A fundwise stochastic discount factor estimator for private equity funds

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Declaration of interest

The author reports no conflict of interest. The author alone is responsible for the content and writing of the paper.

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Abstract

This paper proposes a simple stochastic discount factor estimation methodology suited for private equity fund-level cash flow data. The asymptotic inference framework for our least-mean-distance estimator draws on the notion of a measurable economic distance between distinct private equity funds.

1 Introduction

Do investments in private equity funds offer abnormal returns to fund investors when risk-adjusted to public market factors? Currently, a popular approach to answer this question is to evaluate private equity fund cash flows by Stochastic Discount Factor (SDF) models that draw on public market return covariates. The basic idea for SDF model estimation is that the sum of all discounted fund net cash flows is zero when the true SDF is applied. Unfortunately, there is no conclusion about the best methodology to estimate these SDF models, as a variety of proposal coexists in the academic private equity fund literature (Driessen et al., 2012; Korteweg and Nagel, 2016; Ang et al., 2018; Gredil et al., 2019).

Especially our conclusions from the Driessen et al. (2012) and Korteweg and Nagel (2016) approaches lead us to suggest a new least-mean-distance estimator for SDF models that is applicable to private equity fund-level cash flow data. On the one hand, we provide asymptotic inference formulations that rely on the concept of spatial (near-epoch) dependency between funds comparable to Korteweg and Nagel (2016). In this context, it is paramount to quantify the economic distance between funds by a measure like absolute vintage year difference or cash flow overlap[1]. On the other hand, our least-mean-distance estimator can be regarded as generalization of the Driessen et al. (2012) methodology, where we provide the asymptotic inference framework that was missing in the original paper. In contrast to Driessen et al. (2012), we explicitly do not require the pooling of private equity fund cash flows to form vintage year portfolios. The increased number of cross-sectional units (in a fundwise approach) in turn decreases the asymptotic variance of coefficient estimates compared to the case when just a small number of vintage year portfolios is available.

In the empirical application of our new estimator, we estimate an exponentially affine SDF model that can draw on the five return factors associated with the q^5 investment factor model recently proposed by Hou et al. (2020). We calculate asymptotic standard errors and a Wald test statistic for the model coefficients. Moreover, we assess the small-sample variance of coefficient estimates and the out-of-sample performance of the different SDF models by hv-block cross validation which accounts for the inter-vintage-year dependence of private equity funds (Racine, 2000).

The paper is structured as follows. Section 2 introduces our fundwise least-mean-distance estimator and its corresponding asymptotic inference framework. Section 3 applies the method to estimate SDFs for various private equity fund types by drawing on the public market factors of the q^5 investment factor model of Hou et al. (2020). Section 4 concludes.

[1]However, this economic inter-fund distance refers NOT to the term least-mean-distance estimator.

2 Methodology

2.1 Fundwise least-mean-distance estimator

Let fund $i = 1, 2, \ldots, n$ be characterized by its (net) cash flows $CF_{t,i}$ and its net asset values $NAV_{t,i}$ with discrete time index $t = 1, 2, \ldots, T$. The data generating processes for CF and NAV are left unspecified. The stochastic discount factor $\Psi_{t,\tau}$ can be used to calculate the time-τ present value $P_{t,\tau,i}$ of a time-t cash flow of any given PE fund i

$$P_{t,\tau,i} = \Psi_{t,\tau} \cdot CF_{t,i} \tag{1}$$

As SDFs are commonly parameterized by a vector $\theta \in \mathbb{R}^p$, i.e., $\Psi_{t,\tau} \equiv \Psi_{t,\tau}(\theta)$, our goal is to find an estimation method for the optimal θ. For each fund i and all points τ within a common fund lifetime, the pricing error $\epsilon_{\tau,i}$ of all fund cash flows is calculated as

$$\epsilon_{\tau,i} = \sum_{t=1}^{T} P_{t,\tau,i} \qquad \forall \quad \tau, i \tag{2}$$

