A fundwise stochastic discount factor estimator for private equity funds

Christian Tausch
AssetMetrix GmbH
Theresienhöhe 13, D-80339 Munich
christian.tausch@quant-unit.com

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Declaration of interest

The author reports no conflict of interest. The author alone is responsible for the content and writing of the paper.

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Abstract

This paper proposes a simple stochastic discount factor estimation methodology suited for private equity fund-level cash flow data.

1 Introduction

Do investments in private equity funds offer abnormal returns to fund investors when risk-adjusted to public market factors? Or more profoundly, do we need yet another Stochastic Discount Factor (SDF) approach to evaluate private equity fund cash flows?

To answer these questions, we propose a simple SDF model estimation methodology that incorporates private equity fund-level cash flow data. The basic idea is that the expected value of all discounted fund net cash flows is zero when an appropriate SDF model is applied. Therefore, our estimator is closely related to the well-known approaches of Driessen et al. (2012) and Korteweg and Nagel (2016). Similarly to Korteweg and Nagel (2016), we provide asymptotic inference formulations that rely on a spatial (near-epoch) dependency between funds. On the other hand, our least-mean-distance estimator can be regarded as generalization of the Driessen et al. (2012) methodology, where we provide the asymptotic inference framework that was missing in the original paper. In contrast to Driessen et al. (2012), we explicitly do not require the pooling of private equity fund cash flows to form vintage year portfolios, to increase the number of cross-sectional units (cite advantage of pricing underlying assets in public market literature).

In the empirical application of our new estimator, we estimate a simple linear SDF model that can draw on the five return factors associated with the q^5 investment factor model recently proposed by Hou et al. (2020). In a model selection setup, for various private equity fund types the most relevant subset of q^5 factors are selected by means of lasso-regularization (Tibshirani, 1996) in combination with hv-block bootstrapping (Racine, 2000). Post-model selection, results: best factors by fund types (Preqin asset class classification: Private Equity, Venture Capital, Private Debt).

2 Methodology

2.1 Fundwise least-mean-distance estimator

Let fund $i = 1, 2, \ldots, n$ be characterized by its (net) cash flows $CF_{t,i}$ and its net asset values $NAV_{t,i}$ with discrete time index $t = 1, 2, \ldots, T$. The data generating processes for CF and NAV are left unspecified. The stochastic discount factor $\Psi_{t,\tau}$ can be used to calculate the time-τ present value $P_{t,\tau,i}$ of a time-t cash flow of any given PE fund i

$$P_{t,\tau,i} = \Psi_{t,\tau} \cdot CF_{t,i} \tag{1}$$

As SDFs are commonly parameterized by a vector $\theta \in \mathbb{R}^p$, i.e., $\Psi_{t,\tau} \equiv \Psi_{t,\tau}(\theta)$, our goal is to find an estimation method for the optimal θ. For each fund i and all points τ within a common fund lifetime, the pricing error $\epsilon_{\tau,i}$ of all fund cash flows is calculated as

$$\epsilon_{\tau,i} = \sum_{t=1}^{T} P_{t,\tau,i} \qquad \forall \quad \tau, i \tag{2}$$

We define the w-weighted τ-average fund pricing error as

$$\bar{\epsilon}_i = w_i \cdot \frac{1}{\text{card}(\mathcal{T}_i)} \sum_{\tau \in \mathcal{T}_i} \epsilon_{\tau,i} \qquad \forall \quad i \tag{3}$$

where $\mathcal{T}_i$ gives the set of relevant present value times τ for fund i and, for example, can be thought of all quarterly/yearly dates within the usual fund lifetime of ten to fifteen years. Each fund i is characterized by its vintage year which can be expressed by $v_i = \min(\mathcal{T}_i) \in 1, 2, \ldots, V$, where V denotes the maximum vintage year used in a given data set. Here we implicitly assume that $\mathcal{T}_i$ always contains at least the fund's starting date. Finally, the scalar weighting factor w_i can be (i) one divided by the fund's invested capital for equal weighting of funds, (ii) one divided by the vintage year sum of invested capital for vintage year weighting, (iii) the scalar one for fund-size weighting, or (iv) some macroeconomic deflator.

