Moreover, we show that the optimal prize scheme must be more equally distributed among the participants as eliciting diverse ideas becomes more important.

The remainder of the paper is organized as follows. In Section 2, we review the related literature. In Section 3, we present the data and motivating evidence. We propose our model and provide its analysis in Sections 4 and 5. Section 6 concludes the paper. Technical proofs are relegated to the Appendix.

2 Related Literature

Our paper contributes to the literature on optimal prize allocation in contests with incomplete information and endogenous entry. Liu and Lu (2019) show that after introducing an entry cost to the contest model in Moldovanu and Sela (2001), the winner-take-all prize still maximizes total effort. Hammond et al. (2019) study the contest design problem with endogenous entry but allow the contest organizer to charge an entry fee, which is used to augment the prize budget. Stouras et al. (2022) study innovation contests with costly entry, in which the designer aims to maximize the performance of the highest submission and incurs a penalty when there is no entrant. They characterize conditions under which winner-take-all and multiple prizes are optimal, respectively, which provide a rationale for awarding multiple prizes in many innovation contests. Liu and Lu (2023) characterize the effort-maximizing rank-order prize design when both rewards and punishments can be used. We contribute to this strand of literature by introducing the contest organizer's benefit from obtaining proposals with diverse ideas. We find that the winner-take-all prize scheme is no longer optimal in the presence of both costly entry and a sufficiently high value of diverse ideas.

In the literature, there are alternative ways to rationalize the adoption of multiple prizes in contest design. For example, in contests with incomplete information, the shapes of agents' effort cost function (Moldovanu and Sela, 2001; Zhang, 2024), prize valuation function (Glazer and Hassin, 1988; Polishchuk and Tonis, 2013), endogenous entry and the penalty resulting from an unproductive contest (Stouras et al., 2022), or the distribution of an output shock (Ales et al., 2017) can affect whether the contest organizer should adopt multiple prizes.[4] In addition, the advantage of

[4]Sarne and Lepioshkin (2017) illustrate that a multiple-prize scheme is often optimal when participation is costly

multiple prizes can also be rationalized by the relative strength of contestants' abilities (Szymanski and Valletti, 2005), the distribution of noise in performance (Drugov and Ryvkin, 2020), and the patience of the contest organizer (Korpeoglu et al., 2021). We show that under linear cost function and incomplete information, the value of diverse ideas can be a driving factor of optimal prize structure.

Our paper joins the large and growing literature on innovation contests in Economics (e.g., Taylor, 1995; Che and Gale, 2003; Letina and Schmutzler, 2019; Dong et al., 2023) and Operations Research (e.g., Terwiesch and Xu, 2008; Körpeoğlu and Cho, 2018; Korpeoglu et al., 2021; Gao et al., 2022; Stouras et al., 2022). Ales et al. (2019) and Segev (2020) provide comprehensive surveys of the literature on innovation contests. While the Operations Research community primarily focuses on the winner-take-all contests (Terwiesch and Xu, 2008; Körpeoğlu and Cho, 2018; Korpeoglu et al., 2021; Stouras et al., 2022), we emphasize the value of eliciting diverse ideas, which is aligned with Koh (2017), Letina and Schmutzler (2019), and Erkal and Xiao (2021). Koh (2017) shows that contest organizers want to attract more agents under higher quality uncertainty. Letina and Schmutzler (2019) study the optimal innovation contest in which the quality of an agent's innovation depends on the distance between his approach and an unobservable ideal approach. They show that a simple "bonus" tournament with a high prize and a low prize is optimal, which induces the socially optimal variety of approaches.[5] Differing from Letina and Schmutzler (2019) in which each agent chooses an approach, in our paper, each agent is endowed with a private approach/idea, which determines the marginal cost of preparing his proposal. Moreover, we note that the organizer can approximate the best idea from many proposals, so the quality of all the submitted proposals and the diversity of ideas both matter in our setting. Erkal and Xiao (2021) also assume that each agent is endowed with a private idea and characterize the optimal prize in winner-take-all innovation contests. They propose a new stochastic order to rank idea quality distributions and uncover the relationship between the scarcity of high-quality ideas and the optimal prize. Our study emphasizes that, in some innovation contests, the best idea or approach may not belong to any agent but is developed from several agents' submitted proposals.

The empirical results in our paper also contribute to the small literature on the empirical study of contests. Implementing theoretical insights in practice requires knowledge of the contest environment based on empirical studies. Jia (2008) uses NBA data to estimate the contest success function (CSF) and uses the Bayesian model selection method to compare three popular forms of CSFs. Hwang (2012) and Jia and Skaperdas (2012) use battle data to estimate the CSF and study military conflict technology. Sunde (2003) and Malueg and Yates (2010) use sports data to test some implications of contest theory. Kang (2016) studies lobbying activities in the U.S. energy sector and shows how spending affects voting outcomes. Lemus and Marshall (2021) use data from Kaggle prediction contests to estimate a dynamic model and show that public information

and the agents' efforts are random variables beyond their control.

[5] Letina (2016) also emphasizes the importance of the diversity of research projects and how it interacts with duplication of research effort among firms in the market.

disclosure improves average performance. Huang and He (2021) develop a structural estimation procedure for a Tullock contest and adopt it to study how campaign spending affects U.S. House election outcomes. Using data from software contests, Boudreau et al. (2011) find that having a greater number of competitors reduces the incentive to exert effort but makes it more likely that one competitor obtains an extreme-value solution. The latter effect dominates the former for more uncertain problems. We find a similar result: for projects with higher uncertainty, the contest organizers should induce more entry by offering more prizes.

3 Data and Motivating Evidence

We obtain a sample of 327 design proposal contests from the Guangzhou Public Resource Trading Center (`www.gzggzy.cn`). The data cover all public procurement contests that involve the submission of design proposals from 2014 to 2018 in Guangdong, China. These projects belong to five categories: building and architecture (Obs= 195), urban planning and transportation (Obs= 83), landscaping (Obs= 28), electricity and mechanics (Obs= 12), and water conservancy (Obs= 9). Most procurers (contest organizers) of these projects are governments and state-owned enterprises that are required by the law to conduct procurement through designated procurement platforms. Table 1 provides the summary statistics of the data.

Table 1: Summary Statistics

	All projects Obs=327		Other projects Obs=195		Building projects Obs=132		t test
Variable	Mean	SD	Mean	SD	Mean	SD	p-value
p.design	1193	1225	918	928	1379	1361	**0.000**
p.construct	42461	89816	34396	33267	47921	112845	0.116
proposal.day	42.584	21.575	42.765	20.798	42.462	22.137	0.900
design.day	122.786	98.504	117.682	58.918	126.241	118.031	0.387
D.model	0.278	0.449	0.008	0.087	0.462	0.500	**0.000**
D.prequalify	0.018	0.134	0.000	0.000	0.031	0.173	0.014
area	74,710	125,217	68,206	44,592	79,136	158,129	0.363
N.prize	4.896	2.919	4.265	3.061	5.323	2.746	**0.002**
D.multi.prize	0.725	0.447	0.583	0.495	0.821	0.385	**0.000**
V	19.711	35.029	16.972	33.698	21.564	35.869	0.240
v_1	5.599	9.738	5.225	10.161	5.852	9.460	0.574
v.loser	14.402	26.027	12.171	24.207	15.913	27.148	0.193
v.loser.ratio	0.508	0.328	0.414	0.359	0.572	0.290	**0.000**
v.Gini	0.507	0.289	0.572	0.293	0.464	0.278	**0.001**
n	8.419	4.508	7.205	3.951	9.241	4.683	**0.000**

Note: All monetary variables (*p.design*, *p.construct*, V, v_1, and *v.loser*) are in units of CNY 10,000. *D.multi.prize*, *D.model*, and *D.prequalify* are dummy variables. The last column displays the p-value of a t test between building projects and other projects. Numbers in bold indicate that the test is rejected at the 95% significance level.

In practice, a contest organizer launches a contest by posting an announcement on the procurement platform with the help of a professional procurement agency. Appendix B is a sample contest announcement. The announcement contains information on the project and invites agents to participate in the proposal contest. For each contest, we observe the budget for the design component (*p.design*), the budget for the construction component (*p.construct*),[6] the total work area in square meters (*area*), whether agents need to submit a design model (*D.model*), whether agents need to satisfy prequalification (*D.prequalify*), the number of days to submit the proposal (*proposal.day*), the number of days for the winner to complete the detailed design work (*design.day*), and the city of the project. The projects are located across 19 cities in Guangdong Province.

After agents complete their design proposals, a committee of experts evaluates them and determines a ranking. The agent ranked first is offered the design contract. If the offer is not accepted, the design contract will be offered to the other agents in rank order. For all contests, we observe the prize scheme, $\mathbf{v} = (v_1, v_2, ..., v_8)$, that specifies the design compensation awarded to the agents based on the ranking. The total amount of prizes is $V = \sum_{i=1}^{8} v_i$. We compute three measures for the prize structure: the total amount of prizes for contest losers ($v.loser = \sum_{i=2}^{8} v_i$), the proportion of loser compensation in the total prize ($v.loser.ratio = v.loser/V = (V - v_1)/V$), and the Gini coefficient of prizes ($v.Gini$).[7]

Note that the prizes of the contests are called design compensation because preparing the design proposal is costly for the agents. Design compensation is different from *p.design*. The former specifies direct payments to the winner and losers of the proposal contest. The latter is the monetary payment specified in the design contract awarded to the winner to carry out the complete design work after the contest.

Based on opinions obtained from industry experts, in building and architecture design projects, procurers typically want the design proposals to be more creative and diverse than in other projects (urban planning, transportation, landscaping, electricity, mechanics, and water conservancy). From Table 1, approximately 46% of building projects require the submission of a design model ($D.model$= 1), whereas less than 1% of other projects do. Moreover, the budget of the design component (*p.design*) is also significantly larger for building projects than other projects. This indicates that design work in other projects is relatively standard and does not require a demonstration of innovative ideas. Regarding prize structures, building projects tend to allocate a larger proportion of the compensation budget to losers than other projects. On average, 57.2% of the budget is allocated to the losers in building projects, while in other projects, the figure is 41.4%.

Figure 1-(A) is the histogram of *N.prize*, which is the number of positive prizes offered. In the data, the maximum number of positive prizes is 8. In Table 1, we find that 237 (72.5%) of the 327 contests offer multiple prizes ($D.multi.prize > 1$). This proportion is significantly higher among building projects (82.1%) than other projects (58.3%). On average, a contest offers 4.896 prizes.

[6] Agents engaged in the design contest only work on the design component and not the construction component. The construction component is carried out by other construction companies.

[7] We compute the Gini coefficient of prizes by the method in Gastwirth (1972).

The average number of prizes of building projects (5.323) is significantly greater than that of other projects (4.265). Hence, the contest organizers of building projects are more inclined to adopt multiple prizes. Moreover, Table 1 shows that the amount and proportion of loser compensation in contests for building projects is higher than those for other projects. Building projects also tend to adopt more equal prize schemes (measured by $v.Gini$).

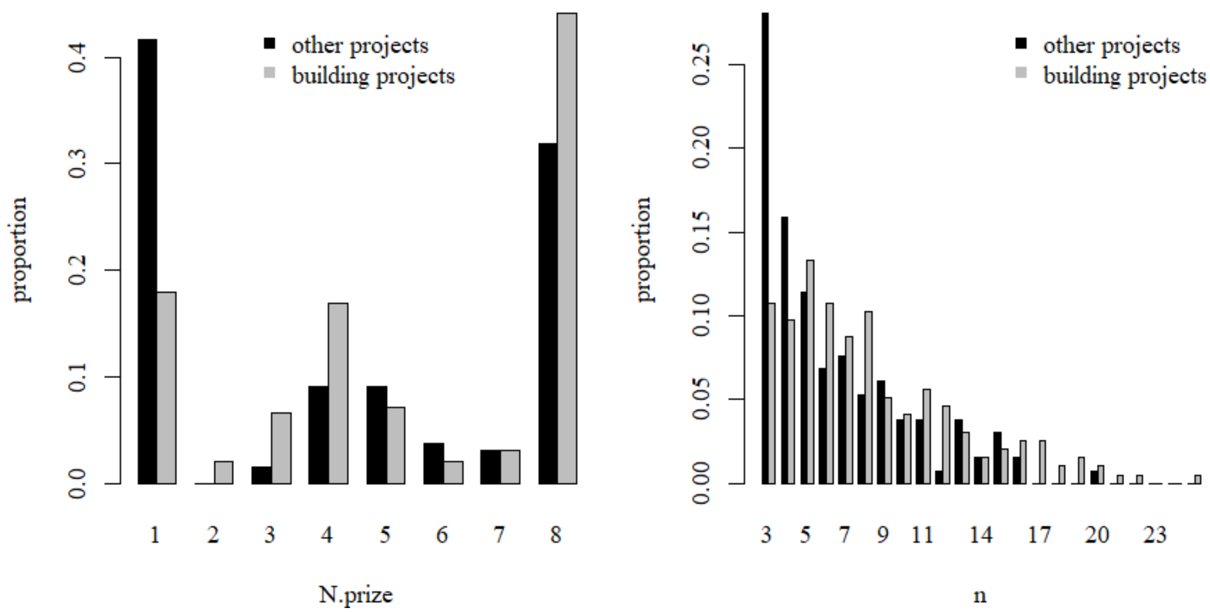


Figure 1: Illustration of Prize Structure

Figure 1-(B) depicts prizes by agent ranking among all contests offering multiple prizes (Obs= 237). Clearly, the amount of prizes decreases in the ranking. The first runner-up obtains an average compensation of CNY 36,992, while the seventh runner-up obtains an average compensation of CNY 10,434. In fact, all observed prize schemes are weakly decreasing without any exception. We observe the number of agents (n) who submit a complete design proposal before the call-for-proposals deadline. On average, a contest attracts 8.4 agents. Figure 2 shows the distribution of n.[8] In general, there are more agents participating in building projects than in others.

Figure 3 depicts the relationship between the prize structure and the number of prizes allocated to losers ($v.loser.ratio$) and the number of participants (n), which suggests that allocating a larger proportion of the budget to compensate losers attracts more agents to submit proposals. This empirical data pattern is consistent with the experimental evidence in Cason et al. (2010), which shows that dividing a fixed prize in proportion to participant contributions induces more entries and greater total effort.

[8]The law requires that at least three agents submit bids (proposals) to maintain the competitiveness of the contest. Otherwise, the procurement fails, and the procurer needs to launch another contest.

