The Sample Space.

Distribution of r balls into n cells.

The below table describes all possible outcomes of the experiment of placing three balls into three cells.

1.	abc	-	-	10.	a	bc	-	19.	-	a	bc
2.	-	abc	-	11.	b	ac	-	20.	-	b	ca
3.	-	-	abc	12.	c	ab	-	21.	-	c	ab
4.	ab	c	-	13.	a	-	bc	22.	a	b	c
5.	ac	b	-	14.	b	-	ac	23.	a	c	b
6.	bc	a	-	15.	c	-	ab	24.	b	a	c
7.	ab	-	c	16.	-	ab	c	25.	b	c	a
8.	ac	-	b	17.	-	ac	b	26.	c	a	b
9.	bc	-	a	18.	-	bc	a	27.	c	b	a

The sample space consists of $3^3 = 27$ points. Each of the arrangements represents a simple event, that is, a sample point.

We list here a number of situations in which the intuitive background varies; all are, however, abstractly equivalent to the scheme of placing r balls into n cells, in the sense that the outcomes differ only in their verbal description.

(b,1) *Birthdays.* The possible configurations of the birthdays of r people correspond to the different arrangements of r balls into $n = 365$ cells.

(b,2) *Accidents.* Classifying r accidents according to the weekdays when they occurred is equivalent to placing r balls into $n = 7$ cells.

(b,3) In *firing* at n targets, the hits correspond to the balls, the targets to the cells.

(b,4) *Sampling.* Let a group of r people be classified according to, say, age or profession. The classes play the role of our cells, the people that of balls.

(b,5) *Irradiation in Biology.* When the cells in the retina of the eye are exposed to light, the light particles play the role of the balls, and the actual cells are the "cells" of our model. Similarly, in the study of the genetic effect of irradiation, the chromosomes correspond to the cells of our model and α-particles to the balls.

(b,6) In *cosmic ray experiments*, the particles hitting the Geiger counters represent the balls, and counters function as cells.

(b,7) *Dice.* The possible outcomes of a throw with r dice correspond to placing r balls into $n = 6$ cells. When *tossing a coin* we are in effect dealing with $n = 2$ cells.

(b,9) *Random Digits.* The possible orderings of a sequence of r digits correspond to the distribution of r balls (= places) into ten cells called $0, 1, 2, \ldots, 9$.

(b,10) The *sex* distribution of r persons. Here we have $n = 2$ cells and r balls.

(b,11) *Coupon collecting.* The different kind of coupons represent the cells, the coupons collected represent the balls.

(b,12) *Aces in a bridge.* The four players represent four cells, and we have $r = 4$ balls.

(b,13) *Gene distribution.* Each descendant of an individual, person, plant or animal inherits from the progenitor certain genes. If a particular gene can appear in n forms $A_1, A_2, \ldots, A_n$, then the descendants may be classified according to the type of the gene. The descendants correspond to the balls, the genotypes $A_1, A_2, \ldots, A_n$ to the cells.

(b,14) *Chemistry.* Suppose that a long chain polymer reacts with oxygen. An individual chain may react with $0, 1, 2 \ldots$ oxygen molecules. Here the reacting oxygen molecules play the role of the balls and the polymer chains the role of the cells into which the balls are put.

(b,15) *Theory of photographic emulsions.* A photographic plate is covered with grains sensitive to light quanta: a grain reacts if it is hit by a certain number r, of quanta. For the theory of black-white contrast we must know, how many cells are likely to be hit by the r quanta. We have here an occupancy problem, where the grains correspond to the cells, and the light quanta to the balls (Actually the situation is more complicated since a plate usually contains grains of different sensitivity).

(b,16) *Misprints.* The possible distributions of r misprints in the n pages of a book correspond to all the different distributions of r balls in n cells, provided r is smaller than the number of letters per page.

The case of indistinguishable balls.

Consider the experiment of distribution $r = 3$ balls into $n = 3$ cells. Suppose that the balls are no longer distinguishable. This means that we no longer distinguish between three arrangements such as $4, 5, 6$. The sample space of the experiment which we call, *placing three indistinguishable balls into three cells* is as follows:

1.	***	-	-
2.	-	***	-
3.	-	-	***
4.	**	*	-
5.	**	-	*

1.	*	**	-
2.	*	-	**
3.	-	**	*
4.	-	*	**
5.	*	*	*

In the scheme above, we have considered indistinguishable balls, but table 2 still refers to a first, second and third cell and their order is indistinguishable.