The Axioms of Cognitive Geometry: A Formal Model of Psychophysical Correlation

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The empirical study of consciousness has matured significantly in recent decades, most notably with regard to the production of increasingly fine-grained experimental data. However, the foundations of theoretical cognitive science remain unsettled, creating a "crisis of falsifiability" (Doerig, Schurger, Hess, et al. 2019; Hanson and Walker 2021). We argue that the crisis of falsifiability in consciousness studies ultimately stems from insufficient logical and mathematical rigor: leading theories of consciousness such as Integrated Information Theory (IIT) and Global Workspace Theory (GWT) are fundamentally incapable of interfacing with standard mathematical physics, making such theories either empirically untestable in principle or logically "pre-falsified" in practice. Thus, drawing on a rigorous formalization of phenomenological structure, we propose a formal axiomatic system which is fully integrated with fundamental physics from the outset. Specifically, we posit an *orthogonal phase-space geometry*: the standard "physical Hilbert space" ($\mathcal{H}_\Psi$) of textbook quantum mechanics, linked to an orthogonal "phenomenal Hilbert space" ($\mathcal{H}_\Phi$) by mathematically well-defined Awareness (A) and Volition (V) operators. Crucially, this orthogonal phase-space geometry yields a novel and falsifiable empirical prediction: a precise $\pi/2$ (90-degree) phase offset between the neural correlates of sensory awareness, and the neural correlates of volitional action, in a sensorimotor loop (Austen Clark 1993; O'Regan 2011; O'Regan 2021). The "Awareness Operator" or $A|\Omega\rangle$ formalism thus offers a comprehensive resolution to the crisis in the foundations of cognitive science: a mathematically precise, physically grounded, and empirically testable model.

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1. Introduction

1.1. The Methodological Challenge

Cognitive science faces a critical methodological challenge: while empirical techniques have produced an abundance of fine-grained experimental data, the theoretical frameworks required to interpret this data remain contested. This dynamic has resulted in what many now recognize as a "crisis of falsifiability" (Doerig, Schurger, and Herzog 2021; Hanson and Walker 2021). At the heart of this crisis lie two powerful critiques—one mathematical, one philosophical—that together threaten the foundations of current theoretical models.

The Mathematical Critique: Representation Dependence

Theoreticians of cognitive science frequently posit a basic distinction between **feedback** and **feed-forward** mechanisms. Systems exhibiting feedback affect their own behavior by "feeding" their own output "back" to themselves as input. Conscious systems clearly affect their own behavior; thus, the existence of feedback is a *necessary* condition for consciousness. The question remains whether it is also *sufficient*.

Causal Structure Theories (CSTs), such as the Integrated Information Theory (IIT) proposed by Tononi and Koch (2015), maintain that the existence of specific feedback structures in a system's network architecture is sufficient to generate consciousness. IIT quantifies consciousness as a mathematical measure (Φ) of the extent to which a system's dynamics exhibit "integrated feedback."[1] This leads to the well-known prediction that even simple feedback systems, such as photodiodes, possess a non-zero degree of consciousness (Koch and Tononi 2013).

While Φ is a rigorous construct, it faces a significant mathematical hurdle: it does not constitute an **invariant measure**. The *unfolding argument* demonstrates that the exact same physical system may be predicted to have $\Phi = 0$ (unconscious) or $\Phi > 0$ (conscious), depending entirely upon the mathematical representation scheme chosen to describe it (Doerig, Schurger, Hess, et al. 2019; Herzog, Schurger, and Doerig 2022).

The unfolding argument establishes that any network graph with recursive feedback may be "unfolded" into an equivalent purely feed-forward representation (e.g., a Markov chain) without altering the system's input-output function. If Φ varies while the physical

[1] Formally, Φ is quantified as the Earth Mover's Distance between the probability distribution of the system's present state and its distribution following a hypothetical "Minimum Information Partition" (Oizumi, Albantakis, and Tononi 2014).

behavior remains constant, then Φ describes an artifact of the model rather than an intrinsic property of the physical system. This has led to the formulation of a sharp falsifiability criterion: a scientific theory of consciousness must make predictions that are **invariant** with respect to transformations that leave the input-output function unchanged (Hanson and Walker 2021). Theories that fail this criterion risk placing their core tenets outside the reach of empirical verification.

The Philosophical Critique: The Scaling Problem

A distinct objection targets the reliance on *information* as a proxy for phenomenal presence. As John Searle (2013) has argued, there is a fundamental asymmetry between consciousness (which is observer-independent) and information (which is generally observer-relative).

Brute physical facts only "contain information" insofar as they are interpreted by a conscious agent. Construed as brute physical facts, there is no difference between the state of a blank hard drive and one containing a complex mathematical proof. Tononi and Koch (2015) claim that a photodiode is conscious because it distinguishes light-on from light-off. However, this creates a scaling problem: just as a mercury thermometer expands or contracts to measure temperature without "knowing" it, Φ may scale with causal complexity without necessarily scaling with phenomenal presence.

The Internal Critique: Dissociative Epiphenomenalism

A third line of criticism, developed by Herzog, Schurger, and Doerig (2022), suggests that CSTs may be *internally self-undermining*. IIT implies that systems with identical input-output functions can have different consciousnesses due to different causal structures. This entails that consciousness—both its magnitude and its content—is fully dissociated from behavior.

As Hanson and Walker (2021) have shown, one can theoretically construct systems that experience X while reporting Y. This has a devastating consequence for the epistemic foundations of the theory: if the content of consciousness is dissociated from reports, then *reports about first-person experience cannot be trusted as indicators of actual phenomenal content*. Yet, the axioms of IIT are explicitly grounded in such introspective reports (Oizumi, Albantakis, and Tononi 2014). Herzog, Schurger, and Doerig (2022) term this **dissociative epiphenomenalism**: if the theory is true, the evidence for its axioms (reports) becomes unreliable.

1.2. The Deeper Problem: Strong Emergence

These critiques share a common root. The unfolding argument shows that Φ is representation-dependent; the scaling argument shows that information is observer-relative. Both problems arise because current frameworks treat consciousness as the ontologically derivative *result* of fundamental physical processes, relying on the assumption of **strong emergence**.

Strong emergence is the thesis that certain higher-level properties are genuinely novel and not derivable, even in principle, from a complete description of lower-level constituents. In cognitive science, this is often treated as the default solution to the hard problem (Chalmers 2010). As we argue in Section A, however, this thesis faces severe logical difficulties. From set-theoretic and category-theoretic perspectives, the claim that a property simultaneously belongs to a system and transcends the derivation of that system creates an unbridgeable explanatory gap.

If consciousness is conceived as something that must "emerge" from physics, any theory faces a dilemma: either consciousness is reducible (dissolving the hard problem), or it is not (leaving it unexplained). A genuinely explanatory theory requires a different starting point. Froese (2024) has recently argued that the interaction between mind and matter cannot be rendered fully intelligible within a framework that treats the mental as derivative. The convergence of independent theoretical programs on similar conclusions suggests that the field may be approaching a necessary paradigm shift.

