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Intelligent Market-Making in Artificial Financial Markets

Sanmay Das

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by

Sanmay Das

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Certified by: Tomaso Poggio
Eugene McDermott Professor
Thesis Supervisor

Accepted by: Arthur C. Smith
Chairman, Department Committee on Graduate Students

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Abstract

This thesis describes and evaluates a market-making algorithm for setting prices in financial markets with asymmetric information, and analyzes the properties of artificial markets in which the algorithm is used. The core of our algorithm is a technique for maintaining an online probability density estimate of the underlying value of a stock. Previous theoretical work on market-making has led to price-setting equations for which solutions cannot be achieved in practice, whereas empirical work on algorithms for market-making has focused on sets of heuristics and rules that lack theoretical justification. The algorithm presented in this thesis is theoretically justified by results in finance, and at the same time flexible enough to be easily extended by incorporating modules for dealing with considerations like portfolio risk and competition from other market-makers. We analyze the performance of our algorithm experimentally in artificial markets with different parameter settings and find that many reasonable real-world properties emerge. For example, the spread increases in response to uncertainty about the true value of a stock, average spreads tend to be higher in more volatile markets, and market-makers with lower average spreads perform better in environments with multiple competitive market-makers. In addition, the time series data generated by simple markets populated with market-makers using our algorithm replicate properties of real-world financial time series, such as volatility clustering and the fat-tailed nature of return distributions, without the need to specify explicit models for opinion propagation and herd behavior in the trading crowd.

Thesis Supervisor: Tomaso Poggio

Title: Eugene McDermott Professor

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Chapter 1

Introduction

In the last decade there has been a surge of interest within the finance community in describing equity markets through computational agent models. At the same time, financial markets are an important application area for the fields of agent-based modeling and machine learning, since agent objectives and interactions tend to be more clearly defined, both practically and mathematically, in these markets than in other areas. In this thesis we consider market-making agents who play important roles in stock markets and who need to optimize their pricing decisions under conditions of asymmetric information while taking into account other considerations such as portfolio risk. This setting provides a rich and dynamic testbed for ideas from machine learning and artificial intelligence and simultaneously allows one to draw insights about the behavior of financial markets.

1.1 Background

The important concepts presented and derived in this thesis are drawn from both the finance and artificial intelligence literatures. The set of problems we are studying with respect to the dynamics of market behavior has been studied in the market microstructure and artificial markets communities, while the approach towards modeling financial markets and market-making presented here is based on techniques from artificial intelligence such as non-parametric probability density estimation and multi-agent based simulation.

1.1.1 Market Microstructure and Market-Making

The detailed study of equity markets necessarily involves examination of the processes and outcomes of asset exchange in markets with explicit trading rules. Price formation in markets occurs through the process of trading. The field of market microstructure is concerned with the specific mechanisms and rules under which trades take place in a market and how these mechanisms impact price formation and the trading process. O'Hara [23] and Madhavan [21] present excellent surveys of the market microstructure literature.

Asset markets can be structured in different ways. The simplest type of market is a standard double auction market, in which competitive buyers and sellers enter their prices and matching prices result in the execution of trades [13]. Some exchanges like the New York Stock Exchange (NYSE) employ *market-makers* for each stock in order to ensure immediacy and liquidity. The market-maker for each stock on the NYSE is obligated to continuously post two-sided quotes (a *bid* quote and an *ask* quote). Each quote consists of a size and a price, and the market-maker must honor a market buy or sell order of that size (or below) at the quoted price, so that customer buy and sell orders can be immediately executed. The NYSE employs monopolistic market-makers. Only one market-maker is permitted per stock, and that market-maker is strictly regulated by the exchange to ensure market quality. Market quality can be measured in a number of different ways. One commonly used measure is the average size of the bid-ask spread (the difference between the bid and ask prices). An exchange like the NASDAQ (National Association of Securities Dealers Automated Quotation System) allows multiple market-makers for each stock with less regulation, in the expectation that good market quality will arise from competition between the market-makers[1].

Theoretical analysis of microstructure has traditionally been an important part of the literature. Models in theoretical finance share some important aspects that we use throughout this thesis. For example, the ability to model order arrival as a stochastic process, following Garman [14] and Glosten and Milgrom [15], is important for the derivations of optimal pricing strategies presented here. The basic concepts of how market-makers minimize risk through inventory control and how this process affects prices in the market are also used in framing the market-maker's decision problem and deriving pricing strategies (see, for example, Amihud and Mendelson [1]).

[1]A detailed exposition of the different types of market structures is given by Schwartz [26].

The main problem with theoretical microstructure models is that they are typically restricted to simple, stylized cases with rigid assumptions about trader behavior. There are two major alternative approaches to the study of microstructure. These are the experimental markets approach ([11, 17] *inter alia*) and the artificial markets approach ([7, 10, 25] *inter alia*). The work presented in this thesis falls into the artificial markets approach, and we briefly review the artificial markets literature.

1.1.2 Artificial Markets

Artificial markets are market simulations populated with artificially intelligent electronic agents that fill the roles of traders. These agents can use heuristics, rules, and machine learning techniques to make trading decisions. Many artificial market simulations also use an evolutionary approach, with agents entering and leaving the market, and agent trading strategies evolving over time. Most research in artificial markets centers on modeling financial markets from the bottom up as structures that emerge from the interactions of individual agents.

Computational modeling of markets allows for the opportunity to push beyond the restrictions of traditional theoretical models of markets through the use of computational power. At the same time, the artificial markets approach allows a fine-grained level of experimental control that is not available in real markets. Thus, data obtained from artificial market experiments can be compared to the predictions of theoretical models and to data from real-world markets, and the level of control allows one to examine precisely which settings and conditions lead to the deviations from theoretical predictions usually seen in the behavior of real markets. LeBaron [18] provides a summary of some of the early work on agent-based computational finance.

There are two major strands of research on agent-based modeling of financial markets. The first of these focuses on the emergent properties of price processes that are generated by the markets. Typically, the goal of research that follows this approach is to replicate observed properties of financial time series in real markets. For example, the recent paper of Raberto *et al* [25] follows this approach, implementing simple traders who place limit orders, along with a model of opinion propagation among agents in the Genoa Artificial Stock Market. The results described by Raberto *et al* show that their model can capture some features of real financial time series, such as volatility clustering and the leptokurtic distribution of returns. Lux [20] also obtains leptokurtic return distributions in his model, which focuses on chaotic properties of the dynamical system derived from traders changing between chartist and fundamentalist trading strategies[2].

The other strand of research in artificial markets focuses more on the algorithms employed by individual traders. This strand attempts to understand the environments in which particular strategies are successful, and the resulting implications for market design. Examples of research that follow this pattern include the reinforcement-learning electronic market-maker designed by Chan and Shelton [6], recent work in the Genoa Artificial Market framework by Cincotti *et al* [8] that studies long-run success of trading strategies, and the NASDAQ-inspired simulations of Darley *et al* [10].

There is a paucity of work on market-making in the artificial markets literature. Some simulations of the NASDAQ stock market have been carried out, but none of them have focused on market-maker behavior or on adaptive agents [10, 4]. With the exception of the work of Chan and Shelton mentioned above, most research on market-making has been in the theoretical finance literature, such as the important paper of Garman [14] which was among the first to explicitly formulate the market-maker's decision problem. Amihud and Mendelson [1] introduced inventory control considerations for market-making. Glosten and Milgrom [15] solve the market-maker's decision problem under information asymmetry. This thesis extends theoretical models of market-making and implements them within the context of our artificial market.

1.1.3 Multi-Agent Simulations and Machine Learning

From the perspective of computer science, both multi-agent based simulation and machine learning have increased their importance as subfields of artificial intelligence over the last decade or so. As LeBaron [18] points out, financial markets are one of the most important applications for agent-based modeling because issues of price and information aggregation and dissemination tend to be sharper in financial settings, where objectives of agents are usually clearer. Further, the availability of massive amounts of real financial data allows for comparison with the results of agent-based simulations.

In general, work on artificial markets incorporates either learning or evolution as a means of adding dynamic structure to the markets.

[2] Interestingly, Lux does not actually implement a multi-agent simulation, but restricts his model to a level of simplicity at which he can model the entire market as a system of nonlinear differential equations.

In settings where the availability of information is a crucial aspect of market dynamics, adaptive agents who can incorporate information and learn from market trends become important players. For example, techniques from classification [22] can be used to predict price movements for chartist agents, and explicit Bayesian learning can be used by decision-theoretic agents to incorporate all available information into the decision-making process. Techniques for tracking a moving parameter [5, 3] are useful in estimating the possibly changing fundamental value of a stock. The price-setting process of market-making essentially forms a control layer on top of an estimation problem, leading to tradeoffs similar to the exploration-exploitation tradeoffs often found in reinforcement learning contexts [28]. Competitive market-making poses its own set of problems that need to be addressed using game-theoretic analysis and considerations of collaborative and competitive agent behavior [12].

