

# Semiparametric SDF Estimators for Pooled, Non-Traded Cash Flows

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## Guiding Question

A black and white photograph of a man with short hair and a mustache, wearing clear-rimmed glasses and a dark leather jacket over a chain necklace. He is looking directly at the camera with a neutral expression. In the background, there is a bonsai tree on the left, a person sitting at a table with food in the middle ground, and another person's back to the camera on the right. The overall lighting is dramatic, with strong highlights and shadows.

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Source: <https://traderepublic.com/en-de/private-markets>

# Guiding Question

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Return



## Go beyond public markets

Invest in **hidden champions** that are not listed on the stock exchange with ELTIFs.

**Balance your portfolio.** Private markets are not connected to public market volatility.

**Your return** depends on the selected investments of the fund managers.

**Past performance is not a reliable indicator** for future results. Investments may result in partial or total loss.

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# Guiding Question

*Are private markets really not connected to public market volatility?*

## **How can we estimate the true public-market exposure of private equity funds?**

- Target: recover latent market exposure from non-traded PE cash flow/NAV data.
- Challenge: stale NAVs, asynchronous cash flows, and overlapping fund lives.
- Plan: naive approach first, then semiparametric SDF estimation.

## **Useful for:**

- Risk-adjusted benchmarks for private equity funds.
- Holistic risk management for public and private portfolios.
- More realistic strategic asset allocation decisions.

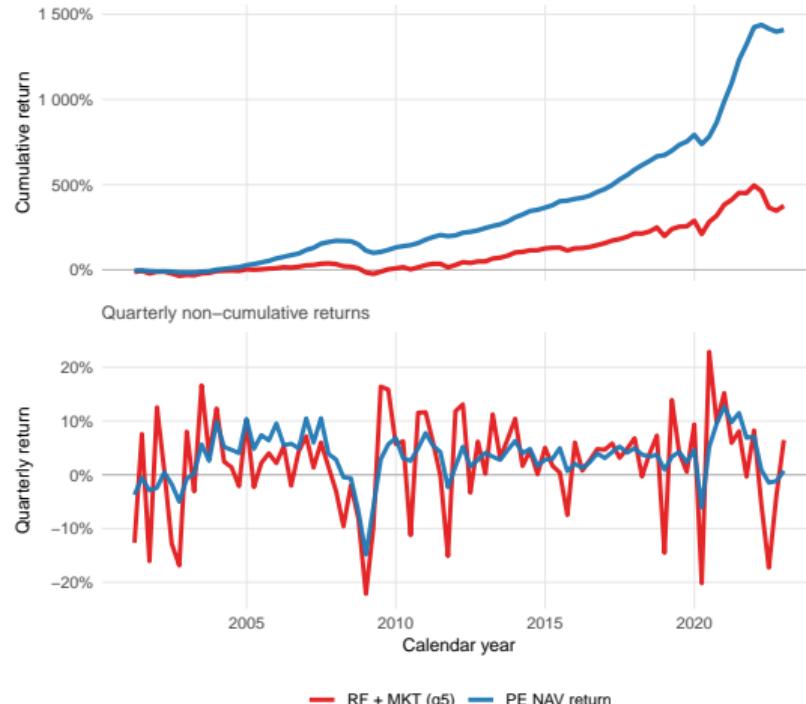
# Agenda

- ① Motivation: NAV-return autocorrelation
- ② Estimation framework
- ③ Simulation evidence
- ④ Empirical PE results
- ⑤ Takeaways