Our least-mean-distance estimator minimizes the average loss of $\bar{\epsilon}$

$$\hat{\theta} = \arg\min_{\theta \in \Theta} \; S_n(\theta) \quad \text{with} \quad S_n(\theta) = \frac{1}{n} \sum_{i=1}^{n} L\left(\bar{\epsilon}_i\right) \tag{4}$$

where L denotes a loss function, e.g., $L(x) = (x - 0)^2$. Throughout the paper, the weighted average fund pricing error $\bar{\epsilon} \equiv \bar{\epsilon}(\theta)$ is regarded as nonlinear random function of the SDF parameter θ.

2.2 Asymptotic framework

2.2.1 Vintage year asymptotics

We employ a spatial framework, where we assume that the (spatial, i.e., economic) distance between cross-sectional units, i.e., private equity funds, can be measured in quantitative way. Here asymptotic results are derived for the case when the number of funds goes to infinity $n \to \infty$. However, identification of model parameters requires a sufficient number of funds from different vintage years in the fund-level data set used for model estimation (Driessen et al., 2012; Korteweg and Nagel, 2016).

Assumption 1. *(i) The number of vintage years $V \to \infty$ as $n \to \infty$. (ii) The number of funds per vintage year is bounded by some positive constant. (iii) The maximal fund lifetime is also bounded by a positive constant. (iv) The economic distance between fund i and j is measured by the vintage year difference $d_{i,j} = v_i - v_j$.*

2.2.2 Law of large numbers

The global moment condition underlying our estimation approach is that the (i-unconditional) expected value of $\bar{\epsilon}$ shall be zero, if we use the optimal SDF parameter θ_0. This also means, instead of applying a time-series law of large numbers, we rely on a spatial (cross-sectional) law of large numbers, but acknowledge the statistical dependence of pricing errors with respect to vintage year differences between funds. Jenish and Prucha (2012) develop an asymptotic inference framework for near-epoch dependent spatial processes that is instructive for our setting.

Assumption 2. *The (i) time-trend and (ii) dependence structure of $\bar{\epsilon}$ shall allow*

$$n^{-1} \sum_{i=1}^{n} \bar{\epsilon}_i \xrightarrow{a.s.} E[\bar{\epsilon}] \quad as \quad V, n \to \infty$$

Specifically, the process $\bar{\epsilon}$ is spatial near-epoch dependent with respect to fund vintage years (Jenish and Prucha, 2012).

To satisfy the time trend part (i) of this law of large number assumption, the weighting factor w, introduced in equation 3, can be used to make $\bar{\epsilon}$ stationary. Spatial near-epoch dependence with respect to fund vintage years formalizes the idea that two funds with a small absolute vintage year difference are supposed to be dependent (due to being exposed to the same macroeconomic condition), whereas two funds with a very large absolute vintage year difference can be assumed to be independent. In our framework, spatial distance is considered as economic distance between funds (represented by $\bar{\epsilon}$); our spatial space is thus of dimension one.

2.2.3 Consistency

The estimator $\hat{\theta}$ shall converge in probability to the true parameter value θ_0 as the number of distinct vintage years in our data set goes to infinity. Multiple funds for a specific vintage year are not necessarily required and are thus considered as additional, stabilizing moment conditions.

Assumption 3. *Consistency of $\hat{\theta}$ requires $\hat{\theta} \xrightarrow{p} \theta_0$ as $V, n \to \infty$. Thus $E[\bar{\epsilon}] = 0$ if and only if $\theta = \theta_0$. The parameter space is compact $\theta \in \Theta$.*

Compactness of Θ can be assured by lower and upper bounds for all parameters that can be justified by economic reasoning in our case, e.g., a market beta factor of ten seems implausible for PE funds because of the implied risk and return expectations.

2.2.4 Central limit theorem

To assess the significance of our parameter estimates, we want to describe the asymptotic distribution of the parameter vector as a normal distribution.