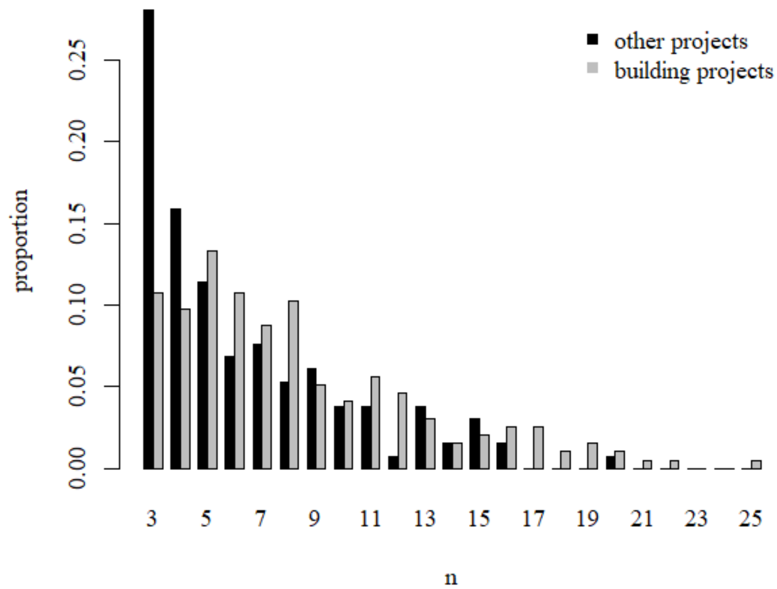


Figure 2: Histogram of Number of agents

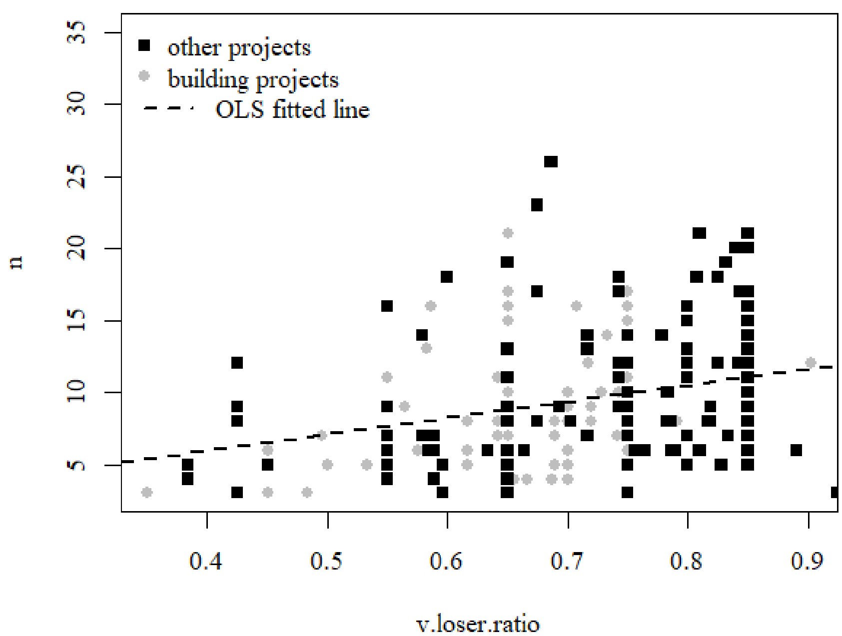


Figure 3: Prize Structure and Agent Entry

Table 2 further demonstrates the endogenous entry pattern of design contests. Regressions (1) and (2) indicate that allocating a larger amount of prizes to contest losers significantly increases the number of participating agents. Regressions (3) and (4) show that allocating a larger proportion of resources to contest losers significantly increases the number of agents. Regressions (5) and (6) show that fewer agents tend to participate in contests with more unequal prize distribution (higher

$v.Gini$). These empirical results are consistent with our theoretical finding in Section 5.

Table 2: Prize Structure and Agent Participation

	Dependent variable: n					
	(1)	(2)	(3)	(4)	(5)	(6)
v.loser	0.064^{***}	0.058^{***}				
	(0.009)	(0.009)				
v.loser.ratio			5.568^{***}	5.580^{***}		
			(0.696)	(0.747)		
v.Gini					-6.305^{***}	-6.101^{***}
					(0.827)	(0.827)
D.building		1.908^{***}		0.807		1.126^{**}
		(0.570)		(0.571)		(0.564)
p.design		0.0001		0.001^{***}		0.0005^{**}
		(0.0002)		(0.0002)		(0.0002)
p.construct		−0.00000		-0.00001^{*}		-0.00001^{*}
		(0.00000)		(0.00000)		(0.00000)
proposal.day		0.002		0.001		0.002
		(0.011)		(0.011)		(0.011)
design.day		0.004		0.006^{**}		0.006^{**}
		(0.003)		(0.003)		(0.003)
D.model		0.296		0.337		0.249
		(0.608)		(0.592)		(0.594)
D.prequalify		−0.978		−1.174		−1.077
		(1.771)		(1.726)		(1.728)
area		0.004^{*}		0.005^{**}		0.005^{**}
		(0.002)		(0.002)		(0.002)
city FE		✓		✓		✓
Observations	327	326	327	326	327	326
R^2	0.135	0.236	0.164	0.274	0.186	0.271

Note: *v.loser* is the total amount of prizes awarded to contest losers. *v.loser.ratio* is the proportion of prize awarded to losers. *v.Gini* is the Gini coefficient of prize distribution among agents. Significance levels are indicated by * ($p < 10\%$), ** ($p < 5\%$), and *** ($p < 1\%$).

In summary, many real-world contest organizers choose to offer multiple prizes, and offering multiple prizes can effectively attract more agents to enter the contest. Moreover, for projects requiring more innovative and diverse ideas (building and architectural design), contest organizers tend to offer more prizes and adopt more equal prize distributions among participating agents. These data patterns serve as motivating evidence for our model in which the contest organizer exhibits a preference for diverse ideas.

4 The Model

4.1 Model Setup

The innovation contest calls for proposals for the design of projects (or solutions to problems). To elicit an innovative project design (or ideal solution), the contest organizer sets up a ranked-order prize scheme given a fixed budget. Agents (solvers) participate in the contest by preparing proposals to demonstrate their ideas. Their proposals are evaluated by the organizer or a group of experts to determine the ranking.[9] In standard contest models, the contest organizer aims to maximize the total effort spent by the agents in preparing their proposals. Our model considers an important and intrinsic feature of innovation contests: the contest organizer is uncertain about the ideal design (or the best solution) and, therefore, cannot fully specify and describe the evaluation criteria in the contest announcement.

Our model is based on the standard contest model with incomplete information (Moldovanu and Sela, 2001). There are N (≥ 2) potential agents. The contest organizer spends a fixed budget $V(>0)$ on a ranked-based prize scheme, $\mathbf{v} = (v_1, v_2, \ldots, v_N)$, with $v_1 \geq v_2 \geq \cdots \geq v_N \geq 0$ and $\sum_{i=1}^{N} v_i \leq V$. Here, v_i is the prize for the agent ranked at the ith position. The contest organizer and all agents are risk-neutral. Retaining the budget does not increase the contest organizer's payoff.

Given $\mathbf{v}$, each agent decides whether to incur a commonly known opportunity cost $c \in (0, V/N)$ to enter the contest.[10] The condition $c < V/N$ ensures that the contest organizer can induce full entry if she wishes. When there are n entrants, these entrants win the respective first n prizes, $(v_1, v_2, \ldots, v_n)$, from $\mathbf{v}$ according to their ranking. The winner receives v_1; the first runner-up receives v_2; ...; the agent ranked at the bottom receives v_n; and all nonparticipating agents obtain zero prizes.

Upon entering the contest, each agent must decide his effort level, e_i. A generic agent i is endowed with an efficiency parameter t_i, which is his privately informed type. Each t_i is independently drawn from the distribution function $F(\cdot)$. The density function $f(\cdot)$ has a compact support $[a, b]$ with $a > 0$. Agent i's cost of effort is given by $c(e_i, t_i) = \frac{e_i}{t_i}$. This effort represents how much the agent spends in preparing the proposal and presenting the idea to fulfill the well-specified evaluation criterion. The contest organizer or the experts will determine the ranking of all submitted proposals based on the written criterion. Hence, the contest outcome and agents' ranking are determined by effort levels.

In addition to the effort, the quality or value of the proposal also depends on the idea and other factors that cannot be clearly specified in the evaluation rules. Suppose that each agent draws

[9] We assume the evaluation process is objective and fair. In practice, this evaluation process may be subject to problems of quality manipulation corruption (Huang, 2019; Huang and Xia, 2019) and uncertainty (Takahashi, 2018). We do not consider these issues in this paper.

[10] Opportunity costs are ubiquitous in innovative contests. For a particular contest, agents need to spend time and resources acquiring project-related information before entering the contest and forgo the opportunity to participate in other contests due to limited capacity.

an idea, x_i, from a uniformly distributed idea space, Uniform$[0, d]$.[11] The variable x_i measures how innovative the idea is or how close the approach is to the ideal solution. The parameter d represents the diversity of ideas or the level of uncertainty. When $d = 0$, it reduces the classical model of contests with incomplete information. The value of the proposal to the contest organizer is $q_i = e_i + x_i$.

In other words, the proposal evaluation is based on the criterion that can be specified, and agents spend effort in preparing the proposal to obtain good scores in the evaluation exercise. However, the contest organizer's actual payoff from the submitted proposal is also affected by factors that cannot be specified in the evaluation rules. This model setting reflects an essential feature of creative industry and innovation activities: novel designs, innovative solutions, and unconventional ideas cannot be easily measured by pre-specified rules.

The contest organizer's objective is to maximize the total value (quality) of the proposals submitted by participating agents. The organizer wants many agents to spend time and effort in developing ideas and demonstrating their thoughts in high-quality proposals; thus, she cares about the quality of all proposals, not just that of the winner. This reasoning aligns with the established literature, where the contest organizer maximizes total effort. Suppose that there are $n \geq 1$ agents entering the contest (agents $1, 2, ..., n$). The organizer's payoff is $\sum_{i=1}^{n}(e_i + x_i)$. As the organizer does not observe participants' types (t_i) and their ideas (x_i), she aims to maximize the expected payoff. The exact expression of the objective function is provided below in expression (O-d).

The timing of the game is as follows.

• Period 0: Each of N agents independently draws his efficiency t and idea x from the distribution F and Uniform$[0, d]$, respectively.

• Period 1: The contest organizer chooses a prize scheme $\mathbf{v}$ and commits to it.

• Period 2: All potential agents simultaneously decide whether to participate in the contest by incurring the opportunity cost c.

• Period 3: Each entrant chooses his effort level after learning the number of entrants n.

• Period 4: The prize allocation is implemented according to $\mathbf{v}$.

4.2 Equilibrium Analysis

We first characterize the equilibrium for any given prize structure $\mathbf{v}$ and then search for the optimal prize allocation rule. Since the contest rule is anonymous and agents are symmetric, it is natural to focus on symmetric equilibria. Throughout the paper, we restrict our attention to monotone equilibria in which the effort level increases in t. We call the scenario with n entrants scenario n for convenience.

We begin the equilibrium analysis from period 2. Consider a symmetric Bayesian Nash equilibrium of the entry decision characterized by a threshold type t^c. Given F and $\mathbf{v}$, an agent will enter

[11] Our main results of the paper hold for a wide class of distribution of ideas.

the contest if and only if his type $t \geq t^c$; otherwise, he will not participate.[12] The participation constraint requires that the expected payoff is no less than the opportunity cost c. Under monotone equilibria, an agent with the threshold type t^c has the lowest type among all entrants and, thus, will be allocated the lowest prize v_n in scenario n, for any n. Moreover, the agent with t^c must choose $e = 0$ in equilibrium and has an expected payoff of c from the contest.[13] Therefore, the threshold type t^c is determined by

$$\sum_{n=1}^{N} p_n(t^c)v_n = c, \tag{1}$$

where $p_n(t^c) = \binom{N-1}{n-1}(1 - F(t^c))^{n-1}F^{N-n}(t^c)$ is the probability that there are exactly $n - 1$ rival agents entering the contest.

Note that the prizes are allocated based on the ranking of submitted proposals evaluated by the specified rules. Therefore, the ranking is based on the equilibrium effort levels of agents, which further depend on their types. A more efficient agent with a larger t_i will spend more effort in equilibrium. An agent's idea, x_i, is not taken into account in the evaluation process, so the idea does not affect the equilibrium effort levels.

In period 3, given the entry threshold $t^c \in [a, b)$ determined by (1), let n be the realized number of entrants. An entrant knows that each of his rivals has a type independently drawn from the truncated distribution function $G(t, t^c) = \frac{F(t)-F(t^c)}{1-F(t^c)}$ with density function $g(t, t^c) = \frac{f(t)}{1-F(t^c)}$ and support $t \in [t^c, b]$. Given t^c, the equilibrium effort of an agent and the total effort of n entrants are as follows.

Proposition 1. *Suppose that the induced entry threshold is t^c. In scenario $n \geq 1$ with prizes $\mathbf{W}_n \equiv (v_1, v_2, \ldots, v_n)$,*
(i) The unique symmetric effort function $e^{(n)}(t, \mathbf{W}_n, t^c)$ for type $t \in [t^c, b]$ is

$$e^{(n)}(t, \mathbf{W}_n, t^c) = tV^{(n)}(t) - \int_{t^c}^{t} V^{(n)}(s)ds - t^c v_n,$$

where

$$V^{(n)}(t) = \sum_{j=1}^{n} v_{n+1-j}\binom{n-1}{j-1}G^{j-1}(t, t^c)(1 - G(t, t^c))^{n-j}$$

is the expected prize that an entrant of type t obtains.
(ii) The corresponding scenario-n total effort is

$$TE^{(n)}(\mathbf{v}, t^c) = n\int_{t^c}^{b} J(t)V^{(n)}(t)g(t, t^c)dt - nt^c v_n, \tag{2}$$

[12] The proof is standard based on Samuelson (1985) and McAfee and McMillan (1987).

[13] An agent with the threshold type may enjoy a payoff strictly larger than c if the prize structure induces full entry. However, this is clearly not optimal, unless the diversity coverage d is infinity (i.e., $d = +\infty$ in (O-d) below).

where $J(t) = t - \frac{1-F(t)}{f(t)}$ is the notion of "virtual efficiency" of type t as in the mechanism design literature (cf. Myerson, 1981).

In period 1, a prize scheme $\mathbf{v}$ induces an entry threshold t^c by the participation constraint (1). Proposition 1 characterizes the equilibrium after the realization of n. Therefore, the expected total effort is the weighted average of the scenario-n expected total effort across all scenarios:

$$TE(\mathbf{v}) = \sum_{n=1}^{N} \binom{N}{n} (1 - F(t^c))^n F^{N-n}(t^c) TE^{(n)}(\mathbf{v}, t^c), \tag{3}$$

where $TE^{(n)}(\mathbf{v}, t^c)$ is given in equation (2). Note that whether the agent knows the number of rivals does not affect the expected total effort or the results of contest design. If agents do not know the number of rival(s), an agent of type $t \geq t^c$ will bid the weighted average of equilibrium effort across all scenarios: $\sum_{n=1}^{N} p_n(t^c) e^{(n)}(t, \mathbf{W}_n, t^c)$. As a result, the expected total effort is the same as that in (3).

5 Optimal Prize Structure

5.1 The Organizer's Problem

The number of entrants endogenously depends on the prize structure $\mathbf{v}$. Suppose that there are $n \geq 1$ entrants (e.g., agents $1, 2, ..., n$ enter the contest). The organizer's payoff is $\sum_{i=1}^{n}(e_i + x_i)$, where e_i and x_i are agent i's effort and idea, respectively. The effort level e_i depends on agent i's type t_i and the prize structure $\mathbf{v}$, which is characterized in Proposition 1. The organizer does not observe each participant's type t_i and their idea x_i. When there are n entrants, the organizer's payoff is

$$E_{t,x}[\sum_{i=1}^{n}(e_i + x_i)] = E_t[\sum_{i=1}^{n} e_i] + E_x[\sum_{i=1}^{n} x_i] = TE^{(n)}(\mathbf{v}, t^c) + \frac{nd}{2},$$

where $TE^{(n)}(\mathbf{v}, t^c)$ is the expected total effort when there are n entrants as in Proposition 1.

When choosing the prize scheme, the number of entrants is not realized, so the organizer's expected payoff is

$$\begin{aligned} P(\mathbf{v}, t^c; d) &= \sum_{n=1}^{N} \binom{N}{n} (1 - F(t^c))^n F^{N-n}(t^c)[TE^{(n)}(\mathbf{v}, t^c) + \frac{nd}{2}] \\ &= TE(\mathbf{v}, t^c) + \sum_{n=1}^{N} N(1 - F(t^c)) \binom{N-1}{n-1} (1 - F(t^c))^{n-1} F^{N-n}(t^c) \times \frac{d}{2} \\ &= TE(\mathbf{v}, t^c) + N(1 - F(t^c))\frac{d}{2}, \end{aligned}$$

where $TE(\mathbf{v}, t^c)$ is the ex-ante expected total effort as in expression (3). In the second term, $1 - F(t^c)$ represents the entry probability of a generic agent, and $d/2$ is the expected value of one additional idea. Hence, $N(1-F(t^c))d/2$ is the expected value of eliciting diverse ideas from agents. As the entry threshold t^c decreases, the value of diverse ideas $N(1-F(t^c))d/2$ increases.

