Semiparametric SDF Estimators for Pooled, Non-Traded Cash Flows

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Guiding Question

How can we estimate the true public-market exposure of private equity funds?

- Target: recover latent market exposure from non-traded PE cash flow/NAV data.
- Challenge: stale NAVs, asynchronous cash flows, and overlapping fund lives.
- Plan: naive benchmark first, then semiparametric SDF estimation.

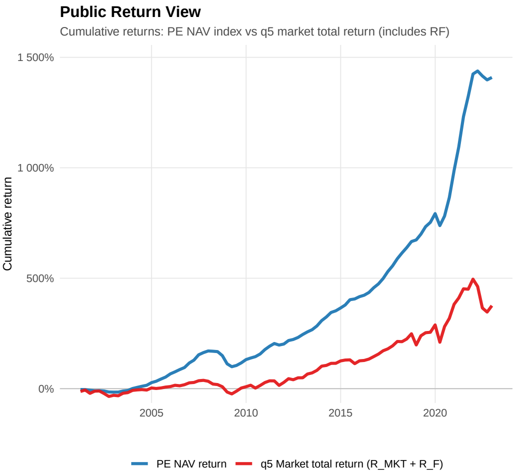
Useful for:

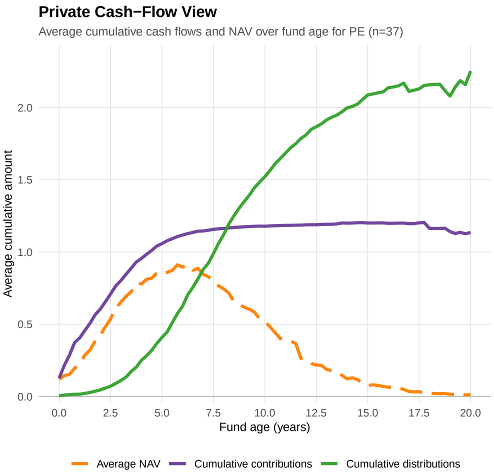
- Risk-adjusted benchmarks for private equity funds.
- Holistic risk management for public and private portfolios.
- More realistic strategic asset allocation decisions.

Agenda

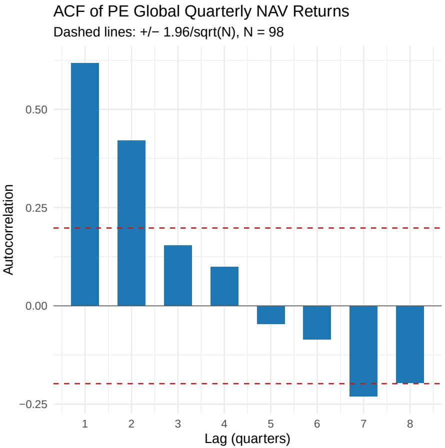
1. Motivation via NAV-return autocorrelation and Dimson beta (6 min)
2. Estimator and inference framework (7 min)
3. Simulation evidence on bias-variance tradeoff (9 min)
4. Empirical PE results and practical implications (6 min)
5. Takeaways and comparison to naive benchmark (2 min)

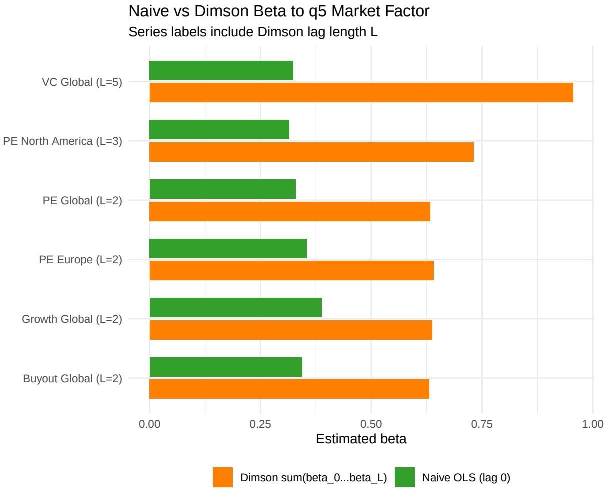
Motivation: Public Returns vs Private Cash Flows





Motivation: [Dimson, 1979] Regression from NAV Returns





Data: Preqin quarterly index levels and q5 monthly R_{MKT} aggregated to quarters (overlap through 2022Q4).