1.2 Contributions

The research described in this thesis serves as a bridge in the literature between the purely theoretical work on optimal market-making techniques such as the paper of Glosten and Milgrom [15], which we use for the theoretical underpinnings of this work, and the more realistic experimental work on market-making that has been carried out by Chan [7] and Darley *et al* [10]. We derive an algorithm for price setting that is theoretically grounded in the optimal price-setting equations derived by Glosten and Milgrom, and generalize the technique to more realistic market settings. The algorithm has many desirable properties in the market environments in which we have tested it, such as the ability to make profits while maintaining a low bid-ask spread.

The market-making algorithm presented in this thesis is flexible enough to allow it to be adapted to different settings, such as monopolistic or competitive market-making settings, and extended with other modules. We present extensive experimental results for the market-making algorithm and extensions such as inventory control. We analyze the effects of competition, volatility and jumps in the underlying value on market-maker profits, the bid-ask spread and the execution of trades.

The data from simulations of markets in which market-makers use the algorithms developed in this thesis yield interesting insights into the behavior of price processes. We compare the time series properties of the price data generated by our simulations to the known characteristics of such data from real markets and find that we are able to replicate some important features of real financial time series, such as the leptokurtic distribution of returns, without postulating explicit, complex models of agent interaction and herd behavior[3] as has previously been done in the literature [20, 25][4].

1.3 Overview

This thesis is structured as follows. Chapter 2 provides necessary background information on market microstructure, introduces the market model, and derives the equations for price setting that the main market-making algorithm uses. It also presents in detail the cornerstone of the market-making algorithm, a technique for online probability density estimation that the market-maker uses to track the true underlying value of the stock.

Chapter 3 describes the practical implementation of the algorithm by taking into account the market-maker's profit motive and desire to control portfolio risk. This chapter also presents empirical analysis of the algorithm in various different market settings, including settings with multiple competitive market-makers, and details the important time series properties of our model. Chapter 4 summarizes the contributions of this thesis and suggests avenues for future work.

[3]Some explicit models of herd behavior are presented in the economics literature by Banerjee [2], Cont and Bouchaud [9] and Orléan [24] *inter alia*.

[4]It is worth noting that the true value process can induce behavior (especially following a jump) similar to that induced by herd behavior through informed traders all buying or selling simultaneously based on superior information. However, the mechanism is a much weaker assumption than the assumption of explicit imitative behavior or mimetic contagion.

Chapter 2

The Market-Making Algorithm

2.1 Market Microstructure Background

The artificial market presented in this thesis is largely based on ideas from the theoretical finance literature[1]. Here we briefly review some of the important concepts. A stock is assumed to have an underlying true value (or fundamental value) at all points in time. The price at which the stock trades is not necessarily close to this value at all times (for example, during a bubble, the stock trades at prices considerably higher than its true value). There are two principal kinds of traders in the market. *Informed traders* (sometimes referred to as fundamentalist traders) are those who know (or think they know) the true value of the stock and base their decisions on the assumption that the transaction price will revert to the true value. Informed traders will try to buy when they think a stock is undervalued by the market price, and will try to sell when they think a stock is overvalued by the market price. Sometimes it is useful to think of informed traders as those possessing inside information. *Uninformed traders* (also referred to as noise traders) trade for reasons exogenous to the market model. Usually they are modeled as buying or selling stock at random (one psychological model is traders who buy or sell for liquidity reasons). Other models of traders are often mentioned in the literature, such as *chartists* who attempt to predict the direction of stock price movement, but we are

[1] For a detailed introduction to the basic concepts of market microstructure see Schwartz [26].

Buy Orders		Sell Orders	
Size	Price ($)	Price ($)	Size
x_1	23.20	23.28	y_1
x_2	23.18	23.30	y_2
x_3	23.15	24.25	y_3
x_4	23.00		

Figure 2.1: An example limit order book

not concerned with such models of trading in this thesis.

There are two main types of orders in stock markets. These are *market orders* and *limit orders*. A market order specifies the size of the order in shares and whether the order is a buy or sell order. A limit order also specifies a price at which the trader placing the order is willing to buy or sell. Market orders are guaranteed execution but not price. That is, in placing a market order a trader is assured that it will get executed within a short amount of time at the best market price, but is not guaranteed what that price will be. Limit orders, on the other hand, are guaranteed price but not execution. That is, they will only get executed at the specified price, but this may never happen if a matching order is not found.

A *double auction market* in the context of stocks can be defined as a market in which limit orders and market orders are present and get executed against each other at matching prices. The limit orders taken together form an order book, in which the buy orders are arranged in decreasing order of price, while the sell orders are arranged in increasing order of price (see figure 2.1 for an example). Orders that match are immediately executed, so the highest buy order remaining must have a lower price than the lowest ask order remaining. Market orders, when they arrive, are executed against the best limit order on the opposite side. So, for example, a market buy order would get executed against the best limit sell order currently on the book.

Double auction markets are effective when there is sufficient liquidity in the stock. There must be enough buy and sell orders for incoming market orders to be guaranteed immediate execution at prices that are not too far away from the prices at which transactions executed recently. Sometimes these conditions are not met, typically for stocks that do not trade in high volume (for obvious reasons) and immediately following particularly favorable or unfavorable news (when everyone wants to be either on the buy or sell side of the market, leading to huge imbalances).

Market-makers are traders designated by markets to maintain immediacy and liquidity in transactions. Market-makers are obligated to continuously post two-sided quotes (*bid* (for buying) and *ask* (for selling) quotes) and honor these quotes. Apart from providing immediacy and liquidity to order execution, market-makers are also expected to smooth the transition when the price of a stock jumps dramatically, so that traders do not believe they received unfair executions, and to maintain a reasonable bid-ask spread. Exchanges with monopolistic market-makers like the NYSE monitor the performance of market-makers on these categories, while markets like NASDAQ use multiple market-makers and expect good market quality to arise from competition between market-makers.

2.2 Detailed Market Model

The market used in this thesis is a discrete time dealer market with only one stock. The market-maker sets bid and ask prices (P_b and P_a respectively) at which it is willing to buy or sell one unit of the stock at each time period (when necessary we denote the bid and ask prices at time period i as P_b^i and P_a^i). If there are multiple market-makers, the market bid and ask prices are the maximum over each dealer's bid price and the minimum over each dealer's ask price. All transactions occur with the market-maker taking one side of the trade and a member of the trading crowd (henceforth a "trader") taking the other side of the trade.

The stock has a true underlying value (or fundamental value) V^i at time period i. All market makers are informed of V^0 at the beginning of a simulation, but do not receive any direct information about V after that[2]. At time period i, a single trader is selected from the trading crowd and allowed to place either a (market) buy or (market) sell order for one unit of the stock. There are two types of traders in the market, uninformed traders and informed traders. An uninformed trader will place a buy or sell order for one unit at random if selected to trade. An informed trader who is selected to trade knows V^i and will place a buy order if $V^i > P_a^i$, a sell order if $V^i < P_b^i$ and no order if $P_b^i \leq V^i \leq P_a^i$.

In addition to perfectly informed traders, we also allow for the presence of noisy informed traders. A noisy informed trader receives a signal of the true price $W^i = V^i + \tilde{\eta}(0, \sigma_W)$ where $\tilde{\eta}(0, \sigma_W)$ represents a sample from a normal distribution with mean 0 and variance σ_W^2. The

[2]That is, the only signals a market-maker receives about the true value of the stock are through the buy and sell orders placed by the trading crowd.

noisy informed trader believes this is the true value of the stock, and places a buy order if $W^i > P_a^i$, a sell order if $W^i < P_b^i$ and no order if $P_b^i \leq W^i \leq P_a^i$.

The true underlying value of the stock evolves according to a jump process. At time $i + 1$, with probability p, a jump in the true value occurs[3]. When a jump occurs, the value changes according to the equation $V^{i+1} = V^i + \tilde{\omega}(0, \sigma)$ where $\tilde{\omega}(0, \sigma)$ represents a sample from a normal distribution with mean 0 and variance σ^2. Thus, jumps in the value can be more substantial at a given point in time than those in a unit random walk model such as the one used by Chan and Shelton [6], but the probability of a change in the true value in our model is usually significantly lower than the probability of a change in the true value in unit random walk models.

This model of the evolution of the true value corresponds to the notion of the true value evolving as a result of occasional news items. For example, jumps can be due to information received about the company itself (like an earnings report), or information about a particular sector of the market, or even information that affects the market as a whole. When a jump occurs, the informed traders are placed in an advantageous position. The periods immediately following jumps are the periods in which informed traders can trade most profitably, because the information they have on the true value has not been disseminated to the market yet, and the market maker is not informed of changes in the true value and must estimate these through orders placed by the trading crowd. The market-maker will not update prices to the neighborhood of the new true value for some period of time immediately following a jump in the true value, and informed traders can exploit the information asymmetry.

