

Financial Derivatives and Zero-Sum Games

1. What are Financial Derivatives?

Financial derivatives are contracts whose value is derived from the performance of underlying assets, indexes, or interest rates. Common underlyings include stocks, bonds, commodities, currencies, interest rates, and market indexes.

Key Characteristics:

- Derive value from underlying instruments.
- Can be used for hedging, speculation, or arbitrage.
- Often traded over-the-counter (OTC) or on exchanges.

2. Types of Financial Derivatives

2.1 Forward Contracts

A forward contract is a customized agreement between two parties to buy or sell an asset at a specified future date for a price agreed upon today.

Payoff:

$$\text{Long Forward Payoff} = S_T - K \quad \text{Short Forward Payoff} = K - S_T$$

Where S_T is the spot price at maturity and K is the forward price.

2.2 Futures Contracts

Standardized forward contracts traded on exchanges. They involve daily settlement (mark-to-market) and margin requirements.

Key Difference: Unlike forwards, futures are regulated and liquid.

2.3 Options

Contracts that give the holder the right (but not the obligation) to buy (Call) or sell (Put) an asset at a specified price before/at a specified date.

$$\text{Call Option Payoff} = \max(S_T - K, 0) \quad \text{Put Option Payoff} = \max(K - S_T, 0)$$

Types:

- *European*: Exercisable only at maturity.
- *American*: Exercisable any time before expiry.

2.4 Swaps

Swaps are agreements to exchange future cash flows. Common types include:

- **Interest Rate Swaps**: Fixed-for-floating rate exchange.
- **Currency Swaps**: Exchange of principal and interest payments in different currencies.

3. Zero-Sum Game in Derivatives

In many derivative contracts (especially futures and forwards), the gain of one party is exactly the loss of the other. This makes them a **zero-sum game**.

Definition:

A zero-sum game is a situation where one participant's gain or loss is exactly balanced by the losses or gains of the other participants.

$$\text{Payoff}_A + \text{Payoff}_B = 0$$

Examples:

- In a forward contract, if the buyer makes \$100, the seller loses \$100.
- In futures trading, the profit of the long position is the exact loss of the short.

4. Financial Games and Strategic Behavior

Derivatives often involve game-theoretic behavior:

- **Hedging Game**: Institutions use options to protect portfolios—insurance-type games.
- **Arbitrage Game**: Traders exploit mispricings in options or forwards—competitive zero-sum outcomes.
- **Regulatory Game**: Institutions may shift derivatives positions in response to capital regulations.

Nash Equilibrium in Derivatives

In large institutional settings, portfolio decisions can resemble strategic games where players reach equilibrium based on their expectations about others' hedging/speculation behavior.

5. Derivatives and Market Completeness

Derivatives help in:

- **Completing Markets:** Allow replication of otherwise unattainable payoffs.
- **Transferring Risk:** From risk-averse hedgers to risk-tolerant speculators.
- **Improving Liquidity:** Derivatives attract diverse participants.

6. Conclusion

Derivatives are fundamental instruments in modern financial markets. While they allow sophisticated risk management and speculation, they also represent a strategic battleground with zero-sum dynamics at their core—making them prime examples of financial games.

CQF/CFA-Style Multiple-Choice Questions on Financial Derivatives

MCQ 1

Which of the following is NOT a derivative instrument?

- (A) Forward contract
- (B) Treasury bond
- (C) Call option
- (D) Swap

Answer: (B)

MCQ 2

The value of a derivative primarily depends on:

- (A) Interest rates alone
- (B) Time to maturity alone
- (C) The value of an underlying asset
- (D) Market capitalization

Answer: (C)

MCQ 3

Which derivative gives the holder the right but not the obligation to buy an asset?

- (A) Short future
- (B) Long forward
- (C) Call option
- (D) Put option

Answer: (C)

MCQ 4

What is the payoff for a European call option at maturity?

- (A) $\max(K - S_T, 0)$
- (B) $\max(S_T - K, 0)$
- (C) $S_T + K$
- (D) $S_T \times K$

Answer: (B)

MCQ 5

Which of the following best describes a forward contract?

- (A) Traded on exchanges and standardized
- (B) Settled daily with margin requirements
- (C) OTC agreement to buy/sell at a future date
- (D) Offers the right but not the obligation to transact

Answer: (C)

MCQ 6

In a zero-sum game, the total payoff of all players:

- (A) Equals zero
- (B) Is always positive
- (C) Is always negative
- (D) Depends on the underlying

Answer: (A)

MCQ 7

A trader expecting a rise in interest rates should:

- (A) Long bond futures
- (B) Short bond futures
- (C) Buy calls on bonds
- (D) Buy puts on bonds

Answer: (B)

MCQ 8

Which of the following is a distinguishing feature of futures over forwards?

- (A) Privately negotiated
- (B) Non-standardized
- (C) Mark-to-market daily
- (D) Delivered only physically

Answer: (C)

MCQ 9

Which is true for a long forward contract at maturity?

- (A) Payoff = $S_T - K$
- (B) Payoff = $K - S_T$
- (C) Payoff = $S_0 - K$
- (D) Payoff = $S_T + K$

Answer: (A)

MCQ 10

An interest rate swap is an agreement to exchange:

- (A) Currency rates
- (B) Option premiums
- (C) Fixed and floating interest payments
- (D) Equity for debt

Answer: (C)

MCQ 11

Which of the following positions will benefit most from high volatility?

- (A) Long stock
- (B) Short put
- (C) Long call
- (D) Short call

Answer: (C)

MCQ 12

Which option strategy involves buying a call and a put with the same strike?

- (A) Bull spread
- (B) Butterfly spread
- (C) Straddle
- (D) Collar

Answer: (C)

MCQ 13

Who gains in a swap if floating rates rise?

- (A) Fixed-rate receiver
- (B) Floating-rate payer
- (C) Fixed-rate payer
- (D) Floating-rate receiver

Answer: (D)

MCQ 14

Which of the following increases the value of a call option (all else equal)?

- (A) Lower underlying volatility
- (B) Shorter time to maturity
- (C) Higher interest rates
- (D) Lower underlying price

Answer: (C)

MCQ 15

Which of the following risks is most associated with OTC derivatives?

- (A) Liquidity risk
- (B) Credit risk
- (C) Operational risk
- (D) Reinvestment risk

Answer: (B)

MCQ 16

A European put option will be in-the-money at expiration if:

- (A) $S_T > K$
- (B) $S_T < K$
- (C) $S_T = K$
- (D) $S_T \geq K$

Answer: (B)

MCQ 17

Derivatives are used by financial institutions for all EXCEPT:

- (A) Hedging exposure
- (B) Capital raising
- (C) Arbitrage
- (D) Speculation

Answer: (B)

MCQ 18

Which of the following best defines margin in futures trading?

- (A) Fee paid for the right to trade
- (B) Deposit as a performance bond
- (C) Net asset value of the position
- (D) Trading commission

Answer: (B)

MCQ 19

If a trader sells a call option, their maximum loss is:

- (A) Unlimited
- (B) Zero
- (C) Limited to the premium received
- (D) Equal to the strike price

Answer: (A)

MCQ 20

Which statement is true regarding a long straddle position?

- (A) Profits only if the stock moves significantly
- (B) Profits only if the stock stays flat
- (C) Has limited profit potential
- (D) Loses when volatility rises

Answer: (A)