Semiparametric SDF Estimators for Pooled, Non-Traded Cash Flows

GOR AG Analytics Workshop, Frankfurt (Deutsche Bahn)

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Guiding Question

Source: `https://traderepublic.com/en-de/private-markets`

Guiding Question

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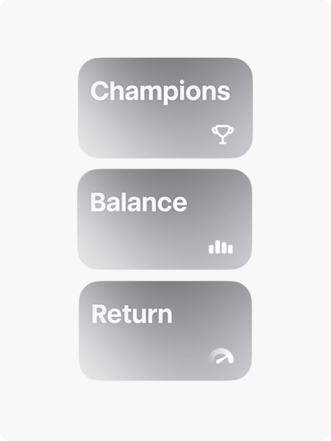

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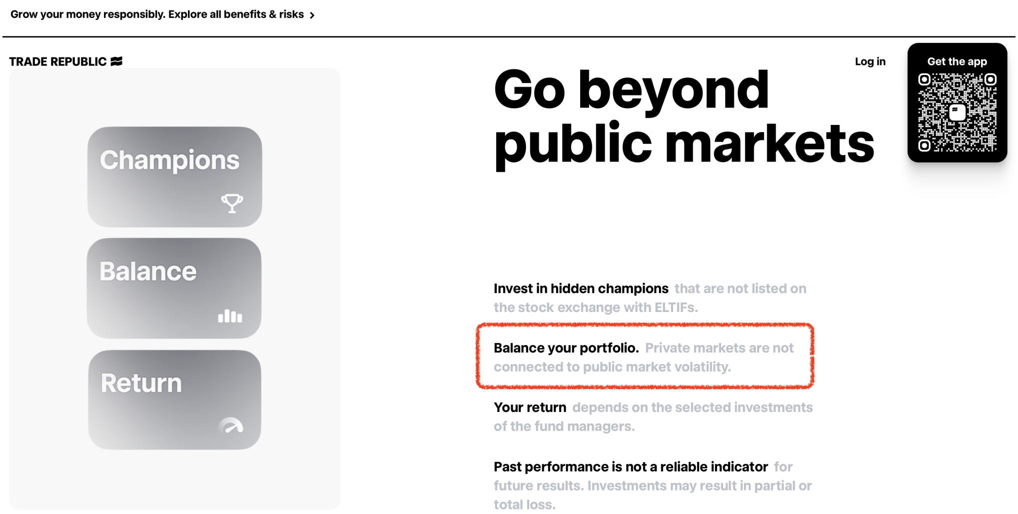
Guiding Question



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Guiding Question

Are private markets really not connected to public market volatility?

How can we estimate the true public-market exposure of private equity funds?

- Goal: recover latent market exposure from non-traded PE cash flow data.
- Plan: naive approach first, then semiparametric SDF estimation.

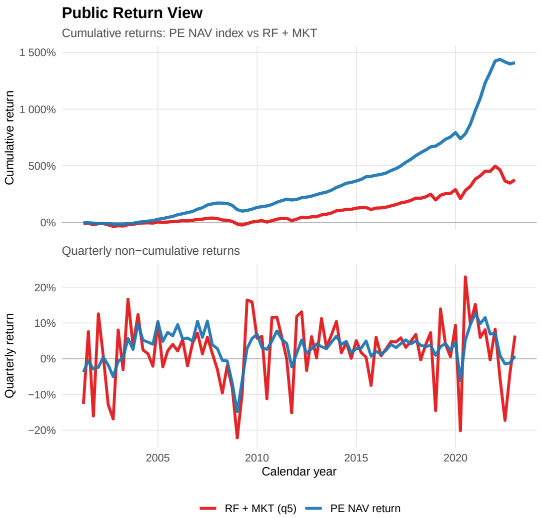
Useful for:

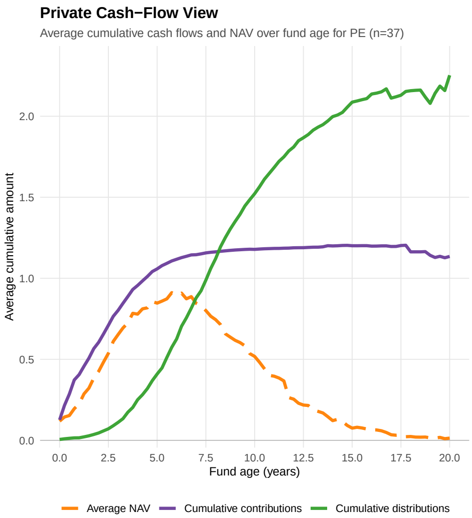
- Risk-adjusted benchmarks for private equity funds.
- Holistic risk management for public and private portfolios.
- More realistic strategic asset allocation decisions.

