

Summary for single convex funciton optimization f

B stands for B -Lipschitz of f , R is the diameter of X , L stands for L -smoothness, μ stands for μ -strongly convexity, and $\kappa := L/\mu$.

name	γ_t	condition	last/average iterate convergence	convergence rate w.r.t T	ϵ optimal iteration
GD / PGD	$\frac{R}{B\sqrt{T}}$	B	Avg	$\mathcal{O}(1/\sqrt{T})$	$\mathcal{O}(1/\epsilon^2)$
GD / PGD / Mirror GD	$\frac{1}{L}$	L	Last	$\mathcal{O}(1/T)$	$\mathcal{O}(1/\epsilon)$
Nesterov Accelerated GD	Addaptive	L	Last	$\mathcal{O}(1/T^2)$	$\mathcal{O}(1/\sqrt{\epsilon})$
GD / PGD	$\frac{1}{L}$	L, μ	Last	$\mathcal{O}\left(\left(1 - \frac{\mu}{L}\right)^T\right)$	$\mathcal{O}\left(\kappa \ln\left(\frac{1}{\epsilon}\right)\right)$
GD	$\frac{1}{L}$	L, μ -PL	Last	$\mathcal{O}\left(\left(1 - \frac{\mu}{L}\right)^T\right)$	$\mathcal{O}\left(\kappa \ln\left(\frac{1}{\epsilon}\right)\right)$
Coorinate GD	$\frac{1}{L_i}$	L, μ	Last	$\mathcal{O}\left(\left(1 - \frac{\mu}{dL}\right)^T\right)$	$\mathcal{O}\left(d\kappa \ln\left(\frac{1}{\epsilon}\right)\right)$
Sub GD / Mirror GD	$\frac{R}{B\sqrt{T}}$	B	Avg	$\mathcal{O}\left(\frac{1}{\sqrt{T}}\right)$	$\mathcal{O}(1/\epsilon^2)$
Sub GD	$\frac{f(x_t)-f^*}{\frac{\ g(x_t)\ _2^2}{2} \cdot \frac{2}{\mu(t+1)}}$	B, μ	Avg	$\mathcal{O}\left(\frac{1}{T}\right)$	$\mathcal{O}(1/\epsilon)$
Frank-Wolfe	$\frac{2}{t+2}$	L	Last	$\mathcal{O}\left(\frac{1}{T}\right)$	$\mathcal{O}(1/\epsilon)$
Newton	Auto	Hessian B - Lipschitz, μ	Last	$\mathcal{O}\left(\left(\frac{1}{2}\right)^{2^T-1}\right)$	$\mathcal{O}(\log \log(1/\epsilon))$
Newton	Auto	L, μ	Last	$\mathcal{O}\left(\left(1 - \frac{\mu^2}{L^2}\right)^t\right)$	$\mathcal{O}\left(\kappa^2 \ln\left(\frac{1}{\epsilon}\right)\right)$

Summary for stochastic funciton optimization of $F = \mathbb{E}[f_\xi]$.

name	γ_t	condition	last/average iterate convergence	convergence rate w.r.t T	ϵ optimal iteration of f or $ \nabla f $
SGD	$\frac{R}{B\sqrt{T}}$	B	Avg	$\mathcal{O}\left(\frac{1}{\sqrt{T}}\right)$	$\mathcal{O}\left(\frac{1}{\epsilon^2}\right)$
SGD	$\frac{1}{2\mu}$	B, μ	Last	$\mathcal{O}\left(\frac{1}{T}\right)$	$\mathcal{O}\left(\frac{1}{\epsilon}\right)$
SGD (!convex)	$\min\left\{\frac{1}{L}, \frac{\gamma}{\sigma\sqrt{T}}\right\}$	$L,$ $\text{var}[\nabla f_\xi] \leq \sigma$	Last	$\mathcal{O}\left(\frac{1}{T^{1/4}}\right)$	$\mathcal{O}\left(\frac{1}{\epsilon^4}\right)$

Summary for stochastic funciton optimization of finite sum $F = \sum_i f_i/n$.

name	γ_t	condition	Type	Error w.r.t T	ϵ -iteration	comp/iter	ϵ -cost
Full GD	$\frac{1}{L_i}$	$\overline{L_i}, \mu$	Last	$\mathcal{O}\left(\left(1 - \frac{\mu}{L_i}\right)^T\right)$	$\mathcal{O}\left(\kappa \ln\left(\frac{1}{\epsilon}\right)\right)$	$\mathcal{O}(n)$	$\mathcal{O}\left(n\kappa \ln\left(\frac{1}{\epsilon}\right)\right)$
SAG/SAGA	$\frac{1}{16L}$	L_i	Last	$\mathcal{O}\left(\left(1 - \min\left\{\frac{C}{n}, \frac{C'}{\kappa}\right\}\right)^T\right)$	$\mathcal{O}\left((n + \kappa) \ln\left(\frac{1}{\epsilon}\right)\right)$	$\mathcal{O}(1)$	$\mathcal{O}\left((n + \kappa) \ln\left(\frac{1}{\epsilon}\right)\right)$
SVRG	$< \frac{1}{2L}$	L_i	Last	$\mathcal{O}(\rho^T)$	$\mathcal{O}\left((n + \kappa) \ln\left(\frac{1}{\epsilon}\right)\right)$	$\mathcal{O}(2)$	$\mathcal{O}\left((n + \kappa) \ln\left(\frac{1}{\epsilon}\right)\right)$
SPIDER (!convex)	$< \frac{1}{2L}$	Avg- $L,$ $\text{var}[\nabla f_\xi] \leq \sigma^2$	Avg	$\mathcal{O}\left(\frac{1}{\sqrt{T}}\right)$	$\mathcal{O}\left(\frac{1}{\epsilon^2}\right)$	$\mathcal{O}\left(\frac{\sigma}{\epsilon}\right)$	$\mathcal{O}\left(\frac{\sigma}{\epsilon^3}\right)$