1.3. The Axiomatic Path Forward

The mathematical structure required to accommodate phenomenal states in our scientific description of reality must be built into its foundations. Instead of attempting to make consciousness "emerge" from physics, we propose extending physical theory to include phenomenal states axiomatically. Accordingly, this paper introduces a system for what we term **Cognitive Geometry**.

We posit that the "hard problem" is not an inherent mystery of nature, but a **structural artifact** of our current formalisms (Kuhn 1962). Drawing on formal structures inspired by phenomenological and contemplative traditions, we introduce a dual-phase space geometry: two orthogonal Hilbert spaces linked by well-defined operators. This provides a formal mathematical bridge between physical states $|\psi\rangle$ and phenomenal states $|\phi\rangle$.

From this axiomatic foundation, we derive:

- A resolution to the stalemate over causal structure theories, grounded in invariant geometric properties rather than representation-dependent measures.
- A formal account of the embodied, perspectival nature of conscious experience.
- A novel, falsifiable empirical prediction: a precise **π/2 (90-degree) phase offset** between the neural processes associated with the *locus* of sensory integration (Awareness) and the *locus* of motor initiation (Volition). This prediction can be tested using established neuroimaging methods focusing on phase-coding in sensorimotor loops (Austen Clark 1993; O'Regan 2011).

The $A|\Omega\rangle$ framework thus offers a comprehensive response to the crisis in theoretical cognitive science: a mathematically precise, physically grounded, and empirically testable model.

2. The Axiomatic Foundations of the $A|\Omega\rangle$ Framework

2.1. The Principle of Minimal Extension: A Dual-Phase Space

The framework we propose is grounded in a *principle of minimal extension*: we do not violate the established laws of physics, but rather extend these in the most minimal manner possible to accommodate consciousness as a fundamental (as opposed to "emergent") feature of reality. As argued in Section 1, attempts to derive phenomenal states from physical states within a single ontological domain face severe logical difficulties. This necessitates an extension of the global state space itself.

We therefore posit the following axiomatic structure:

Axiom 2.1 (Dual Hilbert Spaces)**.** *There exist two Hilbert spaces:*

1. *The **Physical Hilbert Space**, $\mathcal{H}_\Psi$, which is the familiar state space of quantum mechanics. Physical state vectors $|\psi\rangle \in \mathcal{H}_\Psi$ completely describe all possible physical systems.*

2. *The **Phenomenal Hilbert Space**, $\mathcal{H}_\Phi$, which is a distinct state space. Phenomenal state vectors $|\phi\rangle \in \mathcal{H}_\Phi$ completely describe all possible phenomenal experiences. Intuitively, a phenomenal state vector $|\phi\rangle$ may be interpreted as a mathematical formalization of a **phenomenal primitive**: an ontologically irreducible, discrete unit of experience.*

Axiom 2.2 (Tensor Product Structure)**.** *The complete kinematical state space $\mathcal{K}$ is the tensor product:*

$$\mathcal{K} := \mathcal{H}_\Phi \otimes \mathcal{H}_\Psi \tag{1}$$

States in $\mathcal{K}$ are composite vectors of the form $|\phi\rangle \otimes |\psi\rangle$, with inner product:

$$\langle \phi_1 \otimes \psi_1 | \phi_2 \otimes \psi_2 \rangle_{\mathcal{K}} = \langle \phi_1 | \phi_2 \rangle_\Phi \cdot \langle \psi_1 | \psi_2 \rangle_\Psi \tag{2}$$

Axiom 2.3 (Sectoral Independence)**.** *Physical and phenomenal observables act on separate tensor factors:*

$$\mathcal{O}_{phys} = I_\Phi \otimes \hat{O}_\Psi \tag{3}$$

$$\mathcal{O}_{phen} = \hat{O}_\Phi \otimes I_\Psi \tag{4}$$

where I denotes the identity operator on the respective space. Consequently, all physical observables commute with all phenomenal observables:

$$[\mathcal{O}_{phys}, \mathcal{O}_{phen}] = 0 \tag{5}$$

This tensor product structure provides the mathematical foundation for the **mutual incommensurability** of phenomenal experiences and physical properties. The two sectors cannot be compared directly—there is no meaningful sense in which a physical state is "closer to" or "farther from" a phenomenal state—yet they coexist within a unified kinematical framework that permits lawful interaction via the operators introduced below.

2.2. The Dynamics of Physics and Experience: Dual Hamiltonians

In physics, the temporal evolution of a system is governed by its Hamiltonian operator $\hat{H}$. To maintain consistency and formal symmetry, we posit that both sectors of $\mathcal{K}$ possess corresponding dynamical generators.

Axiom 2.4 (Dual Hamiltonians)**.** *There exist two Hamiltonian operators:*

1. *The **Physical Hamiltonian**, H_Ψ, governs the evolution of states within $\mathcal{H}_\Psi$ according to standard quantum mechanics.*

2. *The **Phenomenal Hamiltonian**, H_Φ, governs the evolution of states within $\mathcal{H}_\Phi$.*

On the full state space $\mathcal{K}$, these act as:

$$\mathbf{H}_\Psi = I_\Phi \otimes H_\Psi \tag{6}$$
$$\mathbf{H}_\Phi = H_\Phi \otimes I_\Psi \tag{7}$$

The Awareness and Volition operators introduced in Section 2.4 are *dynamical* operators: they do not commute with the Hamiltonians.

Proposition 2.1 (Dynamical Character of Transfer Operators)**.** *The Awareness operator A and Volition operator V satisfy:*

$$[A, \mathbf{H}_\Psi] \neq 0 \tag{8}$$
$$[V, \mathbf{H}_\Phi] \neq 0 \tag{9}$$

This non-commutativity is essential. If A commuted with $\mathbf{H}_\Psi$, awareness would be a conserved quantity, unchanging in time. Instead, the non-vanishing commutators ensure that awareness and volition are *processes*: transformations that occur *in* time, mediating the causal commerce between physical and phenomenal domains.

2.3. The Ground States of Physics and Experience

A fundamental principle of any physical system is the existence of a lowest-energy state, or **vacuum state**. In $\mathcal{H}_\Psi$, this is the familiar physical vacuum $|0\rangle$. We posit an analogous ground state for the phenomenal sector.

Axiom 2.5 (Phenomenal Vacuum)**.** *The phenomenal Hamiltonian H_Φ possesses a unique ground state $|\Omega\rangle \in \mathcal{H}_\Phi$, the* ***phenomenal vacuum****, satisfying:*

$$H_\Phi|\Omega\rangle = E_\Omega|\Omega\rangle \tag{10}$$

where E_Ω is the minimum eigenvalue of H_Φ.

Like the physical vacuum $|0\rangle$, the phenomenal vacuum $|\Omega\rangle$ is a state of minimal excitation. It may be understood as "pure phenomenality", devoid of any specific phenomenal content $|\phi\rangle$. This formalizes the concept of **minimal phenomenal consciousness** or **pure awareness** found in phenomenological philosophy (Husserl 1913) and contemplative neuroscience (Lutz et al. 2008). It is the background capacity for experience, distinct from the specific contents of experience.