# Motivation: Public Returns vs Private Cash Flows

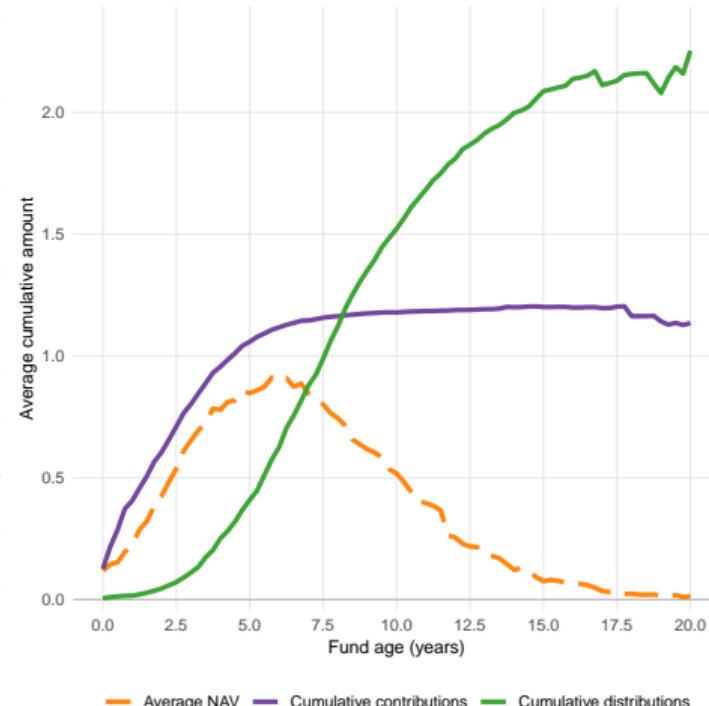
## Public Return View

Cumulative returns: PE NAV index vs RF + MKT

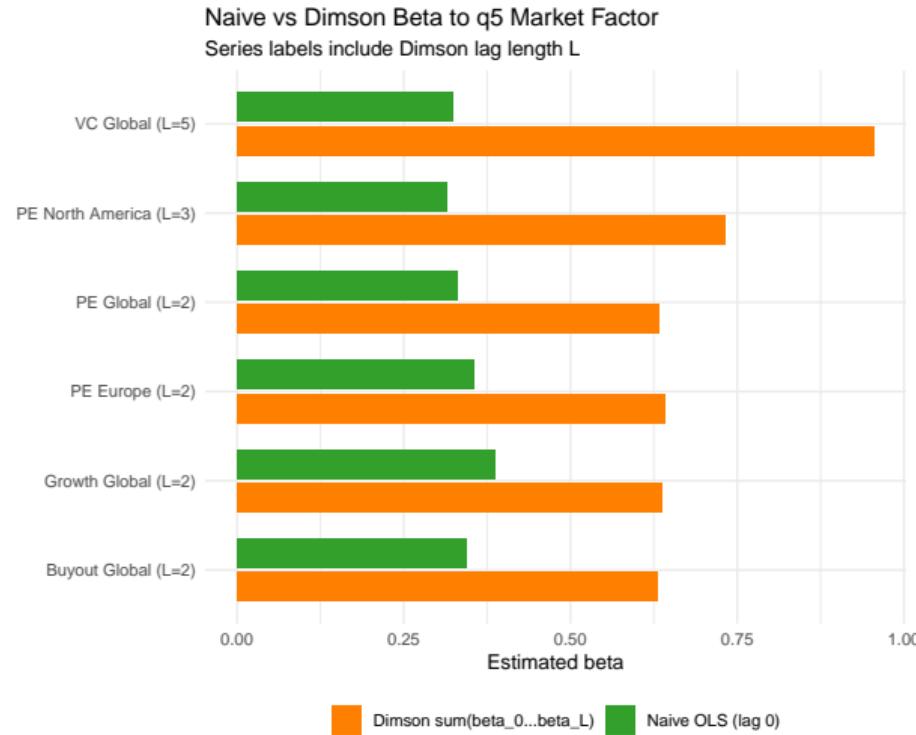
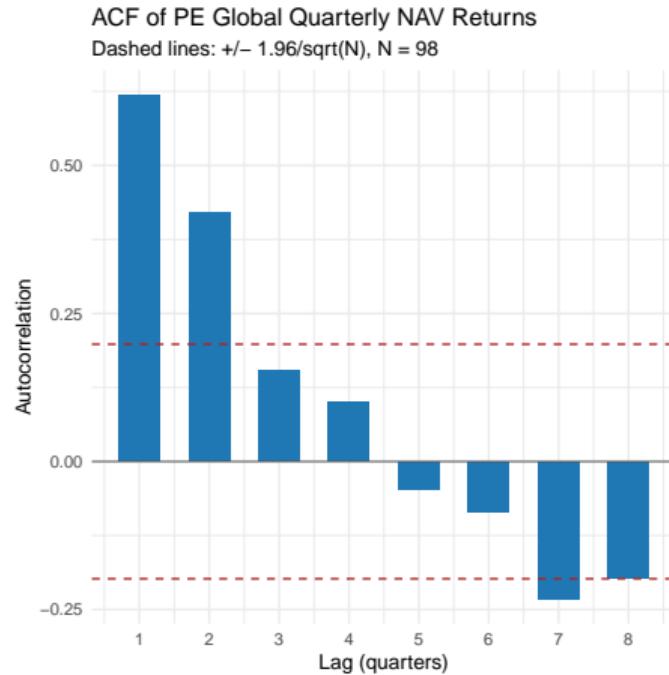


## Private Cash-Flow View

Average cumulative cash flows and NAV over fund age for PE (n=37)



# Motivation: [Dimson, 1979] Regression from NAV Returns



Data: Preqin quarterly index levels and q5 monthly  $R_{MKT}$  aggregated to quarters (overlap through 2022Q4).

# Estimator: Starting Point [Driessen et al., 2012]

For each fund  $i$ , [Driessen et al., 2012] imposes a zero-NPV pricing condition at inception date  $\tau_i^{(0)}$ :

$$\epsilon_i^{\text{DLP}}(\theta) = \sum_t \Psi_{\tau_i^{(0)}, t}(\theta) CF_{i,t}$$

Cross-sectional estimator:

$$\hat{\theta}_{\text{DLP}} = \arg \min_{\theta \in \Theta} \frac{1}{n} \sum_{i=1}^n (\epsilon_i^{\text{DLP}}(\theta))^2$$

## Mapping to our notation

- DLP12 is the special case  $\mathcal{T}_i = \{\tau_i^{(0)}\}$ , so  $|\mathcal{T}_i| = 1$ .
- Then  $\bar{\epsilon}_i(\theta) = \epsilon_i^{\text{DLP}}(\theta)$  and the LMD objective nests DLP12.
- This paper extends DLP12 by allowing multiple discount dates per fund ( $|\mathcal{T}_i| > 1$ ).

Variables:  $i$  fund/portfolio index,  $t$  cash-flow time,  $\tau_i^{(0)}$  inception discount date,  $CF_{i,t}$  net cash flow,  $\Psi_{\tau, t}(\theta)$  SDF ratio,  $\theta$  parameter vector,  $\Theta$  parameter space,  $n$  number of units.