We define the w-weighted τ-average fund pricing error as

$$\bar{\epsilon}_i = w_i \cdot \frac{1}{\text{card}(\mathcal{T}_i)} \sum_{\tau \in \mathcal{T}_i} \epsilon_{\tau,i} \qquad \forall \quad i \tag{3}$$

where $\mathcal{T}_i$ gives the set of relevant present value times τ for fund i, which can be thought of as all quarterly/yearly dates within the usual fund lifetime of ten to fifteen years. Each fund i is characterized by its vintage year which can be expressed by $v_i = \min(\mathcal{T}_i) \in 1, 2, \ldots, V$, where V denotes the maximum vintage year used in a given data set. Here we implicitly assume that $\mathcal{T}_i$ always contains at least the fund's starting date. Finally, the scalar weighting factor w_i can be (i) one divided by the fund's invested capital for equal weighting of funds, (ii) one divided by the vintage year sum of invested capital for vintage year weighting, (iii) the scalar one for fund-size weighting, or (iv) some macroeconomic deflator.

Our least-mean-distance estimator minimizes the average loss of $\bar{\epsilon}$

$$\hat{\theta} = \arg\min_{\theta \in \Theta} \; S_n(\theta) \quad \text{with} \quad S_n(\theta) = \frac{1}{n} \sum_{i=1}^{n} L\left(\bar{\epsilon}_i\right) \tag{4}$$

where L denotes a loss function, e.g., $L(x) = (x - 0)^2$. Throughout the paper, the weighted average fund pricing error $\bar{\epsilon} \equiv \bar{\epsilon}(\theta)$ is regarded as nonlinear random function of the SDF parameter θ.

2.2 Asymptotic framework

2.2.1 Vintage year asymptotics

We employ a spatial framework, where we assume that the (spatial, i.e., economic) distance between cross-sectional units, i.e., private equity funds, can be measured in quantitative way. Here asymptotic results are derived for the case when the number of funds goes to infinity

$n \to \infty$. However, to expose our SDF to enough distinct covariate realizations (economic conditions), identification of model parameters requires a sufficient number of funds from different vintage years in the fund-level data set used for model estimation (Driessen et al., 2012; Korteweg and Nagel, 2016).

Assumption 1. *(i) The number of vintage years $V \to \infty$ as $n \to \infty$. (ii) The number of funds per vintage year is bounded by some positive constant. (iii) The maximal fund lifetime is also bounded by a positive constant. (iv) The economic distance between fund i and j is measured by the vintage year difference $d_{i,j} = v_i - v_j$.*

2.2.2 Law of large numbers

The global moment condition underlying our estimation approach is that the (i-unconditional) expected value of $\bar{\epsilon}$ shall be zero, if we use the optimal SDF parameter θ_0. This also means, instead of applying a time-series law of large numbers, we rely on a spatial (cross-sectional) law of large numbers, but acknowledge the statistical dependence of pricing errors with respect to vintage year differences between funds. Jenish and Prucha (2012) develop an asymptotic inference framework for near-epoch dependent spatial processes that is instructive for our setting.

Assumption 2. *The (i) time-trend and (ii) dependence structure of $\bar{\epsilon}$ shall allow*

$$n^{-1} \sum_{i=1}^{n} \bar{\epsilon}_i \overset{a.s.}{\to} E[\bar{\epsilon}] \quad as \quad V, n \to \infty$$

Specifically, the process $\bar{\epsilon}$ is spatial near-epoch dependent with respect to fund vintage years (Jenish and Prucha, 2012), i.e., two funds with distance $d_{i,j} > D$ are assumed to be independent.

To satisfy the time trend part (i) of this law of large number assumption, the weighting factor w, introduced in equation 3, can be used to make $\bar{\epsilon}$ stationary. Spatial near-epoch dependence with respect to fund vintage years formalizes the idea that two funds with a small absolute vintage year difference are supposed to be dependent (due to being exposed to the same macroeconomic condition), whereas two funds with a very large absolute vintage year difference can be assumed to be independent. In our framework, spatial distance is considered as economic distance between funds (represented by $\bar{\epsilon}$); our spatial space is thus of dimension one.