Estimator: Starting Point [Driessen et al., 2012]

For each fund i, [Driessen et al., 2012] imposes a zero-NPV pricing condition at inception date $\tau_i^{(0)}$:

$$\epsilon_i^{\text{DLP}}(\theta) = \sum_t \Psi_{\tau_i^{(0)},t}(\theta)\, CF_{i,t}$$

Cross-sectional estimator:

$$\hat{\theta}_{\text{DLP}} = \arg\min_{\theta\in\Theta} \frac{1}{n}\sum_{i=1}^{n} \left(\epsilon_i^{\text{DLP}}(\theta)\right)^2$$

Mapping to our notation

- DLP12 is the special case $\mathcal{T}_i = \{\tau_i^{(0)}\}$, so $|\mathcal{T}_i| = 1$.
- Then $\bar{\epsilon}_i(\theta) = \epsilon_i^{\text{DLP}}(\theta)$ and the LMD objective nests DLP12.
- This paper extends DLP12 by allowing multiple discount dates per fund ($|\mathcal{T}_i| > 1$).

Estimator: Least Mean Distance (LMD)

For fund i and discounting date τ:

$$\epsilon_{\tau,i}(\theta) = \sum_t \Psi_{\tau,t}(\theta)\, CF_{i,t}$$

Average over selected discounting dates $\mathcal{T}_i$:

$$\bar{\epsilon}_i(\theta) = \frac{1}{|\mathcal{T}_i|} \sum_{\tau \in \mathcal{T}_i} \epsilon_{\tau,i}(\theta)$$

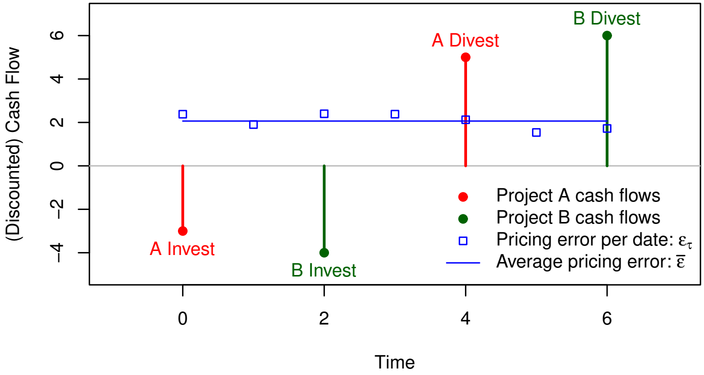
Estimate parameters by nonlinear least-mean-distance:

$$\hat{\theta} = \arg\max_{\theta \in \Theta} \left(-\frac{1}{n} \sum_{i=1}^{n} L(\bar{\epsilon}_i(\theta)) \right), \quad L(x) = x^2$$

Variables: i fund/portfolio index, t cash-flow time, τ discounting date, $CF_{i,t}$ net cash flow, $\Psi_{\tau,t}(\theta)$ SDF ratio, θ parameter vector, $\mathcal{T}_i$ discount-date set.

Estimator: Net Present Value (NPV) vs Net Future Value (NFV)

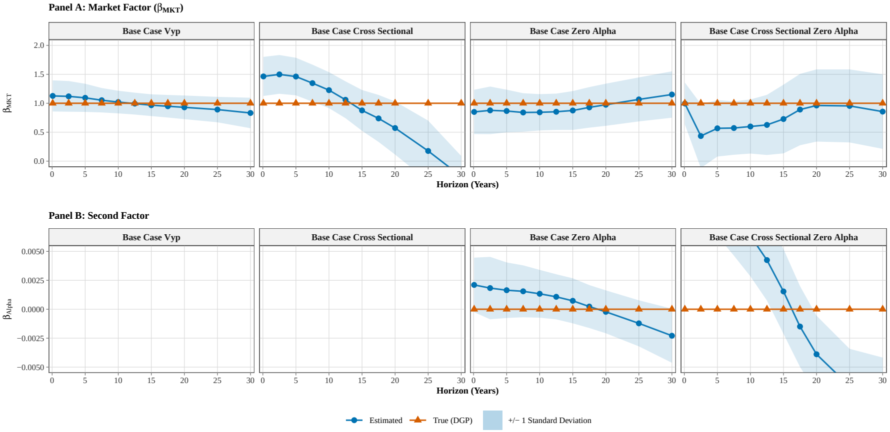
- NPV-only discounting (fund inception) is theoretically unbiased.
- NFV: Adding future-value dates introduces a timing risk **bias** term.
- **But:** Finite-sample performance can improve when averaging across dates.