## Estimator: Least Mean Distance (LMD)

For fund  $i$  and discounting date  $\tau$ :

$$\epsilon_{\tau,i}(\theta) = \sum_t \Psi_{\tau,t}(\theta) CF_{i,t}$$

Average over selected discounting dates  $\mathcal{T}_i$ :

$$\bar{\epsilon}_i(\theta) = \frac{1}{|\mathcal{T}_i|} \sum_{\tau \in \mathcal{T}_i} \epsilon_{\tau,i}(\theta)$$

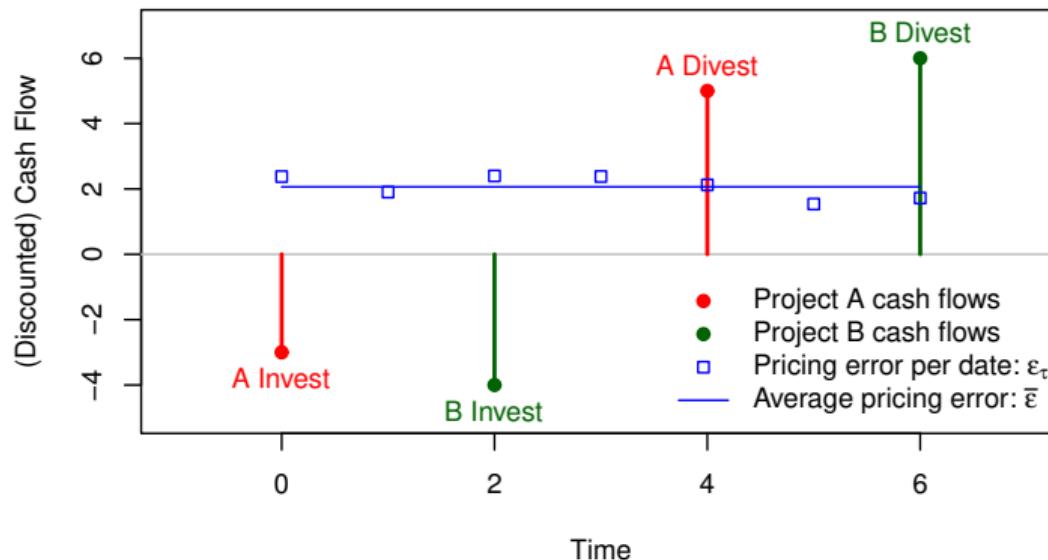
Estimate parameters by nonlinear least-mean-distance:

$$\hat{\theta} = \arg \max_{\theta \in \Theta} \left( -\frac{1}{n} \sum_{i=1}^n L(\bar{\epsilon}_i(\theta)) \right), \quad L(x) = x^2$$

Variables:  $i$  fund/portfolio index,  $t$  cash-flow time,  $\tau$  discounting date,  $CF_{i,t}$  net cash flow,  $\Psi_{\tau,t}(\theta)$  SDF ratio,  $\theta$  parameter vector,  $\mathcal{T}_i$  discount-date set.

# Estimator: Net Present Value (NPV) vs Net Future Value (NFV)

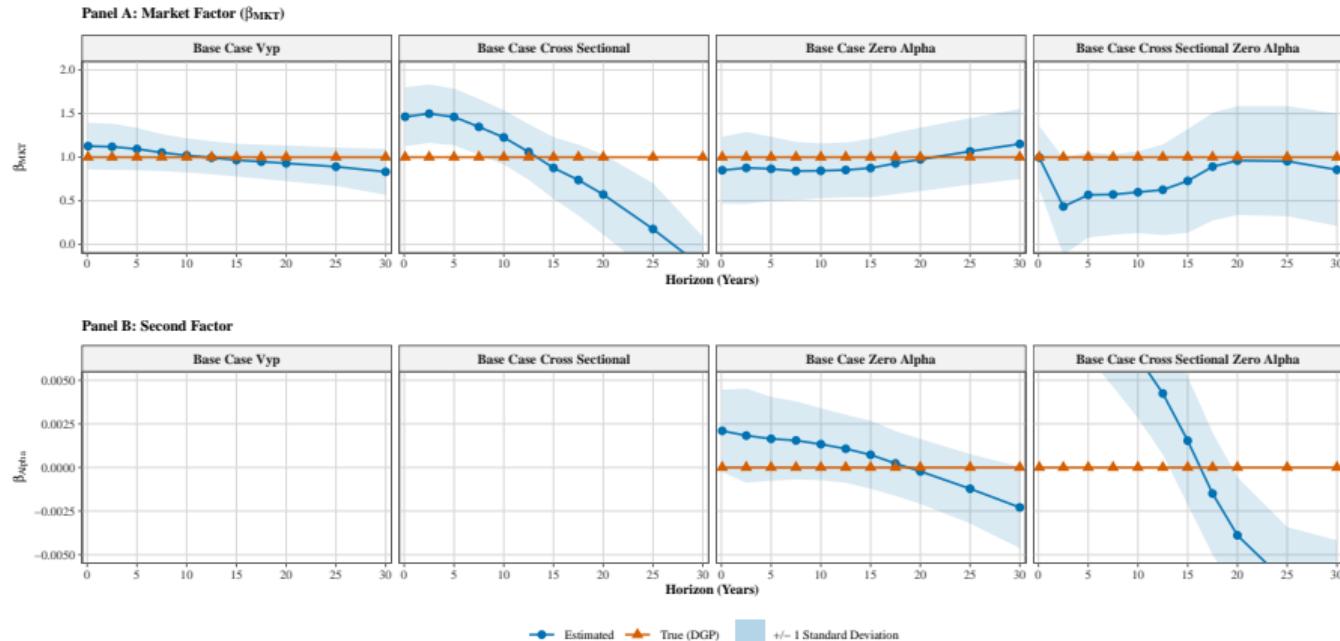
- NPV-only discounting (fund inception) is theoretically unbiased.
- NFV: Adding future-value dates introduces a timing risk **bias** term.
- **But:** Finite-sample performance can improve when averaging across dates.



