Quantitative Finance Notes: Bonds

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1. What is a Bond?

A **bond** is a fixed-income instrument that represents a loan made by an investor to a borrower (typically corporate or governmental). The bond holder receives periodic interest payments (coupons) and the return of principal at maturity.

Mathematically, the price P of a bond paying annual coupons C and face value F maturing in T years with yield r is given by:

$$P = \sum_{t=1}^{T} \frac{C}{(1+r)^t} + \frac{F}{(1+r)^T}$$

2. Types of Bonds

- **Zero-Coupon Bond:** Pays no periodic interest, only principal at maturity.
- **Coupon Bond:** Pays periodic fixed interest (coupons).
- **Floating Rate Bond:** Coupon rate is variable, often linked to a benchmark rate.
- **Government Bond:** Issued by national governments (e.g., US Treasury).
- **Corporate Bond:** Issued by corporations; higher risk than government bonds.
- **Callable Bond:** Can be redeemed early by the issuer.

3. Parties Involved

- **Issuer:** The borrower (government or corporation).
- **Bondholder:** The lender/investor who purchases the bond.
- **Underwriter:** Helps issue the bond in primary market (e.g., investment bank).
- **Rating Agencies:** Assess credit risk (e.g., Moody's, S&P).

4. Why Bonds are Considered Risk-Free (in theory)

- Government bonds (like U.S. Treasuries) are backed by the full faith and credit of the government.
- They are assumed to have no default risk.
- Risk-free rate (used in DCF, CAPM, BSM) is typically derived from short-term treasury yields.

5. CQF-Style Questions and Answers

Q1: What is the formula for the price of a zero-coupon bond?

A:

$$P = \frac{F}{(1+r)^T}$$

Q2: What does the term "duration" refer to?

A: Duration measures the sensitivity of a bond's price to changes in interest rates. It is the weighted average time to receive cash flows.

Q3: What is the difference between yield to maturity and coupon rate?

A: Coupon rate is the fixed annual interest paid by the bond. Yield to maturity (YTM) is the internal rate of return assuming the bond is held to maturity.

Q4: Why might a government bond not be truly risk-free in practice?

A: Due to inflation risk, currency risk (for foreign investors), or sovereign credit events in emerging markets.

Q5: What is the relationship between bond price and interest rates?

A: Inverse. As interest rates rise, bond prices fall, and vice versa.

Next Topics Preview

- Yield Curve and Term Structure
- Credit Spread
- Pricing of Callable Bonds