Crucially, this mathematical separation of *phenomenality as such* from determinate phenomenal *content* avoids the common conflation of "first-personality" or "for-me-ness" (Zahavi and Kriegel 2016) with phenomenal consciousness itself. Our formalism allows for the existence of **pure consciousness events** (Forman 1990)—states in which awareness obtains without determinate perspectival structure.

2.4. The Core Transformation Equations

The dynamics between physical and phenomenal domains are mediated by two fundamental operators: the **Awareness Operator** (A), which maps from the physical to the phenomenal domain, and the **Volition Operator** (V), which maps from the phenomenal to the physical domain.

Axiom 2.6 (Transfer Operators)**.** *There exist bounded linear operators A and V on $\mathcal{K}$ satisfying:*

$$A : \mathcal{H}_\Phi \otimes \mathcal{H}_\Psi \to \mathcal{H}_\Phi \otimes \mathcal{H}_\Psi \tag{11}$$

$$V : \mathcal{H}_\Phi \otimes \mathcal{H}_\Psi \to \mathcal{H}_\Phi \otimes \mathcal{H}_\Psi \tag{12}$$

The action of these operators is captured by the **Core Transformation Equations**:

Definition 2.1 (Awareness Transformation)**.** The generation of phenomenal experience from a physical substrate is given by:

$$A\big(|\Omega\rangle \otimes |\psi_0\rangle\big) = |\phi_0\rangle \otimes |L\rangle \tag{13}$$

Definition 2.2 (Volition Transformation)**.** The causal efficacy of phenomenal states upon physical systems is given by:

$$V\big(|\phi_0\rangle \otimes |L\rangle\big) = |\Omega\rangle \otimes |\psi_0'\rangle \tag{14}$$

The terms in these equations are interpreted as follows:

- $|\psi_0\rangle \in \mathcal{H}_\Psi$: A specific physical state (e.g., a neural configuration).
- $|\Omega\rangle \otimes |\psi_0\rangle$: The **awareness potential**—a physical configuration coupled to the phenomenal ground state.
- $|\phi_0\rangle \in \mathcal{H}_\Phi$: The specific phenomenal content of the resulting experience (e.g., the subjective character of seeing red).
- $|L\rangle \in \mathcal{H}_\Psi$: The **spacetime locus**—a physical state representing the embodied "point of view" or **egocentric reference frame** (Varela, Thompson, and Rosch 1992). This is the geometric origin of the coordinate system defined by the subject's physical location. The locus is uniquely determined by the input state $|\psi_0\rangle$.
- $|\phi_0\rangle \otimes |L\rangle$: The output of the Awareness transformation: phenomenal content ($|\phi_0\rangle$) structurally bound to an embodied point of view ($|L\rangle$).
- $|\psi_0'\rangle \in \mathcal{H}_\Psi$: The physical state resulting from volitional action—reflecting the causal efficacy of mental states upon the physical world.

These equations constitute the axiomatic heart of our model. They posit that subjectivity is not a primitive feature, but the output of a specific transformation acting upon a non-dual ground. The Awareness operator A generates embodied, perspectival experience from the coupling of physical systems to the phenomenal vacuum.

The structural relationship between A and V—specifically, their emergence as complementary off-diagonal components of a unified dynamical operator—directly constrains their temporal signatures. As we develop in Section 5, this geometric constraint yields a precise, falsifiable empirical prediction: neural processes dominated by awareness (A) and those dominated by volition (V) will exhibit a characteristic $\pi/2$ phase offset in their oscillatory dynamics.

3. The Dynamics of Actualization: Priors, Resonance, and Selection

3.1. The Problem of Selection

The framework presented in Section 2 provides the static geometry of the psychophysical relationship. We must now specify the *dynamics*—the laws governing how definite perceptual outcomes emerge from fields of possibility.

Consider a physical system in a state of potentiality (e.g., a neural population encoding multiple competing interpretations of a sensory stimulus):

$$|\psi\rangle = \sum_i c_i |\psi_i\rangle \tag{15}$$

When the Awareness operator A acts on the tensor product $|\Omega\rangle \otimes |\psi\rangle$, the result is a *potential* for all experiences consistent with the physical state. Yet lived experience is definite. We perceive the Necker cube as *either* face-front *or* face-back, not both simultaneously.

The question of **actualization** is therefore: what mechanism selects a single, definite phenomenal outcome from the field of potentialities? A satisfactory theory must provide a dynamical account of this selection process.

3.2. The Formal Mechanism: Phenomenal Priors

We propose that this selection mechanism is governed by the system's accumulated history. In Bayesian approaches to cognition (e.g., Predictive Coding, Active Inference), perception is treated as an inference process constrained by **priors**—learned expectations derived from past experience (Friston 2010; Andy Clark 2013). These priors do not deterministically cause perception, but they condition the landscape of possibility, making certain interpretations more likely than others.

We formalize this insight by identifying the **Phenomenal Hamiltonian** H_Φ as the carrier of these priors. In this framework, H_Φ is not a static operator but a **state-dependent operator**, dynamically shaped by the system's entire experiential history. Formally:

Axiom 3.1 (History-Dependent Phenomenal Hamiltonian)**.** *The phenomenal Hamiltonian $H_\Phi(t)$ is a functional of the system's history:*

$$H_\Phi(t) = H_\Phi^{(0)} + \int_{-\infty}^{t} \mathcal{F}\left[|\phi(\tau)\rangle, \tau\right] d\tau \tag{16}$$

where $H_\Phi^{(0)}$ is the bare Hamiltonian and $\mathcal{F}$ encodes the modification of the dynamical landscape by past states.

This history-dependence defines a **dynamic attractor landscape**. Just as synaptic plasticity shapes the physical connectivity of the brain, this term shapes the geometry of the phenomenal state space. It creates "basins of attraction" that favor familiar or learned experiences. This formalizes the phenomenological insight that our past experiences structure our capacity for present awareness (Merleau-Ponty 1945).

3.3. The Dynamical Cycle: From Potential to Actuality

Dynamics arise not from the Awareness operator A or the Volition operator V acting in isolation, but from their *composition*—the cyclic process $\mathcal{H}_\Psi \to \mathcal{H}_\Phi \to \mathcal{H}_\Psi$.

Proposition 3.1 (Dynamical Generation)**.** *The compositions $V \circ A$ (physical evolution mediated by the phenomenal) and $A \circ V$ (phenomenal evolution mediated by the physical) generate the non-trivial dynamics of the system.*

The actualization process can be described as a three-stage cycle:

1. **Projection**: The Awareness operator A maps the physical state into the phenomenal sector, creating a field of potential experiences.
2. **Resonant Selection**: This field propagates through the prior-conditioned landscape of H_Φ. The system "resonates" with states that match its learned priors (attractors), filtering the superposition into a single definite outcome $|\phi_j\rangle$.
3. **Return**: The Volition operator V carries this actualized state back to the physical sector, binding it to the spacetime locus $|L\rangle$ and initiating physical action $|\psi_0'\rangle$.