2.2.3 Consistency

The estimator $\hat{\theta}$ shall converge in probability to the true parameter value θ_0 as the number of distinct vintage years in our data set goes to infinity. Multiple funds for a specific vintage year are not necessarily required and are thus considered as additional, stabilizing moment conditions.

Assumption 3. *Consistency of $\hat{\theta}$ requires $\hat{\theta} \overset{p}{\to} \theta_0$ as $V, n \to \infty$. Thus $E[\bar{\epsilon}] = 0$ if and only if $\theta = \theta_0$. The parameter space is compact $\theta \in \Theta$.*

Compactness of Θ can be assured by lower and upper bounds for all parameters that can be justified by economic reasoning in our case, e.g., a market beta factor of ten seems implausible for PE funds because of the implied risk and return expectations.

2.2.4 Central limit theorem

To assess the significance of our parameter estimates, we want to describe the asymptotic distribution of the parameter vector as a normal distribution.

Assumption 4. *$\sqrt{n}(\hat{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}(0, \mathbf{\Sigma})$ as $V, n \to \infty$ with covariance matrix $\mathbf{\Sigma}$.*

2.3 Asymptotic inference

In the general (time-series) near-epoch-dependent least-mean-distance literature, $\mathbf{\Sigma}$ can be characterized according to Pötscher and Prucha (1997, Theorem 11.2.b, Theorem H.1):

$$\mathbf{\Sigma} = C^{-1}\Delta(C^{-1})^\top$$

with expected hessian matrix converging to C as $V, n \to \infty$

$$E\left(\nabla_{\theta\theta}S_n\right) \to C$$

and the expected covariance matrix of gradients converging to Δ as $V, n \to \infty$

$$nE\left[\nabla_\theta S_n(\nabla_\theta S_n)^\top\right] \to \Delta$$

Here, the gradient vector $\nabla_\theta S_n$ is denoted as column vector. We define the corresponding finite sample estimators analogously to Pötscher and Prucha (1997, Chapters 12, 13.1) and numerically approximate the first and second derivatives by finite differences. $\hat{C}$ is relatively straightforward

$$\hat{C} = \frac{1}{n}\sum_{i=1}^{n} \nabla_{\theta\theta}L\left(\epsilon_i\right)$$

Due to the (spatial near-epoch) dependence, the intricate part is to consistently estimate $\hat{\Delta}$ by a Spatial Heteroskedasticity and Autocorrelation Consistent (SHAC) covariance matrix estimator (Kim and Sun, 2011, equation 2)

$$\hat{\Delta} = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} k_{i,j}\left[\nabla_\theta L\left(\epsilon_i\right)\left(\nabla_\theta L\left(\epsilon_j\right)\right)^\top\right]$$

We define the kernel weight k as

$$k_{i,j} \equiv K\left(\frac{d_{i,j}}{b_n}\right)$$

with kernel function $K : \mathbb{R} \to [0, 1]$ satisfies $K(0) = 1$, $K(x) = K(-x)$, $\int_{-\infty}^{\infty} K^2(x)dx < \infty$, and $K(\cdot)$ continuous at zero and at all but a finite number of other points. A common choice is the Bartlett kernel $K_{BT}(x) = \max(0, 1 - |x|)$; see equation 2.7 in Andrews (1991) for other popular kernel choices. This means absolute vintage year differences larger than the bandwidth (or truncation) parameter $b_n = D$ are considered independent and are thus excluded from the $\hat{\Delta}$ estimation formula.

In large samples, the parameter standard error vector can thus be estimated by

$$\mathrm{SE}(\hat{\theta}) = \sqrt{\mathrm{diag}\left[n^{-\frac{1}{2}}\hat{C}^{-1}\hat{\Delta}(\hat{C}^{-1})^{\top}(n^{-\frac{1}{2}})^{\top}\right]} = \sqrt{\mathrm{diag}\left[\frac{\hat{C}^{-1}\hat{\Delta}(\hat{C}^{-1})^{\top}}{n}\right]}$$