Agenda

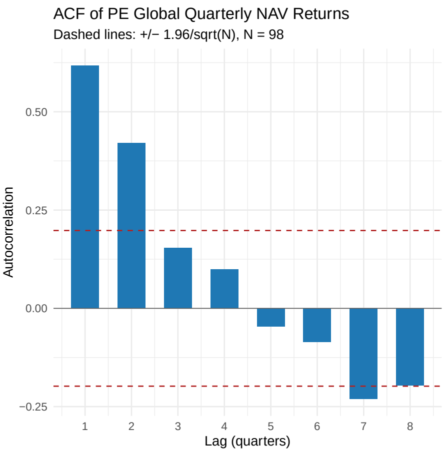
1. Motivation: simple approach
2. Estimation framework
3. Simulation evidence
4. Empirical PE results
5. Conclusion

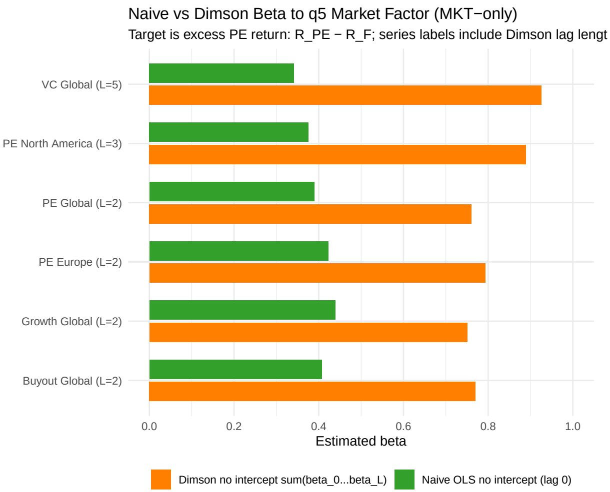
Motivation: Public Returns vs Private Cash Flows





Motivation: [Dimson, 1979] Regression from NAV Returns





Data: Preqin quarterly index levels and q5 monthly R_{MKT} aggregated to quarters (overlap through 2022Q4).

Estimator: Starting Point [Driessen et al., 2012]

For each fund i, [Driessen et al., 2012] imposes a zero-NPV pricing condition at inception date $t_{0,i}$:

$$\epsilon_i^{\text{DLP}}(\theta) = \sum_{t=0}^{T} \Psi_t(\theta)\, CF_{i,t}$$

Cross-sectional estimator:

$$\hat{\theta}_{\text{DLP}} = \arg\min_{\theta \in \Theta} \frac{1}{n} \sum_{i=1}^{n} \left(\epsilon_i^{\text{DLP}}(\theta)\right)^2$$

combine to one combined slide

We propose generalization that nests the [Driessen et al., 2012] estimator as special case.

Variables: i fund/portfolio index, t cash-flow time, $CF_{i,t}$ net cash flow, θ parameter vector, Θ parameter space, n number of units, sample end date T.
Important: $\Psi_t(\theta)$ Stochastic Discount Factor (SDF) from t to time $t_{0,i}$ (=fund inception date).