## Simulation: What Is Tested?

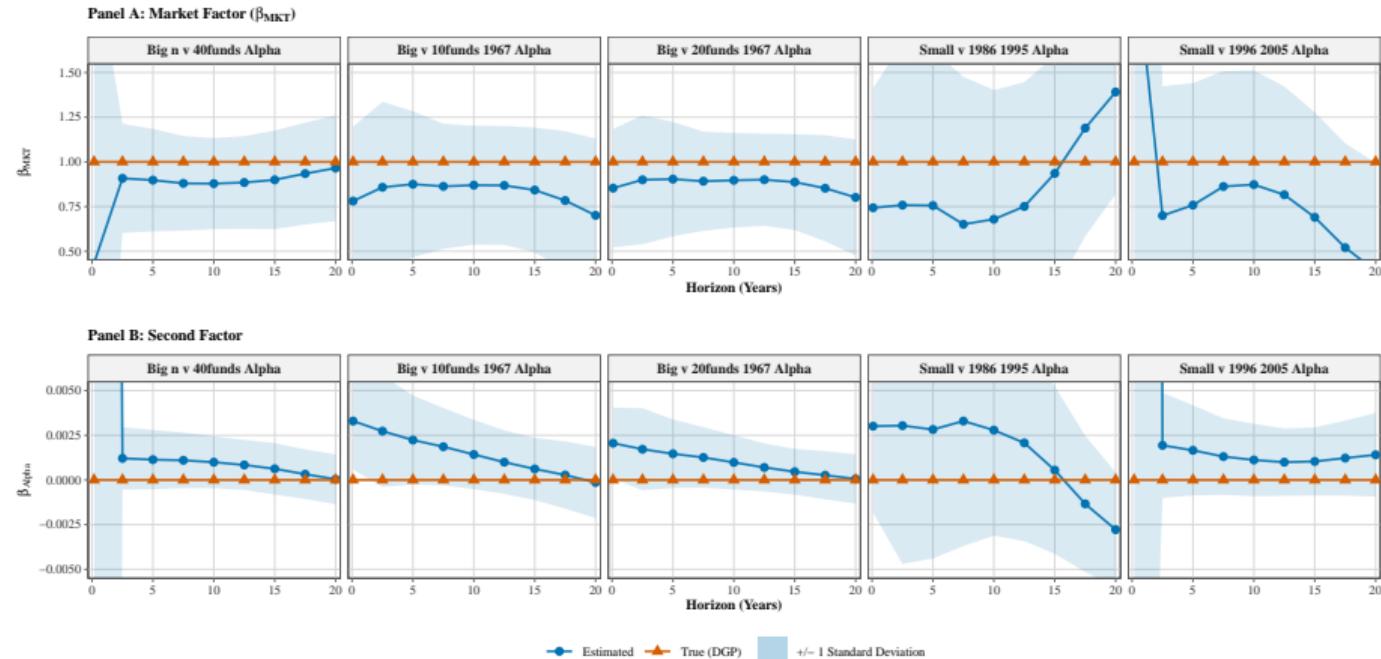
- **Base:** 20 vintages, synthetic fund cash flows generated under known SDF structure.
- **Focus on horizon choice:** size of discounting set  $\mathcal{T}$ .
- **Compare units:** individual funds vs vintage-year portfolios (VYP).
- **Compare sample geometry:** more vintages ( $V$ ) vs larger within-vintage size ( $n/V$ ).
- **Compare model forms:** simple linear vs exponential affine SDF.

# Simulation: 1. Single Funds vs Portfolio Formation



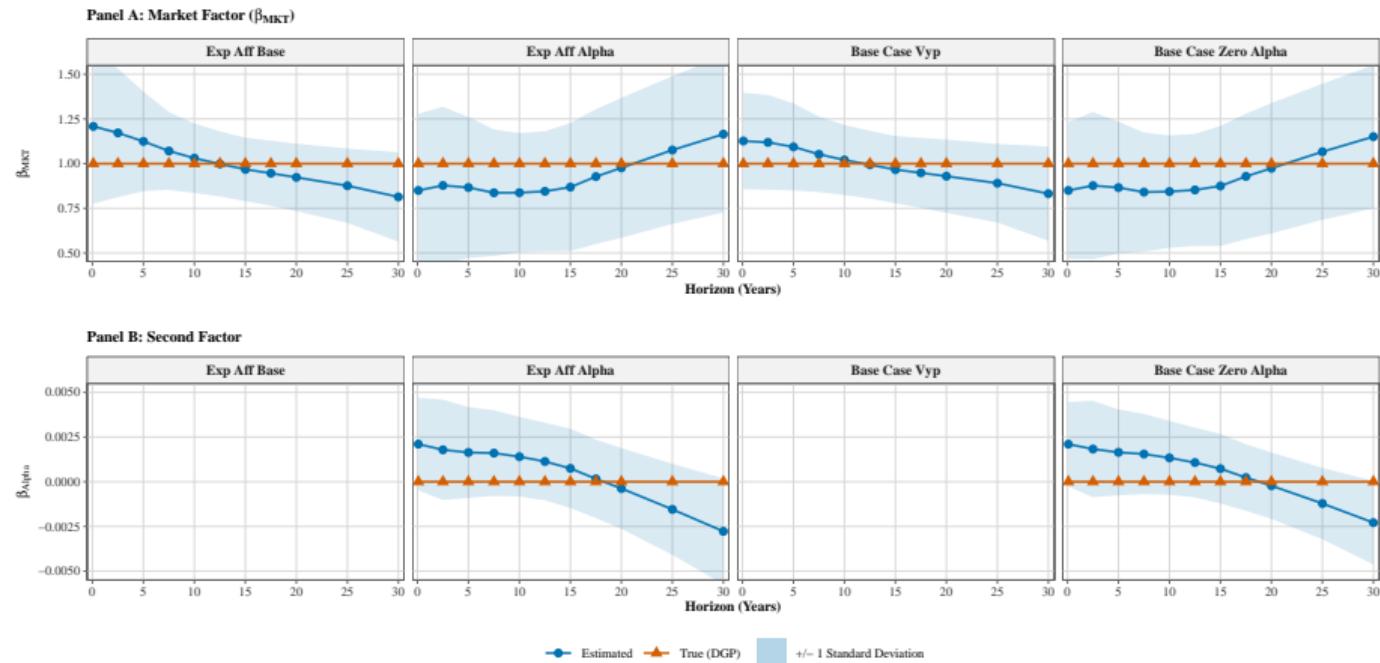
**Takeaway:** vintage-year portfolios materially reduce bias and variance versus single-fund estimation.