Assumption 4. *$\sqrt{n}(\hat{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}(0, \mathbf{\Sigma})$ as $V, n \to \infty$ with covariance matrix $\mathbf{\Sigma}$.*

2.3 Asymptotic inference

In the general (time-series) near-epoch-dependent least-mean-distance literature, $\mathbf{\Sigma}$ can be characterized according to Pötscher and Prucha (1997, Theorem 11.2.b, Theorem H.1):

$$\mathbf{\Sigma} = C^{-1}\Delta(C^{-1})^{\top}$$

with expected hessian matrix converging to C as $V, n \to \infty$

$$E\left(\nabla_{\theta\theta}S_n\right) \to C$$

and the expected covariance matrix of gradients converging to Δ as $V, n \to \infty$

$$nE\left[\nabla_{\theta}S_n(\nabla_{\theta}S_n)^{\top}\right] \to \Delta$$

Here, the gradient vector $\nabla_{\theta}S_n$ is denoted as column vector. We define the corresponding finite sample estimators analogously to Pötscher and Prucha (1997, Chapters 12, 13.1) and approximate the first and second derivatives by finite differences. $\hat{C}$ is relatively straightforward

$$\hat{C} = \frac{1}{n}\sum_{i=1}^{n}\nabla_{\theta\theta}L\left(\epsilon_i\right)$$

Due to the (spatial near-epoch) dependence, the intricate part is to consistently estimate $\hat{\Delta}$ by a Spatial Heteroskedasticity and Autocorrelation Consistent (SHAC) covariance matrix estimator (Kim and Sun, 2011, equation 2)

$$\hat{\Delta} = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}k_{i,j}\left[\nabla_{\theta}L\left(\epsilon_i\right)\left(\nabla_{\theta}L\left(\epsilon_j\right)\right)^{\top}\right]$$

We define the kernel weight k as

$$k_{i,j} \equiv K\left(\frac{d_{i,j}}{b_n}\right)$$

with kernel function $K : \mathbb{R} \to [0,1]$ satisfies $K(0) = 1$, $K(x) = K(-x)$, $\int_{-\infty}^{\infty} K^2(x)dx < \infty$, and $K(\cdot)$ continuous at zero and at all but a finite number of other points. Specifically, we select the Bartlett kernel $K_{BT}(x) = \max(0, 1 - |x|)$; see equation 2.7 in Andrews (1991) for other popular kernel choices. This means absolute vintage year differences larger than the bandwidth (or truncation) parameter b_n are considered independent and are thus excluded from the $\hat{\Delta}$ estimation formula.

In large samples, the parameter standard error vector can thus be estimated by

$$\mathrm{SE}(\hat{\theta}) = \sqrt{\mathrm{diag}\left[n^{-\frac{1}{2}}\hat{C}^{-1}\hat{\Delta}(\hat{C}^{-1})^{\top}(n^{-\frac{1}{2}})^{\top}\right]} = \sqrt{\mathrm{diag}\left[\frac{\hat{C}^{-1}\hat{\Delta}(\hat{C}^{-1})^{\top}}{n}\right]}$$

The Wald test statistic for linear hypotheses $H_0 : R\theta = r$ and $H_1 : R\theta \neq r$ is constructed as

$$W = (R\hat{\theta} - r)^{\top}\left[R\frac{\hat{C}^{-1}\hat{\Delta}(\hat{C}^{-1})^{\top}}{n}R^{\top}\right]^{-1}(R\hat{\theta} - r) \overset{H_0}{\sim} \chi^2_q$$

where $\hat{\theta}$ is the $p \times 1$ parameter vector, R is a $q \times p$ matrix, and r is a $q \times 1$ vector. Usually, we select R as $p \times p$ identity matrix, and r as $p \times 1$ vector (e.g., of zeros). Under the null hypothesis, W is chi-squared distributed with q degrees of freedom. As large values of W indicate the rejection of H_0, the corresponding p-value is calculated as $1 - F_{\chi_q^2}(W)$ where $F_{\chi_q^2}$ is the cumulative distribution function of a chi-squared random variable with q degrees of freedom.

However, in view of the limited amount of available private equity data (typically the oldest vintages start in the 1980s), asymptotic characterizations of $\mathbf{\Sigma}$ and $\mathrm{SE}(\hat{\theta})$ are of limited importance. In empirical applications, the small sample behavior of an estimation method for private equity data is more relevant than its asymptotic theory. Moreover, the standard asymptotic distribution associated with an estimator is generally not valid for post-model-selection inference, i.e., if a model selection procedure is applied to find the best model from a collection of competitors (Leeb and Pötscher, 2005).