Estimator: Our generalization

For fund i and discounting date τ:

$$\epsilon_{\tau,i}(\theta) = \sum_{t=0}^{T} \frac{\Psi_t(\theta)}{\Psi_\tau(\theta)} CF_{i,t}$$

Average over selected compounding dates $\mathcal{T}_i$ (compounding horizons):

$$\bar{\epsilon}_i(\theta) = \frac{1}{|\mathcal{T}_i|} \sum_{\tau \in \mathcal{T}_i} \epsilon_{\tau,i}(\theta)$$

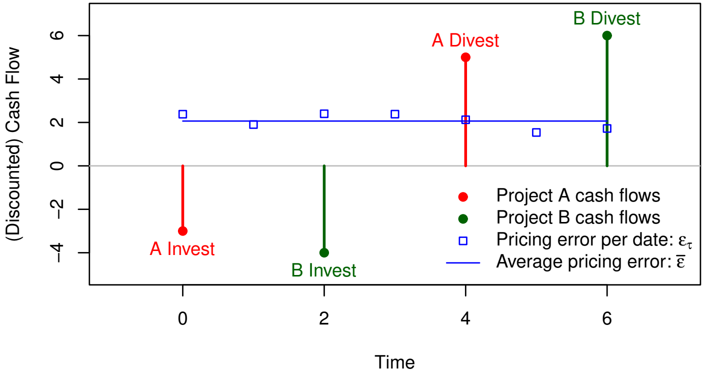
Estimate parameters by nonlinear least-mean-distance:

$$\hat{\theta} = \arg\min_{\theta \in \Theta} \left(\frac{1}{n} \sum_{i=1}^{n} L(\bar{\epsilon}_i(\theta)) \right), \quad L(x) = x^2$$

Variables: i fund/portfolio index, t cash-flow time, τ compounding date, $CF_{i,t}$ net cash flow, $\frac{\Psi_t(\theta)}{\Psi_\tau(\theta)}$ SDF ratio, θ parameter vector, $\mathcal{T}_i$ compounding-date set, L loss function.

Estimator: Net Present Value (NPV) vs Net Future Value (NFV)

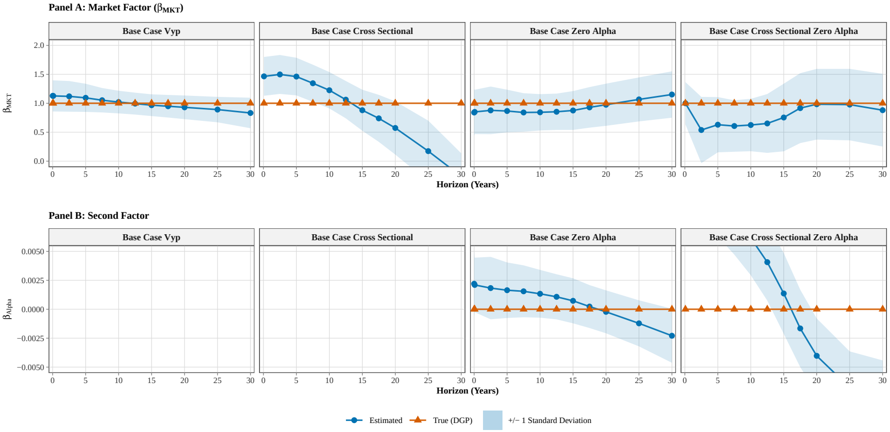
- NPV-only discounting (fund inception) is theoretically unbiased.
- NFV: Adding compounding horizons > 0 introduces a **bias term**.
- **Spoiler:** Finite-sample performance can improve when averaging across future dates.



Simulation: What Is Tested?