2.3 The Market-Making Algorithm

The most important feature of the market-making model presented in this thesis is that the market-maker attempts to track the true value over time by maintaining a probability distribution over possible true values and updating the distribution when it receives signals from the market buy or sell orders that traders place. The true value and the market-maker's prices together induce a probability distribution on the orders that arrive in the market. The market-maker's task is to maintain an online probabilistic estimate of the true value, which is itself a moving target.

[3] p is typically small, of the order of 1 in 1000 in most of our simulations

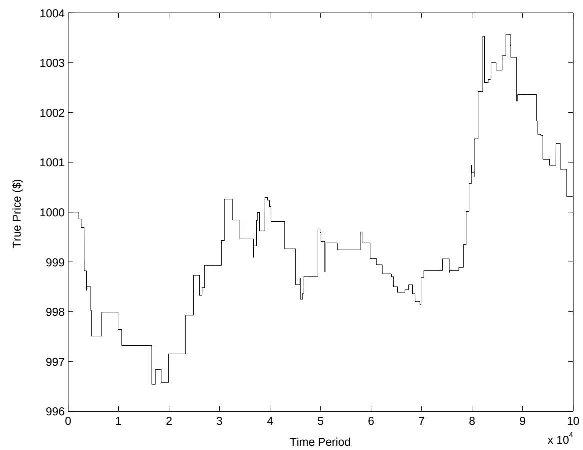

Figure 2.2: Example of the true value over time

Glosten and Milgrom [15] analyze the setting of bid and ask prices so that the market maker enforces a zero profit condition. The zero profit condition corresponds to the Nash equilibrium in a setting with competitive market-makers (or, more generally in any competitive price-setting framework [12]). Glosten and Milgrom suggest that the market maker should set $P_b = E[V|\text{Sell}]$ and $P_a = E[V|\text{Buy}]$. Our market-making algorithm computes these expectations using the probability density function being estimated.

Various layers of complexity can be added on top of the basic algorithm. For example, minimum and maximum conditions can be imposed on the spread, and an inventory control mechanism could form another layer after the zero-profit condition prices are decided. Thus, the central part of our algorithm relates to the density estimation itself. We will describe the density estimation technique in detail before addressing other possible factors that market-makers can take into account in deciding how to set prices. For simplicity of presentation, we neglect noisy informed traders in the initial derivation, and present the updated equations for taking them into account later.

2.3.1 Derivation of Bid and Ask Price Equations

In order to estimate the expectation of the underlying value, it is necessary to compute the conditional probability that $V = x$ given that a particular type of order is received. Taking market sell orders as the example:

$$E[V|\text{Sell}] = \int_0^\infty x \Pr(V = x|\text{Sell})\, dx \tag{2.1}$$

Since we want to explicitly compute these values and are willing to make approximations for this reason, we discretize the X-axis into intervals, with each interval representing one cent. Then we get:

$$E[V|\text{Sell}] = \sum_{V_i=V_{\min}}^{V_i=V_{\max}} V_i \Pr(V = V_i|\text{Sell})$$

Applying Bayes' rule and simplifying:

$$E[V|\text{Sell}] = \sum_{V_i=V_{\min}}^{V_i=V_{\max}} \frac{V_i \Pr(\text{Sell}|V = V_i) \Pr(V = V_i)}{\Pr(\text{Sell})}$$

Since P_b is set by the market maker to $E[V|\text{Sell}]$ and the *a priori* probabilities of both a buy and a sell order are equal to 1/2:

$$P_b = 2 \sum_{V_i=V_{\min}}^{V_i=V_{\max}} V_i \Pr(\text{Sell}|V = V_i) \Pr(V = V_i) \tag{2.2}$$

Since $V_{\min} < P_b < V_{\max}$,

$$P_b = 2 \sum_{V_i=V_{\min}}^{V_i=P_b} V_i \Pr(\text{Sell}|V = V_i) \Pr(V = V_i) + 2 \sum_{V_i=P_b+1}^{V_i=V_{\max}} V_i \Pr(\text{Sell}|V = V_i) \Pr(V = V_i) \tag{2.3}$$

The importance of splitting up the sum in this manner is that the term $\Pr(\text{Sell}|V = V_i)$ is constant within each sum, because of the influence of informed traders. An uninformed trader is equally likely to sell whatever the market maker's bid price. On the other hand, an informed trader will never sell if $V > P_b$. Suppose the proportion of informed traders in the trading crowd is α. Then $\Pr(\text{Sell}|V \le P_b) = \frac{1}{2} + \frac{1}{2}\alpha$ and

$\Pr(\text{Sell}|V > P_b) = \frac{1}{2} - \frac{1}{2}\alpha$. Then the above equation reduces to:

$$P_b = 2(\sum_{V_i=V_{\min}}^{V_i=P_b} (\frac{1}{2} + \frac{1}{2}\alpha)V_i \Pr(V = V_i) + \sum_{V_i=P_b+1}^{V_i=V_{\max}} (\frac{1}{2} - \frac{1}{2}\alpha)V_i \Pr(V = V_i)) \quad (2.4)$$

Using a precisely parallel argument, we can derive the expression for the market-maker's ask price:

$$P_a = 2(\sum_{V_i=V_{\min}}^{V_i=P_a} (\frac{1}{2} - \frac{1}{2}\alpha)V_i \Pr(V = V_i) + \sum_{V_i=P_a+1}^{V_i=V_{\max}} (\frac{1}{2} + \frac{1}{2}\alpha)V_i \Pr(V = V_i)) \quad (2.5)$$

2.3.2 Accounting for Noisy Informed Traders

An interesting feature of the probabilistic estimate of the true value is that the probability of buying or selling is the same conditional on V being smaller than or greater than a certain amount. For example, $\Pr(\text{Sell}|V = V_i, V_i \leq P_b)$ is a constant, independent of V. If we assume that all informed traders receive noisy signals, with the noise normally distributed with mean 0 and variance σ_W^2, and, as before, α is the proportion of informed traders in the trading crowd, then equation 2.3 still applies. Now the probabilities $\Pr(\text{Sell}|V = V_i)$ are no longer determined solely by whether $V_i \leq P_b$ or $V_i > P_b$. Instead, the new equations are:

$$\Pr(\text{Sell}|V = V_i, V_i \leq P_b) = (1-\alpha)\frac{1}{2} + \alpha \Pr(\tilde{\eta}(0, \sigma_W) \leq (P_b - V_i)) \quad (2.6)$$

and:

$$\Pr(\text{Sell}|V = V_i, V_i > P_b) = (1-\alpha)\frac{1}{2} + \alpha \Pr(\tilde{\eta}(0, \sigma_W) \geq (V_i - P_b)) \quad (2.7)$$

The second term in the first equation reflects the probability that an informed trader would sell if the fundamental value were less than the market-maker's bid price. This will occur as long as $W = V +$

$\tilde{\eta}(0, \sigma_W) \leq P_b$. Similarly, the second term in the second equation reflects the same probability, except with the assumption that $V > P_b$.

We can compute the conditional probabilities for buy orders equivalently:

$$\Pr(\text{Buy}|V = V_i, V_i \leq P_a) = (1-\alpha)\frac{1}{2} + \alpha \Pr(\tilde{\eta}(0, \sigma_W) \geq (P_a - V_i)) \tag{2.8}$$

and:

$$\Pr(\text{Buy}|V = V_i, V_i > P_a) = (1-\alpha)\frac{1}{2} + \alpha \Pr(\tilde{\eta}(0, \sigma_W) \leq (V_i - P_a)) \tag{2.9}$$

We can substitute these conditional probabilities back into both the fixed point equations and the density update rule used by the market-maker. First of all, combining equations 2.3, 2.6 and 2.7, we get:

$$P_b = 2 \sum_{V_i = V_{\min}}^{V_i = P_b} \left(\frac{1}{2} - \frac{1}{2}\alpha + \alpha \Pr(\tilde{\eta}(0, \sigma_W) \leq (P_b - V_i))\right) V_i \Pr(V = V_i) +$$
$$2 \sum_{V_i = P_b + 1}^{V_i = V_{\max}} \left(\frac{1}{2} - \frac{1}{2}\alpha + \alpha \Pr(\tilde{\eta}(0, \sigma_W) \geq (V_i - P_b))\right) V_i \Pr(V = V_i) \tag{2.10}$$

Similarly, for the ask price:

$$P_a = 2 \sum_{V_i = V_{\min}}^{V_i = P_a} \left(\frac{1}{2} - \frac{1}{2}\alpha + \alpha \Pr(\tilde{\eta}(0, \sigma_W) \geq (P_a - V_i))\right) V_i \Pr(V = V_i) +$$
$$2 \sum_{V_i = P_a + 1}^{V_i = V_{\max}} \left(\frac{1}{2} - \frac{1}{2}\alpha + \alpha \Pr(\tilde{\eta}(0, \sigma_W) \leq (V_i - P_a))\right) V_i \Pr(V = V_i) \tag{2.11}$$

2.3.3 Approximately Solving the Equations

A number of problems arise with the analytical solution of these discrete equations for setting the bid and ask prices. Most importantly, we have not yet specified the probability distribution for priors on V, and any reasonably complex solution leads to a form that makes analytical solution infeasible. Secondly, the values of $V_{\min}$ and $V_{\max}$ are undetermined. And finally, actual solution of these fixed point equations must be approximated in discrete spaces. We solve each of these problems in turn to construct an empirical solution to the problem and then present experimental results in the next chapter.