This cyclic architecture mirrors the structure of recent computational models of cortical dynamics. Specifically, Sun et al. (2025) have demonstrated that robust long-range order in neural networks is induced by a coupling between fast "activity" variables and slow "memory" (resource) variables. In our framework, this corresponds to the interaction between the fast evolution of the physical state $|\psi\rangle$ and the slower, history-dependent evolution of the phenomenal landscape H_Φ.[2]

[2] The isomorphism is precise. Sun et al. (2025) describe the system via coupled equations: $\dot{\rho} \propto R\rho + D\nabla^2\rho$ (activity) and $\dot{R} \propto -R\rho$ (resource/memory) [cite: 63-64]. In our framework, ρ corresponds to the physical sector state $|\psi\rangle$, and R corresponds to the phenomenal sector operator H_Φ (which acts as a history-dependent filter). The diffusion term $D\nabla^2\rho$ in their model corresponds to the second-order differential structure generated by the composition of our transfer operators, $V \circ A \propto \Box\psi$ (see Theorem B.3). The "timescale separation" they identify as the cause of long-range order is the temporal manifestation of the metric orthogonality between the physical and phenomenal sectors.

3.4. Stochastic Resonance and Probability

The selection process is probabilistic, governed by a mechanism of **stochastic resonance**. This process involves three components:

1. **Signal**: The physical amplitude $|c_i|^2$ provides the driving intensity.
2. **Filter**: The history-dependent H_Φ acts as a nonlinear filter (the prior).
3. **Noise**: Irreducible indeterminacy acts as the stochastic element.

The interaction of these elements selects a phenomenal outcome. Crucially, to maintain consistency with physical law, the resulting probability distribution must match the predictions of quantum mechanics.

3.5. The Born Rule Locus Correspondence

The **Born Rule Locus Correspondence (BRLC)** guarantees this consistency. It states that the probability of a phenomenal actualization matches the physical probability defined by the Born Rule:

Theorem 3.1 (Born Rule Locus Correspondence)**.** *The phenomenal probability distribution satisfies:*

$$P(\phi_i) = |\langle \psi_i | L \rangle|^2 = P(\psi_i) \tag{17}$$

This theorem ensures that our extended framework remains empirically consistent with standard physics. The "Locus" $|L\rangle$ acts as the reference frame for these probabilities, formally defining the **embodied perspective** from which the measurement is made.

3.6. Temporal Structure: The Origin of the Phase Relationship

The algebraic structure of the transfer operators imposes a strict constraint on the timing of this cycle. As derived in Section B, the operators A and V emerge as components of a unified time operator proportional to the Pauli matrix σ_y:

$$A \sim \sigma_y(-i\partial_E), \qquad V \sim \sigma_y(+i\partial_E) \tag{18}$$

The matrix $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ generates rotations. Specifically, its action corresponds to a multiplication by $\pm i$, which is geometrically equivalent to a rotation by 90 degrees ($\pi/2$) in the complex phase plane.

This implies that the *processes* of Awareness (physical $\rightarrow$ phenomenal) and Volition (phenomenal $\rightarrow$ physical) are not merely sequential but are orthogonal in phase.

Proposition 3.2 (Quadrature Relationship)**.** *Neural oscillations associated with awareness-dominated processing and neural oscillations associated with volition-dominated processing will exhibit a systematic* $\boldsymbol{\pi/2}$ *(90-degree) phase offset.*

This is a **geometric invariant** of the theory. It predicts that the neural correlates of sensory integration and motor initiation will be "locked" in quadrature. This provides a robust, falsifiable signature of the underlying cognitive geometry.

3.7. Implications for Neural Dynamics

This framework reframes the "collapse" of the wavefunction not as a metaphysical mystery, but as a **decision process**. The selection of a definite outcome is driven by the interaction between the current physical state and the system's history (priors). The emergence of definiteness is a dynamical process with a specific temporal signature—the $\pi/2$ phase offset—which can be empirically verified.

4. Phenomenological Implications: The Structure of Embodied Experience

The axiomatic framework developed in Sections 2 and 3 has profound implications for understanding the structure of lived experience. In this section, we demonstrate that the $A|\Omega\rangle$ framework provides: (1) a formal basis for the perspectival, embodied nature of experience; (2) a resolution to the distinction between minimal awareness and high-level subjectivity; and (3) an account of how learning and practice transform the geometry of experience.

The $A|\Omega\rangle$ framework offers a rigorous formalization of embodied phenomenology. By anchoring phenomenal space to a physical locus ($|L\rangle$) and distinguishing primitive awareness ($|\Omega\rangle$) from constructed subjectivity, it resolves longstanding conceptual ambiguities. Most importantly, it translates these phenomenological structures into geometric constraints (the σ_y phase relationship) that generate testable neuroscientific predictions.

4.1. A Formal Basis for Embodiment and Perspectival Experience

A central insight of the phenomenological tradition, from Husserl (1913) to contemporary enactivism (Varela, Thompson, and Rosch 1992; Zahavi 2005), is the distinction between the body as a physical object (*Körper*) and the body as lived from the first-person perspective (*Leib*). The lived body is the pre-reflective "zero-point of orientation" from which all experience is structured. We experience the world from *here*, where "here" is constituted by our embodied situation.

Recall the Awareness Transformation (Definition 2.1):

$$A\big(|\Omega\rangle \otimes |\psi_0\rangle\big) = |\phi_0\rangle \otimes |L\rangle \tag{19}$$

The **spacetime locus** $|L\rangle \in \mathcal{H}_\Psi$ provides the formal representation of this zero-point. Crucially, $|L\rangle$ resides in the physical Hilbert space. It represents the actual spatiotemporal coordinates of the organism. However, the Awareness Transformation produces the tensor product $|\phi_0\rangle \otimes |L\rangle$: phenomenal content *bound to* a physical locus. This binding is the formal correlate of **embodied perspective**. The framework models the generation of *situated* experience—experience that is always localized in a body, oriented from a point of view.

Signals about limb position and visceral states can be understood as processes that *stabilize* the locus state $|L\rangle$. These physical processes ground the geometry of phenomenal space in the biology of the organism. The locus $|L\rangle$ naturally defines the system boundary, resolving the ambiguity often found in causal structure theories. The boundary is physically determined by the spatiotemporal extent of the integrated organism.

4.2. The Problem of Subjectivity

We now turn to the relationship between *subjectivity* and *consciousness itself*. In much of cognitive science, these are treated as equivalent: to be conscious is to have a sense of "mineness" or a "self" (Zahavi and Kriegel 2016). However, this conflation renders unintelligible a wide range of experiencesfrom flow states to contemplative absorptionwhere the sense of a narrative self dissolves while vivid awareness persists (Lutz et al. 2008; Metzinger 2003).