Simulation: What Is Tested?

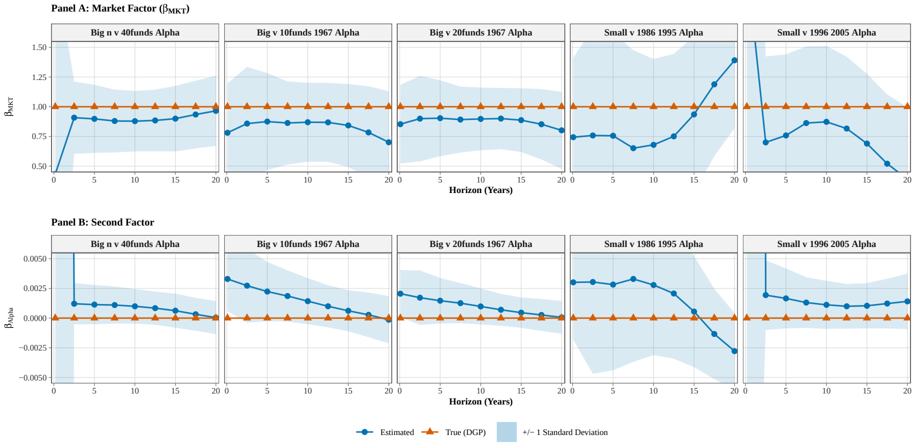
- **Base:** 20 vintages, synthetic fund cash flows generated under known SDF structure.
- **Focus on horizon choice:** size of discounting set $\mathcal{T}$.
- **Compare units:** individual funds vs vintage-year portfolios (VYP).
- **Compare sample geometry:** more vintages (V) vs larger within-vintage size (n/V).
- **Compare model forms:** simple linear vs exponential affine SDF.

Simulation: 1. Single Funds vs Portfolio Formation



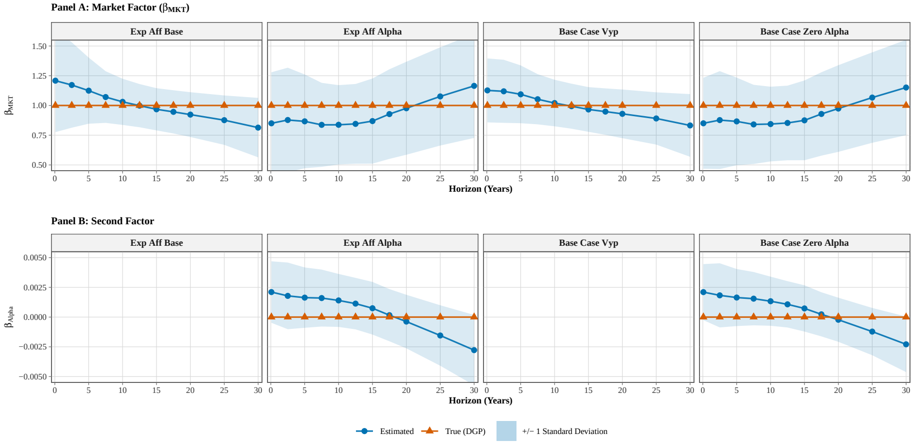
Takeaway: vintage-year portfolios materially reduce bias and variance versus single-fund estimation.

Simulation: 2. More Vintages or More Funds per Vintage?



Takeaway: increasing funds per vintage is more powerful for variance reduction than only extending the time span.