# Simulation: 2. More Vintages or More Funds per Vintage?



**Takeaway:** increasing funds per vintage is more powerful for variance reduction than only extending the time span.

# Simulation: 3. Linear vs Exponential Affine SDF



**Takeaway:** no robust finite-sample superiority of exponential affine specification in this setting.  
[Korteweg and Nagel, 2016]

## Simulation: Synthesis

- ① Use portfolio aggregation to stabilize estimation.
- ② Prioritize richer cross-sections per vintage when possible (more funds per moment).
- ③ Prefer parsimonious factor structure in current PE data regime.
- ④ Horizon choice (size of  $T$ ) is a first-order control for finite-sample performance.

**Interpretation:** practical estimator quality is dominated by finite-sample bias-variance tradeoffs.

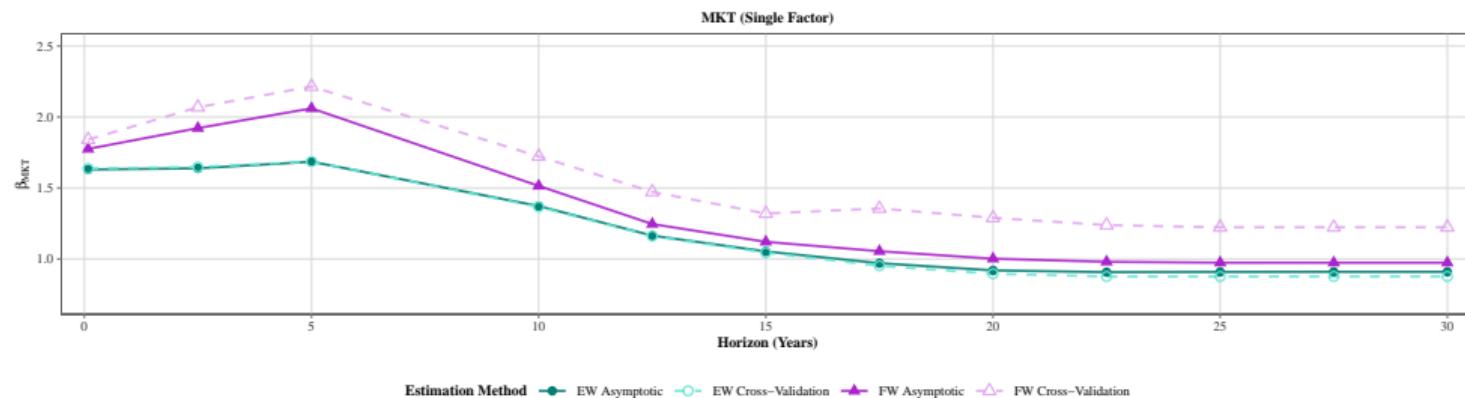
## Empirical: Preqin PE Data & $q^5$ Public Factors

- Data snapshot: 2745 PE funds, vintages 1983–2019.
- Primary unit for estimation: vintage-year portfolios (equal- and value-weighted).
- Factors:  $q^5$  family; focus on MKT and simple two-factor extensions.
- Horizon selected from simulation guidance: 15-year baseline.
- Benchmark for comparison: NAV-based naive/Dimson market-exposure estimates from the motivation section.