We assume that the market-making agent is aware of the true value at time 0, and from then onwards the true value infrequently receives random shocks (or jumps) drawn from a normal distribution (the variance of which is known to the agent). Our market-maker constructs a vector of prior probabilities on various possible values of V as follows.

If the initial true value is V_0 (when rounded to an integral value in cents), then the agent constructs a vector going from $V_0 - 4\sigma$ to $V_0+4\sigma-1$ to contain the prior value probabilities. The probability that $V = V_0 - 4\sigma + i$ is given by the ith value in this vector[4]. The vector is initialized by setting the ith value in the vector to $\int_{-4\sigma+i}^{-4\sigma+i+1} \mathcal{N}(0,\sigma)\,dx$ where $\mathcal{N}$ is the normal density function in x with specified mean and variance. The reason for selecting 4σ as the range is that it contains 99.9% of the density of the normal, which we assume to be a reasonable number of entries. The vector is also maintained in a normalized state at all times so that the entire probability mass for V lies within it.

The fixed point equations 2.10 and 2.11 are approximately solved by using the result from Glosten and Milgrom that $P_b \leq E[V] \leq P_a$ and then, to find the bid price, for example, cycling from E[V] downwards until the difference between the left and right hand sides of the equation stops decreasing. The fixed point real-valued solution must then be closest to the integral value at which the distance between the two sides of the equation is minimized.

2.3.4 Updating the Density Estimate

The market-maker receives probabilistic signals about the true value. With perfectly informed traders, each signal says that with a certain probability, the true value is lower (higher) than the bid (ask) price. With noisy informed traders, the signal differentiates between different possible true values depending on the market-maker's bid and ask quotes. Each time that the market-maker receives a signal about the true value by receiving a market buy or sell order, it updates the posterior on the value of V by scaling the distributions based on the type of order. The Bayesian updates are easily derived. For example, for $V_i \leq P_a$ and market buy orders:

$$\Pr(V = V_i|\text{Buy}) = \frac{\Pr(\text{Buy}|V = V_i)\Pr(V = V_i)}{\Pr(\text{Buy})}$$

[4] It is important to note that the true value can be a real number, but for all practical purposes it ends up getting truncated to the floor of that number.

The prior probability $V = V_i$ is known from the density estimate, the prior probability of a buy order is $1/2$, and $\Pr(\text{Buy}|V = V_i, V_i \leq P_a)$ can be computed from equation 2.8. We can compute the posterior similarly for each of the cases.

An interesting note is that in the case of perfectly informed traders, the signal only specifies that the true value is higher or lower than some price, and not how much higher or lower. In that case, the update equations are as follows. If a market buy order is received, this is a signal that with probability $\frac{1}{2}(1-\alpha)+\alpha = \frac{1}{2}+\frac{1}{2}\alpha, V > P_a$. Similarly, if a market sell order is received, the signal indicates that with probability $\frac{1}{2}+\frac{1}{2}\alpha, V < P_b$. In the former case, all probabilities for $V = V_i, V_i > P_a$ are multiplied by $\frac{1}{2} + \frac{1}{2}\alpha$, while all the other discrete probabilities are multiplied by $1 - (\frac{1}{2} + \frac{1}{2}\alpha)$. Similarly, when a sell order is received, all probabilities for $V = V_i, V_i < P_b$ are multiplied by $\frac{1}{2} + \frac{1}{2}\alpha$, and all the remaining discrete probabilities are multiplied by $1 - (\frac{1}{2} + \frac{1}{2}\alpha)$ before renormalizing.

These updates lead to less smooth density estimates than the updates for noisy informed traders, as can be seen from figure 2.3 which shows the density functions 5, 10 and 15 steps after a jump in the underlying value of the stock. The update equations that consider noisy informed traders serve to smoothly transform the probability distribution around the last transaction price by a mixture of a Gaussian and a uniform density, whereas the update equations for perfectly informed traders discretely shift all probabilities to one side of the transaction price in one direction and on the other side of the transaction price in the other direction. The estimates for perfectly informed traders also tend to be more susceptible to noise, as they do not restrict most of the mass of the probability density function to as small an area as the estimates for noisy informed traders.

From figure 2.4 we can see that the market-maker successfully tracks the true value over the course of an entire simulation. Another interesting feature of the algorithm is that the bid-ask spread reflects the market-maker's uncertainty about the true value — for example, it is typically higher immediately after the true value has jumped.

In the next chapter we present empirical results from applying this algorithm in different market settings and also extend the basic algorithm for market-making presented here to take into account other factors like inventory control and profit motive.

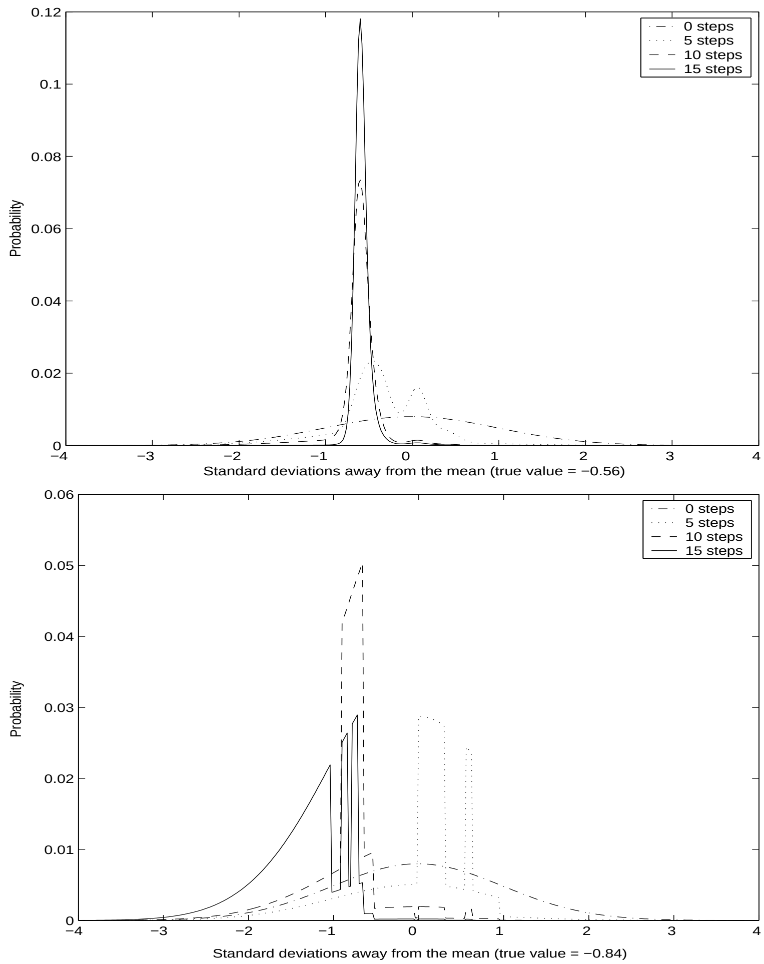

Figure 2.3: The evolution of the market-maker's probability density estimate with noisy informed traders (above) and perfectly informed traders (below)

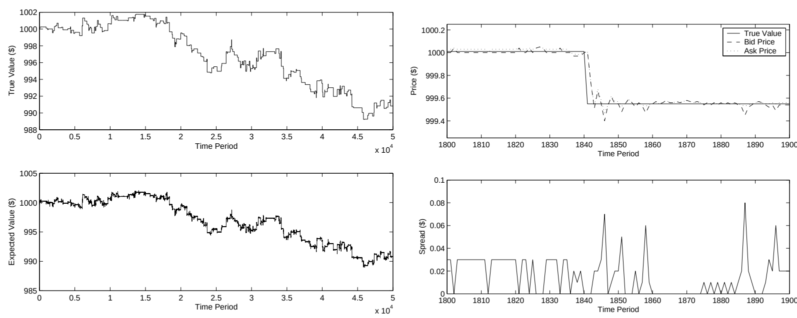

Figure 2.4: The market-maker's tracking of the true price over the course of the simulation (left) and immediately before and after a price jump (right)

Chapter 3

Inventory Control, Profit Motive and Transaction Prices

3.1 A Naïve Market-Maker

At this stage, it is necessary to introduce a simple algorithm for market-making. There are two main reasons to study such an algorithm. First, it helps to elucidate the effects of some extensions to the main algorithm presented in the last chapter (which we shall sometimes refer to as the "sophisticated" algorithm), and second, it provides a basis for comparison. This naïve market-maker "surrounds" the last transaction price with its bid and ask quotes while maintaining a fixed spread at all times. At the first time period, the market-maker knows the initial true value and sets its bid and ask quotes around that price. So, for example, if the last transaction price was P_h and the market-maker uses a fixed spread δ, it would set its bid and ask quotes at $P_h - \frac{\delta}{2}$ and $P_h + \frac{\delta}{2}$ respectively.