Simulation: 3. Linear vs Exponential Affine SDF



Takeaway: no robust finite-sample superiority of exponential affine specification in this setting. [Korteweg and Nagel, 2016]

Simulation: Synthesis

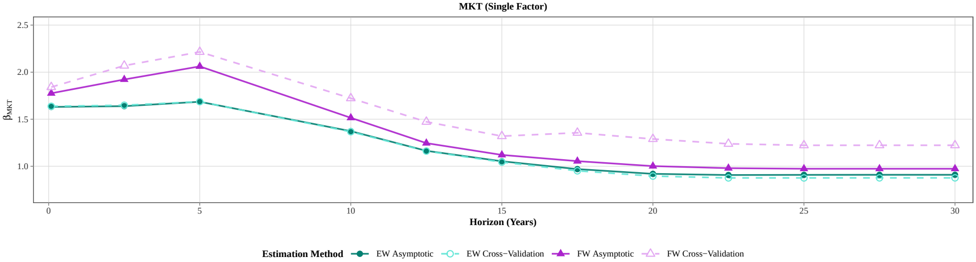
1. Use portfolio aggregation to stabilize estimation.
2. Prioritize richer cross-sections per vintage when possible (more funds per moment).
3. Prefer parsimonious factor structure in current PE data regime.
4. Horizon choice (size of $\mathcal{T}$) is a first-order control for finite-sample performance.

Interpretation: practical estimator quality is dominated by finite-sample bias-variance tradeoffs.

Empirical: Preqin PE Data & q^5 Public Factors

- Data snapshot: 2745 PE funds, vintages 1983–2019.
- Primary unit for estimation: vintage-year portfolios (equal- and value-weighted).
- Factors: q^5 family; focus on MKT and simple two-factor extensions.
- Horizon selected from simulation guidance: 15-year baseline.
- Benchmark for comparison: NAV-based naive/Dimson market-exposure estimates from the motivation section.

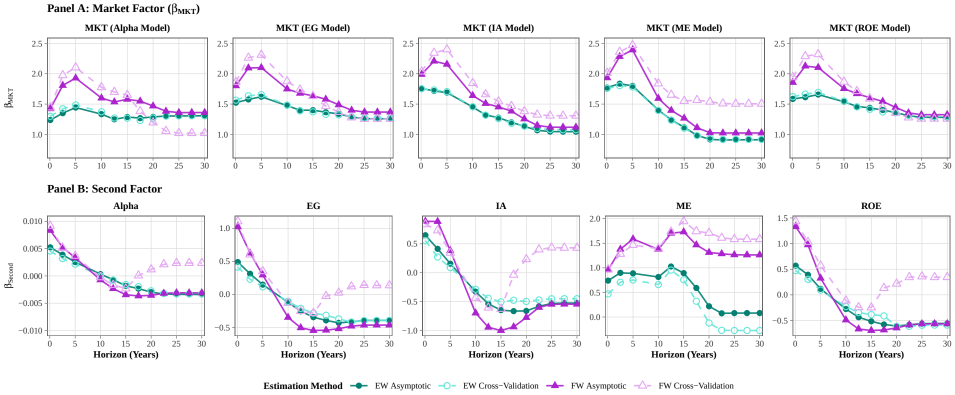
Singe-factor models: Dimson beta as lower bound.
Two-factor models: Apply machine-learning methods to form “stronger learner.”



Reading:

- Short-horizon MKT betas are high and decline with horizon.
- Betas stabilize near 1 at long horizons.
- CV inference is much more stable than asymptotic t-statistics.
- Relative to naive contemporaneous NAV betas, SDF estimates are materially closer to full market exposure.