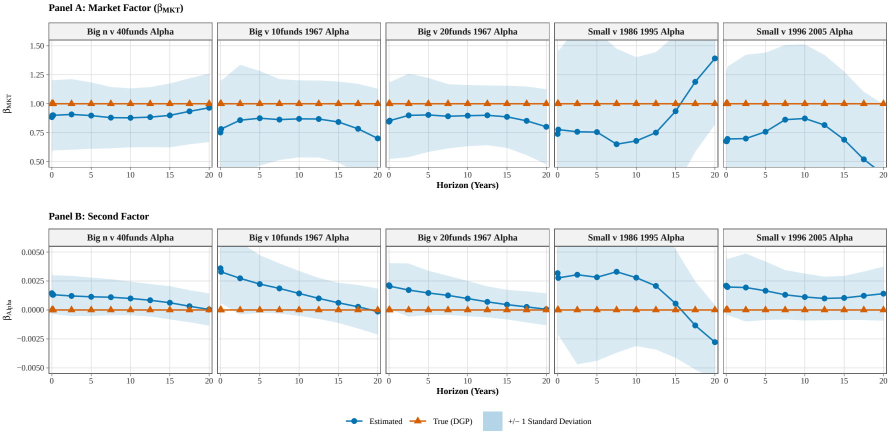
- **Base simulation setup:**
  - Synthetic fund cash flows generated under known SDF structure.
  - 15 equally-sized deals per fund.
  - 5 year initial investment period per fund.
  - 10 year maximum holding period per deal.
  - 20 vintages years from 1986 to 2005 using this macro environment.
  - Monthly returns: $R_i = \alpha + R_{\mathbf{riskfree}} + \beta_{\mathbf{MKT}} R_{\mathbf{MKT}} + \beta_2 R_2 + \cdots + e_i$ (with i.i.d. e_i)
- **Main focus on horizon choice:** size of compounding horizon set $\mathcal{T}$.
- **Compare units:** individual funds vs vintage-year portfolios (VYP).
- **Compare sample geometry:** more vintages (V) vs larger within-vintage size (n/V).
- **Compare SDF model forms:**
  - Exponential affine vs simple linear SDF.
  - Single-factor vs multi-factor models.

Simulation: 1. Single Funds vs Portfolio Formation



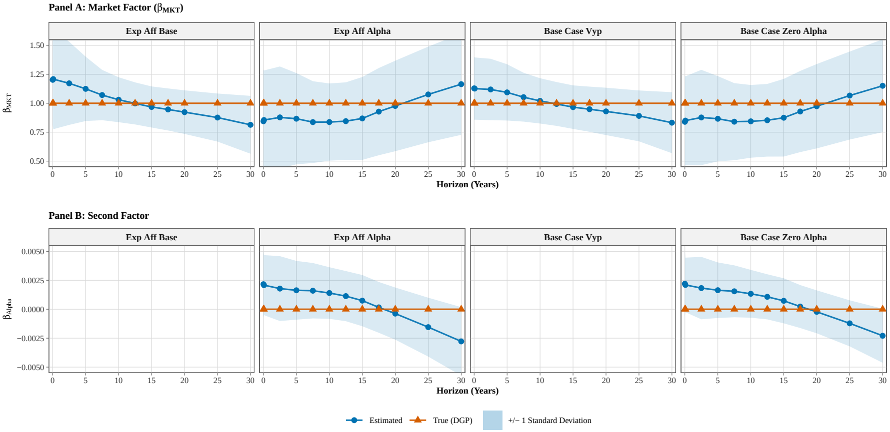
Takeaway: Vintage-year portfolios reduce bias and variance versus single-fund estimation.
Horizon on x-axis corresponds to $\mathcal{T}$-averaging in all charts. Horizon 0 is base case [Driessen et al., 2012].

Simulation: 2. More Vintages or More Funds per Vintage?



Takeaway: Increasing funds per vintage is more powerful for bias/variance reduction than only extending the time span.

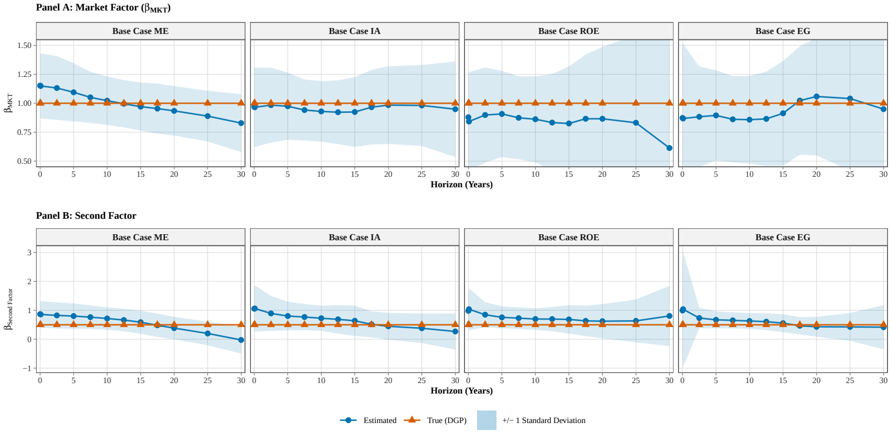
Simulation: 3. Exponential Affine vs Linear SDF



Takeaway: No robust finite-sample superiority of exponential affine SDF in this setting.