The Wald test statistic for linear hypotheses $H_0 : R\theta = r$ and $H_1 : R\theta \neq r$ is constructed as

$$W = (R\hat{\theta} - r)^{\top}\left[R\frac{\hat{C}^{-1}\hat{\Delta}(\hat{C}^{-1})^{\top}}{n}R^{\top}\right]^{-1}(R\hat{\theta} - r) \overset{H_0}{\sim} \chi^2_q$$

where $\hat{\theta}$ is the $p \times 1$ parameter vector, R is a $q \times p$ matrix, and r is a $q \times 1$ vector. Usually, we select R as $p \times p$ identity matrix, and r as $p \times 1$ vector (e.g., of zeros). Under the null hypothesis, W is chi-squared distributed with q degrees of freedom. As large values of W indicate the rejection of H_0, the corresponding p-value is calculated as $1 - F_{\chi^2_q}(W)$ where $F_{\chi^2_q}$ is the cumulative distribution function of a chi-squared random variable with q degrees of freedom.

However, in view of the limited amount of available private equity data (typically the oldest vintages start in the 1980s), asymptotic characterizations of $\mathbf{\Sigma}$ and $\mathrm{SE}(\hat{\theta})$ are of limited importance. In empirical applications, the small sample behavior of an estimation method for private equity data is more relevant than its asymptotic theory. Moreover, the standard asymptotic distribution associated with an estimator is generally not valid for post-model-selection inference, i.e., if a model selection procedure is applied to find the best model from a collection of competitors (Leeb and Pötscher, 2005).

2.4 Comparison to other approaches

The estimator in equation 4 exhibits a cross-sectional nature, since it sums over all funds rather than constructing vintage year based time-series. This means, we intentionally opt against a framework compatible with classical, time-series GMM that requires the construction of stationary, ergodic time-series of moment conditions (Hansen, 1982, 2012). These time-series (of random functions) are used to empirically estimate the expected value of pricing errors in equation 2. The stationarity requirement of classical time-series GMM restrict us with respect to (i) more elaborate weighting-schemes for w, like fund-size weighting, and (ii) the usage of fund cash flows from non-realized vintages.

In contrast to our approach, the closely related method of Driessen et al. (2012) forms vintage year portfolios instead of using individual fund moments. Further, they discount all fund cash flows just to the first cash flow date (like in a classical net present value calculation) and we additionally average over all dates within W_i to alleviate the exploding alpha issue mentioned in Driessen et al. (2012). Although Driessen et al. (2012) describe their estimator as one-step GMM approach, it can be regarded very closely related to our least-mean-distance estimator. Especially, the asymptotic assumptions from subsection 2.2

shall be likewise apply to their setting. Due to the fundwise nature of our approach it is more stable but slower than the vintage year pooling conducted in Driessen et al. (2012). This stability results in smaller asymptotic standard errors for the coefficients.

Similar to our ansatz, Korteweg and Nagel (2016) draw on a spatial framework to handle cross-sectional dependence between funds (Conley, 1999). However, they ultimately utilize a classical GMM estimator, thus a time-series law of large numbers. To obtain their Generalized Public Market Equivalent (GPME) metric they opt for pricing public market cash flows that shall replicate PE funds instead of directly pricing the observed fund cash flows. Time-series GMM estimators always bear the risk of under-identification, if the corresponding time series are constructed by pooling all fund cash flows from a given fund type, since you end up with just one moment condition per fund type.

3 Empirical estimation

3.1 Data

We use the Preqin cash flow data set as of 26th February 2020. We pool all regions and analyze the following fund types (using the Preqin asset class classification): PE ("Private Equity"; 2474 distinct funds in data set), VC ("Venture Capital"; 985), RE ("Real Estate"; 810), PD ("Private Debt"; 488), INF ("Infrastructure", 174), NATRES ("Natural Resources", 167). For these fund types we extract all equal-weighted cash flow series. For non-liquidated funds we treat the latest net asset value as final cash flow. We explicitly refrain from excluding the most recent vintage years.

The public market factors that enter our SDF draw on the US data set of the recently popularized q^5 investment factor model sourced from `http://global-q.org/factors.html` (Hou et al., 2015, 2020). Their five factor model includes: MKT the market excess return, ME the size factor, IA the investment factor, ROE the return on equity factor, and EG the expected growth factor.