We require a framework that distinguishes **Minimal Phenomenal Consciousness** (bare awareness) from **High-Level Subjectivity** (the constructed self-model).

4.3. Phenomenological Distinctions: Reflexivity vs. Representation

Phenomenological research distinguishes two aspects of cognition often conflated:

1. **Pre-Reflective Self-Awareness**: The fundamental, "reflexive" nature of awareness itself. This is the bare fact that an experience is happening, prior to any reflective subject-object structuring (Zahavi 2005).
2. **The Phenomenal Self-Model (PSM)**: The constructed appearance of a "self" or "ego" facing the world (Metzinger 2003). This is a complex representational structure, not a primitive feature of awareness.

Ordinary experience involves both. However, contemplative neuroscience suggests these can dissociate: in certain states, the self-model (subjectivity) dissolves, while pre-reflective awareness remains (Lutz et al. 2008).

4.4. The Formal Resolution

The $A|\Omega\rangle$ framework maps these distinct concepts onto its core mathematical structures:

Formal Object	Cognitive Function	Phenomenological Gloss
$\|\Omega\rangle$	Background Field	Pre-Reflective Awareness
$\|\phi_0\rangle \otimes \|L\rangle$	Egocentric Reference Frame	Embodied Subjectivity (PSM)
$\|\phi_0\rangle$	Phenomenal Content	Sensory Qualia
$\|L\rangle$	Somatic Anchor	The Body-Schema

This mapping reveals that **Subjectivity is Generated, Not Primitive**. The sense of a perspectival self ($|\phi_0\rangle \otimes |L\rangle$) is produced by the Awareness Transformation. It is not present in the input ($|\Omega\rangle$). This aligns with Metzinger's (2003) argument that the "self" is a process, not an entity.

4.5. Plasticity of the Phenomenal Field

The history-dependent Phenomenal Hamiltonian H_Φ (introduced in Section 3.2) implies that the *geometry* of phenomenal space exhibits plasticity. Just as synaptic weights change with learning, the "attractor landscape" of H_Φ evolves based on the system's history.

This provides a formal mechanism for **contemplative neuroplasticity**. Practices that cultivate specific states (e.g., open monitoring) systematically reshape H_Φ, altering the probability landscapes for future experiences. Advanced practitioners may stabilize access to states approximating $|\Omega\rangle$—awareness with minimal binding to the egoic locus—consistent with reports of "non-dual" awareness (Lutz et al. 2008).

4.6. Convergence with Enactive Approaches

Our bidirectional operator structure converges with recent work in enactive cognitive science. Froese (2024) proposes an "irruption theory" of mind-matter interaction involving two processes: **Absorption** (matter $\to$ mind) and **Irruption** (mind $\to$ matter).
The mapping to our framework is direct:

$$A \text{ (Awareness Operator)} \longleftrightarrow \text{Absorption} \tag{20}$$

$$V \text{ (Volition Operator)} \longleftrightarrow \text{Irruption} \tag{21}$$

Crucially, our algebraic formulation adds specificity to this model. The σ_y structure of the operators (derived from Krein space commutation relations) predicts that these two processes are not merely distinct but are **orthogonal in phase**.

This leads directly to our central empirical prediction: neural oscillations associated with sensory reception (Absorption/Awareness) and motor initiation (Irruption/Volition) should exhibit a systematic **π/2 phase offset**.

5. A Novel and Falsifiable Empirical Prediction

An axiomatic framework moves from philosophical system to scientific theory when it makes contact with the empirical world through novel, falsifiable predictions. The $A|\Omega\rangle$ framework derives its central prediction not from contingent modeling choices but from a *geometric invariant* of the underlying mathematical structure. This prediction offers a clear, quantitative experimental target that distinguishes the framework from competing theories.

5.1. The Bidirectional Mind-Matter Interface

Any adequate theory of consciousness must account for the bidirectional relationship between mind and matter. As developed in Section 4.6, our framework converges with independent phenomenological analysis (Froese 2024) in positing two fundamental processes:

- **Absorption** (world-to-mind): The process by which physical states give rise to phenomenal content; formalized by the Awareness operator A.
- **Irruption** (mind-to-world): The process by which phenomenal states causally affect physical states; formalized by the Volition operator V.

While phenomenological analysis suggests these processes should be temporally distinct, our framework derives a much stronger constraint: they must be in **quadrature**.

5.2. The Geometric Origin of the $\pi/2$ Phase Offset

As introduced in Section 3.6, the transfer operators A and V emerge as components of a unified time operator $\mathcal{T}$ with σ_y structure:

$$T_{\pm} \;\propto\; \sigma_y\big(\mp i\partial_E\big), \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{22}$$

The Pauli matrix σ_y generates rotations by $\pi/2$ in the complex plane. This implies that the transition from physical to phenomenal (A) and the transition from phenomenal to physical (V) induce conjugate phase shifts.

Consequently, the two operators are in **quadrature**—offset by exactly 90 degrees. This is a geometric invariant. Any system instantiating this specific psychophysical geometry will necessarily exhibit this phase relationship.

5.3. Operational Hypothesis

To translate the theoretical $\pi/2$ prediction into testable experiments, we require methods capable of quantifying the phase relationship between functionally distinct neural signals. A critical methodological distinction must be made here: the prediction concerns *oscillatory* neural activity, not slow DC shifts.

For example, the Readiness Potential (RP) is a slow negative shift preceding voluntary movement. Because it is a non-oscillatory trend, it lacks a well-defined phase. However, motor preparation is also accompanied by robust oscillatory signatures, specifically **Event-Related Desynchronization (ERD)** in the Mu (8–13 Hz) and Beta (12–30 Hz) bands (Pfurtscheller and Lopes da Silva 1999). Our prediction applies to these oscillatory components.

Working Hypothesis 1. Neural oscillations associated with awareness-dominant periods (e.g., sensory integration) and neural oscillations associated with volition-dominant periods (e.g., motor preparation ERD) will exhibit a systematic $\boldsymbol{\pi/2}$ (90-degree) phase offset.

This hypothesis can be tested using established neuroimaging paradigms:

1. **Cross-Frequency Phase-Amplitude Coupling (PAC)**: In volitional tasks, the phase of low-frequency driver oscillations (e.g., Theta/Alpha in executive regions) modulates the amplitude of high-frequency activity (Gamma in sensory regions). We predict that the optimal phase for this coupling will cluster at $\pi/2$, distinguishable from the 0 or π phase typically found in purely sensory processing (Canolty and Knight 2010; Tort et al. 2010).
2. **Perceptual Threshold Dynamics**: In "Hit" vs. "Miss" trials near perceptual threshold, successful conscious access involves a transition from pre-conscious processing to global ignition. We predict that the locking of sensory oscillations to the executive "pacing" rhythm will show a quadrature phase shift specifically on trials where the subject reports awareness.
3. **Sensorimotor Synchronization**: In tasks requiring synchronization of perception and action (e.g., rhythmic tapping), the phase lag between auditory cortex (sensory) and motor cortex (volitional) is often assumed to be optimized for zero-lag synchrony. We predict that a fine-grained analysis of the LFP phase difference will reveal a fundamental $\pi/2$ offset component that organizes the timing of the loop.