Given that we do not consider transaction sizes in this thesis, the above algorithm is actually surprisingly effective for market-making, as it adjusts its prices upwards or downwards depending on the kinds of orders entering the market. The major problem with the algorithm is that it is incapable of adjusting its spread to react to market conditions or to competition from other market-makers, so, as we shall demonstrate, it does not perform as well (relatively speaking) in competitive environments or under volatile market conditions as algorithms that take market events into account more explicitly.

3.2 Experimental Framework

Unless specified otherwise, it can be assumed that all simulations take place in a market populated by noisy informed traders and uninformed traders. The noisy informed traders receive a noisy signal of the true value of the stock with the noise term being drawn from a Gaussian distribution with mean 0 and standard deviation 5 cents. The standard deviation of the jump process for the stock is 50 cents, and the probability of a jump occurring at any time step is 0.005. The market-maker is informed of when a jump occurs, but not of the size or direction of the jump. The market-maker uses an inventory control function (defined below) and increases the spread by lowering the bid price and raising the ask price by a fixed amount (this is done to ensure profitability and is also explained below). We report average results from 50 simulations, each lasting 50,000 time steps.

3.3 Inventory Control

Stoll analyzes dealer costs in conducting transactions and divides them into three categories [27]. These three categories are portfolio risk, transaction costs and the cost of asymmetric information. In the model we have presented so far, following Glosten and Milgrom [15], we have assumed that transactions have zero execution cost and developed a pricing mechanism that explicitly attempts to set the spread to account for the cost of asymmetric information.

A realistic model for market-making necessitates taking portfolio risk into account as well, and controlling inventory in setting bid and ask prices. In the absence of consideration of trade size and failure conditions, portfolio risk should affect the placement of the bid and ask prices, but not the size of the spread[1] [1, 27, 16]. If the market-maker has a long position in the stock, minimizing portfolio risk is achieved by lowering both bid and ask prices (effectively making it harder for the market-maker to buy stock and easier for it to sell stock), and if the market-maker has a short position, inventory is controlled by raising both bid and ask prices.

Inventory control can be incorporated into the architecture of our market-making algorithm by using it as an adjustment parameter ap-

[1] One would expect spread to increase with the trade size.

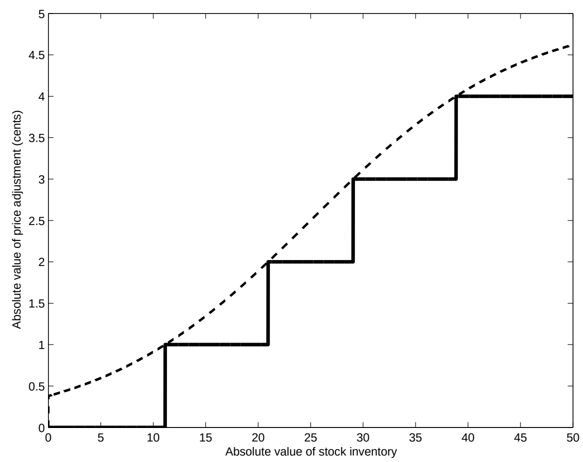

Figure 3.1: Step function for inventory control and the underlying sigmoid function

MM Type	No IC	IC
Naive	158.95	7.85
Sophisticated	39.46	8.99

Table 3.1: Average of absolute value of market-maker's inventory holdings at the end of a simulation

plied after bid and ask prices have been determined by equations 2.10 and 2.11. An example of the kind of function we can use to determine the amount of the shift is a sigmoid function. The motivation for using a sigmoid function is to allow for an initial gradual increase in the impact of inventory control on prices, followed by a steeper increase as inventory accumulates, while simultaneously bounding the upper limit by which inventory control can play a factor in price setting. Of course, the upper bound and slope of the sigmoid can be adjusted according to the qualities desired in the function.

For our simulations, we use an inventory control function that uses the floor of a real valued sigmoid function with a ceiling of 5 cents as the integer price adjustment (in cents). The step function for the adjustment and the underlying sigmoid are shown in figure 3.1.

Figure 3.2 is a scatter plot that shows the effects of using the above

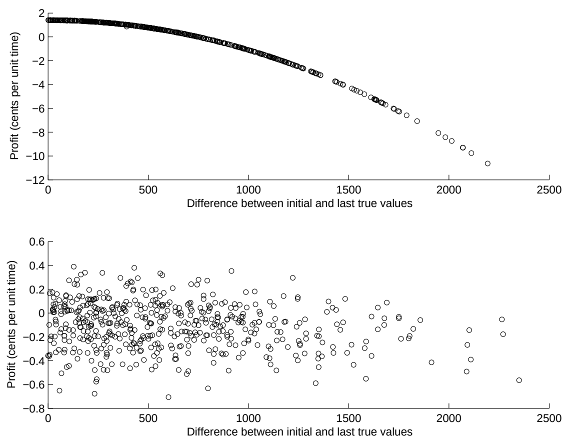

Figure 3.2: Naïve market-maker profits as a function of market volatility without (above) and with (below) inventory control

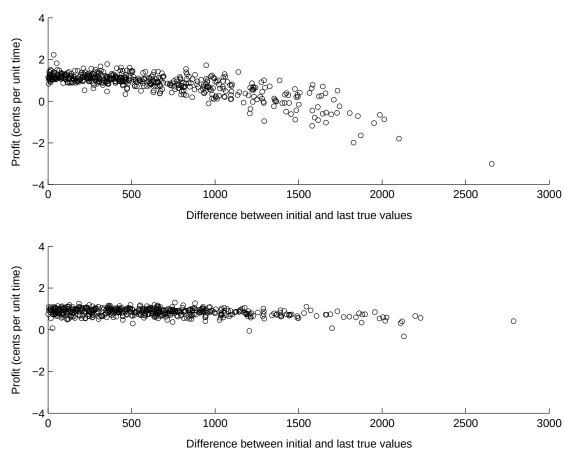

Figure 3.3: Sophisticated market-maker profits as a function of market volatility without (above) and with (below) inventory control

MM Type	No IC	IC
Naïve	−0.9470	−0.1736
Sophisticated	−0.7786	−0.3452

Table 3.2: Correlation between market volatility and market-maker profit for market-makers with and without inventory control

MM Type	No IC	IC
Naïve	−0.1672	−0.1033
Sophisticated	0.8372	0.8454

Table 3.3: Average profit (in cents per time period) for market-makers with and without inventory control

inventory control function for a naïve market-maker using a δ value of 8 cents (note that the Y axes are on different scales for the two parts of the figure). Figure 3.3 shows the effects for a market-maker using the sophisticated algorithm[2]. Table 3.1 shows the average absolute value of inventory held by the market-maker at the end of each simulation for the different cases. The figures use the absolute value of the difference between last true value and initial true value as a proxy for estimating market volatility, as this difference provides a measure of how much a large inventory could affect profit for a particular simulation. 500 simulations were run for each experiment, and 70% of the traders were noisy informed traders, while the rest were uninformed.

The results in figures 3.2 and 3.3 and tables 3.2 and 3.3 demonstrate that without inventory control, market-maker profits are highly correlated with volatility, and the inventory control module we have suggested successfully removes the dependence of profit on volatility without reducing expected profit. The differences in profit for the inventory control and no inventory control cases are not statistically significant for either the naïve or the sophisticated market-maker. In fact, it is somewhat surprising that average profit is not reduced by inventory control, since adding inventory control is similar to adding additional constraints to an optimization problem. This effect could be

[2]For this experiment, the market-maker was modified to increase the spread beyond the zero profit condition by lowering the bid price by 3 cents and increasing the ask price by 3 cents. The motivation for this is to use a profitable market-maker, as will become clear in the next section, and to perform a fair comparison with a naïve market-maker that uses a fixed spread of 8 cents.

due to the fact that our algorithm is not in fact performing exact optimization, and the inventory control module may help to adjust prices in the correct direction in volatile markets. Another interesting fact is that the sophisticated market-making algorithm is less susceptible to the huge losses that the naïve market-maker incurs in very volatile market environments, even without inventory control. This suggests that the sophisticated algorithm is adapting to different environments more successfully than the naïve algorithm.

3.4 Profit Motive

The zero-profit condition of Glosten and Milgrom is expected from game theoretic considerations when multiple competitive dealers are making markets in the same stock. However, since our method is an approximation scheme, the zero profit method is unlikely to truly be zero-profit. Further, the market-maker is not always in a perfectly competitive scenario where it needs to restrict the spread as much as possible. In this section, we investigate some possibilities for increasing the spread to ensure profitability conditions for the market-making algorithm.