The contest organizer's goal is to find a prize scheme $\mathbf{v}$ (and hence t^c) to maximize her payoff:

$$\text{(O-}d\text{)} \qquad \max_{t^c\in[a,b],\ \mathbf{v}} P(\mathbf{v},t^c;d) = TE(\mathbf{v},t^c) + N(1-F(t^c))\frac{d}{2}$$

$$\text{subject to } \sum_{n=1}^{N} v_n \le V \text{ and } \sum_{n=1}^{N} p_n(t^c)v_n = c.$$

Note that once $\mathbf{v}$ is given, t^c is pinned down correspondingly by equation (1). Including t^c as a choice variable in the contest organizer's problem is purely for notational convenience.[14] The two constraints in the optimization problem are the contest organizer's budget constraint and the participation constraint for the threshold type t^c. The parameter d captures the value of diverse ideas or the uncertainty of ideal design. Across different kinds of contests, contest organizers may value diverse ideas differently. We will show that it is this difference in the value of diverse ideas that drives the optimality of winner-take-all or multiple prizes observed in the data.

The organizer's problem (O-d) is a generalization of existing contest models in the literature. when $d=0$ and $c=0$, our model reduces to the one in Moldovanu and Sela (2001); when $d=0$ and $c>0$, our model corresponds to the setting of Liu and Lu (2019) except that the prize allocation rule in Liu and Lu (2019) can be contingent on the number of entrants n. However, we follow Moldovanu and Sela (2001) and do not allow this contingency, because most prize allocation rules in reality are not contingent on n. This is also the case for the observed proposal contests in Section 3. In their analysis of both contingent and non-contingent prize schemes, Liu and Lu (2023) demonstrate that establishing the optimum in a restricted class of prize schemes can be much more technically challenging because of the presence of more constraints.

5.2 Rewriting the Objective Function

The expected total effort $TE(\mathbf{v},t^c)$ in the objective function of problem (O-d) is quite complicated, as it involves various binomials and their summations. It turns out that the objective function can be greatly simplified. Let $G_{(\ell,n)}(t,t^c)$ denote the CDF of the ℓth order statistics of n independent random variables, each following the CDF $G(t,t^c)$. The CDF of the ℓth order statistics is

$$G_{(\ell,n)}(t,t^c) = \sum_{j=\ell}^{n} \binom{n}{j} G^j(t,t^c)(1-G(t,t^c))^{n-j},$$

[14] When $t^c=b$, the participation constraint (1) may not hold. However, in this case, there is no entry, which means $TE(\mathbf{v},t^c)=0$ and $1-F(t^c)=0$, so $t^c=b$ can never be optimal.

and its density function is

$$g_{(\ell,n)}(t,t^c) = n\binom{n-1}{\ell-1} G^{\ell-1}(t,t^c)(1-G(t,t^c))^{n-\ell} g(t,t^c).$$

When $t^c = a$, G reduces to F, and we can write

$$\begin{aligned} G_{(\ell,n)}(t,a) &= F_{(\ell,n)}(t) = \sum_{j=\ell}^{n} \binom{n}{j} F^j(t)(1-F(t))^{n-j}, \\ g_{(\ell,n)}(t,a) &= f_{(\ell,n)}(t) = n\binom{n-1}{\ell-1} F^{\ell-1}(t)(1-F(t))^{n-\ell} f(t). \end{aligned} \tag{4}$$

It turns out that one can use these order statistics to rewrite the expected total effort $TE(\mathbf{v}, t^c)$ in problem (O-d), which leads to the following result:

Lemma 1. *The expected total effort $TE(\mathbf{v}, t^c)$, i.e., expression (3), can be rewritten as*

$$TE(\mathbf{v}, t^c) = N(1-F(t^c)) \left[\sum_{n=1}^{N} \frac{p_n(t^c)}{n} \sum_{j=1}^{n} v_{n+1-j} \left(\int_{t^c}^{b} J(t) g_{(j,n)}(t,t^c) dt \right) - ct^c \right].$$

Lemma 1 implies that the expected total effort can be expressed as a linear function in all N prizes, with coefficients related to order statistics across different scenarios and orders. That is,

$$TE(\mathbf{v}, t^c) = \sum_{k=1}^{N} \beta_k(t^c) v_k - Nc(1-F(t^c))t^c,$$

where

$$\beta_k(t^c) = N(1-F(t^c)) \sum_{n=k}^{N} \frac{p_n(t^c)}{n} \left(\int_{t^c}^{b} J(t) g_{(n+1-k,n)}(t,t^c) dt \right), \; k = 1, 2, \ldots, N. \tag{5}$$

As shown in Liu and Lu (2023), the coefficients $\beta_k(t^c)$ can be simplified as follows, which will greatly facilitate our analysis of the optimal prize scheme.

Lemma 2. *The coefficients $\beta_k(t^c)$ in the objective function can be rewritten as*

$$\beta_k(t^c) = \int_{t^c}^{b} J(t) f_{(N-k+1,N)}(t) dt, \; k = 1, 2, \ldots, N, \tag{6}$$

where $f_{(N-k+1,N)}(t)$ is the density function of the $(N-k+1)$th order statistics of N random draws that follow CDF $F(\cdot)$, as defined in (4).

5.3 The Analysis

Given Proposition 1, the contest organizer's problem can be decomposed into two steps: In the first step, we fix the entry threshold t^c to characterize properties of optimal prize schemes that induce t^c; in the second step, we vary across all entry thresholds $t^c \in [a, b]$ to determine the optimal entry threshold and, consequently, the corresponding optimal prize scheme.

First Step: Fixing the Entry Threshold

Let us start with the first step. For any fixed $t^c \in [a, b]$ and d, the value of diverse ideas in the objective function (O-d) is a constant. Hence, in this step, the contest organizer's problem is equivalent to

$$\max_{\mathbf{v}} TE(\mathbf{v}, t^c), \text{ subject to } \sum_{n=1}^{N} v_n \leq V \text{ and } \sum_{n=1}^{N} p_n(t^c) v_n = c. \tag{7}$$

Denote the value function of the above problem as $TE^*(t^c)$ and the set of optimal solution(s) as

$$S^*(t^c) = \{\mathbf{v} \in S(t^c) : TE(\mathbf{v}, t^c) = TE^*(t^c)\}, \tag{8}$$

where $S(t^c)$ is the feasible set of problem (7), i.e.,

$$S(t^c) = \{\mathbf{v} \in \mathbb{R}^N : v_1 \geq v_2 \geq \cdots \geq v_N \geq 0, \ \sum_{n=1}^{N} v_n \leq V, \text{ and } \sum_{n=1}^{N} p_n(t^c) v_n = c\}.$$

Because $TE(\mathbf{v}, t^c)$ is continuous in $\mathbf{v}$ and the feasible set is compact, the existence of optimal solutions and the value function are guaranteed by the extreme value theorem.

By equation (1), winner-take-all ($\mathbf{v} = (V, 0, \ldots, 0)$) induces the cutoff $t_1 = F^{-1}\left[\left(\frac{c}{V}\right)^{\frac{1}{N-1}}\right]$. The following lemma shows that when ignoring the value of diverse ideas, winner-take-all is the unique optimal prize scheme, which echoes Liu and Lu (2019).

Lemma 3. *$TE^*(t^c)$ reaches its unique optimum over $t^c \in [a, b]$ at $t_1 \equiv F^{-1}\left[\left(\frac{c}{V}\right)^{\frac{1}{N-1}}\right]$. Furthermore, $S^*(t_1) = \{(V, 0, \ldots, 0)\}$; that is, winner-take-all is the unique optimum.*[15]

Note that when the entry threshold t^c is fixed, as shown in Lemma 2, the contest organizer's problem is linear programming in the sense that all functions in the problem—the objective function, the budget constraint, and the participation constraint—are linear in the N prizes. As a linear programming, it is important to investigate the properties of its coefficients to find its optimum. However, as shown in Liu and Lu (2023), coefficients in this linear programming problem vary in a highly intractable and nonlinear way as the induced entry threshold t^c changes, which makes the

[15] As mentioned in footnote 14, when $t^c = b$, there is no entry, so it can never be optimal.

analysis quite challenging. For example, the number of positive prizes varies with the entry threshold; this issue is further complicated by the restriction imposed by the participation constraint: For a fixed t^c, only a certain restrictive subset of prize schemes induces it.[16]

The arbitrariness and intractability of coefficients complicate the characterization of the optimum for any given entry threshold, making an explicit characterization of the optimum infeasible. Liu and Lu (2023) solve their problem in an indirect manner that does not require finding the optimum for any given entry threshold. However, their approach does not apply to our model, because prizes here have to be nonnegative (more constraints) and the objective function has an additional term (the value of diverse ideas). Therefore, our problem is even more challenging than that in the literature. Fortunately, we are still able to characterize the essential feature of the optimal prize scheme brought by the value of diversity, which is detailed in the second-step analysis below.

Second Step: Varying the Entry Threshold

In the second step, we further pin down the optimal entry threshold t^{c*} by

$$\max_{t^c \in [a,b]} P^*(t^c; d) = TE^*(t^c) + N(1 - F(t^c))\frac{d}{2}.$$

Note that for any $t^c > t_1$, it cannot be the solution to the above problem, because by Lemma 3,

$$P^*(t^c; d) = TE^*(t^c) + N(1 - F(t^c))\frac{d}{2} < TE^*(t_1) + N(1 - F(t_1))\frac{d}{2} = P^*(t_1; d).$$

Hence, we can restrict our attention to the domain $t^c \in [a, t_1]$ and solve the following problem:

$$\text{(O2-}d\text{)} \qquad \max_{t^c \in [a,t_1]} P^*(t^c; d).$$

For a given d, let $t^{c*}(d)$ denote the set of optimal entry thresholds in problem (O-d). Using (8), the optimal prize scheme for problem (O-d) can be denoted as $S^*(t^{c*}(d)) = \{\mathbf{v} : \mathbf{v} \in S^*(t^c),\ \forall t^c \in t^{c*}(d)\}$. Note that $S^*(t^{c*}(d))$ may not be a singleton, and any $\tilde{\mathbf{v}} \in S^*(t^{c*}(d))$ satisfies

$$(9) \qquad P(\tilde{\mathbf{v}}, t^*(d); d) = P^*(t^*(d); d),\ \forall t^*(d) \in t^{c*}(d),\ \forall d \in [0, +\infty).$$

Lemma 3 implies that $t^{c*}(0) = \{t_1\}$ and $S^*(t^{c*}(0)) = \{(V, 0, \ldots, 0)\}$, i.e., winner-take-all is the unique optimum when $d = 0$. It follows that the optimality of winner-take-all in problem (O-d) can be characterized by the induced entry threshold in problem (O2-d).

Lemma 4. *The winner-take-all prize scheme is a solution to problem (O-d) if and only if t_1 is a solution to problem (O2-d). Furthermore, winner-take-all is the unique solution to problem (O-d) if and only if t_1 is the unique solution to problem (O2-d).*

[16] For example, winner-take-all induces a unique entry threshold, so any other entry thresholds cannot be supported by winner-take-all.

Lemma 4 further implies the following result.

Lemma 5. *(i) If there is some $d_0 > 0$ such that winner-take-all is the unique solution to problem (O-d_0), then winner-take-all remains the unique optimum of problem (O-d) for any $d \in [0, d_0]$.*
(ii) If there is some $d_1 > 0$ such that winner-take-all is not a solution to problem (O-d_1), then winner-take-all is not a solution to problem (O-d) for any $d \in [d_1, +\infty)$.
(iii) If there is some $d_2 > 0$ such that winner-take-all is a solution, but not the unique solution, to problem (O-d_2), then such d_2 must be unique.

This result reveals the relationship between the winner-take-all prize and the uncertain value of ideas. Whenever winner-take-all is optimal for some value of diverse ideas, the contest organizer should continue using winner-take-all when she values diverse ideas less; conversely, whenever using multiple prizes is optimal for some value of diverse ideas, she should still offer multiple prizes when she values diverse ideas more.

Recall Lemma 3 that when the organizer does not value diversity ($d = 0$), winner-take-all is the unique optimum (equivalently, t_1 is the unique optimal entry threshold). The following result implies that winner-take-all can still be optimal even when the organizer values diversity.

Lemma 6. *Denote $S_1 = \{d \in [0, +\infty)$: t_1 is the unique solution to problem (O2-d)$\}$, then $S_1 \neq \{0\}$.*

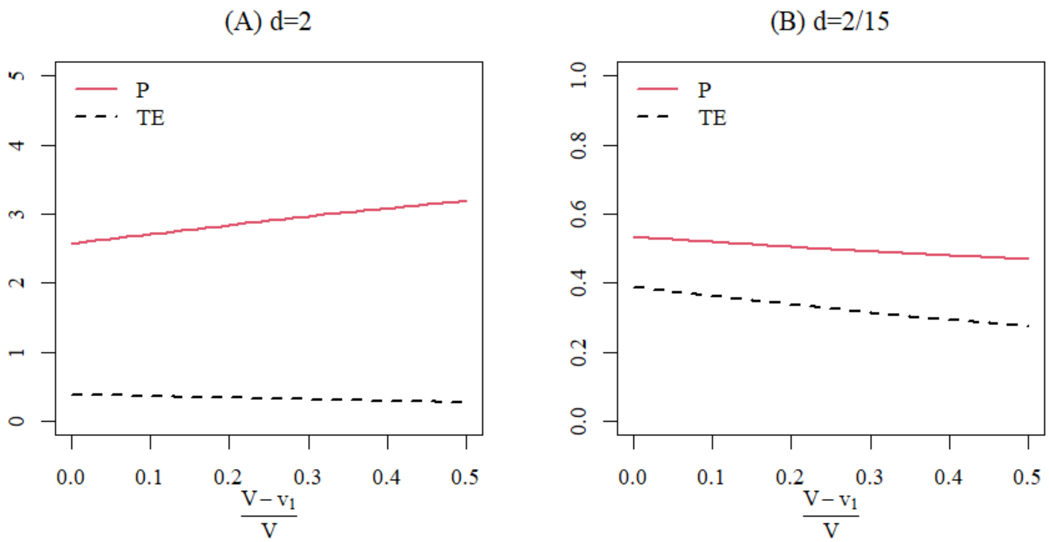
Lemma 6 reveals that winner-take-all is not only the optimal prize scheme when $d = 0$ but also for some $d > 0$.

5.4 Structure of the Optimal Prize Scheme

Combining all the above analyses, we arrive at the main result regarding the optimal prize scheme.

Proposition 2. *There exists a unique $\tilde{d} > 0$, such that the following holds:*
(i) when $d \in [0, \tilde{d})$, the winner-take-all prize structure—that is, $\mathbf{v} = (V, 0, \ldots, 0)$—is the unique optimal prize structure;
(ii) when $d = \tilde{d}$, the winner-take-all prize structure is optimal but may not be unique; and
(iii) when $d \in (\tilde{d}, +\infty)$, the winner-take-all prize structure is not optimal, and multiple positive prizes must arise at the optimum.

Figure 4 illustrates Proposition 2 using a numerical example. In the example, we consider uniformly distributed types and the prize scheme with two positive prizes, $\mathbf{v} = (v_1, V - v_1, 0, \ldots, 0)$. In Panel (A), the value of diversity is high ($d = 2$), and allocating half of the budget as the second prize is optimal. In contrast, Panel (B) shows that when the value of diversity is small ($d = 2/15$), the winner-take-all prize scheme maximizes the objective function in (O-d). Note that the total effort decreases in the prize offered to the runner-up, which is consistent with the result in Liu and Lu (2019).



Note: $t \sim U[0,1]$, $V = 0.01$, $c = 0.001$, $N = 5$ and $\mathbf{v} = (v_1, V - v_1, 0, 0, 0)$.

Figure 4: Total Effort and Contest Organizer's Payoff

Proposition 2 reveals that the value of diverse ideas and the corresponding optimal prize scheme can explain the prize allocation patterns observed in the data. Winner-take-all is the unique optimum if eliciting ideas from many agents does not generate high value ($d < \tilde{d}$). However, when the project involves high uncertainty and inducing participation is sufficiently valuable ($d > \tilde{d}$), the contest organizer should offer multiple prizes. These observations align with empirical findings that a significantly larger proportion of contests for building projects offer multiple prizes compared to those for other projects, as procurers of building and architectural design typically value more diverse and creative ideas.