4. **Traveling Wave Gradients**: As noted by Zhang et al. (2018), low-frequency oscillations often propagate as traveling waves. If awareness and volition occupy orthogonal phase spaces, the phase gradient of these waves as they traverse the frontoparietal network should exhibit discontinuities or shifts clustering near 90°.

This prediction aligns with recent computational models of cortical dynamics. Sun et al. (2025) demonstrate that "time non-locality" (memory) induces a robust phase of long-range order characterized by a separation of timescales between neural activity and resource dynamics. In our framework, this separation corresponds to the interaction between the fast physical state evolution and the slower, history-dependent phenomenal landscape, resulting in the predicted quadrature phase relationship.

5.4. Open Specifications

While the $\pi/2$ offset is a geometric invariant, the specific carrier frequencies are empirical variables:

- **Frequency Bands**: Candidates include Theta (4–8 Hz) for long-range integration and Beta (12–30 Hz) for motor control.
- **Anatomical Loci**: The relevant "Locus" $|L\rangle$ varies by task (e.g., visual cortex for sight, motor cortex for action).

The prediction is robust against these variables: *systematic quadrature* is the signature of the psychophysical operator structure, regardless of the specific frequency band used to implement it.

5.5. Distinctiveness from Competing Theories

This prediction distinguishes the $A|\Omega\rangle$ framework from other leading theories:

- **Global Workspace Theory (GWT)** typically predicts long-range **synchrony** (0° phase lag) during conscious access ("ignition").
- **Integrated Information Theory (IIT)** predicts high Φ but does not specify phase relationships between functional subnetworks.
- **Recurrent Processing Theory** predicts re-entrant loops but does not specify a quadrature constraint.

A finding of systematic $\pi/2$ phase clustering would therefore effectively falsify synchrony-based models in favor of the geometric operator model proposed here.

6. Conclusion: A New Axiomatic Path for Consciousness Science

This paper began by identifying a critical vulnerability in the theoretical foundations of cognitive science: the lack of mathematically rigorous, invariant models has created a crisis of falsifiability, leaving the field's most prominent theories susceptible to being untestable in principle. We argued that a path forward requires moving beyond the search for mere neural correlates and developing formal, generative models that are fully integrated with the principles of fundamental physics. The theory of **Cognitive Geometry**—the $A|\Omega\rangle$ framework—was introduced to meet this challenge.

We have shown that this framework, built upon the minimal axiomatic extension of a dual-phase space geometry, provides a self-consistent and powerful language for describing the psychophysical relationship. Its core principles—the Awareness and Volition operators, the Core Transformation Equation, and the Born Rule Locus Correspondence—work in concert to form a coherent, generative model. By anchoring phenomenal space to a physical spacetime locus ($|L\rangle$), the model provides a formal mathematical basis for **embodied neurophenomenology** (Varela 1996), resolving deep ambiguities concerning the relationship between pre-reflective awareness and constructed subjectivity.

Crucially, the framework's axiomatic structure yields a novel, quantitative, and falsifiable prediction—the **π/2 phase offset** in the sensorimotor loop. This prediction is grounded in the fundamental geometry of the operator algebra (σ_y), not in ad-hoc computational assumptions. This makes the theory robust against the "pre-falsification" critiques that have challenged causal structure theories, satisfying the rigorous criteria for a scientific theory outlined by Hanson and Walker (2021).

The $A|\Omega\rangle$ framework is therefore proposed not merely as another competing theory of consciousness, but as a new foundational paradigm. By unifying the mathematical language of quantum physics with the structural insights of phenomenology, this approach offers the tools to move beyond the descriptive search for correlates toward a generative science capable of deriving the structure of lived experience from first principles.

A. Formal Constraints on Strong Emergence

The main text argues that a formal theory of consciousness requires an axiomatic foundation rather than relying on the concept of strong emergence. This appendix provides the mathematical arguments motivating this choice. We demonstrate that within standard mathematical frameworks (set theory, group theory, category theory), the properties traditionally ascribed to "strongly emergent" phenomena cannot be formalized without introducing discontinuities or breaking the closure of the underlying system. This motivates the **Principle of Minimal Extension** and the dual-phase space geometry of the $A|\Omega\rangle$ framework.

A.1. Set-Theoretic Constraints

From a set-theoretic perspective, the claim of strong emergence faces a closure problem.

Definition A.1 (Physical System and Property Closure)**.** A physical system S is a structured set of elements subject to laws $\mathcal{L}$. The *property closure* $\mathrm{Cl}(S, \mathcal{L})$ is the set of all properties that can be derived from the complete description of S under the laws $\mathcal{L}$.

The thesis of strong emergence asserts the existence of a property P of the system S such that P is not derivable from the description of S under $\mathcal{L}$.

Theorem A.1 (Extension Necessity)**.** *If a property P belongs to system S but $P \notin \mathrm{Cl}(S, \mathcal{L})$, then the system S as defined by $\mathcal{L}$ is incomplete.*

Proof. If P is a property of S, and $\mathrm{Cl}(S, \mathcal{L})$ contains all properties derivable from S through $\mathcal{L}$, then the condition $P \notin \mathrm{Cl}(S, \mathcal{L})$ implies that P cannot be accounted for without modifying either the elements of S or the laws $\mathcal{L}$. Therefore, strong emergence implies that the physical description is mathematically insufficient to capture P, necessitating an axiomatic extension of the state space. □

A.2. Group-Theoretic Constraints

In physics, the invariants of a system are determined by its symmetry groups.

Definition A.2 (Symmetry Group and Observable Space)**.** A *symmetry group* G of a physical system S defines the set of transformations that leave the laws governing S invariant. The *observable space* $\mathcal{O}(G)$ is the set of all physical observables compatible with these symmetries.

Theorem A.2 (Conservation of Invariants). *No dynamical process governed solely by group G can generate observables corresponding to invariants outside of $\mathcal{O}(G)$.*

Proof. While spontaneous symmetry breaking can alter the *state* of a system (hiding symmetries), it does not generate new fundamental Casimir invariants. The observables of the broken phase are still constructed from the representations of the original group G. Therefore, a genuinely novel property that does not transform according to any representation of G requires the introduction of a new symmetry group G', not merely dynamical evolution under G. □

A.3. Category-Theoretic Constraints

Category theory allows us to analyze the structural mappings between domains. We consider the relationship between a category of physical entities, $\mathcal{C}_{\text{phys}}$, and a category of phenomenal entities, $\mathcal{C}_{\text{phen}}$.