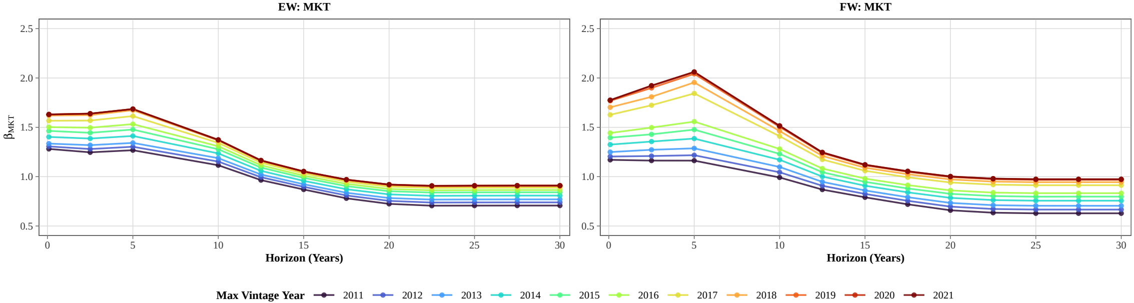
Empirical: 2. Two-Factor Models



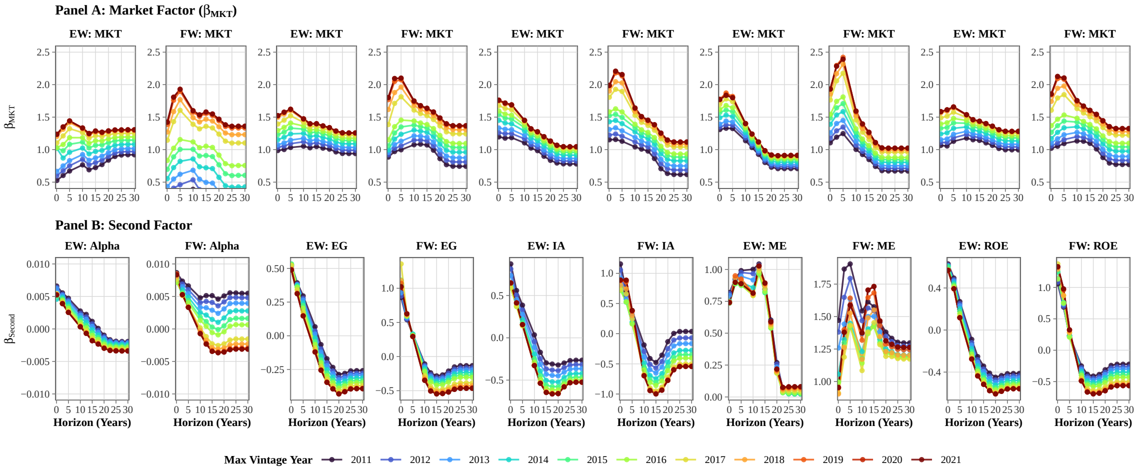
Reading:

- Second-factor loadings vary strongly with horizon and weighting scheme.
- Most non-MKT factors do not show robust incremental signal.

Empirical: 3. Vintage Cutoffs for Single-Factor Model



Reading: including newer vintages tends to increase estimated market exposure, with noisier asymptotic uncertainty.
Dimson beta as lower bound!


Panel A: Market Factor (β_{MKT})
EW: MKT
FW: MKT
EW: MKT
FW: MKT
EW: MKT
FW: MKT
EW: MKT
FW: MKT
EW: MKT
FW: MKT
β_{MKT}
Panel B: Second Factor
EW: Alpha
FW: Alpha
EW: EG
FW: EG
EW: IA
FW: IA
EW: ME
FW: ME
EW: ROE
FW: ROE
β_{Second}
Horizon (Years)
Max Vintage Year
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020
2021

Conclusion: What This Means for Practice and Research

For practitioners

- Start with parsimonious SDFs (MKT-first), then add complexity cautiously.
- Treat asymptotic significance alone as insufficient in sparse PE samples.
- Use dependence-aware validation (e.g., *hv*-block CV) as a default diagnostic.

For researchers

- Finite-sample design choices can dominate asymptotic elegance.
- Data architecture (portfolio formation, horizon design) is part of identification.

Conclusion: Machine-Learning Ensembles

- Two-factor models are messy.
- Idea: Combine multiple weak learners
- Additionally estimate error term [Tausch and Pietz, 2024]

Conclusion: Main Takeaways

1. A semiparametric LMD framework can price pooled non-traded cash flows directly.
2. The central empirical issue is finite-sample stability, not asymptotic theory alone.
3. Portfolio aggregation and horizon design are key levers for usable inference.
4. Current evidence supports single-factor MKT models as the robust baseline for PE.
5. Naive contemporaneous NAV betas understate exposure; lag-aware methods partially recover it.
6. Outlook: Multi-factor models can be stabilized by machine-learning techniques.

Questions and Discussion

Literature

- Dimson, E. (1979).
  Risk measurement when shares are subject to infrequent trading.
  Journal of Financial Economics, 7(2):197–226.
- Driessen, J., Lin, T.-C., and Phalippou, L. (2012).
  A new method to estimate risk and return of nontraded assets from cash flows: the case of private equity.
  Journal of Financial and Quantitative Analysis, 47(3):511–535.
- Korteweg, A. and Nagel, S. (2016).
  Risk-adjusting the returns to venture capital.
  Journal of Finance, 71(3):1437–1470.
- Tausch, C. and Pietz, M. (2024).
  Machine learning private equity returns.
  The Journal of Finance and Data Science, 10:100141.