Note that the threshold is strictly greater than zero ($\tilde{d} > 0$). This means that a low value of diverse ideas does not necessarily lead to the multiplicity of prizes. According to the law, the procurer can use the ideas in the losing proposals by paying design compensation to the losers. It seems that as long as the contest organizer cares even slightly about diversity, she should always offer a small prize to all losers to claim their property rights. However, this is certainly inconsistent with our data, which shows that there are also many contest rules that offer a single prize. Our model nicely explains this: in creative industries that value idea diversity, the contest organizers may still adopt winner-take-all even if they care about diversity.

We want to emphasize that the two factors introduced in the model—opportunity cost (or equivalently, endogenous entry) and the value of diverse ideas—are both essential in justifying the empirical observations in innovation contests. The value of diverse ideas alone cannot justify the use of multiple prizes. This is obvious: In Moldovanu and Sela (2001), full entry is assumed (i.e., t^c is fixed at a), and the winner-take-all prize scheme maximizes both the total effort and full participation. On the other hand, if there is only endogenous entry ($d = 0$), Liu and Lu (2019) show that winner-take-all is the unique optimum, and thus, it cannot rationalize why contest organizers offer multiple prizes.

In addition to the main results above, we also identify the following features regarding the structure of the optimal prize scheme. Recall that $t^{c*}(d)$ denotes the set of the optimal entry threshold(s) given d.

Proposition 3. *$t^{c*}(d)$ is weakly decreasing in d in the sense of strong set order.*[17]

If $t^{c*}(d)$ is a singleton for all d, then this reduces to the usual monotonicity of real numbers. Proposition 3 implies that the organizer should induce more entry if the value of diverse ideas is large. We can also show that the budget constraint must be binding at the optimum.

Lemma 7. *Let $\mathbf{v}^*(d) = (v_1^*(d), \ldots, v_N^*(d))$ be an optimal prize structure when the diversity is d. Then $\sum_{j=1}^N v_j^*(d) = V$, for any d.*

In general, the pattern of the magnitude of the coefficients $\beta_1(t^c), \ldots, \beta_N(t^c)$ can be arbitrary (Liu and Lu, 2023). For instance, it is possible that for some t^c, $\beta_2(t^c) > \beta_3(t^c)$; while for another $t^{c\prime}$, $\beta_2(t^{c\prime}) < \beta_3(t^{c\prime})$. Therefore, it is hard to characterize the exact prize scheme. However, we can further pin down the structure of the optimal prize scheme by making the following assumption regarding the magnitude of the N linear coefficients in (6).

Assumption 1. *$\beta_1(t^c) > \beta_2(t^c) > \cdots > \beta_N(t^c)$ for any $t^c \in [a, b]$.*

Recall Lemma 2 that $TE(\mathbf{v}, t^c) = \sum_{k=1}^N \beta_k(t^c) v_k - Nc(1 - F(t^c))t^c$, so $\beta_k(t^c)$ can be interpreted as the "marginal revenue" of the kth prize, v_k, when the entry threshold is t^c. Thus, Assumption 1 says that the marginal revenue of all N prizes diminishes in ranks for any entry threshold. As such, intuitively, the "revenue-maximizing" (i.e., payoff-maximizing) prize scheme should be in the most "unequal" possible way that makes the prize at a higher rank as large as possible, subject to the budget constraint and the participation constraint at t^c. But if the organizer wants to encourage entry (i.e., induce a lower t^c), a more equal optimal prize scheme that allocates more budget to low-rank prizes should be used. As Proposition 3 implies that the optimal entry threshold t^c decreases in d, intuitively, a lower value of diverse ideas cannot lead to a more equal optimal prize structure. The following result confirms this intuition and formally establishes it.

Proposition 4. *Let $\mathbf{v}^*(d) = (v_1^*(d), \ldots, v_N^*(d))$ be an optimal prize structure when the diversity is d. Then, under Assumption 1, when $d_1 < d_2$, $\mathbf{v}^*(d_2)$ cannot majorize $\mathbf{v}^*(d_1)$,*[18] *unless $\mathbf{v}^*(d_1) = \mathbf{v}^*(d_2)$.*[19]

[17] Let A and B be two subsets of $\mathbb{R}$. Then A dominates B in strong set order, denoted as $A \geq_{SSO} B$, if for any $x \in A$ and $y \in B$, we have $\max\{x, y\} \in A$ and $\min\{x, y\} \in B$. Thus, $t^{c*}(d)$ being weakly decreasing in d means that for any $d < d'$, $t^{c*}(d) \geq_{SSO} t^{c*}(d')$.

[18] Let $\mathbf{v} = (v_1, \ldots, v_N)$ and $\tilde{\mathbf{v}} = (\tilde{v}_1, \ldots, \tilde{v}_N)$ be two N-vectors with $v_1 \geq \ldots \geq v_N$, $\tilde{v}_1 \geq \ldots \geq \tilde{v}_N$, and $\sum_{i=1}^N v_i = \sum_{i=1}^N \tilde{v}_i$. We say that $\mathbf{v}$ majorizes $\tilde{\mathbf{v}}$, if $\sum_{i=1}^j v_i \geq \sum_{i=1}^j \tilde{v}_i$ for any $j = 1, 2, \ldots, N$. Intuitively, when $\mathbf{v}$ majorizes $\tilde{\mathbf{v}}$, $\tilde{\mathbf{v}}$ is "more equal" or "more spread out" than $\mathbf{v}$.

[19] For simplicity, we focus on the prize structure that corresponds to the largest element in the set $t^{c*}(d)$ for each d in Proposition 4.

Propositions 3 and 4 are consistent with our empirical finding in Table 2. The value of diverse ideas (d) in contests for building and architectural designs tends to be higher than those for other projects. Hence, we observe building projects allocate more prizes to losers, adopt more equal prize scheme, and attract more participants.

Because Assumption 1 is not easy to verify, we characterize two assumptions that are much easier to verify and are sufficient conditions for Assumption 1.

Assumption 2. *The virtual efficiency function* $J(t) = t - \frac{1-F(t)}{f(t)}$ *is strictly increasing in* t.

Assumption 3. $\beta_{N-1}(a) = \int_a^b J(t) f_{(2,N)}(t)dt \geq 0$.

Assumption 2 is a standard regularity assumption, which is widely adopted in the mechanism design literature. It is satisfied, for example, when the hazard rate $(1 - F(t))/f(t)$ is decreasing in t. Assumption 3 is satisfied if, for instance, $J(t) \geq 0$ for any $t \in [a, b]$. Under Assumption 2, $J(t) \geq 0$ for any $t \in [a, b]$ is equivalent to $J(a) \geq 0$, that is, $af(a) \geq 1$. When F is the uniform distribution on $[a, b]$, Assumption 2 is automatically satisfied, and Assumption 3 is satisfied when $2a \geq b$.

Lemma 8. *Assumptions 2 and 3 imply Assumption 1.*

5.5 Optimal Prize Scheme under Restrictions

Two-Prize Settings

We first restrict the class of prizes to at most two positive prizes, i.e., $(v_1, V - v_1, 0, 0, ..., 0)$ with $v_1 \in [V/2, V]$. In this case, the participation constraint (1) becomes

$$p_1(t^c)v_1 + p_2(t^c)(V - v_1) = c.$$

Rewrite this constraint as v_1 being a function of the entry threshold t^c,

$$v_1 = \frac{c - (N-1)VF^{N-2}(t^c)(1 - F(t^c))}{F^{N-2}(t^c)(NF(t^c) - N + 1)} \equiv M(t^c).$$

Denote $t_0 \in [a, t_1)$ as the unique threshold such that $p_1(t_0) + p_2(t_0) = 2c/V$ when $N \geq 3$ and $t_0 = a$ when $N = 2$. Any threshold lower than t_0 cannot be induced by a two-prize vector. Finally, $M(t^c)$ is strictly increasing, so $M'(t^c) > 0$ for any $t^c \in [t_0, t_1]$. The proof of these properties is in the Appendix.

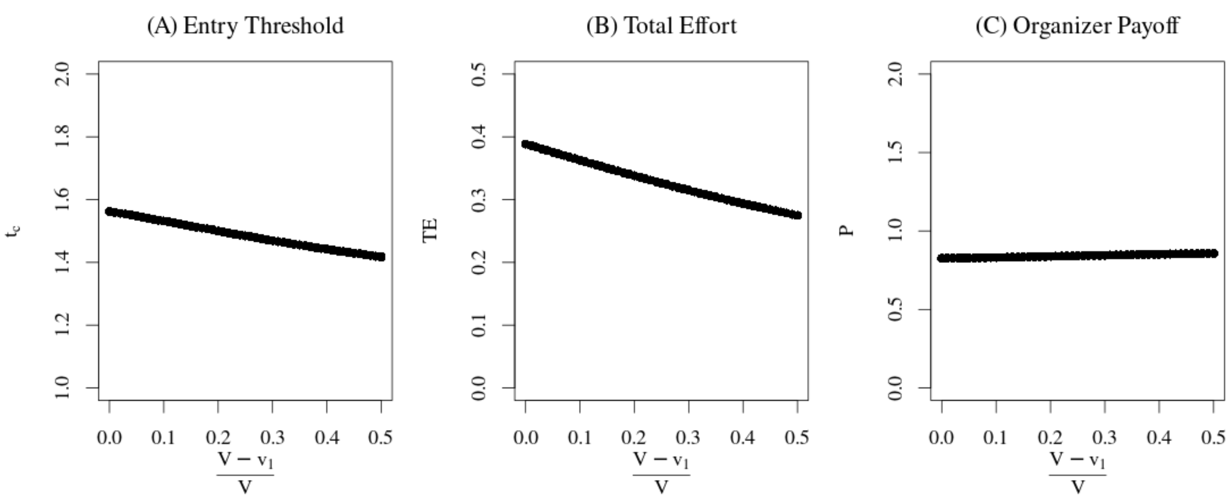
As in problem (O2-d), the optimal entry threshold is the solution to

$$\begin{aligned}\max_{t^c \in [a, t_1]} P^*(t^c; d) &= \max_{t^c \in [t_0, t_1]} P^*(t^c; d) = \max_{t^c \in [t_0, t_1]} \left[TE^*(t^c) + \frac{N(1 - F(t^c))d}{2} \right] \\ &= \max_{t^c \in [t_0, t_1]} \left[\beta_1(t^c)M(t^c) + \beta_2(t^c)(V - M(t^c)) - Nc(1 - F(t^c))t^c + \frac{N(1 - F(t^c))d}{2} \right].\end{aligned}$$

It is shown in the Appendix that $\partial P^*/\partial t^c = (\beta_1(t^c) - \beta_2(t^c))M'(t^c) - Nf(t^c)d/2$, which gives us the corresponding first-order condition characterizing the optimal entry threshold. The following proposition summarizes the optimum under the two-prize setting.

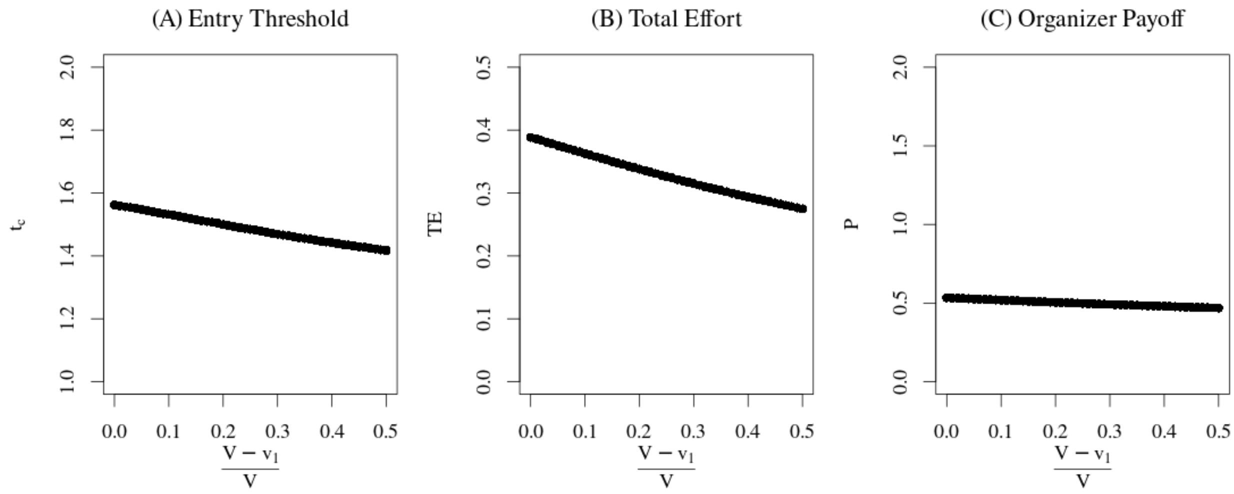
Proposition 5. *Under the two-prize setting, the optimal entry threshold $t^*(d)$ is characterized by the first-order condition (17) (in the Appendix), and the corresponding optimal prize vector is $\mathbf{v}^*(d) = (M(t^*(d)), V - M(t^*(d)), 0, \ldots, 0)$. In particular,*
(i) $t^*(d)$ is decreasing in d;
(ii) the optimal first prize $v_1^*(d)$ is decreasing in d;
(iii) for any $d_1 < d_2$, $\mathbf{v}^*(d_1)$ majorizes $\mathbf{v}^*(d_2)$, which means that a larger diversity level corresponds to a more equal prize design;
(iv) there exists a strictly positive constant $\hat{d}_{\min}$ such that winner-take-all is optimal when $d < \hat{d}_{\min}$;
(v) and there exists a strictly positive constant $\hat{d}_{\max}$ such that when $d > \hat{d}_{\max}$, the optimal prize vector $\mathbf{v}^*(d)$ is the most equal one—that is, prize sharing $(V/2, V/2, 0 \ldots, 0)$ if $N \geq 3$ or $(V - c, c)$ if $N = 2$.

Figure 5 illustrates a numerical example under the two prize setting. When $d = 2/5$, $P(\mathbf{v}, t^c; d)$ is increasing in the second prize, $V - v_1$. The optimal prize scheme is $(V/2, V/2, 0, 0, 0)$. However, when the value of diverse ideas decreases to $d = 2/15$, the winner-take-all prize scheme becomes optimal (Figure 6). Examples in Figures 5 and 6 imply that the threshold $\hat{d}_{\max} \in (2/15, 2/5)$.



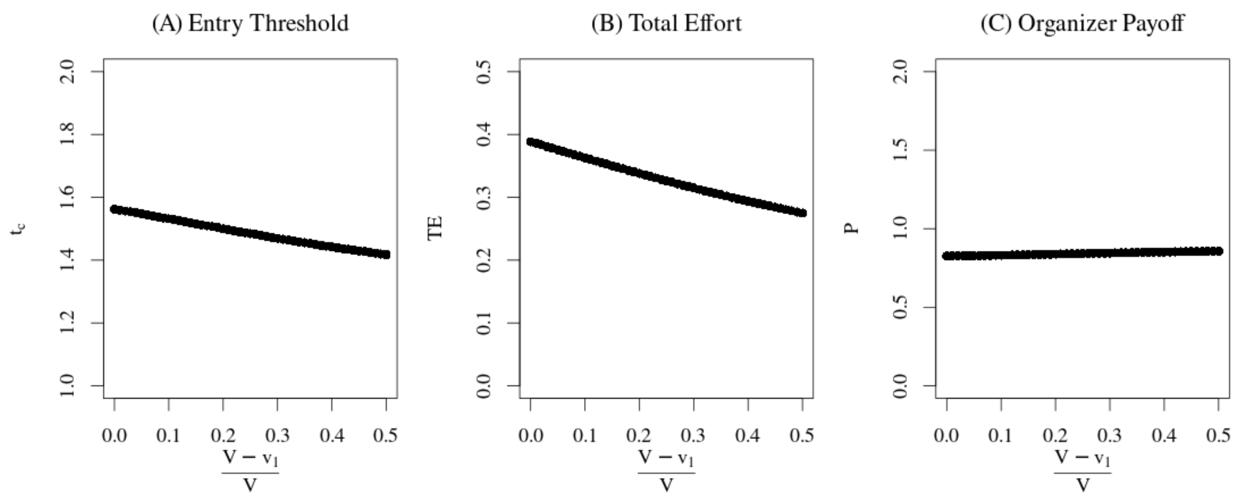
Note: $t \sim U[0, 1]$, $V = 0.01$, $c = 0.001$, $d = 2/5$, and $N = 5$.