Theorem A.3 (Non-Functoriality of Strong Emergence). *A mapping $E : \mathcal{C}_{phys} \to \mathcal{C}_{phen}$ that exhibits strong emergence cannot be a functor.*

Proof Sketch. A functor must preserve the structure of objects and morphisms (composition and identity). Strong emergence posits that $\mathcal{C}_{\text{phen}}$ contains structure not present in or derivable from $\mathcal{C}_{\text{phys}}$. If E were a functor, the structure of the image in $\mathcal{C}_{\text{phen}}$ would be fully determined by the structure of the domain $\mathcal{C}_{\text{phys}}$. The claim that the output contains novel, underivable structure implies that the mapping fails to preserve compositionality or identity, and thus is not a functor. This suggests that the relationship between physics and consciousness cannot be modeled as a structure-preserving map within a single category, but requires a product category structure ($\mathcal{C}_{\text{phys}} \times \mathcal{C}_{\text{phen}}$), as proposed in our dual-phase space model. □

B. Mathematical Structure of the Dual-Phase Space

This appendix defines the core mathematical structures of the $A|\Omega\rangle$ framework. While the main text focuses on the neuroscientific implications, the formal machinery here provides the rigorous basis for those predictions. We employ an **indefinite-metric Krein space** geometry, which allows for a precise modeling of the relationship between two distinct but interacting domains (physical and phenomenal).

B.1. The Kinematical State Space

The foundational innovation of this framework is the use of a tensor product structure to combine physical and phenomenal degrees of freedom.

Definition B.1 (The Kinematical Krein Space)**.** The total state space is the tensor product:

$$\mathcal{K} = \mathcal{H}_\Phi \otimes \mathcal{H}_\Psi \tag{1}$$

where $\mathcal{H}_\Psi$ is the Physical Hilbert Space and $\mathcal{H}_\Phi$ is the Phenomenal Hilbert Space. The space carries an **indefinite inner product** defined by the Fundamental Symmetry operator J:

$$[\Phi_1, \Phi_2] = \langle \Phi_1 | J | \Phi_2 \rangle, \qquad J = J_\Phi \otimes I_\Psi \tag{2}$$

where J_Φ is the metric operator on the phenomenal sector ($J_\Phi^2 = I$, $J_\Phi^\dagger = J_\Phi$).

Remark B.1 (Why Indefinite Metric?)**.** This indefinite metric is essential for modeling the ontological distinctness of the two sectors. Just as the time dimension in relativity has a distinct metric signature from spatial dimensions, the phenomenal sector has a distinct metric signature from the physical. This ensures that subjective experience cannot be simply "reduced" to physical extension, even while they interact.

B.2. Sectoral Structure and Observables

Proposition B.1 (Sectoral Independence)**.** *Physical observables ($I_\Phi \otimes \mathcal{O}_\Psi$) and phenomenal observables ($\mathcal{O}_\Phi \otimes I_\Psi$) commute:*

$$\left[I_\Phi \otimes \mathcal{O}_\Psi,\ \mathcal{O}_\Phi \otimes I_\Psi \right] = 0 \tag{3}$$

Remark B.2 (Interpretation)**.** This formalizes the principle that **observation is distinct from introspection**. Measuring a neuron's voltage does not directly collapse a qualia state (without passing through the transfer operators). This ensures the framework respects the causal closure of physics at the level of simple observation.

B.3. The Transfer Operators

The interaction between domains is mediated by **transfer operators** which constitute the off-diagonal blocks of the generalized time operator $\mathcal{T}$:

$$\mathcal{T} = \begin{pmatrix} 0 & A \\ V & 0 \end{pmatrix} \tag{4}$$

Theorem B.1 (Nilpotency)**.** *The operators are nilpotent: $A^2 = 0$ and $V^2 = 0$.*

Remark B.3 (The Necessity of the Cycle)**.** Nilpotency implies that cognition is inherently **cyclic**. One cannot have "awareness of awareness" ($A \circ A$) as a primitive operation; awareness must trigger a physical update before it can be re-accessed. This mirrors the **Action-Perception Cycle** in enactive cognitive science (Noë 2004) and the **Perception-Action (PA) Cycle** in systems neuroscience (Fuster 2003).

Theorem B.2 (σ_y Structure)**.** *Consistency conditions in Krein space force the operators to take the form:*

$$A, V \;\propto\; \sigma_y\big(\mp i\partial_E\big) \qquad \textit{where} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{5}$$

Corollary B.1 (The Empirical Prediction)**.** *The matrix σ_y generates rotations by $90°$ ($\pi/2$). This is the mathematical source of our central empirical prediction: the neural correlates of Awareness and Volition must be phase-offset by $\pi/2$.*

B.4. Ground States and the Core Transformation

Definition B.2 (Core Transformation)**.** The **Awareness Transformation** generates embodied experience:

$$A\big(|\Omega\rangle \otimes |\psi_0\rangle\big) = |\phi_0\rangle \otimes |L\rangle \tag{6}$$

The **Volition Transformation** effects mental causation:

$$V\big(|\phi_0\rangle \otimes |L\rangle\big) = |\Omega\rangle \otimes |\psi_0'\rangle \tag{7}$$

Here, $|L\rangle$ is the **Spacetime Locus**—the physical state representing the organism's egocentric reference frame.

B.5. Dynamical Consequences

Theorem B.3 (Emergence of Wave Dynamics). *The composition of Awareness and Volition generates second-order dynamics:*

$$V \circ A \ \propto \ \Box \quad \textit{(Wave Operator)} \tag{8}$$

Proposition B.2 (Effective Unitarity). *Although $\mathcal{K}$ has an indefinite metric, the transfer operators satisfy **effective unitarity** when restricted to the physical sector. This ensures that probability is conserved ($P(\phi) = P(\psi)$), providing the rigorous basis for the **Born Rule Locus Correspondence** (Theorem 3.1).*

Mathematical Object	Neuroscientific Correlate	Phenomenological Correlate
$\|\Omega\rangle$ (Phenomenal Vacuum)	Resting State / Background	Pre-Reflective Awareness
$\|L\rangle$ (Spacetime Locus)	Body Schema / Egocentric Frame	Embodied Point of View
A (Awareness Operator)	Sensory Integration / Binding	Appearance of Content
V (Volition Operator)	Motor Initiation / Executive Control	Sense of Agency
σ_y (Operator Structure)	$\pi/2$ Phase Offset	Temporal Flow of Experience

C. The Born Rule Locus Correspondence

This appendix establishes the **Born Rule Locus Correspondence** (BRLC), which ensures that the structure of phenomenal experience maps isomorphically onto the structure of physical states. This correspondence is essential for empirical adequacy: it guarantees that the statistical properties of neural dynamics (measured physically) perfectly predict the statistical properties of subjective reports (measured phenomenologically).

C.1. The Correspondence Principle

The BRLC addresses the fundamental question of **Psychophysical Isomorphism**: how are the probabilities of various phenomenal experiences related to the probabilities of corresponding neural states?

Definition C.1 (Born Rule Locus Correspondence)**.** For a physical state $|\psi\rangle = \sum_i c_i |\psi_i\rangle$ and its phenomenal counterpart $|\phi\rangle$, the probability of experiencing phenomenal content $|\phi_i\rangle$ equals the probability of the physical system occupying state $|\psi_i\rangle$:

$$P(\phi_i) = |c_i|^2 = P(\psi_i) \tag{9}$$

This correspondence is computed relative to the **Spacetime Locus** $|L\rangle$—the physical state of the experiencing organism.