Backup: Comparison to DLP12 and KN16

	DLP12	KN16	This paper
Estimator	Cross-sectional NLS	Time-series GMM (public SDF)	Nonlinear LMD
Cash flows priced	PE fund cash flows	Public replicating portfolios	PE fund cash flows
Discount dates	Inception only	Inception only	Flexible via $\mathcal{T}_i$
Asymptotics	Infill	$V \to \infty$	Increasing domain
Inference	Bootstrap	SHAC	SHAC + CV focus

Motivation: Why Measuring Risk Is Hard in Private Markets

- PE funds generate **cash flow sequences**, not continuously traded returns.
- Fund lives overlap across vintages, creating dependence beyond standard panel assumptions.
- Fund valuation relies on reported NAVs, which can be stale/smoothed.
- Standard return-based factor models are not directly applicable.

Implication: we need a cash-flow-native SDF estimator with robust dependence-aware inference.

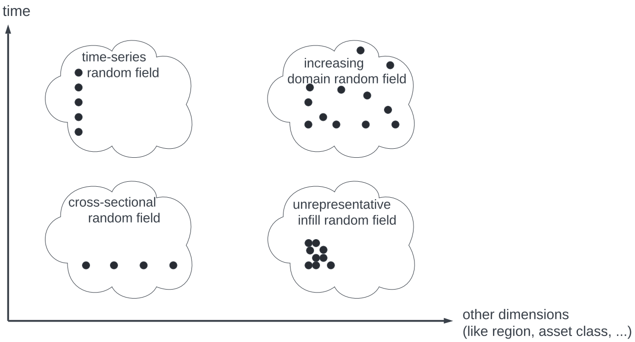
Backup: Significant ACF Lags and Dimson Lag Choice

Series	Significant lags in 1–8	Consecutive from lag 1	Dimson lag L
PE Global	3	2	2
PE North America	5	3	3
PE Europe	3	2	2
Buyout Global	3	2	2
Growth Global	2	2	2
VC Global	5	5	5

Naive benchmark result: contemporaneous beta is low ($\approx$ 0.32–0.39), while Dimson beta increases to $\approx$ 0.63–0.96.

Dependence Structure: Random Field View

- Cross-sectional unit: fund or vintage-year portfolio.
- Dependence driven by economic proximity (here: vintage-year distance).
- Asymptotics: increasing domain ($V \to \infty$), bounded units per vintage.

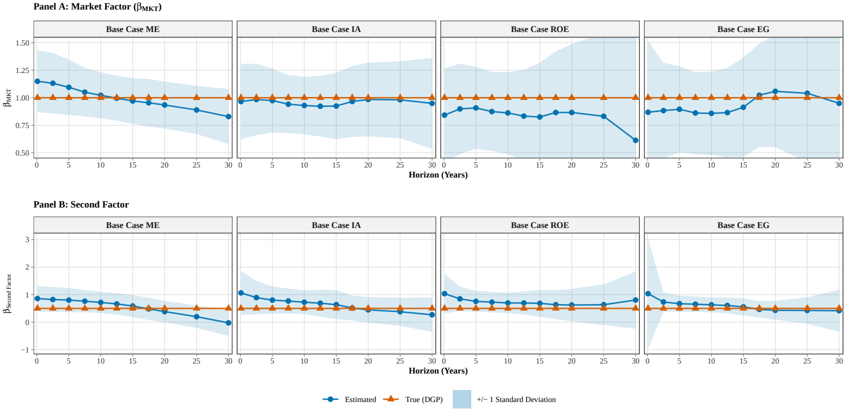


Estimator: Inference Strategy

- Asymptotic covariance: sandwich form $\Sigma = H^{-1}\Lambda H^{-1}$.
- Long-run dependence handled by SHAC (spatial HAC) with vintage-distance kernel.
- Small-sample reliability checked via *hv*-**block cross-validation**.

Reason: asymptotic approximations are fragile with only 20–40 vintage portfolios.

Simulation 4: Two-Factor q-Factor Models



Takeaway: two-factor estimates are horizon-sensitive and high-variance; multivariate identification is fragile.