**Singe-factor models:** Dimson beta as lower bound.

**Two-factor models:** Apply machine-learning methods to form “stronger learner.”

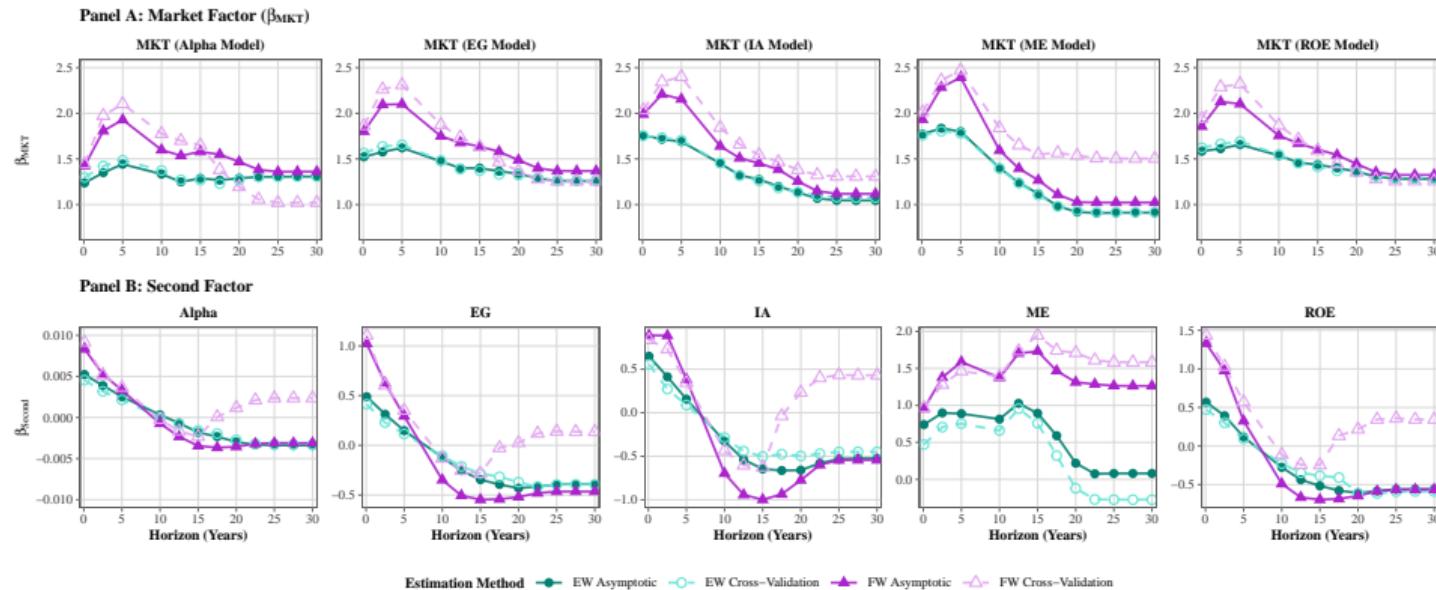
# Empirical: 1. Single-Factor MKT Model



## Reading:

- Short-horizon MKT betas are high and decline with horizon.
- Betas stabilize near 1 at long horizons.
- CV inference is much more stable than asymptotic  $t$ -statistics.
- Relative to naive contemporaneous NAV betas, SDF estimates are materially closer to full market exposure.

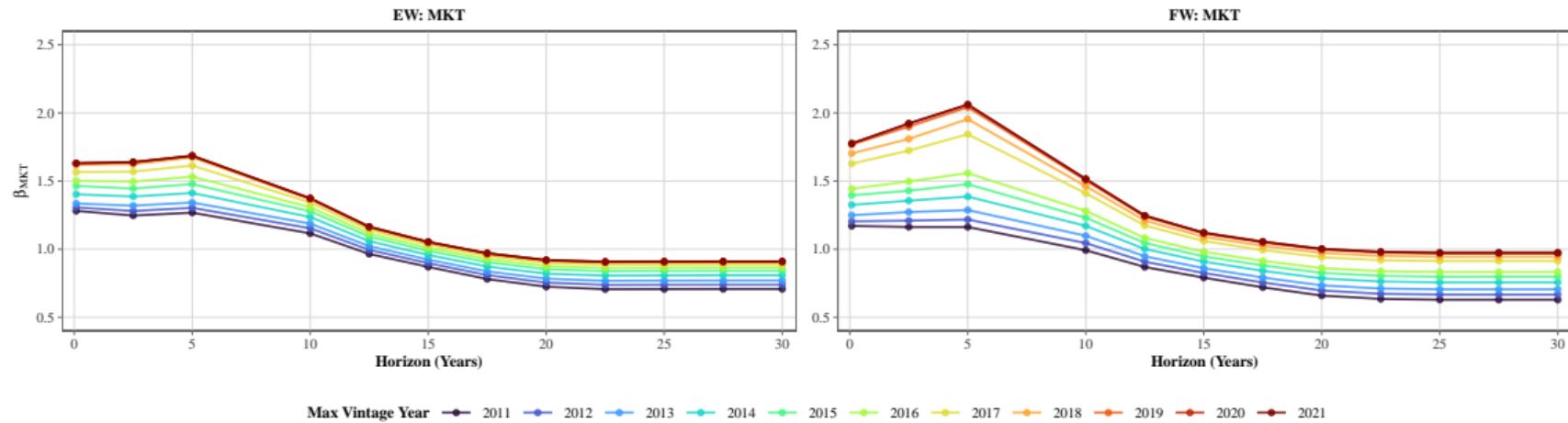
# Empirical: 2. Two-Factor Models



## Reading:

- Second-factor loadings vary strongly with horizon and weighting scheme.
- Most non-MKT factors do not show robust incremental signal.