2.4 Comparison to other approaches

The estimator in equation 4 exhibits a cross-sectional nature, since it sums over all funds rather than constructing vintage year based time-series. This means, we intentionally opt against a framework compatible with classical, time-series GMM that requires the construction of stationary, ergodic time-series of moment conditions (Hansen, 1982, 2012). These time-series (of random functions) are used to empirically estimate the expected value of pricing errors in equation 2. The stationarity requirement would prevent us from (i) using funds from non-liquidated vintages for model estimation and (ii) weighting vintages by cumulative fund sizes or invested capital. Additionally, our fundwise approach automatically down-weights non-fully-invested funds and preserves the unbalanced data structure of empirical single fund data. As empirically the number of funds increases with time, more recent vintages are overweighted in our naive procedure with $w = 1$.

In contrast to our approach, the closely related method of Driessen et al. (2012) forms vintage year portfolios instead of using individual fund moments. Further, they discount all fund cash flows just to the first cash flow date (like in a classical net present value calculation) and we additionally sum over all dates within W_i to alleviate the exploding alpha issue mentioned in Driessen et al. (2012). Although Driessen et al. (2012) describe their estimator as one-step GMM approach, it can be regarded very closely related to our least-mean-distance estimator. Especially, the asymptotic assumptions from subsection 2.2 shall be likewise apply to their setting.

Similar to our ansatz, Korteweg and Nagel (2016) draw on a spatial framework to handle cross-sectional dependence between funds (Conley, 1999). However, they utilize a classical GMM estimator, thus a time-series law of large numbers. To obtain their Generalized Public Market Equivalent (GPME) metric they opt for pricing public market cash flows that shall replicate PE funds instead of directly pricing the observed fund cash flows.

Due to the fundwise nature of our approach it is simple and stable but slow.

3 Empirical estimation

3.1 Data

We use the Preqin cashflow data set as of 26th February 2020. We pool all regions and analyze the following fund types (using the Preqin asset class classification): PE ("Private Equity"; 2474 distinct funds in data set), VC ("Venture Capital"; 985), RE ("Real Estate"; 810), PD ("Private Debt"; 488), INF ("Infrastructure", 174), NATRES ("Natural Resources", 167). For these fund types we extract all unweighted cash flow series, which corresponds to fund-size weighting. For non-liquidated funds we treat the latest net asset value as final cash flow.

The public market factors that enter our SDF draw on the US data set of the recently popularized q^5 investment factor model sourced from `http://global-q.org/factors.html` (Hou et al., 2015, 2020). Their five factor model includes: MKT the market excess return, ME the size factor, IA the investment factor, ROE the return on equity factor, and EG the expected growth factor.

3.2 Model selection by lasso and blockwise cross-validation

To select the most predictive public factors for a given private equity fund SDF model, we perform model selection by the lasso (least absolute shrinkage and selection operator) and estimate the regularization hyper-parameter by blockwise cross-validation. Lasso estimators can be used for variable selection due to their desirable property to produce sparse models by shrinking irrelevant variable coefficients to exactly zero. For a recent asset pricing application of lasso regularization see Feng et al. (2020), and for a brief retrospective of the general lasso approach refer to Tibshirani (2011).

Specifically, our lasso-regularized version of the estimator from equation 4 is given by

$$\hat{\theta}_{\text{lasso}} = \arg\,\min_{\theta \in \Theta} \; \frac{1}{n} \left(\sum_{i=1}^{n} w_i^2 \epsilon_i^2 \right)^{\frac{1}{2}} + \lambda \sum_{j \in J} |\theta_j| \tag{5}$$

where the MKT coefficient is not subject to any regularization, $J = \{\text{ME}, \text{IA}, \text{ROE}, \text{EG}\}$. By not penalizing the MKT coefficient, we preferably explain private equity returns by the market return factor.