SDF model comparison: [Korteweg and Nagel, 2016] vs [Driessen et al., 2012]

Simulation 4: Two-Factor q-Factor Models



Takeaway: Multivariate identification is fragile. Estimate ≈ 1 instead of 0.5 for 2nd factors.
q-factor model from [Hou et al., 2021] is competitor to [Fama and French, 2015].

Simulation: Synthesis

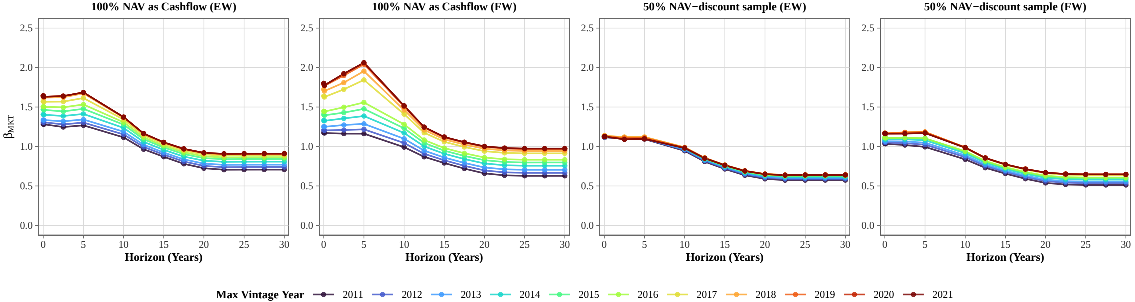
1. Use portfolio formation to stabilize estimation.
2. Increase number of funds per vintage (if possible).
3. Use simple SDF models given limited amount of PE data (avoid multi-factor models).
4. Compounding horizon choice (size of $\mathcal{T}$) is a control for finite-sample bias.

Interpretation: practical estimator quality is dominated by finite-sample bias-variance tradeoffs.

Empirical: Preqin PE Data & q^5 Public Factors

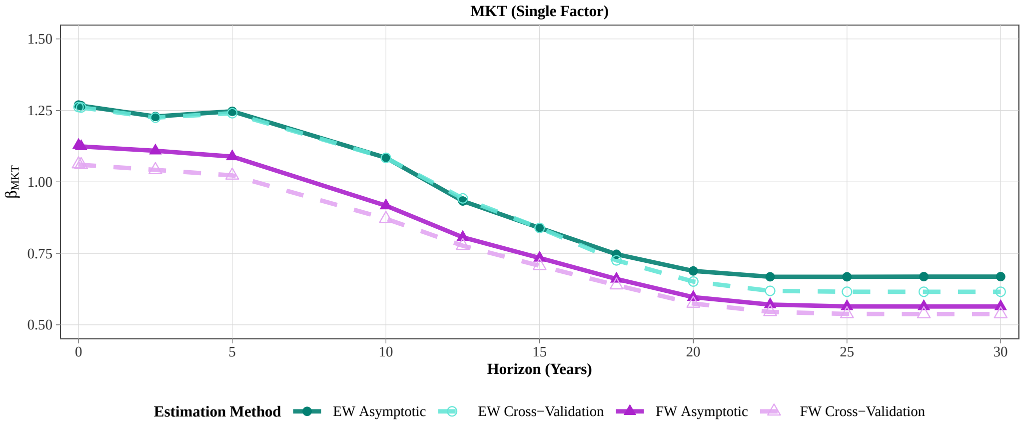
- Preqin Data: 3005 PE funds, vintages 1983–2021.
- Primary unit for estimation: vintage-year portfolios (equal- and value-weighted).
- Factors: q^5 family; focus on MKT and simple two-factor extensions.
- Compounding horizon selected from simulation guidance: 15-year baseline.
- Benchmark for comparison: NAV-based naive/Dimson market-exposure estimates from the motivation section.