Figure 5: Equilibrium and Organizer's Payoff with $(v_1, V - v_1, 0, 0, 0)$



Note: $t \sim U[0,1]$, $V = 0.01$, $c = 0.001$, $d = 2/15$, and $N = 5$.

Figure 6: Equilibrium and Organizer's Payoff with $(v_1, V - v_1, 0, 0, 0)$

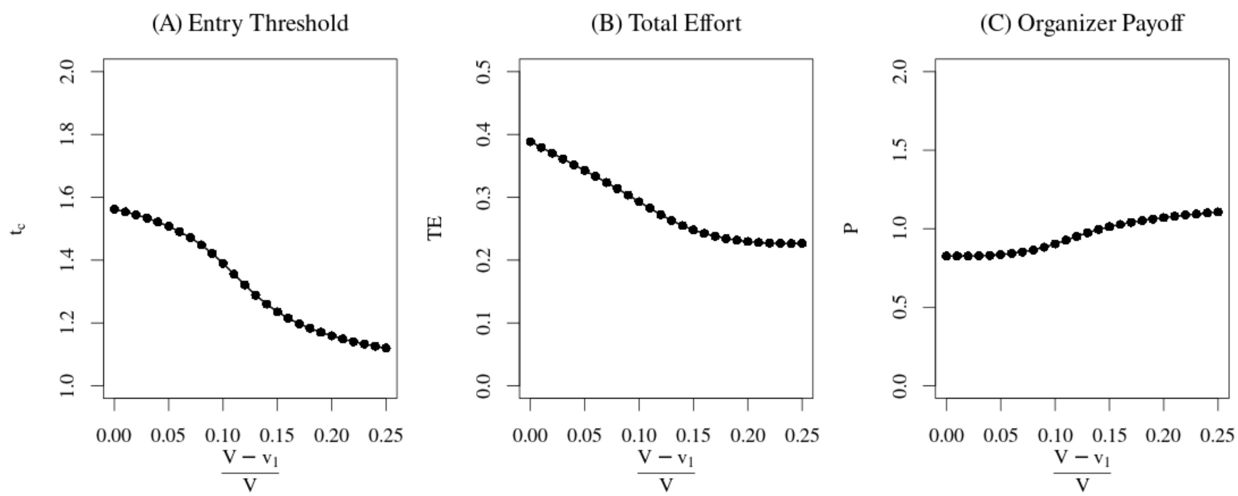


Note: $t \sim U[0,1]$, $V = 0.01$, $c = 0.001$, $d = 2/5$, and $N = 5$.

Figure 7: Equilibrium and Organizer's Payoff with $(v_1, \frac{V-v_1}{2}, \frac{V-v_1}{2}, 0, 0)$

Compensation for Top Losers

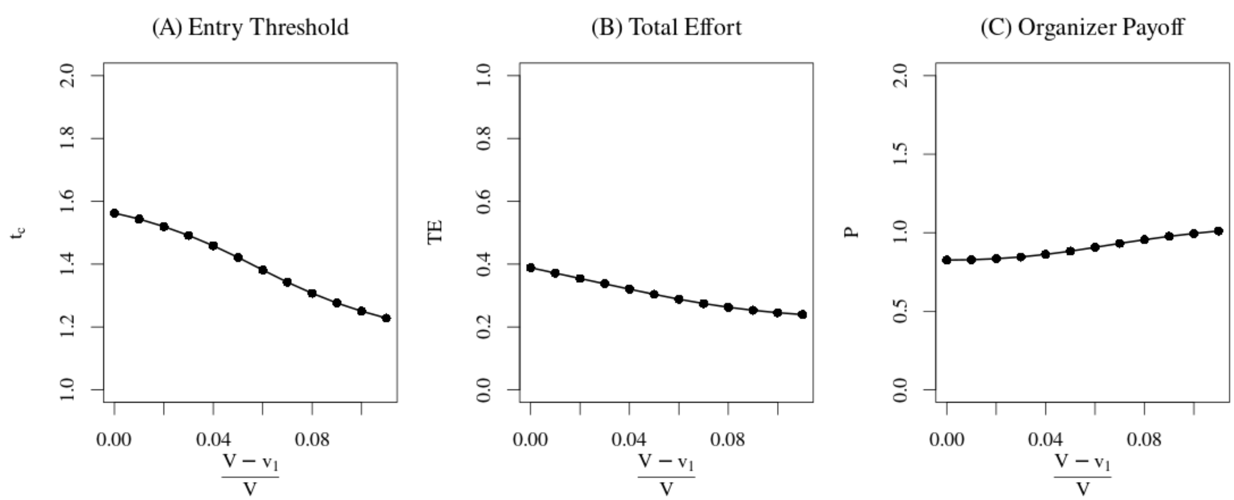
We consider another class of prize schemes in which the contest provides uniform prizes to several highest-ranked losers. We first consider an example that the runner-up and the second runner-up receive uniform compensations. The prize scheme is $(v_1, \frac{V-v_1}{2}, \frac{V-v_1}{2}, \cdots)$ with $v_1 \in [V/3, V]$. Figure 7 illustrates the result of a numerical example. When $d = 2/5$, the optimal prize scheme is $(V/3, V/3, V/3, 0, 0)$, and the organizer's payoff reaches its maximum. Note that the organizer's payoff is higher compared to the case with two prizes. Figure 8 shows that the organizer obtains an even higher payoff with the optimal prize scheme in the class of $(v_1, \frac{V-v_1}{3}, \frac{V-v_1}{3}, \frac{V-v_1}{3}, 0))$. These examples illustrate our main finding that, with a sufficiently high value of diverse ideas, the organizer benefits from allocating equal prizes to more agents.



Note: $t \sim U[0,1]$, $V = 0.01$, $c = 0.001$, $d = 2/5$, and $N = 5$.

Figure 8: Equilibrium and Organizer's Payoff with $(v_1, \frac{V-v_1}{3}, \frac{V-v_1}{3}, \frac{V-v_1}{3}, 0)$

Moreover, we find that the maximum payoff in Figure 8 is higher than that in Figure 9, which is an example of a decreasing prize structure. This comparison shows that offering descending prizes may not yield higher payoffs to the organizer than offering equal prizes to top losers.



Note: $t \sim U[0,1]$, $V = 0.01$, $c = 0.001$, $d = 2/5$, and $N = 5$.

Figure 9: Equilibrium and Organizer's Payoff with $(v_1, \frac{3(V-v_1)}{4}, \frac{2(V-v_1)}{4}, \frac{V-v_1}{4}, 0)$

6 Conclusion

In innovation contests, the ideal design or the best solution is usually uncertain to the contest organizer, so the evaluation criteria cannot be fully specified. As a result, eliciting ideas from agents with diverse backgrounds can be quite valuable. Offering compensation to contest losers can stimulate agents to participate and present their ideas. We establish a contest model with endogenous entry, in which the contest organizer values both the total effort and the value of diverse ideas. Multiple prizes are optimal only if an opportunity cost of entry and a large value of diverse ideas are present. This result explains why winner-take-all is still used in some industries that value diverse ideas. Moreover, under certain restrictions, a higher value of diverse ideas implies that the optimal prize scheme should be more equally distributed among the agents.

The design of innovation contests is a complex yet essential task that requires careful consideration of the overall goals of the contest and the incentive of agents to participate, spend effort, and present their ideas. The insights gained from this research can be applied across various industries. Whether in technology, architecture, or product design, the principles of prize structure, participation, and the value of diverse ideas can improve the effectiveness of innovation contests. Organizers should select the prize structure with caution: While diverse ideas are valuable, their impact on achieving the organizer's objectives may be limited. Blindly adopting a multiple-prize design could therefore undermine the contest's effectiveness. Our findings offer practical guidelines for procurers and managers aiming to leverage contests as a strategic tool for innovation.

First, contest organizers should tailor their prize structures based on the specific goals of the contest and the nature of the project. Companies or governments often seek to explore diverse possibilities and achieve unexpected scientific or technological breakthroughs but may lack the capacity to bear the risks associated with conducting innovation themselves. Our findings suggest that multi-prize contests are better at attracting submission of high-potential proposals and reducing the burden on participating agents. Conversely, when contests are organized to leverage external innovation resources to implement the organizers' ideas, achieve pre-specified and clear goals, or increase efficiency with well-defined evaluation criteria, a winner-take-all approach may be more appropriate.

Second, to stimulate new ideas in solving problems with high uncertainty, managers should reward teammates with relatively equal prizes. This approach fosters a more inclusive environment, motivating a broader range of participants to develop their ideas.

Third, contest organizers or managers should analyze data from past contests to identify trends and patterns in prize allocation and participant behavior. This data-driven approach can inform future contest designs and strike a better balance between prize distribution and participant engagement. Contest participants shall pay particular attention to past contests, the current prize structure, and the historical data shared by the organizers (if available). These pieces of information can devise their strategies when pursuing innovative ideas and capitalize on the financial incentives in the process.

Lastly, we want to emphasize that the theoretical framework proposed in the paper generalizes the standard contest model with a new objective function. The framework is not only applicable to contests for creative designs or innovative solutions. Even for contests with clear evaluation criteria, the optimal prize scheme may not be winner-take-all. For example, in a sales contest, the contest organizer may offer multiple prizes to encourage those who are inexperienced and left behind such that they remain with the job until they become experienced. We expect that our modeling framework with a wider class of objective functions can be applied to the analysis of other competitive environments.

References

Ales, L., S.-H. Cho, and E. Körpeoğlu (2017). Optimal award scheme in innovation tournaments. *Operations Research 65*(3), 693–702.

Ales, L., S.-H. Cho, and E. Körpeoğlu (2019). *Innovation and crowdsourcing contests*. Springer.

Boudreau, K. J., N. Lacetera, and K. R. Lakhani (2011). Incentives and problem uncertainty in innovation contests: An empirical analysis. *Management Science 57*(5), 843–863.

Cason, T. N., W. A. Masters, and R. M. Sheremeta (2010). Entry into winner-take-all and proportional-prize contests: An experimental study. *Journal of Public Economics 94*(9), 604–611.

Che, Y.-K. and I. Gale (2003). Optimal design of research contests. *American Economic Review 93*(3), 646–671.

Dong, X., Q. Fu, M. Serena, and Z. Wu (2023). Research contest design with resource allocation and entry fees. *Working Paper*.

Drugov, M. and D. Ryvkin (2020). Tournament rewards and heavy tails. *Journal of Economic Theory 190*, 105116.

Erkal, N. and J. Xiao (2021). Scarcity of ideas and optimal prizes in innovation contests. *Working Paper*.

Fu, Q. and Z. Wu (2019). Contests: Theory and topics. In *Oxford Research Encyclopedia of Economics and Finance*.

Gao, P., X. Fan, Y. Huang, and Y.-J. Chen (2022). Resource allocation among competing innovators. *Management Science 68*(8), 6059–6074.

Gastwirth, J. L. (1972). The estimation of the lorenz curve and gini index. *Review of Economics and Statistics*, 306–316.

Glazer, A. and R. Hassin (1988). Optimal contests. *Economic Inquiry 26*(1), 133–143.

Hammond, R. G., B. Liu, J. Lu, and Y. E. Riyanto (2019). Enhancing effort supply with prize-augmenting entry fees: Theory and experiments. *International Economic Review 60*(3), 1063–1096.

Huang, Y. (2019). An empirical study of scoring auctions and quality manipulation corruption. *European Economic Review 120*, 103322.

Huang, Y. and M. He (2021). Structural analysis of tullock contests with an application to us house of representatives elections. *International Economic Review*.

Huang, Y. and J. Xia (2019). Procurement auctions under quality manipulation corruption. *European Economic Review 111*, 380–399.

Hwang, S.-H. (2012). Technology of military conflict, military spending, and war. *Journal of Public Economics 96*(1), 226–236.

Jia, H. (2008). An empirical study of contest success functions: Evidence from the nba. *Working Paper*.

Jia, H. and S. Skaperdas (2012). Technologies of conflict. *The Oxford Handbook of the Economics of Peace and Conflict*, 449.

Kalra, A. and M. Shi (2001). Designing optimal sales contests: A theoretical perspective. *Marketing Science 20*(2), 170–193.

Kang, K. (2016). Policy influence and private returns from lobbying in the energy sector. *Review of Economic Studies 83*(1), 269–305.

Kaplan, T., I. Luski, A. Sela, and D. Wettstein (2002). All–pay auctions with variable rewards. *Journal of Industrial Economics 50*(4), 417–430.

Koh, Y. (2017). Incentive and sampling effects in procurement auctions with endogenous number of bidders. *International Journal of Industrial Organization 52*, 393–426.

Korpeoglu, C. G., E. Körpeoğlu, and S. Tunç (2021). Optimal duration of innovation contests. *Manufacturing & Service Operations Management 23*(3), 657–675.

Körpeoğlu, E. and S.-H. Cho (2018). Incentives in contests with heterogeneous solvers. *Management Science 64*(6), 2709–2715.

Lakhani, K. R., L. B. Jeppesen, P. A. Lohse, and J. A. Panetta (2007). The value of openness in scientific problem solving. *Harvard Business School Working Paper*.

Le, T. T., Z. Andreadakis, A. Kumar, R. G. Roman, S. Tollefsen, M. Saville, and S. Mayhew (2020). The covid-19 vaccine development landscape. *Nature Reviews Drug Discovery 19*(5), 305–306.

Lemus, J. and G. Marshall (2021). Dynamic tournament design: Evidence from prediction contests. *Journal of Political Economy 129*(2), 383–420.

Letina, I. (2016). The road not taken: competition and the R&D portfolio. *RAND Journal of Economics 47*(2), 433–460.

Letina, I. and A. Schmutzler (2019). Inducing variety: A theory of innovation contests. *International Economic Review 60*(4), 1757–1780.

Liu, B. and J. Lu (2019). The optimal allocation of prizes in contests with costly entry. *International Journal of Industrial Organization 66*, 137–161.

Liu, B. and J. Lu (2023). Optimal orchestration of rewards and punishments in rank-order contests. *Journal of Economic Theory 208*, 105594.

Liu, X. and J. Lu (2014). The effort-maximizing contest with heterogeneous prizes. *Economics Letters 125*(3), 422–425.

Malueg, D. A. and A. J. Yates (2010). Testing contest theory: Evidence from best-of-three tennis matches. *Review of Economics and Statistics 92*(3), 689–692.

Matros, A. and D. Armanios (2009). Tullock's contest with reimbursements. *Public Choice*, 49–63.

McAfee, R. P. and J. McMillan (1987). Auctions with a stochastic number of bidders. *Journal of Economic Theory 43*(1), 1–19.

Milgrom, P. and C. Shannon (1994). Monotone comparative statics. *Econometrica: Journal of the Econometric Society*, 157–180.

Moldovanu, B. and A. Sela (2001). The optimal allocation of prizes in contests. *American Economic Review*, 542–558.

Myerson, R. B. (1981). Optimal auction design. *Mathematics of operations research 6*(1), 58–73.

Polishchuk, L. and A. Tonis (2013). Endogenous contest success functions: a mechanism design approach. *Economic Theory 52*(1), 271–297.

Samuelson, W. F. (1985). Competitive bidding with entry costs. *Economics Letters 17*(1-2), 53–57.

Sarne, D. and M. Lepioshkin (2017). Effective prize structure for simple crowdsourcing contests with participation costs. In *Proceedings of the AAAI Conference on Human Computation and Crowdsourcing*, Volume 5, pp. 167–176.