The lasso parameter λ is determined by hv-block cross validation to account for the dependency in our data set introduced by overlapping fund cash flows for funds from adjacent vintage years (Racine, 2000). Therefore, we form three partitions for several vintage year groups. As larger validation sets are preferred for model selection, the validation set (v-block) always contains funds of three neighboring vintage years (e.g. 2000, 2001, 2002). To reduce the dependency between training and validation set, we remove all funds from three-year-adjacent vintage years, i.e., the h-block (e.g. 1997, 1998, 1999, 2003, 2004, 2005). Funds from the remaining vintage years enter the training set and are thus used for model estimation (e.g. 1985-1996, 2006-2019). We apply ten-fold cross validation using the ten validation sets described in table 3.2. Here we test in steps of 0.02 all λ values between 0 and 0.1; the optimal λ generally depends on the scaling and variability of fund cash flows.

training.before	h.block.before	validation	h.block.after	training.after
start-1984	1985,1986,1987	1988,1989,1990	1991,1992,1993	1994-end
start-1987	1988,1989,1990	1991,1992,1993	1994,1995,1996	1997-end
start-1990	1991,1992,1993	1994,1995,1996	1997,1998,1999	2000-end
start-1993	1994,1995,1996	1997,1998,1999	2000,2001,2002	2003-end
start-1996	1997,1998,1999	2000,2001,2002	2003,2004,2005	2006-end
start-1999	2000,2001,2002	2003,2004,2005	2006,2007,2008	2009-end
start-2002	2003,2004,2005	2006,2007,2008	2009,2010,2011	2012-end
start-2005	2006,2007,2008	2009,2010,2011	2012,2013,2014	2015-end
start-2008	2009,2010,2011	2012,2013,2014	2015,2016,2017	2018-end
start-2011	2012,2013,2014	2015,2016,2017	2018,2019,2020	2021-end

Table 1: Partitions used for *hv*-block cross validation.

3.3 Post-model selection

Post-model selection in combination with a lasso estimator (Belloni and Chernozhukov, 2013). Common pitfalls associated with post-model selection are discussed in Leeb and Pötscher (2005), i.e., basically that a valid parameter covariance matrix cannot be estimated in the post-model selection step alone without accounting for the model selection step. Tibshirani et al. (2018) propose a selective pivotal inference framework for sequential regression procedures that does not rely on a simple regression setup in combination with normality.

3.4 Results

We apply the simple linear SDF

$$\Psi_{t,\tau}(\theta) = \prod_{h=\tau}^{t} \left(1 + r_f + \sum_{j \in J} \theta_{j,h} \cdot F_{j,h}\right)^{-1} \tag{6}$$

with risk-free return r_f and zero-net-investment portfolio returns F_j.

Our general approach is to estimate an ensemble of SDF models by varying the set of relevant fund lifetimes W_i and the maximum vintage year of funds that enter estimation. We use the variation of estimated parameters within the ensemble to gauge their significance. Specifically, we regard relevant fund lifetimes between 40 and 60 quarters, and maximum vintage years between year 2010 and 2015. To avoid overfitting our SDF ensemble just considers two-factor models that contain $\{\text{MKT}\}$ and $\{\text{ME or IA or ROE or EG}\}$.

$\mathcal{T}_i$ includes quarterly τ horizons smaller than $\{40, 60\}$ quarters.

4 Public market factor style analysis

Assume we want to assess the performance of a given (liquidated) fund, and we estimated a total of N competing SDF models for that given fund type and region $\{\Psi^{(i)}, i = 1, 2, \ldots, N\}$. After we compute the set of (possible) net present values $\{\text{NPV}^{(i)}, i = 1, 2, \ldots, N\}$, we can select the subset of n SDFs that exhibit the smallest squared net present value errors

$\{j : (\mathrm{NPV}^{(j)})^2 < \alpha_n\}$. In the next step, we linearly combine the collection of n underlying factor models to obtain one factor model that describes the investment style of that given fund in terms of public market factors (cf. model averaging vs. combination, bucket of models, bake-off contest).