C.2. Amplitude Preservation

Although the transfer operators operate in an indefinite-metric space, they preserve information structure.

Theorem C.1 (Amplitude Preservation)**.** *The Awareness operator A preserves probability amplitudes. If a neural state has the expansion $|\psi\rangle = \sum_i c_i |\psi_i\rangle$, then the corresponding phenomenal state $|\phi\rangle$ retains the* same *weighting coefficients c_i.*

Proof. This follows from the **effective unitarity** of the operator A when restricted to the physical sector (see Proposition B.2). The operator maps the physical superposition into a correlated phenomenal superposition without distorting the relative weights of the components. □

C.3. The Locus as Egocentric Reference Frame

The locus $|L\rangle$ plays a crucial role in defining the **perspective** of the conscious system.

Proposition C.1 (Locus as Reference Frame)**.** *The spacetime locus $|L\rangle$ defines the **egocentric reference frame** from which probabilities are computed. The Born Rule applies to the physical state $|\psi\rangle$ as measured from the perspective of the locus.*

This formalizes the insight that consciousness is always **situated**. Probabilities are not absolute; they are relative to the observer's embodiment.

Remark C.1 (Biological Constraints on the Basis)**.** This result explains why experience is structured by specific sensory modalities (e.g., vision, audition) rather than arbitrary quantum superpositions. The **preferred basis** $\{|\psi_i\rangle\}$ is determined by the physical configuration of the locus—the specific sensory apparatus and neural architecture of the organism. A bat, having a different locus (echolocation), will have a different natural basis for experience than a human.

C.4. Formal Statement

Theorem C.2 (Born Rule Locus Correspondence — Full Statement)**.** *Let $|\psi\rangle \in \mathcal{H}_\Psi$ be a neural state with expansion $|\psi\rangle = \sum_i c_i|\psi_i\rangle$ in a basis determined by the organism's sensory architecture (the locus $|L\rangle$). Let $|\phi\rangle \in \mathcal{H}_\Phi$ be the corresponding phenomenal state generated by Awareness:*

$$A\big(|\Omega\rangle \otimes |\psi\rangle\big) = |\phi\rangle \otimes |L\rangle \tag{10}$$

Then:

1. *The phenomenal state inherits the expansion $|\phi\rangle = \sum_i c_i|\phi_i\rangle$.*
2. *The probability of experiencing phenomenal content $|\phi_i\rangle$ is $P(\phi_i) = |c_i|^2$.*
3. *This equals the probability of measuring neural state $|\psi_i\rangle$ from the reference frame of the body $|L\rangle$.*

The BRLC ensures that the subjective statistics of experience (e.g., psychometric functions) match the objective statistics of neural firing, preserving the consistency of the science of consciousness.

D. Conservation Laws and the Unmarking Theorem

This appendix establishes the conservation laws governing the relationship between physical and phenomenal states. We prove the **Unmarking Theorem**, which resolves the tension between mental causation and the causal closure of physics. The key insight is that phenomenal states act as **hidden variables** in the global state space—causally efficacious, yet transparent to purely physical measurements.

D.1. Tensor Product Norm Structure

Theorem D.1 (Tensor Product Norm Factorization)**.** *For normalized states $|\phi\rangle \in \mathcal{H}_\Phi$ and $|\psi\rangle \in \mathcal{H}_\Psi$, the norm factorizes: $\||\phi\rangle \otimes |\psi\rangle\|^2 = \||\phi\rangle\|^2 \cdot \||\psi\rangle\|^2 = 1$.*

Corollary D.1 (Ground State Coupling)**.** *Coupling a physical state to the phenomenal ground state does not change its norm: $\||\Omega\rangle \otimes |\psi\rangle\| = \||\psi\rangle\|$.*

D.2. Conservation Under Awareness

Theorem D.2 (Norm Conservation)**.** *The Awareness transformation preserves the norm of the kinematical state: $\|A\big(|\Omega\rangle \otimes |\psi\rangle\big)\| = \||\Omega\rangle \otimes |\psi\rangle\|$.*

Proof. This follows from the effective unitarity of A in the physical sector (see Proposition B.2). $\square$

D.3. The Unmarking Theorem

This theorem explains why consciousness eludes detection by physical instruments.

Theorem D.3 (Unmarking Theorem)**.** *Physical observables cannot distinguish between a physical state in isolation and the same physical state coupled to phenomenal content. For any physical observable $\mathcal{O}_\Psi$:*

$$\langle \phi \otimes L | (I_\Phi \otimes \mathcal{O}_\Psi) | \phi \otimes L \rangle = \langle L | \mathcal{O}_\Psi | L \rangle \tag{11}$$

Remark D.1 (Physical Interpretation)**.** The Unmarking Theorem states that consciousness is **physically transparent**. No measurement performed solely within the physical sector ($I_\Phi \otimes \mathcal{O}_\Psi$) can detect the presence of phenomenal content $|\phi\rangle$. This provides a formal measurement-theoretic definition for the **Explanatory Gap** (Levine 1983), which we term the **"Hard Problem" of Detection**: the structural inability of reductive physical probes to register global relational properties.

D.4. Resolution of Causal Closure

Proposition D.1 (Causal Efficacy Without Violation)**.** *Phenomenal states are causally efficacious via the Volition operator V, yet this efficacy does not violate the conservation laws of physics.*

The resolution relies on the fact that the interaction occurs within the full kinematical space $\mathcal{K}$:

1. The Volition operator maps $|\phi\rangle \otimes |L\rangle \to |\Omega\rangle \otimes |\psi'\rangle$. The phenomenal content $|\phi\rangle$ determines the outcome $|\psi'\rangle$.
2. However, because V is effectively unitary, all physical conservation laws (energy, momentum) are respected across the transformation.
3. To an external observer measuring only physical variables, the system appears to evolve according to probabilistic physical laws. The "hidden variable" determining the specific outcome is the phenomenal state.

Remark D.2 (Analogy to Gauge Invariance)**.** This structure is analogous to **Gauge Invariance** in physics. Gauge potentials (A_μ) are real and necessary for the theory, but they are not directly observable; only field strengths ($F_{\mu\nu}$) are observable. Similarly, phenomenal states are real and necessary for the psychophysical dynamics, but they are not directly observable by physical instruments.

D.5. Summary

Result	**Implication**
Norm factorization	Phenomenal coupling is non-perturbative
Norm conservation	Awareness is a lossless transformation
Unmarking Theorem	Consciousness is transparent to physical probes
Causal efficacy	Mental causation compatible with physical laws
Gauge Analogy	Phenomenology acts as a hidden sector

The Unmarking Theorem explains the historical difficulty of the scientific study of consciousness: the object of study is mathematically guaranteed to be invisible to the standard instruments of physical science, requiring the extended axiomatic framework proposed here.

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