3.2 Model and estimator specifications

We use the exponential affine SDF analogously to Korteweg and Nagel (2016)

$$\Psi_{t,\tau}(\theta) = \exp\left[-\sum_{h=\tau}^{t}\left(1 + r_f + \sum_{j\in J}\theta_{j,h}\cdot F_{j,h}\right)\right] \tag{5}$$

with risk-free return r_f and zero-net-investment portfolio returns F_j. To avoid overfitting, we just test five simple SDF models that contain {MKT} alone or {MKT} plus {ME or IA or ROE or EG}. We test two different sets for $\mathcal{T}_i$: they include all quarterly τ horizons smaller than $\{40, 60\}$ quarters, respectively. In equation 4, we use the quadratic loss function $L(x) = x^2$.

To assess the parameter significance, we compute the asymptotic standard errors as outlined in subsection 2.3. Moreover, we perform a Wald test for the null hypothesis of a MKT coefficient of one and zeros for the other coefficients (ME, IA, ROE, EG). The

Bartlett kernel's bandwidth $b_n = D$ is selected as 12 years, i.e., funds with absolute vintage year differences larger than 12 years are assumed to be independent.

Additionally, we want to test the finite - or more honestly small - sample parameter significance and the out-of-sample performance of our SDF models. Here we rely on hv-block cross validation to account for the dependency in our data set introduced by overlapping fund cash flows for funds from adjacent vintage years (Racine, 2000). Therefore, we form three partitions for several vintage year groups. As larger validation sets are preferred for model selection, the validation set (v-block) always contains funds of three neighboring vintage years (e.g. 2000, 2001, 2002). To reduce the dependency between training and validation set, we remove all funds from three-year-adjacent vintage years, i.e., the h-block (e.g. 1997, 1998, 1999, 2003, 2004, 2005). Funds from the remaining vintage years enter the training set and are thus used for model estimation (e.g. 1985-1996, 2006-2019). We apply ten-fold cross validation using the ten validation sets described in table 3.2.

training.before	h.block.before	validation	h.block.after	training.after
start-1984	1985,1986,1987	1988,1989,1990	1991,1992,1993	1994-end
start-1987	1988,1989,1990	1991,1992,1993	1994,1995,1996	1997-end
start-1990	1991,1992,1993	1994,1995,1996	1997,1998,1999	2000-end
start-1993	1994,1995,1996	1997,1998,1999	2000,2001,2002	2003-end
start-1996	1997,1998,1999	2000,2001,2002	2003,2004,2005	2006-end
start-1999	2000,2001,2002	2003,2004,2005	2006,2007,2008	2009-end
start-2002	2003,2004,2005	2006,2007,2008	2009,2010,2011	2012-end
start-2005	2006,2007,2008	2009,2010,2011	2012,2013,2014	2015-end
start-2008	2009,2010,2011	2012,2013,2014	2015,2016,2017	2018-end
start-2011	2012,2013,2014	2015,2016,2017	2018,2019,2020	2021-end

Table 1: Partitions used for hv-block cross validation.

3.3 Results

4 Conclusion

Equipped with a case specific (economic) distance measure, our least-mean-distance estimator can be easily generalized to estimate SDF models for all kinds of non-traded cash flows.

$$\begin{aligned} \hat{\Delta} &= k_{i,i}\left[\frac{1}{n}\sum_{i=1}^{n}\nabla_\theta L\left(\epsilon_i\right)\left(\nabla_\theta L\left(\epsilon_i\right)\right)^T\right] + \\ &+ \sum_{j=1}^{n-1} k_{i,j}\{\frac{1}{n}\sum_{i=1}^{n-j}[\nabla_\theta L\left(\epsilon_i\right)\left(\nabla_\theta L\left(\epsilon_{i+j}\right)\right)^T + \\ &+ \nabla_\theta L\left(\epsilon_{i+j}\right)\left(\nabla_\theta L\left(\epsilon_i\right)\right)^T]\} \end{aligned} \tag{6}$$