Segev, E. (2020). Crowdsourcing contests. *European Journal of Operational Research 281*(2), 241–255.

Stouras, K. I., J. Hutchison-Krupat, and R. O. Chao (2022). The role of participation in innovation contests. *Management Science 68*(6), 4135–4150.

Sunde, U. (2003). Potential, prizes and performance: Testing tournament theory with professional tennis data. *IZA Discussion Paper No. 947*.

Szymanski, S. and T. M. Valletti (2005). Incentive effects of second prizes. *European Journal of Political Economy 21*(2), 467–481.

Takahashi, H. (2018). Strategic design under uncertain evaluations: structural analysis of design-build auctions. *RAND Journal of Economics 49*(3), 594–618.

Taylor, C. R. (1995). Digging for golden carrots: An analysis of research tournaments. *The American Economic Review*, 872–890.

Terwiesch, C. and Y. Xu (2008). Innovation contests, open innovation, and multiagent problem solving. *Management Science 54*(9), 1529–1543.

Tullock, G. (1980). *Efficient Rent Seeking. In J.M. Buchanan, R.D. Tollison and G. Tullock (Eds.), Towards a Theory of the Rent-seeking Society.* Texas A&M University Press.

Zhang, M. (2024). Optimal contests with incomplete information and convex effort costs. *Theoretical Economics 19*(1), 95–129.

Zhu, F. (2019). Creative contests: Theory and experiment. *Working Paper*.

Appendix

A. Proofs

Proof of Proposition 1. See Lemma 1 in Liu and Lu (2019).

Proof of Lemma 1. By Proposition 1,

$$\begin{aligned} TE^{(n)}(\mathbf{v},t^c) &= n\int_{t^c}^{b} J(t)V^{(n)}(t)g(t,t^c)dt - nt^c v_n \\ &= n\int_{t^c}^{b} J(t)[\sum_{j=1}^{n} v_{n+1-j}\binom{n-1}{j-1}G^{j-1}(t,t^c)(1-G(t,t^c))^{n-j}]g(t,t^c)dt - nt^c v_n \\ &= \sum_{j=1}^{n} v_{n+1-j}\left(\int_{t^c}^{b} J(t)g_{(j,n)}(t,t^c)dt\right) - nt^c v_n. \end{aligned}$$

Therefore,

$$\begin{aligned} TE(\mathbf{v},t^c) &= \sum_{n=1}^{N}\binom{N}{n}(1-F(t^c))^n F^{N-n}(t^c)TE^{(n)}(\mathbf{v},t^c) \\ &= \sum_{n=1}^{N}\binom{N}{n}(1-F(t^c))^n F^{N-n}(t^c)\left[\sum_{j=1}^{n} v_{n+1-j}\left(\int_{t^c}^{b} J(t)g_{(j,n)}(t,t^c)dt\right) - nt^c v_n\right] \\ &= (1-F(t^c))\sum_{n=1}^{N}\frac{Np_n(t^c)}{n}\left[\sum_{j=1}^{n} v_{n+1-j}\left(\int_{t^c}^{b} J(t)g_{(j,n)}(t,t^c)dt\right) - nt^c v_n\right] \\ &= N(1-F(t^c))\sum_{n=1}^{N} p_n(t^c)\left[\frac{\sum_{j=1}^{n} v_{n+1-j}}{n}\left(\int_{t^c}^{b} J(t)g_{(j,n)}(t,t^c)dt\right) - t^c v_n\right] \\ &= N(1-F(t^c))\left[\sum_{n=1}^{N}\frac{p_n(t^c)}{n}\sum_{j=1}^{n} v_{n+1-j}\left(\int_{t^c}^{b} J(t)g_{(j,n)}(t,t^c)dt\right) - ct^c\right], \end{aligned}$$

where the last equality uses (1). □

Proof of Lemma 2. See Lemma 4 in Liu and Lu (2023). □

Proof of Lemma 3. Liu and Lu (2019) analyze a similar contest design problem with entry costs. The only difference between their model and ours is that the prize allocation rule can be contingent on the number of entrants in their model. Specifically, in their paper, the contest rule is a set of scenario prize vectors $\mathbf{W} = \{\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_N\}$, where in vector $\mathbf{W}_n = (w_{n,1}, w_{n,2}, \ldots, w_{n,n}) \in \mathbb{R}^n_+$, $w_{n,1} \geq w_{n,2} \geq \ldots \geq w_{n,n} \geq 0$ and $\sum_{j=1}^{n} w_{n,j} \leq V$.[20] Here, $w_{n,j}$ is the jth prize in scenario n,

[20] Liu and Lu (2019) normalize the prize budget $V = 1$. It is clear that the contest organizer's problem is linear in V, so their result applies to any $V > 0$, as mentioned in footnote 3 in their paper. Specifically, normalizing every prize

which is for the jth highest effort. Thus, when fixing the entry threshold at $t^c \in [a, b)$, the contest organizer's problem, as in the first paragraph of Section 3.2 on page 141 of Liu and Lu (2019), can be equivalently written as

$$\text{(O1-}d\text{-contingent)} \qquad \max_{\mathbf{W}} TE(\mathbf{W}, t^c) = \sum_{n=1}^{N} \binom{N}{n} (1 - F(t^c))^n F^{N-n}(t^c) TE^{(n)}(\mathbf{W}_n, t^c)$$

$$\sum_{j=1}^{n} w_{n,j} \leq V, \; w_{n,1} \geq w_{n,2} \geq \ldots \geq w_{n,n} \geq 0, \forall n, \; \text{and} \; \sum_{n=1}^{N} p_n(t^c) w_{n,n} = c.$$

where $TE^{(n)}(\mathbf{W}_n, t^c)$ is characterized in Lemma 1 of Liu and Lu (2019) (or equivalently, Proposition 1 in this paper). We call this problem (O1-d-contingent) because its objective function is the same as (7) when restricting the prize allocation rule to be independent of n. In problem (O1-d-contingent), we ignore the choice variable $\mathbf{V}$ in Liu and Lu (2019). This clearly does not change anything because once the allocation rule $\mathbf{W}$ is fixed, the budget V_n in scenario n is pinned down correspondingly—just as they mention in footnote 7 in Liu and Lu (2019).

Denote the value function, the feasible set, and the set of solutions of problem (O1-d-contingent) as $TE^{**}(t^c)$, $\tilde{S}(t^c)$, and $\tilde{S}^*(t^c)$, respectively. The feasible set is

$$\tilde{S}(t^c) = \{\{\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_N\} : \sum_{j=1}^{n} w_{n,j} \leq V, \forall n; \; w_{n,1} \geq w_{n,2} \geq \ldots \geq w_{n,n} \geq 0, \; \forall n; \; \text{and} \; \sum_{n=1}^{N} p_n(t^c) w_{n,n} = c\}.$$

Recall that $S(t^c)$ is the feasible set of problem (7). Because problem (O1-d-contingent) allows the allocation rule to be contingent on n, but problem (7) does not, it is clear that for any $t^c \in [a, b)$, $S(t^c) \subseteq \tilde{S}(t^c)$. Moreover, for any $t^c \in [a, b)$, $TE^*(t^c) \leq TE^{**}(t^c)$.[21] Nevertheless, Liu and Lu (2019) show that[22]

$$TE^{**}(t_1) > TE^{**}(t^c), \; \forall t^c \in [a, b) \backslash \{t_1\} \tag{10}$$

and $\tilde{S}^*(t_1)$ is a singleton with

$$\tilde{S}^*(t_1) = \{\{\mathbf{W}_1^*, \mathbf{W}_2^*, \ldots, \mathbf{W}_N^*\}\} = \{\{(V), \; (V, 0), \; (V, 0, 0), \; \cdots, \; (V, \underbrace{0, \ldots, 0}_{N-1 \text{ times}})\},$$

i.e., $\mathbf{W}_n^* = (V, \underbrace{0, \ldots, 0}_{n-1 \text{ times}})\}$, for all $n \geq 1$.

and the entry cost by V—i.e., dividing $w_{n,j}$ by V and dividing c by V—returns to their model. The only difference is that the total effort elicitable for budget V is then V times the total quality when the budget is 1 dollar, as is clear from the linear nature revealed in Lemma 1 and Proposition 1 in their paper.

[21] As mentioned above, $TE^{**}(t^c)$ is equal to the highest effort level in Liu and Lu (2019) multiplied by V, as Liu and Lu (2019) normalize the budget to 1.

[22] Note that t_1 is defined as $F^{-1}(c^{\frac{1}{N-1}})$ in Liu and Lu (2019). As mentioned in footnote 20, the entry cost c in their paper corresponds to c/V in our paper after normalization.

This optimal prize allocation rule does not depend on n, so it is also feasible in the noncontingent contest design problem in this paper. It is the same as $\mathbf{v}^* = (V, 0, \ldots, 0)$, which implies that $TE^*(t_1) = TE^{**}(t_1)$. As such, given that $TE^*(t^c) \leq TE^{**}(t^c)$ for any $t^c \in [a, b]$ and (10), Lemma 3 follows directly. $\square$

Proof of Lemma 4. If winner-take-all is a solution to problem (O-d), then it must satisfy the participation constraint, $p_1(t^c)V = c$. However, this implies that $t^c = t_1$, which, by (9), further implies that t_1 solves problem (O2-d). Conversely, if t_1 solves problem (O2-d), then it means that for problem (7), when the threshold is t_1, the set of optimal solutions is $S^*(t_1)$. However, Lemma 3 implies that $S^*(t_1)$ is a singleton, that is, $S^*(t_1) = \{(V, 0, \ldots, 0)\}$.

Note that the only possible entry threshold that winner-take-all induces is t_1. Therefore, the above argument for the first statement of Lemma 4 obviously applies to the uniqueness statement of Lemma 4. $\square$

Proof of Lemma 5. We first prove (i). Suppose that there is some $d_0 > 0$ such that winner-take-all is the unique solution to problem (O-d_0). Then, by Lemma 4, t_1 is the unique solution to problem (O2-d_0). Therefore, $P^*(t_1; d_0) > P^*(t^c; d_0)$, for any $t^c \in [a, t_1)$, which further implies

$$TE^*(t_1) - TE^*(t^c) > \frac{Nd_0}{2}(F(t_1) - F(t^c)), \text{ for any } t^c \in [a, t_1).$$

Then, for any $d \in [0, d_0]$ and any $t^c \in [a, t_1)$,

$$TE^*(t_1) - TE^*(t^c) > \frac{Nd_0}{2}(F(t_1) - F(t^c)) \geq \frac{Nd}{2}(F(t_1) - F(t^c)).$$

Therefore, for any $d \in [0, d_0]$,

$$P^*(t_1; d) > P^*(t^c; d), \text{ for any } t^c \in [a, t_1),$$

which means that t_1 is the unique solution to problem (O2-d) for any $d \in [0, d_0]$. By Lemma 4, this implies that winner-take-all is the unique solution to problem (O-d) for any $d \in [0, d_0]$.

Now we turn to (ii). Suppose that there is some $d_1 > 0$ such that winner-take-all is not a solution to problem (O-d_1). By Lemma 4, t_1 is not a solution to problem (O2-d_1). Therefore, $P^*(t_1; d_1) < P^*(t_2^c; d_1)$, for some $t_2^c \in [a, t_1)$, which further implies

$$TE^*(t_1) - TE^*(t_2^c) < \frac{Nd_1}{2}(F(t_1) - F(t_2^c)).$$

Then, for any $d \geq d_1$:

$$TE^*(t_1) - TE^*(t_2^c) < \frac{Nd_1}{2}(F(t_1) - F(t_2^c)) \leq \frac{Nd}{2}(F(t_1) - F(t_2^c)),$$

which further implies that

$$P^*(t_1; d) < P^*(t_2^c; d).$$

This means that t_1 is not a solution to problem (O2-d) for any $d \geq d_1$. By Lemma 4, this implies that winner-take-all is not a solution to problem (O-d) for any $d \geq d_1$.

For (iii), suppose that there is some $d_2 > 0$ such that winner-take-all is a solution, but not the unique solution, to problem (O-d_2). Then, by Lemma 4, there is some $t_3^c \in [a, t_1)$ such that both t_1 and t_3^c are solutions to problem (O2-d_2). Since both t_1 and t_3^c are solutions to problem (O2-d_2), $P^*(t_1; d_2) = P^*(t_3^c; d_2)$, which is equivalent to

$$TE^*(t_1) - TE^*(t_3^c) = \frac{Nd_2}{2}(F(t_1) - F(t_3^c)). \tag{11}$$

We first show that for any $d > d_2$, winner-take-all is not a solution to problem (O-d). Again, by Lemma 4, it suffices to show that t_1 is not a solution to problem (O2-d) when $d > d_2$. In fact, suppose to the contrary that t_1 is a solution to problem (O2-d) for some $d' > d_2$. Then, $P^*(t_1; d') \geq P^*(t^c; d')$, for any $t^c \in [a, t_1]$. In particular, $P^*(t_1; d') \geq P^*(t_3^c; d')$, which is equivalent to

$$TE^*(t_1) - TE^*(t_3^c) \geq \frac{Nd'}{2}(F(t_1) - F(t_3^c)).$$

However, since $d' > d_2$,

$$TE^*(t_1) - TE^*(t_3^c) \geq \frac{Nd'}{2}(F(t_1) - F(t_3^c)) > \frac{Nd_2}{2}(F(t_1) - F(t_3^c)) = TE^*(t_1) - TE^*(t_3^c),$$

which is a contradiction.

We next show that for any $d \in [0, d_2)$, winner-take-all is the unique solution to problem (O-d), which would then complete the proof of (iii). By Lemma 4, this is equivalent to showing that t_1 is the unique solution to problem (O2-d) when $d \in [0, d_2)$. We first argue that t_1 must be a solution to problem (O2-d) when $d \in [0, d_2)$. To this end, one needs to show that $P^*(t_1; d) \geq P^*(t^c; d)$, for any $t^c \in [a, t_1]$. This is equivalent to

$$TE^*(t_1) - TE^*(t^c) \geq \frac{Nd}{2}(F(t_1) - F(t^c)), \text{ for all } t^c \in [a, t_1] \text{ and all } d \in [0, d_2). \tag{12}$$

Since t_1 is a solution to problem (O2-d_2),

$$TE^*(t_1) - TE^*(t^c) \geq \frac{Nd_2}{2}(F(t_1) - F(t^c)), \text{ for all } t^c \in [a, t_1]. \tag{13}$$

Thus, for any $d \in [0, d_2)$ and any $t^c \in [a, t_1]$, one has

$$TE^*(t_1) - TE^*(t^c) \geq \frac{Nd_2}{2}(F(t_1) - F(t^c)) \geq \frac{Nd}{2}(F(t_1) - F(t^c)).$$

Therefore, (12) holds.

We next argue that t_1 is the unique solution to problem (O2-d) when $d \in [0, d_2)$. When $d = 0$, this is obvious, following directly from Lemma 3. Now, suppose, to the contrary, that $t_3' \neq t_1$ is also a solution to problem (O2-d) for some $d' \in (0, d_2)$. However, this is impossible. We just showed above that if for some $\hat{d} > 0$ winner-take-all is one, but not the unique, solution to problem (O2-d), then winner-take-all is not a solution to problem (O2-d) for any $d > \hat{d}$. This implies that winner-take-all is not a solution to problem (O2-d) for any $d > d'$. However, this contradicts the fact that winner-take-all is a solution to problem (O-d_2). $\square$

Proof of Lemma 6. Note that $TE^*(t^c)$, the value function of problem (7), is continuous in $[a, t_1]$ by the maximum theorem because in problem (7), the objective function $TE(\mathbf{v}, t^c)$ is continuous in $(\mathbf{v}, t^c) \in \mathbb{R}_+^N \times [a, t_1]$ and the feasible set $S(t^c)$ is continuous in t^c and compact-valued. Therefore, the objective function of problem (O2-d),

$$P^*(t^c; d) = TE^*(t^c) + \frac{Nd}{2}(1 - F(t^c))$$

is continuous in $(t^c, d) \in [a, t_1] \times [0, +\infty)$. Again, the maximum theorem implies that the optimal solution set $t^{c*}(d)$ of problem (O2-d) is compact and is upper hemicontinuous in $d \in [0, +\infty)$.