Empirical: 1. Vintage Cutoffs for Single-Factor Model



Reading: including newer vintages tends to increase estimated market exposure, but totally depending on final NAV $\rightarrow$ Use vintage 2010 cutoff in subsequent analysis.

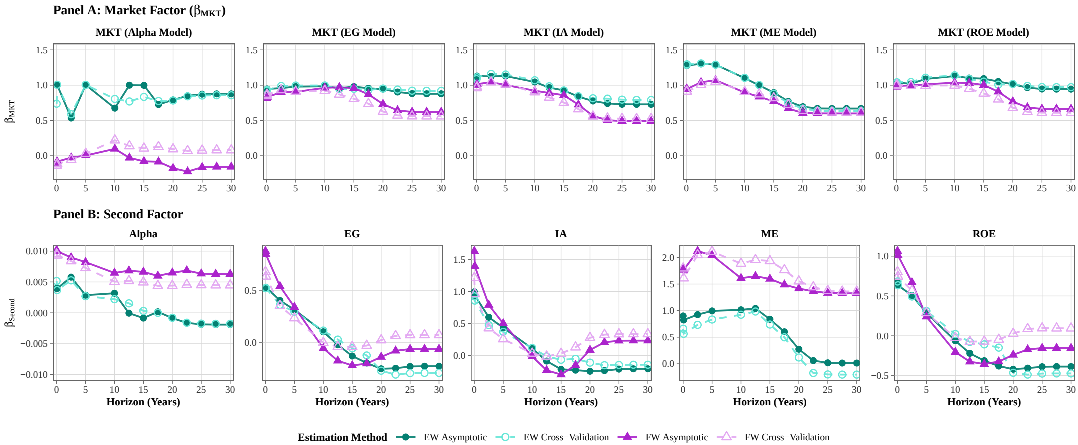
Empirical: 2. Single-Factor MKT Model



Singe-factor models: Dimson beta as lower bound approximation.

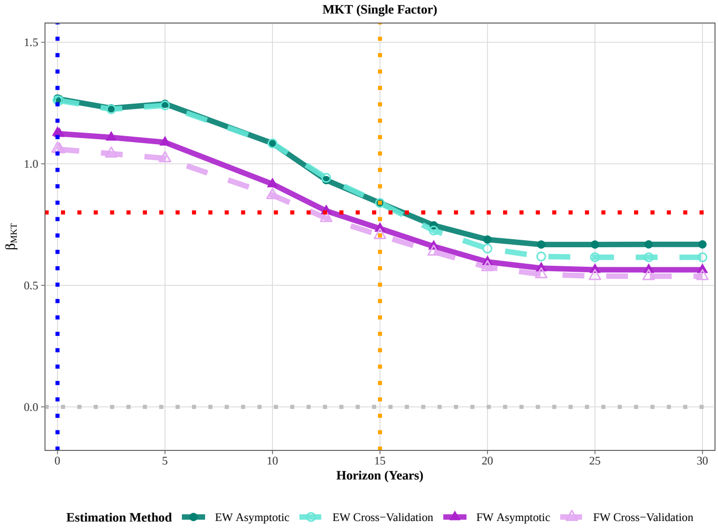
We use empirical vintages until 2010.

Empirical: 3. Two-Factor Models



Two-factor models: Apply machine-learning methods to form “stronger learner.”
We use empirical vintages until 2010.

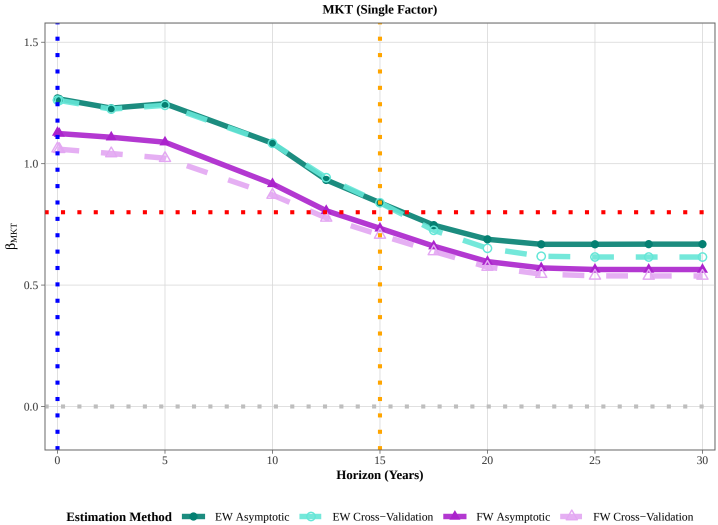
Empirical: Synthesis



Use *MKT*-factor model.
Don't use unrealized NAVs.