# Empirical: 3. Vintage Cutoffs for Single-Factor Model

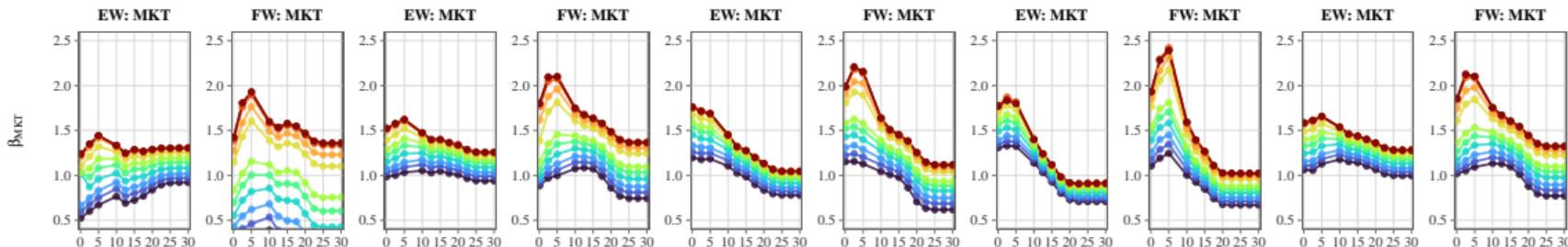


**Reading:** including newer vintages tends to increase estimated market exposure, with noisier asymptotic uncertainty.

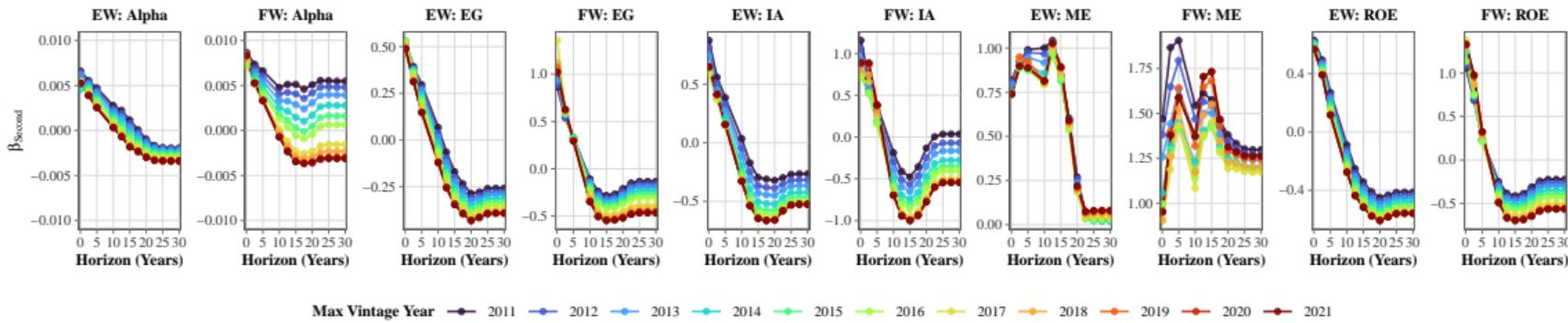
**Dimson beta as lower bound!**

# Empirical: 4. Vintage Cutoffs for Two-Factor Models

Panel A: Market Factor ( $\beta_{MKT}$ )



Panel B: Second Factor



Max Vintage Year    ● 2011    ● 2012    ● 2013    ● 2014    ● 2015    ● 2016    ● 2017    ● 2018    ● 2019    ● 2020    ● 2021

# Conclusion: What This Means for Practice and Research

## For practitioners

- Start with parsimonious SDFs (MKT-first), then add complexity cautiously.
- Treat asymptotic significance alone as insufficient in sparse PE samples.
- Use dependence-aware validation (e.g.,  $hv$ -block CV) as a default diagnostic.

## For researchers

- Finite-sample design choices can dominate asymptotic elegance.
- Data architecture (portfolio formation, horizon design) is part of identification.

## Conclusion: Main Takeaways

- ① A semiparametric LMD framework can price pooled non-traded cash flows directly.
- ② The central empirical issue is finite-sample stability, not asymptotic theory alone.
- ③ Portfolio aggregation and horizon design are key levers for usable inference.
- ④ Current evidence supports single-factor MKT models as the robust baseline for PE.
- ⑤ Naive contemporaneous NAV betas underestimate exposure; lag-aware methods partially recover it.
- ⑥ Outlook: Multi-factor models can be stabilized by machine-learning techniques.

## Questions and Discussion

# Literature

-  Dimson, E. (1979).  
Risk measurement when shares are subject to infrequent trading.  
*Journal of Financial Economics*, 7(2):197–226.
-  Driessen, J., Lin, T.-C., and Phalippou, L. (2012).  
A new method to estimate risk and return of nontraded assets from cash flows: the case of private equity.  
*Journal of Financial and Quantitative Analysis*, 47(3):511–535.
-  Korteweg, A. and Nagel, S. (2016).  
Risk-adjusting the returns to venture capital.  
*Journal of Finance*, 71(3):1437–1470.
-  Tausch, C. and Pietz, M. (2024).  
Machine learning private equity returns.  
*The Journal of Finance and Data Science*, 10:100141.

## Backup: Comparison to DLP12 and KN16

	DLP12	KN16	This paper
Estimator	Cross-sectional NLS	Time-series GMM (public SDF)	Nonlinear LMD
Cash flows priced	PE fund cash flows	Public replicating portfolios	PE fund cash flows
Discount dates	Inception only	Inception only	Flexible via $\mathcal{T}_i$
Asymptotics	Infill	$V \rightarrow \infty$	Increasing domain
Inference	Bootstrap	SHAC	SHAC + CV focus

## Motivation: Why Measuring Risk Is Hard in Private Markets

- PE funds generate **cash flow sequences**, not continuously traded returns.
- Fund lives overlap across vintages, creating dependence beyond standard panel assumptions.
- Fund valuation relies on reported NAVs, which can be stale/smoothed.
- Standard return-based factor models are not directly applicable.