5 Conclusion

$$
\begin{aligned}
\hat{\Delta} &= k_{i,i}\left[\frac{1}{n}\sum_{i=1}^{n}\nabla_\theta L\left(\epsilon_i\right)\left(\nabla_\theta L\left(\epsilon_i\right)\right)^T\right]+ \\
&+\sum_{j=1}^{n-1}k_{i,j}\{\frac{1}{n}\sum_{i=1}^{n-j}[\nabla_\theta L\left(\epsilon_i\right)\left(\nabla_\theta L\left(\epsilon_{i+j}\right)\right)^T+ \\
&+\nabla_\theta L\left(\epsilon_{i+j}\right)\left(\nabla_\theta L\left(\epsilon_i\right)\right)^T]\}
\end{aligned}
\tag{7}
$$

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Type	MKT.mean	MKT.sd	Factor	Second.mean	Second.sd
INF	2.00	0.73	MKT		
INF	1.27	1.14	ME	1.03	0.74
INF	2.01	0.65	IA	-0.55	1.26
INF	1.36	0.68	ROE	4.57	2.07
INF	1.52	0.91	EG	3.78	0.78
NATRES	2.03	0.42	MKT		
NATRES	1.01	0.80	ME	1.80	0.63
NATRES	1.83	0.49	IA	1.79	0.87
NATRES	1.61	0.52	ROE	2.49	1.37
NATRES	1.67	0.47	EG	2.26	0.40
PD	1.50	0.46	MKT		
PD	0.50	0.29	ME	1.67	0.15
PD	1.15	0.33	IA	3.13	0.52
PD	1.16	0.53	ROE	2.05	0.36
PD	1.13	0.40	EG	3.22	0.48
PE	2.79	0.20	MKT		
PE	1.72	0.37	ME	1.57	0.24
PE	2.42	0.24	IA	2.71	0.40
PE	2.03	0.25	ROE	2.29	0.44
PE	2.14	0.19	EG	2.84	0.26
RE	1.39	0.79	MKT		
RE	0.38	0.64	ME	1.52	0.67
RE	1.18	0.92	IA	1.85	0.62
RE	0.77	0.43	ROE	2.94	1.03
RE	0.87	0.67	EG	3.48	0.47
VC	2.81	0.58	MKT		
VC	3.39	0.76	ME	-1.30	1.23
VC	2.69	0.57	IA	-1.08	1.34
VC	2.86	0.79	ROE	-0.48	1.61
VC	2.54	0.84	EG	1.44	1.58

Table 2: Cross validation with equal-weighting and max quarter 40.

Type	MKT.mean	MKT.sd	Factor	Second.mean	Second.sd
INF	1.95	0.76	MKT		
INF	1.18	1.20	ME	1.08	0.76
INF	1.90	0.75	IA	-0.21	1.24
INF	1.38	0.69	ROE	4.50	2.20
INF	1.56	1.00	EG	3.67	0.88
NATRES	1.70	0.37	MKT		
NATRES	0.90	0.83	ME	1.31	0.56
NATRES	1.50	0.46	IA	1.82	0.91
NATRES	1.36	0.44	ROE	2.05	1.36
NATRES	1.38	0.40	EG	2.15	0.39
PD	1.24	0.58	MKT		
PD	0.32	0.34	ME	1.49	0.19
PD	0.91	0.45	IA	3.05	0.43
PD	0.94	0.60	ROE	1.72	0.32
PD	0.92	0.50	EG	2.82	0.45
PE	2.52	0.25	MKT		
PE	1.61	0.32	ME	1.29	0.24
PE	2.17	0.21	IA	2.52	0.40
PE	1.87	0.29	ROE	1.86	0.34
PE	1.96	0.24	EG	2.51	0.24
RE	1.27	0.79	MKT		
RE	0.37	0.58	ME	1.33	0.59
RE	1.07	0.89	IA	1.70	0.60
RE	0.68	0.45	ROE	2.70	0.96
RE	0.78	0.66	EG	3.36	0.46
VC	2.29	0.69	MKT		
VC	3.10	0.79	ME	-1.70	1.24
VC	2.11	0.67	IA	-1.95	2.12
VC	2.56	0.82	ROE	-1.27	1.55
VC	2.14	0.90	EG	-0.15	1.98

Table 3: Cross validation with equal-weighting and max quarter 60.