3.4.1 Increasing the Spread

The simplest solution to the problem of making profit is to increase the spread by pushing the bid and ask prices apart after the zero-profit bid and ask prices have been computed using the density estimate obtained by the market-making algorithm. The major effect of this on the density estimation technique is that the signals the market-maker receives and uses to update its density estimate are determined by transaction prices, which are in turn determined by the bid and ask prices the market-maker has set. The precise values of the bid and ask prices are quite important to the sampling of the distribution on trades induced by the true value.

Figure 3.4 shows the profit obtained by a single monopolistic market-maker in markets with different percentages of noisy informed traders. The numbers on the X axis show the amount (in cents) that is subtracted from (added to) the zero-profit bid (ask) price in order to push the spread apart (we will refer to this number as the shift factor). It is important to note that market-makers can make reasonable profits with low average spreads – an example is given at the end of the section on competitive market-making.

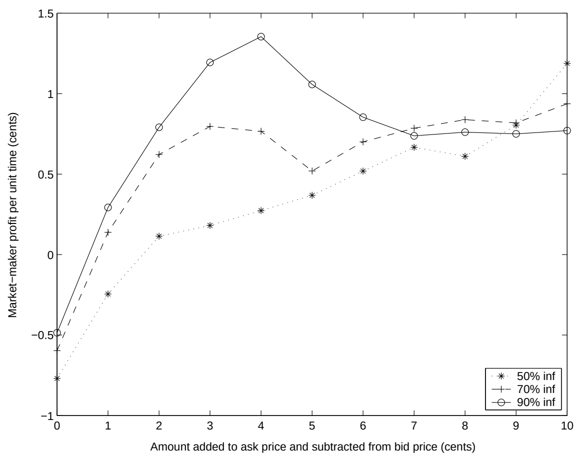

Figure 3.4: Market-maker profits as a function of increasing the spread

With lower spreads, most of the market-maker's profits come from the noise factor of the informed traders, whereas with a higher spread, most of the market-maker's profits come from the trades of uninformed traders. Different percentages of informed traders lead to differently shaped curves. With only 50% of the traders being informed, the market-maker's profit keeps increasing with the size of the spread. However, increasing the spread beyond a point is counterproductive if there are enough noisy informed traders in the markets, because then the market-maker's prices are far enough away from the true value that even the noise factor cannot influence the informed traders to make trades. With 90% of the traders being informed, a global maximum (at least for reasonable spreads) is attained with a low spread, while with 70% of the traders being informed, a local maximum is attained with a fairly low spread, although the larger number of uninformed traders allows for larger profits with rather large spreads.

Another point worth mentioning is that the market-maker's probability density estimates tend to be more concentrated with more noisy informed traders in the markets, because each trade provides more information. This leads to the empirical results being closer to theoretical predictions. For example, the prices leading to zero profit for the 70% informed and 90% informed cases fall between the 0 and 1 points on the X axis, which is close to what one would expect from the theory, whereas with perfectly informed traders zero profit is not obtained

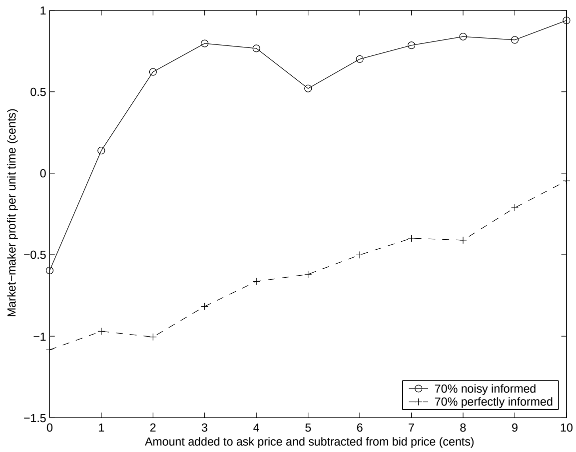

Figure 3.5: Market-maker profits with noisy informed traders and perfectly informed traders

without using a large spread.

Figure 3.5 compares the profits obtained by the market-maker with 70% noisy informed traders as opposed to 70% perfectly informed traders. In the latter case, there is no advantage to be gained by having a smaller spread as there is with noisy informed traders. However, the market-maker's inability to make any profit even with a high spread seems surprising. This is partly attributable to the fact that the point at which the distribution is sampled is more important in the perfectly informed case because the signals only inform the market-maker of the probabilities that the true value is greater than or less than the last transaction price, instead of smoothly morphing points around the last transaction price by a mixture of a Gaussian distribution and a uniform distribution.

3.4.2 Active Learning

The above observation on how the smoothing effect of noisy informed traders on the posterior distribution helps to maintain, in some senses, a "good" density estimate points us in an interesting direction for the case of perfectly informed traders. We can see from figure 3.5 that, counterintuitively, profits increase slowly with increasing spread. Since the market-maker sets "controls" at P_b and P_a, it may not be getting

Bid/Ask Adjustment	Perfectly Informed	Noisy Informed
4	-0.7382	0.8075
4r	-0.5119	0.5594

Table 3.4: Average profit (in cents per time period) for market-makers with and without randomization in the sampling strategy

"good" samples for estimating the probability density for values in the region between P_b and P_a, which is the most important part of the distribution, since the true value probably lies in that range.

One way of dealing with this problem is to occasionally sample from the distribution between P_b and P_a by not increasing the spread and just using the zero-profit condition prices some percentage of the time. We tested the effectiveness of this method in markets with informed traders constituting 50% of the trading crowd, and the market-maker decreasing the bid price and increasing the ask price by 4 cents. In the randomized case, the market-maker increased the spread 60% of the time, leaving it untouched otherwise. The results from this experiment are shown in table 3.4.

"4r" represents the randomized algorithm, and we can see that it outperforms the non-randomized version (in fact, it also outperforms versions that use higher spreads) in markets with perfectly informed traders. The same does not hold true for markets with noisy informed traders, where the loss incurred by not pushing the spread up at each opportunity dominates any benefits gained from improving the density estimate. This is because of the smoother nature of the noisy estimate, as discussed above. There may be an interesting connection between this behavior and the "exploration-exploitation" tradeoff as thought of in reinforcement learning [28] — a market-maker willing to sacrifice profits temporarily in order to improve its estimate can make more profit in the long term. The online nature of the problem decrees that the probability of sampling more aggressively by not increasing the spread cannot be smoothly decreased over time, but perhaps more sophisticated algorithms for sampling the important areas of the distribution might help performance even more in terms of profit in the perfectly informed case. Perhaps some of the ideas for sampling from the distribution more effectively can be adapted to the noisy case. This is an interesting direction for future work.

3.4.3 Competitive Market-Making

The most important aspect of competitive market-making within the framework in which we view it is that market-makers are not guaranteed to execute trades just by being in the market. Instead the highest bid price quoted by any market-maker and the lowest ask price quoted by any market-maker become the effective market bid-ask quotes, and the market-makers compete with each other for trades. A market-maker who does not make a sufficient number of trades will lose out to a market-maker who makes substantially more trades even if the latter makes less profit per trade.

This effect is particularly obvious in the first experiment shown in table 3.5. In this experiment, two market-makers who both use the market-making algorithm presented in this thesis compete with each other, with the difference that one uses a shift factor of 2 and the other a shift factor of 3 for increasing the spread after the zero-profit bid and ask prices have been determined. If neither were using an inventory control mechanism, the market-maker using a shift factor of 3 would in fact make no trades, because the market-maker using a shift factor of 2 would always have the inside quotes for both the bid and the ask. The addition of inventory control allows the market-maker using a shift factor of 3 to make some trades, but this market-maker makes considerably less profit than the one using a shift factor of 2. In a monopolistic environment the market-maker using a shift factor of 3 outperforms the market-maker using a shift factor of 2 and the difference in magnitude of executed trades is not as large.

It is interesting to compare two different strategies for market-making in competitive and monopolistic environments. The naïve market-making algorithm outperforms the sophisticated algorithm in a monopolistic framework with 70% of the traders being perfectly informed and the rest uninformed. However, it is outperformed by the sophisticated algorithm in the same environment when they are in direct competition with each other. Although both are incurring losses, the sophisticated algorithm is making more trades than the naïve algorithm under competition, so the improved performance is not a function of simply not making trades. This is the third experiment reported in table 3.5. The second experiment in table 3.5 shows the performance of the two algorithms with 70% of the trading crowd consisting of noisy informed traders and the remaining 30% consisting of uninformed traders. Again, the presence of competition severely degrades the naïve market-maker's performance without significantly hurting the sophisticated market-maker. This suggests that our algorithm for market-making

70% noisy informed traders

MM Type	Competitive		Monopolistic	
	Profit	# Trades	Profit	# Trades
Soph (shift = 2)	0.6039	38830	0.6216	39464
Soph (shift = 3)	0.0157	594	0.8655	34873

70% noisy informed traders

MM Type	Competitive		Monopolistic	
	Profit	# Trades	Profit	# Trades
Naive (spread = 8)	-0.8020	17506	-0.0840	35176
Soph (shift = 3)	0.7687	20341	0.8655	34873

70% perfectly informed traders

MM Type	Competitive		Monopolistic	
	Profit	# Trades	Profit	# Trades
Naive (spread = 8)	-0.9628	12138	-0.5881	23271
Soph (shift = 3)	-0.6379	16331	-0.8422	27581

Table 3.5: Market-maker profits (in cents per time period) and average number of trades in simulations lasting 50,000 time steps in monopolistic and competitive environments

is robust with respect to competition.

