Chapter 3. Basic Topology of R

3.1 Discussion: The Cantor Set.

What follows is a fascinating mathematical construction, due to Georg Cantor, which is extremely useful for extending the horizons of our intuition about the nature of subsets of the real line. Cantor's name has already appeared in the first chapter in our discussion of uncountable sets. Indeed, Cantor's proof that $\mathbf{R}$ is uncountable occupies another spot on the short list of the most significant contrubutions towards the understanding of the mathematical infinite. In the words of the mathematician David Hilbert, "No one shall expel us from the paradise that Cantor has created for us."

Let C_0 be the closed interval $[0,1]$ and define C_1 to be the set that results when the open middle one third is removedl that is,

$$C_1 = C_0 \setminus \left(\frac{1}{3}, \frac{2}{3}\right) = \left[0, \frac{1}{3}\right] \cup \left[\frac{2}{3}, 1\right]$$

Now, construct C_2 in a similar way by removing the open middle third of each of the two components of C_1: