MOOC Econometrics

# Test Exercise 3

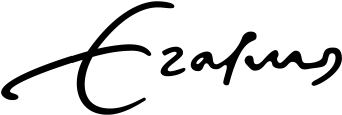
**Notes:**

* See website for how to submit your answers and how feedback is organized.

**Goals and skills being used:**

* Apply the Akaike Information Criterion (AIC) for model selection.
* Examine large-sample behavior of AIC.
* Link AIC to F-test discussed in multiple regression lectures.

**Questions**

This test exercise is of a theoretic nature. The exercise is based on Exercise 5.2c of ‘Econometric Methods with Applications in Business and Economics’. The question of interest is how the decision whether or not to include a group of variables differs based on AIC from that based on the F-test. We will stepwise show that for large samples selection based on AIC corresponds to an F-test with a critical value of approximately 2.

1. Consider the usual linear model, where *y* = *Xβ* + *ε*. We now compare two regressions, which differ in how many variables are included in the matrix *X*. In the full (unrestricted) model *p*1 regressors are included. In the restricted model only a subset of *p*0 *< p*1 regressors are included.

Show that the smallest model is preferred according to the AIC if

1. Argue that for very large values of *n* the inequality of (a) is equal to the condition

Use that *ex* ≈ 1 + *x* for small values of *x*.

1. Show that for very large values of *n* the condition in (b) is approximately equal to

where *eR* is the vector of residuals for the restricted model with *p*0 parameters and *eU* the vector of residuals for the full unrestricted model with *p*1 parameters.

1. Finally, show that the inequality from (c) is approximately equivalent to an F-test with critical value 2, for large sample sizes.

**HW3:**

1. In the full (unrestricted) model, p1 regressors are included, while in the restricted model, only a subset of p0 < p1 regressors are included. In other words, the coefficients of (p1 -p0) variables are 0 in the restricted model.  
     
   As given in lecture 3.2, , where k is the number of regressors.  
     
   So, for the unrestricted model,  
      
     
   For the restricted model,  
      
   The smallest model is preferred if AIC(R) < AIC(U), or  
     
      
     
   move the log item together,

combine the item,

Since the exponential fucntion ex is monotic and struictly increasing, we get

1. When n is very large, will be very small.  
   For small values of *x*, we can use the Taylor expansion, *ex* ≈ 1 + *x* .  
   Then,   
      
   Combined with (a),  
      
   move 1 to LHS,
2. Considering that the standard error of regression s2 = e’e/(n-k), where k is the number of regressors.  
   From (b),   
      
     
   When n is very large, ,   
     
   Therefore,
3. As shown in 2.4.2, F can be computed from residual sums of squares:

In the contex of this question,

Then,  
   
  
When n is very large, from (c) we got  
  
   
  
Thus, the inequality from (c) is approximately equivalent to an F-test with critical value 2.