Type	MKT	SE.MKT	Factor	Coef	SE.Coef	Wald.p.value.MKT_1
PE	2.76	1.33	MKT			0.02
PE	1.63	3.02	ME	1.48	3.17	0.00
PE	2.34	1.08	IA	2.64	7.28	0.00
PE	2.08	2.25	ROE	2.22	3.57	0.00
PE	2.16	0.08	EG	2.83	0.05	0.99
VC	2.87	1.45	MKT			0.01
VC	3.71	5.72	ME	-1.78	6.43	0.00
VC	2.79	1.40	IA	-1.66	4.14	0.00
VC	3.09	0.58	ROE	-1.09	0.43	0.27
VC	2.82	1.38	EG	0.96	5.53	0.00
PD	1.60	5.22	MKT			0.00
PD	0.46	14.43	ME	1.68	5.94	0.00
PD	1.17	6.95	IA	3.18	10.60	0.00
PD	1.26	4.71	ROE	1.98	8.58	0.00
PD	1.20	7.89	EG	3.24	3.20	0.00
RE	1.19	1.37	MKT			0.80
RE	0.13	7.29	ME	1.35	4.78	0.00
RE	0.95	2.32	IA	1.41	7.35	0.00
RE	0.76	3.13	ROE	2.75	3.57	0.00
RE	0.69	3.49	EG	3.69	6.54	0.00
NATRES	1.90	1.73	MKT			0.12
NATRES	0.84	1.49	ME	1.68	1.79	0.01
NATRES	1.64	1.52	IA	1.91	16.03	0.00
NATRES	1.56	1.15	ROE	2.18	4.29	0.00
NATRES	1.52	1.64	EG	2.28	18.23	0.00
INF	1.72	2.46	MKT			0.08
INF	0.75	11.62	ME	1.27	3.52	0.00
INF	1.73	3.79	IA	-0.02	11.57	0.01
INF	1.08	8.99	ROE	4.65	23.92	0.00
INF	1.09	7.84	EG	3.97	4.13	0.00

Table 4: Asymptotic inference with equal-weighting and max quarter 40.

Type	MKT	SE.MKT	Factor	Coef	SE.Coef	Wald.p.value.MKT_1
PE	2.52	1.81	MKT			0.01
PE	1.59	3.43	ME	1.23	4.68	0.00
PE	2.15	2.34	IA	2.45	7.87	0.00
PE	1.94	3.56	ROE	1.81	5.17	0.00
PE	1.99	3.37	EG	2.48	6.69	0.00
VC	2.33	1.00	MKT			0.18
VC	3.48	5.71	ME	-2.18	6.32	0.00
VC	2.20	1.05	IA	-2.82	4.64	0.00
VC	2.77	1.55	ROE	-1.92	3.49	0.00
VC	2.40	1.00	EG	-0.89	5.65	0.00
PD	1.38	10.19	MKT			0.00
PD	0.36	0.15	ME	1.51	0.15	0.95
PD	0.98	13.26	IA	3.18	13.83	0.00
PD	1.07	6.03	ROE	1.68	10.56	0.00
PD	1.02	9.89	EG	2.86	5.77	0.00
RE	1.11	3.07	MKT			0.74
RE	0.16	0.49	ME	1.19	0.60	0.53
RE	0.89	2.52	IA	1.31	7.36	0.00
RE	0.67	3.27	ROE	2.50	5.06	0.00
RE	0.62	4.35	EG	3.57	7.32	0.00
NATRES	1.61	1.23	MKT			0.45
NATRES	0.88	5.09	ME	1.17	7.58	0.00
NATRES	1.36	1.28	IA	1.96	15.52	0.00
NATRES	1.34	1.36	ROE	1.74	6.94	0.00
NATRES	1.27	1.54	EG	2.18	17.55	0.00
INF	1.65	2.45	MKT			0.11
INF	0.65	11.79	ME	1.32	3.48	0.00
INF	1.62	2.09	IA	0.21	5.14	0.18
INF	1.07	8.98	ROE	4.67	23.98	0.00
INF	1.09	7.85	EG	3.91	4.20	0.00

Table 5: Asymptotic inference with equal-weighting and max quarter 60.