For a market with 70% of the trading crowd consisting of noisy informed traders and the remaining 30% consisting of uninformed traders, our algorithm, using inventory control and a shift factor of 1, achieves an average profit of 0.0074 ± 0.0369 cents per time period with an average spread of 2.2934 ± 0.0013 cents. These parameter settings in this environment yield a market-maker that is close to a Nash equilibrium player, and it is exceedingly unlikely that any algorithm would be able to outperform this one in direct competition in such an environment given the low spread. It would be interesting to compare the performance of other sophisticated market-making algorithms to this one in competitive scenarios. Another interesting avenue to explore is the possibility of adaptively changing the shift factor depending on the level of competition in the market. Clearly, in a monopolistic setting, a market-maker is better off using a high shift factor, whereas in a competitive setting it is likely to be more successful using a smaller one. An algorithm for changing the shift factor based on the history of other

σ	Shift	Spread	Profit
100	1	2.7366	-0.7141
100	2	5.0601	-0.1410
50	1	2.2934	0.0074
50	2	4.4466	0.6411

Table 3.6: Market-maker average spreads (in cents) and profits (in cents per time period) as a function of the standard deviation of the jump process

p	Shift	Spread	Profit
0.005	1	2.2934	0.0074
0.005	2	4.4466	0.6411
0.0001	1	2.0086	0.8269
0.0001	2	4.0154	1.4988

Table 3.7: Market-maker average spreads (in cents) and profits (in cents per time period) as a function of the probability of a jump occurring at any point in time

market-makers' quotes would be a useful addition.

3.5 The Effects of Volatility

Volatility of the underlying true value process is affected by two parameters. One is the standard deviation of the jump process, which affects the variability in the amount of each jump. The other is the probability with which a jump occurs. Table 3.6 shows the result of changing the standard deviation σ of the jump process and table 3.7 shows the result of changing the probability p of a jump occurring at any point in time. As expected, the spread increases with increased volatility, and profit decreases. A higher average spread needs to be maintained to get the same profit in more volatile markets.

3.6 Accounting for Jumps

The great advantage of our algorithm for density estimation and price setting is that it quickly restricts most of the probability mass to a relatively small region of values/prices, which allows the market-maker to quote a small spread and still break even or make profit. The other side of this equation is that once the probability mass is concentrated in one area, the probability density function on other points in the space becomes vanishingly small. In some cases, it is not possible to seamlessly update the estimate through the same process if a price jump occurs. Another problem is that a sequence of jumps could lead to the value leaving the $[-4\sigma, 4\sigma]$ window used by the density estimation technique[3].

If the market-maker is in some way explicitly informed of when a price jump has occurred (perhaps the market-maker gets a signal whenever news arrives or may have arrived, like right before an earnings report is released), although not of the size or direction of the jump, the problem can be solved by recentering the distribution around the current expected value and reinitializing in the same way in which the prior distribution on the value is initially set up. In the "unknown jump" case the problem is more complicated. We tested certain simple rules relating to order imbalance which utilize the fact that the cost of not recentering when a jump has occurred is significantly higher than the cost of recentering if a jump has not occurred. An example of such a rule is to recenter when there have been k more buy orders than sell orders (or vice versa) in the last n time steps. Table 3.8 shows the results obtained using different n and k values, where the loss of the expectation is defined as the average of the absolute value of the difference between the true value and the market-maker's expectation of the true value at each point in time. Clearly there is a tradeoff between recentering too often and not recentering often enough. Although there is a loss to be incurred by waiting for too long after a price jump to recenter, it can be even worse to recenter too aggressively (such as the $n = 5$, $k = 3$ case). An interesting avenue for future work, especially if trade sizes are incorporated into the model, is to devise a classifier that is good at predicting when a price jump has occurred. Perhaps there are particular types of trades that commonly occur following price jumps, especially when limit orders and differing trade sizes are permitted. Sequences of such trades may form patterns that predict the occurrence of jumps in the underlying value.

[3]For cases with perfectly informed traders the first of these problems is typically not critical, but since the probabilities away from the expected value still represent a significant probability mass, estimates can become degraded if the expected value is sufficiently skewed away from the mean.

n	k	Profit	Loss of expectation
Known		0.8259	4.2261
10	5	0.2327	5.5484
10	6	0.3069	5.1544
10	7	0.2678	5.3254
5	3	-0.4892	6.8620

Table 3.8: Average profit (in cents per time period) and loss of expectation for market-makers using different parameters for recentering the probability distribution

3.7 Time Series Properties of Transaction Prices

Liu *et al* present a detailed analysis of the time series properties of returns in a real equity market (they focus on the S&P 500 and component stocks) [19]. Their major findings are that return distributions are leptokurtic and fat-tailed, volatility clustering occurs (that is, big price changes are more likely to be followed by big price changes and small price changes are more likely to be followed by small price changes)[4] and that the autocorrelation of absolute values of returns decays according to a power law, and is persistent over large time scales, as opposed to the autocorrelation of raw returns, which disappears rapidly.

Raberto *et al* are able to replicate the fat tailed nature of the distribution of returns and the clustered volatility observed in real markets [25]. However, the Genoa Artificial Stock Market explicitly models opinion propagation and herd behavior among trading agents in a way that we do not[5]. Nevertheless, our model is also able to replicate the important stylized facts of real financial time series, including the leptokurtic distribution of returns, clustered volatility and persistence of the autocorrelation of absolute returns.

A return over a particular time period is defined as the ratio of the prices at which two transactions occur which are separated by that period in time. In our model, a one step return is the ratio of the prices

[4] Liu *et al* are certainly not the first to discover these properties of financial time series. However, they summarize much of the work in an appropriate fashion and provide detailed references, and they present novel results on the power law distribution of volatility correlation.

[5] A jump in the true value will lead informed traders in our model to make the same decisions on whether to buy or sell, but not because of imitative behavior among the agents themselves.

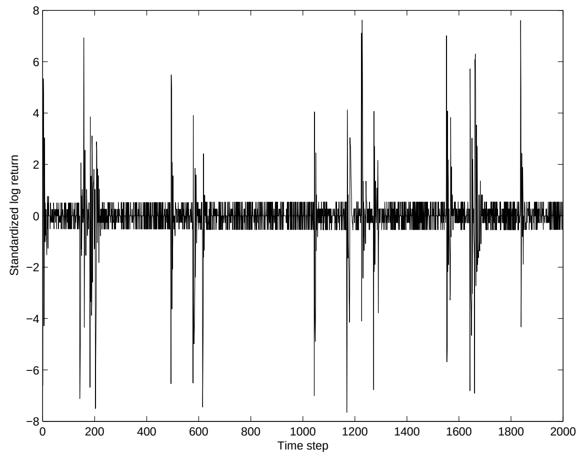

Figure 3.6: Standardized log returns over time

at which two successive transactions occur. We record all transaction prices and assume that the intervals between transactions are the same. All experiments in this section are in a market with 70% noisy informed traders, 30% uninformed traders, and a market-maker using a shift factor of 1 (which results in the market-maker's profit being close to zero). We work with log returns in this model, where the log return is $\log P_{n+1} - \log P_n$. Figure 3.6 shows the standardized log returns over time. Standardized log returns are log returns detrended by the mean and rescaled by the standard deviation. The clustering of volatility and the sharp tails are evident from figure 3.6. Figure 3.7 demonstrates the leptokurtic nature of the distribution of returns. The fat tail is evident from the right half of the graph, where the area being covered by the distribution of returns "pokes out" from the area covered by a normal distribution. The kurtosis for this experiment was 28.7237.