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Type	MKT	SE.MKT	Factor	Coef	SE.Coef	Wald.p.value.MKT_1
PE	2.76	1.33	MKT			0.02
PE	1.63	3.02	ME	1.49	3.17	0.00
PE	2.34	1.08	IA	2.64	7.27	0.00
PE	2.08	2.25	ROE	2.22	3.57	0.00
PE	2.16	2.07	EG	2.83	6.61	0.00
VC	2.87	1.45	MKT			0.01
VC	3.71	5.72	ME	-1.78	6.43	0.00
VC	2.79	1.40	IA	-1.65	4.14	0.00
VC	3.09	1.69	ROE	-1.09	3.57	0.00
VC	2.82	1.38	EG	0.95	5.50	0.00
PD	1.60	5.22	MKT			0.00
PD	0.46	14.43	ME	1.68	5.94	0.00
PD	1.17	6.95	IA	3.18	10.60	0.00
PD	1.26	4.71	ROE	1.98	8.58	0.00
PD	1.20	7.89	EG	3.24	3.20	0.00
RE	1.19	2.87	MKT			0.59
RE	0.13	7.29	ME	1.35	4.78	0.00
RE	0.95	2.32	IA	1.42	7.35	0.00
RE	0.76	3.13	ROE	2.75	3.57	0.00
RE	0.69	3.49	EG	3.69	6.54	0.00
NATRES	1.90	1.73	MKT			0.12
NATRES	0.84	1.49	ME	1.68	1.79	0.01
NATRES	1.64	1.52	IA	1.91	16.04	0.00
NATRES	1.56	1.15	ROE	2.18	4.29	0.00
NATRES	1.52	1.64	EG	2.28	18.23	0.00
INF	1.72	2.46	MKT			0.08
INF	0.75	11.62	ME	1.27	3.52	0.00
INF	1.73	3.79	IA	-0.02	11.57	0.01
INF	1.08	8.99	ROE	4.65	23.92	0.00
INF	1.09	7.84	EG	3.97	4.13	0.00

Table 2: Asymptotic inference with max quarter 40.

Type	MKT	SE.MKT	Factor	Coef	SE.Coef	Wald.p.value.MKT_1
PE	2.52	1.81	MKT			0.01
PE	1.59	3.43	ME	1.23	4.68	0.00
PE	2.15	2.34	IA	2.45	7.87	0.00
PE	1.94	3.56	ROE	1.81	5.17	0.00
PE	1.99	3.37	EG	2.48	6.69	0.00
VC	2.33	1.00	MKT			0.19
VC	3.48	5.71	ME	-2.18	6.32	0.00
VC	2.20	1.05	IA	-2.82	4.64	0.00
VC	2.77	1.55	ROE	-1.92	3.49	0.00
VC	2.40	1.00	EG	-0.89	5.65	0.00
PD	1.38	10.19	MKT			0.00
PD	0.36	28.10	ME	1.51	14.36	0.00
PD	0.98	13.30	IA	3.18	13.83	0.00
PD	1.07	6.03	ROE	1.68	10.56	0.00
PD	1.02	9.89	EG	2.86	5.77	0.00
RE	1.11	3.07	MKT			0.74
RE	0.16	$0.02^{!!!}$	ME	1.19	$0.03^{!!!}$	$1.00^{!!!}$
RE	0.89	2.52	IA	1.31	7.36	0.00
RE	0.67	3.27	ROE	2.50	5.06	0.00
RE	0.62	4.35	EG	3.57	7.32	0.00
NATRES	1.61	1.23	MKT			0.45
NATRES	0.88	5.09	ME	1.17	7.58	0.00
NATRES	1.36	1.28	IA	1.96	15.53	0.00
NATRES	1.34	1.36	ROE	1.74	6.94	0.00
NATRES	1.27	1.54	EG	2.18	17.57	0.00
INF	1.65	2.44	MKT			0.11
INF	0.65	11.78	ME	1.32	3.48	0.00
INF	1.62	2.09	IA	0.21	5.14	0.18
INF	1.07	8.98	ROE	4.67	23.97	0.00
INF	1.09	7.85	EG	3.91	4.20	0.00

Table 3: Asymptotic inference with max quarter 60. Numerical issues within the asymptotic covariance matrix estimation seem to cause the dubious values with superscript $x^{!!!}$; the corresponding estimates for columns MKT and Coef are not affected and stay valid.