**Implication:** we need a cash-flow-native SDF estimator with robust dependence-aware inference.

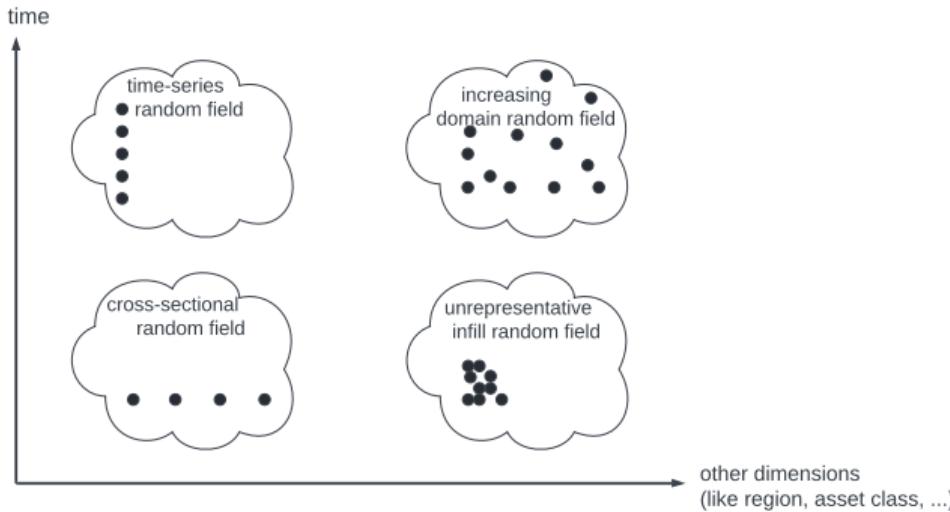
## Backup: Significant ACF Lags and Dimson Lag Choice

Series	Significant lags in 1–8	Consecutive from lag 1	Dimson lag $L$
PE Global	3	2	2
PE North America	5	3	3
PE Europe	3	2	2
Buyout Global	3	2	2
Growth Global	2	2	2
VC Global	5	5	5

**Naive benchmark result:** contemporaneous beta is low ( $\approx 0.32\text{--}0.39$ ), while Dimson beta increases to  $\approx 0.63\text{--}0.96$ .

# Dependence Structure: Random Field View

- Cross-sectional unit: fund or vintage-year portfolio.
- Dependence driven by economic proximity (here: vintage-year distance).
- Asymptotics: increasing domain ( $V \rightarrow \infty$ ), bounded units per vintage.

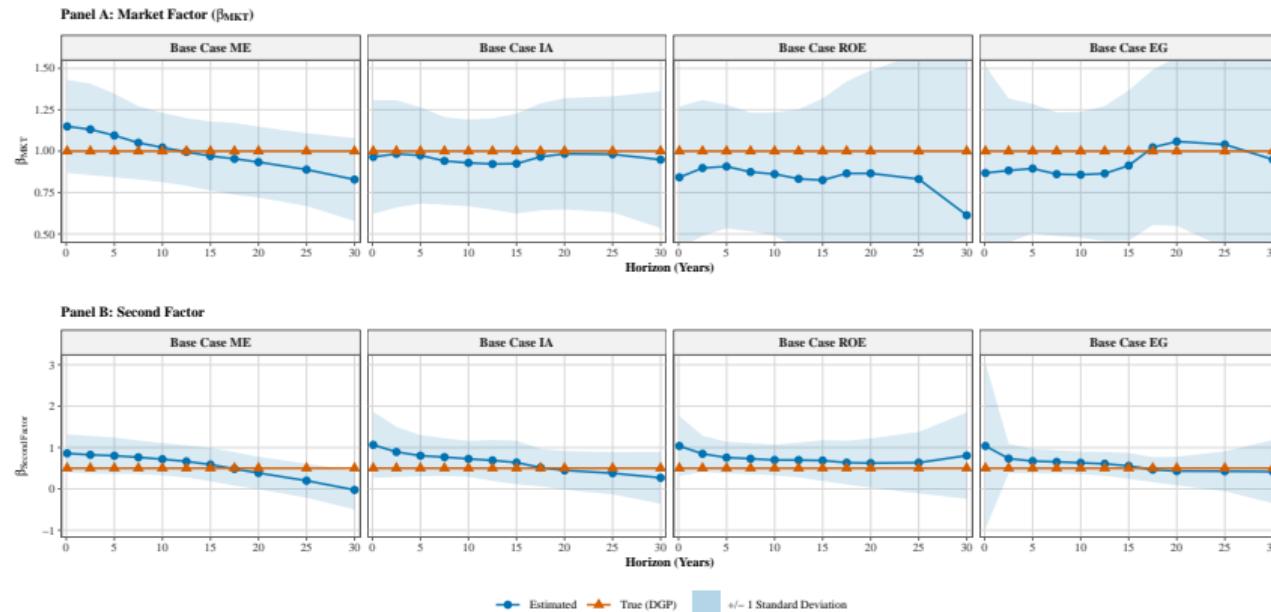


## Estimator: Inference Strategy

- Asymptotic covariance: sandwich form  $\Sigma = H^{-1}\Lambda H^{-1}$ .
- Long-run dependence handled by SHAC (spatial HAC) with vintage-distance kernel.
- Small-sample reliability checked via ***hv-block cross-validation***.

**Reason:** asymptotic approximations are fragile with only 20–40 vintage portfolios.

# Simulation 4: Two-Factor q-Factor Models



**Takeaway:** two-factor estimates are horizon-sensitive and high-variance; multivariate identification is fragile.

## Conclusion: Machine-Learning Ensembles

- Two-factor models are messy.
- Idea: Combine multiple weak learners
- Additionally estimate error term [Tausch and Pietz, 2024]