The other important features of real financial time series that our market also shows are the long-range persistence of the autocorrelation of absolute returns and the clustering of volatility (figure 3.8 and figure 3.6 respectively). Interestingly, the decay of autocorrelation appears to be linear, which is in contrast with the power law decay observed by Liu *et al.* If we look more closely, the decay is linear on a log-log scale for the first 25 lags (indicative of a power law decay) (figure 3.9). The long range persistence of autocorrelation is an important feature of real financial markets [19], but in comparison, Raberto *et*

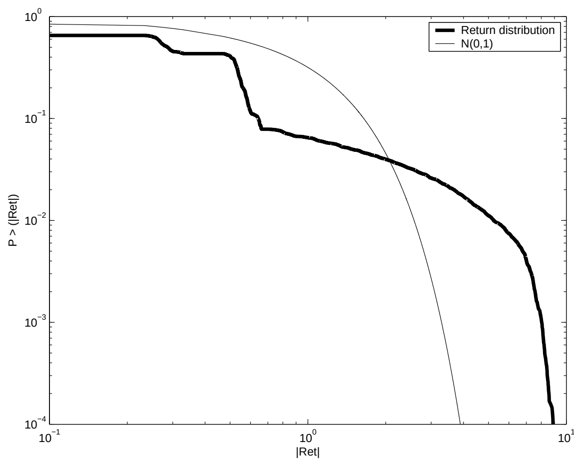

Figure 3.7: Leptokurtic distribution of absolute standardized log returns

al fail to observe persistence of autocorrelation beyond 80 lags in the Genoa market, and they do not see decay consistent with a power law at any scale. Real markets, the Genoa market and our artificial market all show quick decay of the autocorrelation of raw returns (figure 3.10).

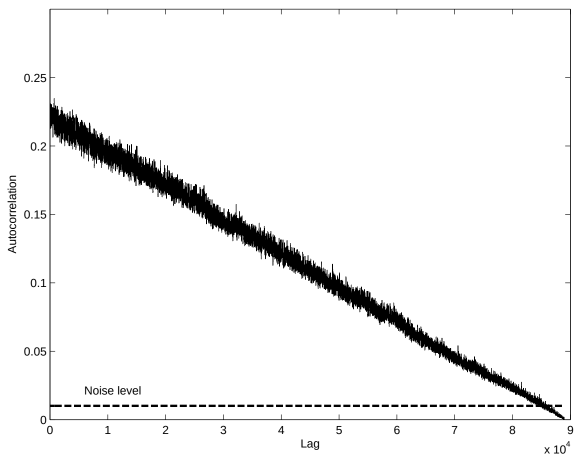

Figure 3.8: Autocorrelation of absolute returns

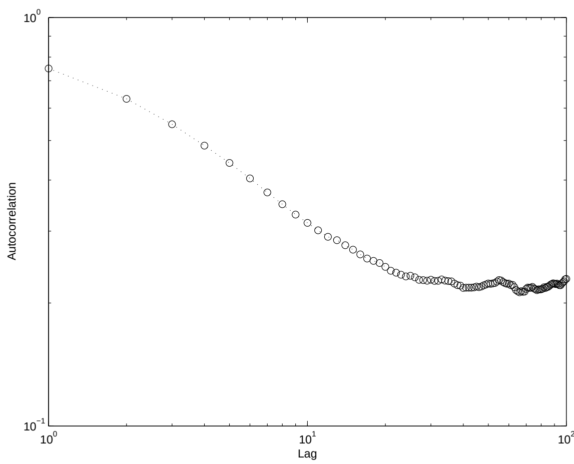

Figure 3.9: Autocorrelation of absolute returns on a log-log scale for lags of 1-100

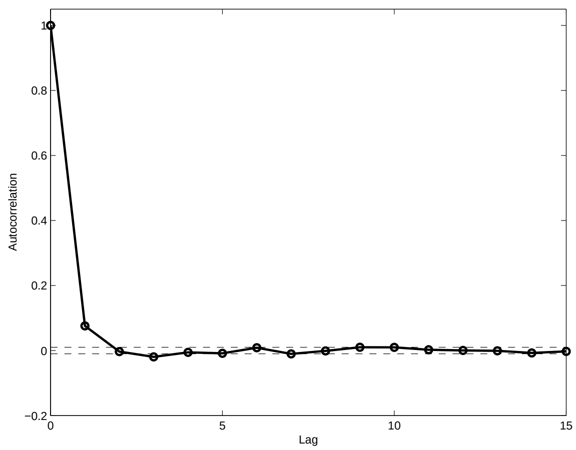

Figure 3.10: Autocorrelation of raw returns

Chapter 4

Conclusions and Future Work

4.1 Summary

The major contribution of this thesis is the presentation of an algorithm for market-making under conditions of asymmetric information in markets with informed and uninformed traders. Glosten and Milgrom derive the basic concept of setting bid and ask prices to be the conditional expectations of the true value given that a sell or buy order is received, but do not extend the concept beyond toy problems [15]. On the other side of the spectrum, Chan and Shelton develop a reinforcement learning algorithm for market-making that is fairly complex and attempts to deal with multiple objectives like profit and inventory control simultaneously, but needs many training episodes and has a hard time approaching profitability, even in markets simpler than the ones we study here [6]. The price setting equations of our market-making algorithm are theoretically grounded in the work of Glosten and Milgrom, and the density estimation technique is essentially explicit Bayesian learning. Modules for inventory control and for increasing profit by increasing the spread can be added to the algorithm after solution of the expected value equations for price-setting.

Our market-making algorithm displays many qualities that one would expect from any reasonable market-maker. It increases the spread when it is more uncertain about the true value (for example, following a jump in the underlying value) and tends to maintain a higher spread in more volatile markets. Our market-maker also allows us to gain insights into the structure of simple markets. For example, in markets with large numbers of noisy informed traders, increasing the spread is counterproductive beyond a point even in the absence of competition because it no longer allows the market-maker to make profits from the errant estimates of the noisy informed traders. In competitive dealer markets, as one would expect, market-makers who execute more trades tend to benefit even if they make less profit per trade, because their quotes are on the inside more often.

Simple artificial markets populated by the kinds of trading crowds and market-makers we describe are capable of replicating some of the important time series phenomena of real financial markets. For example, the leptokurtic distribution of returns and the persistence of the autocorrelation of absolute returns along with the rapid decay of the autocorrelation of raw returns are important phenomena in financial time series [19]. These phenomena are replicated to some extent in the artificial markets described by Lux [20] and Raberto *et al* [25] among others, but only with explicit models of opinion propagation and evolutionary behavior in the trading crowd. The fact that our model does not need to postulate such behavior, instead relying on the simple interaction between informed and uninformed traders, may point to an important underlying regularity of such time series phenomena.

In terms of learning, this thesis describes a nonparametric density estimation algorithm that is very successful in the application domain. The importance of smoothness of the density function for good performance is demonstrated by the far superior performance of the algorithm with noisy informed traders (it is worth noting that the gains are not primarily from the noise itself, but are mostly due to the improved estimation – using the perfectly informed estimates with noisy informed traders does not lead to performance as good as that achieved using the noisy estimates). The importance of maintaining a good estimate rather than just increasing the spread is apparent from the success of the "active learning" algorithm that aggressively samples from the distribution in the perfectly informed case.

4.2 Future Directions

Deriving a model for market-making is just a starting point for much exciting research in agent-based modeling of financial markets. The research directions we believe to be particularly interesting in this regard are focused on the learning aspects and on creating richer market models.

We have briefly touched on how active learning can be used to improve the density estimate and hence the performance of the market-making algorithm. However, the method we present is naïve and has not been studied in depth. It would be worthwhile to investigate how to sample the distribution to achieve the best estimate possible while still maintaining a high enough spread to achieve profit. This is particularly interesting in the competitive framework where an agent cannot compensate for loss at one time by raising the spread exorbitantly at another.

Perhaps the most interesting issues will arise in the context of multi-agent learning. One point of departure is to consider the shift factor and have competitive market-makers using the same basic algorithm try to learn the optimal shift factor to use when competing against each other. It is conceivable that this could give rise to cooperative (or collusive) behavior without the need for explicit communication.

The method of dealing with changes in the underlying value presented in chapter 3 is simplistic. Perhaps it is possible to learn a classifier either online or offline that predicts when a jump has occurred (or when the probability of a jump having occurred is high enough to warrant the cost of recentering the distribution). Lastly in terms of learning, our model makes many assumptions about the market-maker being aware of certain parameters like the percentage of informed traders and the standard deviations of the jump process and the noise terms. What if the market-maker had to estimate these instead of knowing them upfront[1]?

There are also some fascinating directions for future work in terms of the market structure and model. Among these is more detailed examination of the time series properties of returns and an analysis of why they differ from real markets in the characteristics in which they do differ. We are also interested in calibrating the artificial market parameters to real markets. For example, the probability of a jump or the standard deviation of the jump could be usefully linked to occurrences in real markets. This would give more meaning to the precise values derived from the experimental results, like market-maker bid-ask spreads. Finally, it is important to investigate richer, more complex market-models. The first step in this direction is to incorporate consideration of different trade sizes. Following this, we would also like to explore markets with different types of traders, including traders who are capable of placing limit orders.

[1] In preliminary experiments, the profitability and low spread seem fairly robust to the market-maker using a wrong estimate of the number of informed traders, or of the variance of the noise factor.

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