- Unbiased estimator
- Best biased estimator
- Dimson benchmark
- Luciano bound

Empirical: Synthesis



Use *MKT*-factor model.
Don't use unrealized NAVs.

- Unbiased estimator
- Best biased estimator
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Conclusion

Private markets are connected to public market volatility.

Main results

- Our approach can better control finite-sample bias than [Driessen et al., 2012].
- Our approach yields empirical β_{MKT} estimates similar to [Dimson, 1979].
- Given amount of **currently available** private market data:
  Cash-flow based estimation results are close to NAV-return-based estimation results.
- Portfolio formation and compounding horizons are key for cash-flow-based inference.

Additional topics (for cash-flow-based inference)

- Use dependence-aware cross validation instead of asymptotic standard errors.
- Outlook: Multi-factor models must be stabilized (e.g. by machine-learning techniques).

Literature

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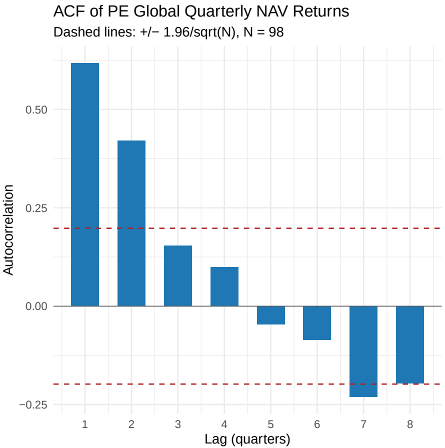
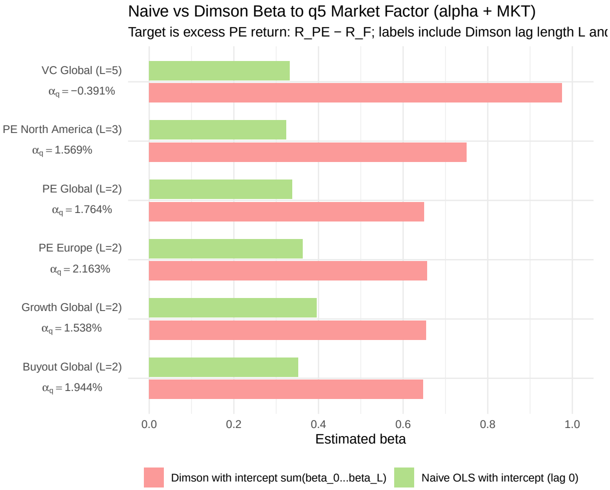
Motivation: [Dimson, 1979] Regression from NAV Returns (Alpha+MKT)





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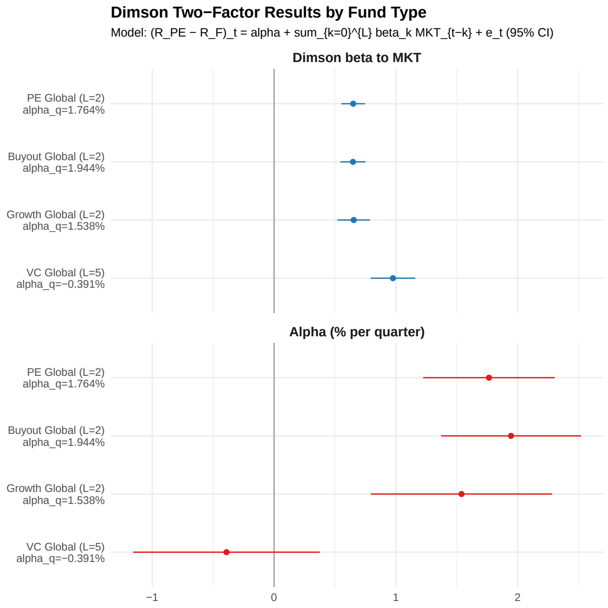