Now we are ready to prove Lemma 6. Suppose, to the contrary, that $S_1 = \{0\}$. Note that $t^{c*}(0) = \{t_1\}$. Since $t^{c*}(d)$ is upper hemicontinuous in d and $t^{c*}(0) = \{t_1\}$, there exists a sequence $\{d_k\}_k$ with $d_k > 0$ and $\lim_{k\to\infty} d_k = 0$, such that there exists some $t^c(d_k) \in t^{c*}(d_k)$ for each k with $\lim_{k\to\infty} t^c(d_k) = t_1$ (note that without loss of generality, one can assume that the sequence $\{t^c(d_k)\}_k$ is convergent, as this sequence is in the compact set $[a, t_1]$). Note further that $t_1 \notin t^{c*}(d_k)$ for each k, by part (i) of Lemma 5 (otherwise, $S_1 \neq \{0\}$). Thus, $t^c(d_k) < t_1$ for all k.

Since $t_1 \notin t^{c*}(d_k)$ for each k, $P^*(t^c(d_k); d_k) > P^*(t_1; d_k)$, $\forall k$, which is equivalent to

$$TE^*(t^c(d_k)) - TE^*(t_1) > \frac{Nd_k}{2}(F(t^c(d_k)) - F(t_1)), \text{ for any } k.$$

Since $t^c(d_k) < t_1$, we have

$$\frac{TE^*(t^c(d_k)) - TE^*(t_1)}{F(t^c(d_k)) - F(t_1)} < \frac{Nd_k}{2} \text{ for any } k,$$

or equivalently,

$$\frac{TE^*(t_1) - TE^*(t^c(d_k))}{F(t_1) - F(t^c(d_k))} < \frac{Nd_k}{2} \text{ for any } k.$$

Recall that in the proof of Lemma 3, we showed that $TE^*(t^c) \leq TE^{**}(t^c)$, for any $t^c \in [a, t_1]$, with equality when $t^c = t_1$. Here, $TE^{**}(t^c)$ is the value function of the contest organizer's problem in Liu and Lu (2019), by treating the budget as V in their paper. It then follows that

$$\frac{TE^{**}(t_1) - TE^{**}(t^c(d_k))}{F(t_1) - F(t^c(d_k))} \leq \frac{TE^*(t_1) - TE^*(t^c(d_k))}{F(t_1) - F(t^c(d_k))} < \frac{Nd_k}{2} \text{ for any } k,$$

which implies that,

$$\frac{TE^{**}(t_1) - TE^{**}(t^c(d_k))}{F(t_1) - F(t^c(d_k))} < \frac{Nd_k}{2} \text{ for any } k.$$

This inequality is the same as

$$\frac{TE^{**}(t_1) - TE^{**}(t^c(d_k))}{t_1 - t^c(d_k)} \cdot \frac{t_1 - t^c(d_k)}{F(t_1) - F(t^c(d_k))} < \frac{Nd_k}{2} \text{ for any } k. \tag{14}$$

In their proof of Lemma 2, Liu and Lu (2019) show that $TE^{**}(t^c)$ is differentiable in $t^c \in [t_2, t_1)$, where $t_2 \in (a, t_1)$. In fact, from the proof there, it is clear that the left derivative of $TE^{**}(t^c)$ at $t^c = t_1$ exists and is strictly positive.[23] Recall that, by construction, $t^c(d_k) < t_1$ for all k, $\lim_{k\to\infty} d_k = 0$, and $\lim_{k\to\infty} t^c(d_k) = t_1$. Thus, letting $k \to \infty$ in (14),

$$\lim_{k\to\infty} \left[\frac{TE^{**}(t_1) - TE^{**}(t^c(d_k))}{t_1 - t^c(d_k)} \cdot \frac{t_1 - t^c(d_k)}{F(t_1) - F(t^c(d_k))} \right] \leq \lim_{k\to\infty} \frac{Nd_k}{2} = 0.$$

However, note that the left-hand side of the above inequality is precisely $1/f(t_1)$ mulipltied by the left derivative of $TE^{**}(t^c)$ at $t^c = t_1$, both of which are strictly positive. This is clearly a contradiction. $\square$

Proof of Proposition 2. Recall that the maximum theorem implies that the optimal solution set $t^{c*}(d)$ of problem (O2-d) is compact and is upper hemicontinuous in $d \in [0, +\infty)$. Denote $S_1 = \{d \in [0, +\infty) : t_1 \text{ is the unique solution to problem (O2-}d)\}$. Lemma 3 implies that $0 \in S_1$. Lemma 6 reveals that S_1 cannot be a singleton. Denote $S_2 = \{d \in [0, +\infty) : t_1 \text{ is a solution, but not the unique solution, to problem (O2-}d)\}$ and $S_3 = \{d \in [0, +\infty) : t_1 \text{ is not a solution to problem (O2-}d)\}$. Obviously, $S_1 \cup S_2 \cup S_3 = [0, +\infty)$. Furthermore, Lemma 5 implies that if $S_2 \neq \emptyset$, then S_2 must be a singleton. Therefore, by Lemmas 5 and 6, there exists some $\tilde{d} > 0$ such that $[0, \tilde{d}) \subseteq S_1$ and $(\tilde{d}, +\infty) \subseteq S_3$. Moreover, $\tilde{d} \in S_1$ or $\tilde{d} \in S_2$. By construction, $\tilde{d}$ is unique. To see why $\tilde{d} \notin S_3$, suppose, to the contrary, that $\tilde{d} \in S_3$. Consider a convergent sequence $\{d_k\}_k$ with $d_k \in S_1$ and $\lim_{k\to\infty} d_k = \tilde{d}$. By definition, $t^{c*}(d_k) = \{t_1\}$ for all k. Thus, $\lim_{k\to\infty} t^{c*}(d_k) = \{t_1\}$. However, $t_1 \notin t^{c*}(\tilde{d})$, which violates the upper hemicontinuity of $t^{c*}(d)$ at $d = \tilde{d}$.

Finally, Proposition 2 then follows by applying Lemma 4. $\square$

[23] To be clear, we use their notations in their proof of Lemma 2. They show on page 152 that when $t^c \in [t_2, t_1)$ (by letting $n = 2$),

$$\frac{dTE^{**}}{dt^c} = Nf(t^c)[\beta(t^c)(\varphi(2, t^c) - \varphi(1, t^c)) + (c - p_1(t^c))\varphi(2, t^c)].$$

(Note that the notation TE^* in their proof has been changed to TE^{**} to be consistent with the notation used in this paper when referring to their paper.) It is clear from their proof that the left derivative of $TE^{**}(t^c)$ at $t^c = t_1$ exists and can be obtained by letting $t^c \to t_1^-$ in the above equation. Thus, the left derivative at t_1 is equal to

$$Nf(t_1)[\beta(t_1)(\varphi(2, t_1) - \varphi(1, t_1)) + (c - p_1(t_1))\varphi(2, t_1)].$$

By the definition of these notations in their paper, one has $Nf(t_1) > 0$, $c - p_1(t_1) = 0$, $\beta(t_1) = p_2(t_1) + 2p_1(t_1) - 2c = p_2(t_1) > 0$, and $\varphi(2, t_1) - \varphi(1, t_1) > 0$. Therefore, the left derivative at t_1 is strictly positive.

Proof of Proposition 3. Recall that $t^{c*}(d)$ is set of the solutions to Problem (O2-d)

$$\max_{t^c \in [a,t_1]} P^*(t^c; d) = TE^*(t^c) + \frac{Nd}{2}(1 - F(t^c)).$$

It is obvious that $P^*(t^c; d)$ has increasing difference in $(t^c, -d)$. Thus, by Milgrom and Shannon (1994), $t^{c*}(d)$ is weakly decreasing in d in strong set order. □

Proof of Lemma 7. We first show the following result.

Lemma 9. *$\beta_1(t^c) > 0$ and $\beta_1(t^c) > \beta_j(t^c)$ for any $t^c \in [a, b)$ and any $j = 2, \ldots, N$.*

In fact, Lemma 4 in Liu and Lu (2019) shows that for any $n \geq 1$ and any $t^c < b$, $\int_{t^c}^b J(t) g_{(n,n)}(t, t^c) dt > \int_{t^c}^b J(t) g_{(k,n)}(t, t^c) dt$ for any k with $1 \leq k \leq n - 1$; moreover, $\int_{t^c}^b J(t) g_{(n,n)}(t, t^c) dt > 0$. Thus, by the definition of $\beta_k(t^c)$ in (5),

$$\beta_1(t^c) = N(1 - F(t^c)) \sum_{n=1}^{N} \frac{p_n(t^c)}{n} \left(\underbrace{\int_{t^c}^{b} J(t) g_{(n,n)}(t, t^c) dt}_{>0} \right) > 0;$$

and when $k \geq 2$, we have

$$\begin{aligned}
\beta_1(t^c) &= N(1 - F(t^c)) \sum_{n=1}^{N} \frac{p_n(t^c)}{n} \left(\int_{t^c}^{b} J(t) g_{(n,n)}(t, t^c) dt \right) \\
&> N(1 - F(t^c)) \sum_{n=k}^{N} \frac{p_n(t^c)}{n} \left(\int_{t^c}^{b} J(t) g_{(n,n)}(t, t^c) dt \right) \\
&> N(1 - F(t^c)) \sum_{n=k}^{N} \frac{p_n(t^c)}{n} \left(\int_{t^c}^{b} J(t) g_{(n+1-k,n)}(t, t^c) dt \right) \\
&= \beta_k(t^c).
\end{aligned}$$

We next show that $p_1(t^c) \leq p_n(t^c)$ for any $t^c \in [a, t_1]$ and $n \geq 2$. Clearly, $p_1(a) \leq p_n(a)$ for any n. Now we focus on the case when $t^c \in (a, t_1]$ because the optimal entry threshold cannot exceed t_1. By definition, we need to show that

$$F^{N-1}(t^c) \leq \binom{N-1}{n-1} (1 - F(t^c))^{n-1} F^{N-n}(t^c), \text{ when } t^c \in (a, t_1],$$

which is equivalent to

$$\binom{N-1}{n-1} \left(\frac{1 - F(t^c)}{F(t^c)}\right)^{n-1} \geq 1, \text{ when } t^c \in (a, t_1].$$

Clearly, by binomial coefficients' symmetry and monotonicity in n and the fact that $(\frac{1-F(t^c)}{F(t^c)})^{n-1}$

is increasing in n, it suffices to show this for $n = 2$. When $n = 2$, the inequality becomes

$$F(t^c) \leq \frac{N-1}{N}, \; t^c \in (a, t_1].$$

Therefore, one only needs to show that $F(t_1) \leq 1 - 1/N$. Note that by the definition of t_1 and the fact that $c \leq V/N$, we have

$$F(t_1) = (\frac{c}{V})^{\frac{1}{N-1}} \leq (\frac{1}{N})^{\frac{1}{N-1}}.$$

Therefore, the problem boils down to showing that $(\frac{1}{N})^{\frac{1}{N-1}} \leq \frac{N-1}{N}$ for any $N \geq 2$. This inequality is trivially true when $N = 2$. When $N \geq 3$, notice that

$$\begin{aligned} (\frac{1}{N})^{\frac{1}{N-1}} &\leq \frac{N-1}{N} \Leftrightarrow -\frac{\ln N}{N-1} \leq \ln(N-1) - \ln N \\ &\Leftrightarrow \frac{\ln N}{N-1} \leq \frac{\ln(N-1)}{N-2}. \end{aligned}$$

Since the function $h(x) = \frac{\ln(x+1)}{x}$ is decreasing in $x \in [1, +\infty)$, the above inequality holds.

Now we are ready to prove Lemma 7. For the prize vector $\mathbf{v}^*(d)$, denote the corresponding entry threshold as $t^c(d)$. If $t^c(d) = a$, then this is the full-entry case which reduces to Moldovanu and Sela (2001), so Lemma 7 holds.

Let us turn to the case $t^c(d) > a$. Suppose that the budget constraint is slack for $\mathbf{v}^*(d)$, then $\sum_{j=1}^{N} v_j^*(d) < V$. Let m be the largest integer such that $v_m^*(d) > 0$. Obviously, $m \geq 2$. Consider the prize structure

$$\mathbf{v}^{**}(d) = (v_1^*(d) + \frac{\varepsilon}{p_1(t^c(d))}, v_2^*(d), \dots, v_{m-1}^*(d), v_m^*(d) - \frac{\varepsilon}{p_m(t^c(d))}, \underbrace{v_{m+1}^*(d), \dots, v_N^*(d)}_{\text{all are } 0}),$$

where $\varepsilon > 0$ is small enough such that $\mathbf{v}^{**}(d)$ still satisfies the budget constraint and $v_m^*(d) - \frac{\varepsilon}{p_m(t^c(d))} \geq 0$. Clearly, by construction, the prizes in $\mathbf{v}^{**}(d)$ decreases with ranks, and $\mathbf{v}^{**}(d)$ satisfies the budget constraint and the participation constraint.

Notice that

$$\frac{\beta_1(t^c(d))}{p_1(t^c(d))} - \frac{\beta_m(t^c(d))}{p_m(t^c(d))} > 0,$$

because if $\beta_m(t^c(d)) \leq 0$, then it trivially holds (as $\beta_1(t^c(d)) > 0$); otherwise, since $\beta_1(t^c(d)) > \beta_m(t^c(d)) > 0$ and $0 < p_1(t^c(d)) \leq p_m(t^c(d))$, we still have the above inequality. However, this means $\mathbf{v}^{**}(d)$ gives a strictly higher payoff to the organizer than $\mathbf{v}^*(d)$ does, because the payoff under $\mathbf{v}^{**}(d)$ minus that under $\mathbf{v}^*(d)$ is

$$\left[\underbrace{\frac{\beta_1(t^c(d))}{p_1(t^c(d))} - \frac{\beta_m(t^c(d))}{p_m(t^c(d))}}_{>0}\right]\varepsilon > 0.$$

This completes the proof. $\square$

Proof of Proposition 4. Step 1: By the argument in the proof of Lemma 1 in Liu and Lu (2023), we have that: For any prize structure $\mathbf{v} = (v_1, \ldots, v_N) \neq v\mathbf{e}$ with $v_1 \geq \cdots \geq v_N$, where $v \in \mathbb{R}$ and $e = (1, \ldots, 1)$ is the N-vector with all its elements being 1,

$$\sum_{j=1}^{N} p_j(t^c)v_j \text{ is strictly increasing in } t^c \in [a, b].$$

Step 2: We show that for any two prizes $\mathbf{v}$ and $\mathbf{v}'$ satisfying the budget constraints and monotonicity in prizes, if $\mathbf{v}$ majorizes $\mathbf{v}'$ and $\mathbf{v} \neq \mathbf{v}'$, then

$$\sum_{j=1}^{N} \beta_j(t^c)v_j > \sum_{j=1}^{N} \beta_j(t^c)v'_j.$$

This is because

$$\sum_{j=1}^{N} \beta_j(t^c)v_j = \sum_{j=1}^{N-1} \left[(\beta_j(t^c) - \beta_{j+1}(t^c)) \sum_{i=1}^{j} v_j \right] + \beta_N(t^c) \sum_{i=1}^{N} v_j.$$

Thus,

$$\begin{aligned} & \sum_{j=1}^{N} \beta_j(t^c)v_j - \sum_{j=1}^{N} \beta_j(t^c)v'_j \\ = \quad & \sum_{j=1}^{N-1} \left[\left(\underbrace{\beta_j(t^c) - \beta_{j+1}(t^c)}_{>0} \right) \left(\underbrace{\sum_{i=1}^{j} v_j - \sum_{i=1}^{j} v'_j}_{\geq 0} \right) \right] + \beta_N(t^c) \left(\underbrace{\sum_{i=1}^{N} v_j - \sum_{i=1}^{N} v'_j}_{=0} \right) \\ > \quad & 0. \end{aligned}$$

The inequality is strict, because $\sum_{i=1}^{j} v_j - \sum_{i=1}^{j} v'_j > 0$ for at least one $j = 1, \ldots, N$ as $\mathbf{v} \neq \mathbf{v}'$.