Type	MKT.mean	MKT.sd	Factor	Second.mean	Second.sd	validation.error
INF	2.00	0.73	MKT			3186*
INF	1.27	1.14	ME	1.03	0.74	3333
INF	2.01	0.65	IA	-0.55	1.26	3221
INF	1.36	0.68	ROE	4.57	2.07	3043
INF	1.52	0.91	EG	3.78	0.78	4307
NATRES	2.03	0.42	MKT			7340
NATRES	1.01	0.80	ME	1.80	0.63	9146
NATRES	1.83	0.49	IA	1.79	0.87	7670
NATRES	1.61	0.52	ROE	2.49	1.37	8183
NATRES	1.67	0.47	EG	2.26	0.40	6933*
PD	1.50	0.46	MKT			3098
PD	0.50	0.29	ME	1.67	0.15	2998
PD	1.15	0.33	IA	3.13	0.52	2896
PD	1.16	0.53	ROE	2.05	0.36	2756*
PD	1.13	0.40	EG	3.22	0.48	2906
PE	2.79	0.20	MKT			1108
PE	1.72	0.37	ME	1.57	0.24	1053
PE	2.42	0.24	IA	2.71	0.40	1013*
PE	2.03	0.25	ROE	2.29	0.44	1297
PE	2.14	0.19	EG	2.84	0.26	1039
RE	1.39	0.79	MKT			1893
RE	0.38	0.64	ME	1.52	0.67	1723
RE	1.18	0.92	IA	1.85	0.62	1767
RE	0.77	0.43	ROE	2.94	1.03	1963
RE	0.87	0.67	EG	3.48	0.47	1221*
VC	2.81	0.58	MKT			926*
VC	3.39	0.76	ME	-1.30	1.23	1110
VC	2.69	0.57	IA	-1.08	1.34	961
VC	2.86	0.79	ROE	-0.48	1.61	1126
VC	2.54	0.84	EG	1.44	1.58	984

Table 4: Cross validation with max quarter 40.

Type	MKT.mean	MKT.sd	Factor	Second.mean	Second.sd	validation.error
INF	1.95	0.76	MKT			3234*
INF	1.18	1.20	ME	1.08	0.76	3400
INF	1.90	0.75	IA	-0.21	1.24	3265
INF	1.38	0.69	ROE	4.50	2.20	3494
INF	1.56	1.00	EG	3.67	0.88	4872
NATRES	1.70	0.37	MKT			8055
NATRES	0.90	0.83	ME	1.31	0.56	10577
NATRES	1.50	0.46	IA	1.82	0.91	8414
NATRES	1.36	0.44	ROE	2.05	1.36	9469
NATRES	1.38	0.40	EG	2.15	0.39	7782*
PD	1.24	0.58	MKT			3334
PD	0.32	0.34	ME	1.49	0.19	3403
PD	0.91	0.45	IA	3.05	0.43	3276
PD	0.94	0.60	ROE	1.72	0.32	3173*
PD	0.92	0.50	EG	2.82	0.45	3458
PE	2.52	0.25	MKT			1199
PE	1.61	0.32	ME	1.29	0.24	1217
PE	2.17	0.21	IA	2.52	0.40	1132*
PE	1.87	0.29	ROE	1.86	0.34	1380
PE	1.96	0.24	EG	2.51	0.24	1155
RE	1.27	0.79	MKT			1993
RE	0.37	0.58	ME	1.33	0.59	1871
RE	1.07	0.89	IA	1.70	0.60	1879
RE	0.68	0.45	ROE	2.70	0.96	1953
RE	0.78	0.66	EG	3.36	0.46	1262*
VC	2.29	0.69	MKT			1017*
VC	3.10	0.79	ME	-1.70	1.24	1206
VC	2.11	0.67	IA	-1.95	2.12	1101
VC	2.56	0.82	ROE	-1.27	1.55	1275
VC	2.14	0.90	EG	-0.15	1.98	1165

Table 5: Cross validation with max quarter 60.