Dimson Two−Factor Results by Fund Type
Model: (R_PE − R_F)_t = alpha + sum_{k=0}^{L} beta_k MKT_{t−k} + e_t (95% CI)
Dimson beta to MKT
PE Global (L=2)
alpha_q=1.764%
Buyout Global (L=2)
alpha_q=1.944%
Growth Global (L=2)
alpha_q=1.538%
VC Global (L=5)
alpha_q=−0.391%
Alpha (% per quarter)
PE Global (L=2)
alpha_q=1.764%
Buyout Global (L=2)
alpha_q=1.944%
Growth Global (L=2)
alpha_q=1.538%
VC Global (L=5)
alpha_q=−0.391%
−1
0
1
2

Backup: Comparison to DLP12 and KN16

	DLP12	KN16	This paper
Estimator	Cross-sectional NLS	Time-series GMM (public SDF)	Nonlinear LMD
Cash flows priced	PE fund cash flows	Public replicating portfolios	PE fund cash flows
Discount dates	Inception only	Inception only	Flexible via $\mathcal{T}_i$
Asymptotics	Infill	$V \to \infty$	Increasing domain
Inference	Bootstrap	SHAC	SHAC + CV focus

Motivation: Why Measuring Risk Is Hard in Private Markets

- PE funds generate **cash flow sequences**, not continuously traded returns.
- Fund lives overlap across vintages, creating dependence beyond standard panel assumptions.
- Fund valuation relies on reported NAVs, which can be stale/smoothed.
- Standard return-based factor models are not directly applicable.

Implication: we need a cash-flow-native SDF estimator with robust dependence-aware inference.

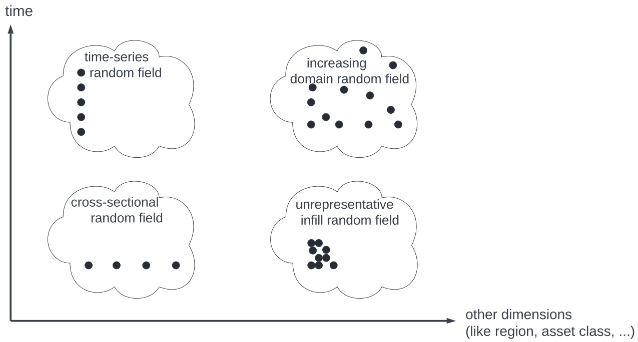
Backup: Significant ACF Lags and Dimson Lag Choice

Series	Significant lags in 1–8	Consecutive from lag 1	Dimson lag L
PE Global	3	2	2
PE North America	5	3	3
PE Europe	3	2	2
Buyout Global	3	2	2
Growth Global	2	2	2
VC Global	5	5	5

Naive benchmark result: contemporaneous beta is low (≈ 0.32–0.39), while Dimson beta increases to ≈ 0.63–0.96.

Dependence Structure: Random Field View

- Cross-sectional unit: fund or vintage-year portfolio.
- Dependence driven by economic proximity (here: vintage-year distance).
- Asymptotics: increasing domain ($V \to \infty$), bounded units per vintage.



Estimator: Inference Strategy

- Asymptotic covariance: sandwich form $\Sigma = H^{-1}\Lambda H^{-1}$.
- Long-run dependence handled by SHAC (spatial HAC) with vintage-distance kernel.
- Small-sample reliability checked via *hv*-**block cross-validation**.

Reason: asymptotic approximations are fragile with only 20–40 vintage portfolios.

Conclusion: Machine-Learning Ensembles

- Two-factor models are messy.
- Idea: Combine multiple weak learners

Conclusion: Main Takeaways

1. Start with parsimonious SDFs (MKT-first), then add complexity cautiously.
2. Finite-sample design choices can dominate asymptotic elegance.
3. Treat asymptotic significance alone as insufficient in sparse PE samples.
4. Data architecture (portfolio formation, horizon design) is part of identification.

Questions and Discussion