Step 3: Let $d_1 < d_2$, and suppose, to the contrary, that $\mathbf{v}^*(d_2)$ majorizes $\mathbf{v}^*(d_1)$. Assume that $\mathbf{v}^*(d_j)$ induces the entry threshold $t^c(d_j)$, for $j = 1, 2$. There are two cases.

Case 1: $t^c(d_1) = t^c(d_2)$. Then both $\mathbf{v}^*(d_1)$ and $\mathbf{v}^*(d_2)$ are feasible prize vectors when the parameter is d_1 or d_2. If $\mathbf{v}^*(d_1) \neq \mathbf{v}^*(d_2)$, the result in Step 2 implies that $\mathbf{v}^*(d_2)$ leads to a strictly higher payoff to the organizer than $\mathbf{v}^*(d_1)$ does. This contradicts the optimality of $\mathbf{v}^*(d_1)$. Thus, $\mathbf{v}^*(d_1) = \mathbf{v}^*(d_2)$ in Case 1.

Case 2: $t^c(d_1) \neq t^c(d_2)$. By Proposition 3, $t^c(d_1) > t^c(d_2)$. Since $\mathbf{v}^*(d_2)$ satisfies the participation constraint when the cutoff is $t^c(d_2)$, we have $\sum_{j=1}^{N} p_j(t^c(d_2))v^*_j(d_2) = c$. Clearly, $\mathbf{v}^*(d_2) \neq$

$(V/N, \ldots, V/N)$, so Step 1 implies that

$$\sum_{j=1}^{N} p_j(t^c(d_1))v_j^*(d_2) > c. \tag{15}$$

By Lemma 7, $\mathbf{v}^*(d_2)$ also satisfies the budget constraint $\sum_{j=1}^{N} v_j^*(d_2) = V$. Suppose k is the largest integer in $\{1, \ldots, N\}$ such that $v_k^*(d_2) > 0$. It is obvious that there is no loss of generality to assume $k \geq 2$. In the proof of Lemma 7 we showed that $p_1(t^c(d_1)) \leq p_j(t^c(d_1))$ for any j and $t^c \leq t_1$. Thus, when we increase $v_1^*(d_2)$ by $\varepsilon_1 > 0$ and decrease $v_k^*(d_2)$ by ε_1 (and holding the rest $N-2$ prizes unchanged), the new prize vector $\mathbf{v}(\varepsilon_1)$ still satisfies the budget constraint and the prize monotonicity constraint. If ε_1 is small enough, the new prize vector would have

$$\sum_{j=1}^{N} p_j(t^c(d_1))v_j(\varepsilon_1) > c.$$

Thus, case 2.1: $\sum_{j=1}^{N} p_j(t^c(d_1))v_j(\varepsilon_1) \leq c$ when $\varepsilon_1 = v_k^*(d_2)$. In this case, there exists a unique $\tilde{\varepsilon}_1 \in (0, v_1^*(d_2)]$, such that

$$\sum_{j=1}^{N} p_j(t^c(d_1))v_j(\tilde{\varepsilon}_1) = c.$$

Then the prize vector $\mathbf{v}(\tilde{\varepsilon}_1)$ is a prize vector that satisfies all constraints when the cutoff is $t^c(d_1)$. By definition, $\mathbf{v}(\tilde{\varepsilon}_1)$ majorizes $\mathbf{v}^*(d_2)$ and $\mathbf{v}(\tilde{\varepsilon}_1) \neq \mathbf{v}^*(d_2)$. Since $\mathbf{v}^*(d_2)$ majorizes $\mathbf{v}^*(d_1)$, we have that $\mathbf{v}(\tilde{\varepsilon}_1)$ majorizes $\mathbf{v}^*(d_1)$ and they are different prize vectors. By Step 2, this implies that $\mathbf{v}(\tilde{\varepsilon}_1)$ yields a strictly higher payoff to the organizer than $\mathbf{v}^*(d_1)$ does, which contradicts the optimality of $\mathbf{v}^*(d_1)$.

Case 2.2: $\sum_{j=1}^{N} p_j(t^c(d_1))v_j(\varepsilon_1) > c$ when $\varepsilon_1 = v_k^*(d_2)$. In this case, we look at the prize vector

$$\mathbf{v}^{**}(d_2) = (v_1^*(d_2) + v_k^*(d_2), v_2^*(d_2), \ldots, v_{k-1}^*(d_2), 0, \ldots, 0).$$

Note that $\mathbf{v}^{**}(d_2)$ majorizes $\mathbf{v}^*(d_2)$. Since $\sum_{j=1}^{N} p_j(t^c(d_1))v_j^{**}(d_2) > c$, we can repeat the procedure in case 2.1 by checking whether $\sum_{j=1}^{N} p_j(t^c(d_1))v_j^{**}(\varepsilon_2) \leq c$ when $\varepsilon_2 = v_{k-1}^*(d_2)$, where

$$\mathbf{v}^{**}(\varepsilon_2) = (v_1^*(d_2) + v_k^*(d_2) + \varepsilon_2, v_2^*(d_2), \ldots, v_{k-1}^*(d_2) - \varepsilon_2, 0, \ldots, 0).$$

If yes, like in case 2.1 we can find a new prize vector which satisfies all constraints and majorizes $\mathbf{v}^*(d_1)$, leading to the same contradiction. If not, we can repeat the procedure in case 2.1 by shifting the $(k-1)$th prize to the first prize, and the do the same exercise.

This procedure must stop in at most $k-1$ steps, because $p_1(t^c(d_1))V \leq c$ as $t^c(d_1) \leq t_1$. Therefore, the contradiction in Case 2.1 must arise. This completes the proof. □

Proof of Lemma 8. By Lemma 5 in Liu and Lu (2023), $\beta_{N-1}(a) \geq 0$ and $J(t)$ being increasing implies that

$$\beta_1(t^c) > \beta_2(t^c) > \cdots > \beta_{N-1}(t^c) \geq 0, \text{ for any } t^c \in [a, b).$$

Moreover, if $\beta_N(t^c) < 0$ for some t^c, then trivially

$$\beta_1(t^c) > \beta_2(t^c) > \cdots > \beta_N(t^c).$$

If $\beta_N(t^c) \geq 0$ for some t^c, then Lemma 5 in Liu and Lu (2023) again implies

$$\beta_1(t^c) > \beta_2(t^c) > \cdots > \beta_N(t^c).$$

This completes the proof. $\square$

Proof of Properties of $M(t^c)$. Define $K(t^c, v_1) = p_1(t^c)v_1 + p_2(t^c)(V - v_1),\ t^c \in [a, t_1]$. When $N = 2$, the participation constraint $K(t^c, v_1) = c$ becomes $v_1 F(t^c) + (V - v_1)(1 - F(t^c)) = c$. Clearly, this defines v_1 as a function of t^c when $t^c \in [a, t_1]$:

$$v_1 = M(t^c) = \frac{c - V(1 - F(t^c))}{2F(t^c) - 1}. \tag{16}$$

Note that by the definition of t_1 and $c < V/N = V/2$, we have $F(t_1) = c/V < 1/2$. Therefore, $M(t^c)$ is strictly increasing in t^c, $M(a) = V - c$, and $M(t_1) = V$.

Now we turn the case of $N \geq 3$. Let us first rewrite K as follows:

$$\begin{aligned} K(t^c, v_1) &= p_1(t^c)v_1 + p_2(t^c)(V - v_1) \\ &= (V - v_1)(p_1(t^c) + p_2(t^c)) + (2v_1 - V)p_1(t^c). \end{aligned}$$

When $N \geq 3$, for any fixed $v_1 \in [V/2, V]$, $K(t^c, v_1)$ is strictly increasing in $t^c \in [a, b]$. This is because $V - v_1 \geq 0$, $2v_1 - V \geq 0$, $p_1'(t^c) > 0$ when $t^c \in (a, b)$, and $p_1'(t^c) + p_2'(t^c) > 0$ for any $t^c \in (a, b)$.

Notice that for any fixed $v_1 \in [V/2, V]$, $K(a, v_1) = 0 < c$. Moreover, as shown in the proof of Lemma 7, $p_1(t^c) < p_2(t^c)$ for any $t^c \in (a, t_1]$, so we have

$$K(t_1, v_1) = p_1(t_1)v_1 + p_2(t_1)(V - v_1) \geq p_1(t_1)v_1 + p_1(t_1)(V - v_1) = p_1(t_1)V = c,$$

where the inequality is strictly when $v_1 < V$. Combining with the fact that $K(t^c, v_1)$ is strictly increasing in $t^c \in [a, b]$, we obtain that for any $v_1 \in [V/2, V]$, there is a unique $t^c = \tilde{M}(v_1) \in [a, t_1]$ such that $K(\tilde{M}(v_1), v_1) = c$.

Moreover, since $p_1(t^c) < p_2(t^c)$ for any $t^c > a$, $K(t^c, v_1)$ is strictly decreasing in v_1 for any fixed $t^c \in (a, t_1]$. This further implies that $\tilde{M}(v_1)$ is strictly increasing in v_1. Thus, $\tilde{M}(v_1)$ is invertible; so we let $M(t^c)$ be the inverse of $\tilde{M}$. It is clear that $M(t^c)$ is strictly increasing in $t^c \in [\tilde{M}(V/2), \tilde{M}(V)]$. Here, $\tilde{M}(V) = t_1$, whereas $\tilde{M}(V/2)$ is the unique threshold $t_0 \in (a, t_1)$ such that $p_1(t_0) + p_2(t_0) = 2c/V$. $\square$

Proof of Proposition 5. For parts (i) to (iii), notice that $\frac{\partial^2 P^*}{\partial t^c \partial d} = -Nf(t^c)/2 < 0$, so P^* has increasing difference in $(t^c, -d)$, which implies that t^* is decreasing in d, which is consistent with Proposition 3. Because $M(t^c)$ is strictly increasing in t^c, this further implies that the optimal first prize is decreasing in d, which is consistent with Proposition 4. Since there are only two prizes, this means that the second prize increases with d, so Proposition 4 can be strengthened to part (iii) of Proposition 5.

Now we turn to the rest of the claims of Proposition 5. Taking derivative with respect to t^c, we have

$$\begin{aligned}
\frac{\partial P^*}{\partial t^c} &= \beta_1'(t^c)M(t^c) + \beta_2'(t^c)(V - M(t^c)) + (\beta_1(t^c) - \beta_2(t^c))M'(t^c) + Ncf(t^c)J(t^c) - \frac{Nf(t^c)d}{2} \\
&= -J(t^c)Nf(t^c)\left[p_1(t^c)M(t^c) + p_2(t^c)(V - M(t^c))\right] + (\beta_1(t^c) - \beta_2(t^c))M'(t^c) \\
&\quad + Ncf(t^c)J(t^c) - \frac{Nf(t^c)d}{2} \\
&= (\beta_1(t^c) - \beta_2(t^c))M'(t^c) - \frac{Nf(t^c)d}{2}, \\
&= f(t^c)[\underbrace{(\beta_1(t^c) - \beta_2(t^c))\frac{M'(t^c)}{f(t^c)}}_{\delta(t^c)} - \frac{Nd}{2}].
\end{aligned}$$

The above derivation uses the participation constraint that $p_1(t^c)M(t^c) + p_2(t^c)(V - M(t^c)) = c$.

Note that since $\beta_1(t^c) - \beta_2(t^c) > 0$ for any $t^c \in [a, t_1]$ by Lemma 9, $\delta(t^c) > 0$ for any $t^c \in [t_0, t_1]$. Therefore, the optimal entry threshold t^* is characterized by the following first-order condition:

$$\delta(t^*) - \frac{Nd}{2} \begin{cases} \geq 0, \text{ if } t^* = t_1 \\ \leq 0, \text{ if } t^* = t_0 \\ = 0, \text{ if } t^* \in (t_0, t_1) \end{cases}. \tag{17}$$

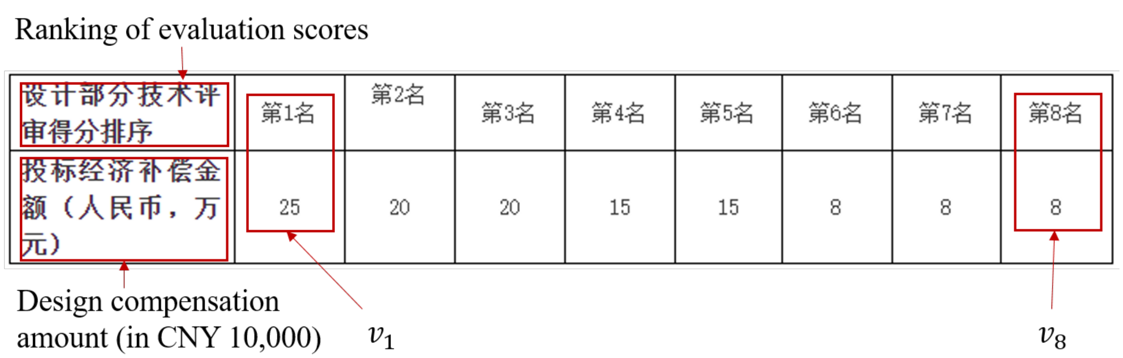
In particular, let $\hat{d}_{\min} = \min_{t^c \in [t_0, t_1]} \delta(t^c)$, which is strictly positive. Then, when $d \in [0, \hat{d})$, $\delta(t^c) - \frac{Nd}{2} > 0$, so P^* is strictly increasing in $t^c \in [t_0, t_1]$, which means that the optimal entry threshold is t_1. In this case, the unique optimal prize structure is winner-take-all. Note that $\hat{d}_{\min} > 0$, this illustrates (i) of Proposition 2 that the cutoff $\tilde{d}$ is strictly positive (clearly, $\tilde{d} \geq \hat{d}_{\min}$). When d gets larger, an interior optimal t^* may arise, which is characterized by the interior first-order condition $\delta(t^*) = Nd/2$.

On the other hand, when $d > \hat{d}_{\max} = \max_{t^c \in [t_0, t_1]} \delta(t^c)$, $\delta(t^c) - \frac{Nd}{2} < 0$, so P^* is strictly decreasing in $t^c \in [t_0, t_1]$, which means that the optimal entry threshold is t_0. Recall that t_0 is the lowest possible threshold inducible, which is achieved by using the most equal prize scheme. This implies that when the value of diverse ideas is sufficiently high, the optimal prize structure is the most equal one—that is, prize sharing $(V/2, V/2, 0 \ldots, 0)$ when $N \geq 3$. When $N = 2$, the designer is able to induce full entry by letting $v_2 = c$. She can still induce full entry by letting $v_2 > c$, but this is clearly not optimal, because she should leave the highest possible budget to the first prize to induce greater effort from the winner. Therefore, when $N = 2$, the most equal prize structure is $(V - c, c)$. $\square$

B. A Sample Contest Announcement

Let us see a contest announcement for a building project from our data. The original announcement is posted on the website of the procurement platform (`www.gzggzy.cn/cms/wz/view/index/layout3/index.jsp?siteId=1&channelId=503&infoId=537554.`). In case the website removes this announcement, we saved a copy of it (`www.dropbox.com/s/mugalp5y780np8i/contest.announcement.ch.pdf?dl=0`) and its English translation (`www.dropbox.com/s/mh4z7py9uuj01jr/contest.announcement.en.pdf?dl=0`).

The procurer is Sun Yat-sen University (`www.sysu.edu.cn`) in Guangzhou, China. The contest is intended to procure design proposals for an apartment building for faculty housing. One important part of the announcement is the prize scheme shown in Figure 10. For this contest, the prize scheme is $\mathbf{v} = (25, 20, 20, 15, 15, 8, 8, 8, 0, 0, ...)$.



设计部分技术评审得分排序	第1名	第2名	第3名	第4名	第5名	第6名	第7名	第8名
投标经济补偿金额（人民币，万元）	25	20	20	15	15	8	8	8

Figure 10: Prize Scheme in the Contest Announcement