SKA Entropy as a Local Field

Bouarfa Mahi
Quantiota
Email: info@quantiota.org

January 3, 2026

This note serves as a preparatory reading before engaging with the full Riemannian SKA Neural Fields framework. Understanding how entropy emerges as a local field—and how it implicitly encodes neuron density—is essential for grasping how the entropy gradient later shapes the geometry of learning space.

From Global to Local Entropy

In classical information theory, entropy measures global uncertainty over an entire distribution. SKA takes a different approach: entropy is defined *locally*, at each point in the neural medium, enabling learning as a spatially distributed field process.

The Local Entropy Density

Consider a volume element at position $\mathbf{r}$ containing $n(\mathbf{r})$ neurons. Let $\mathbf{z}(\mathbf{r}) \in \mathbb{R}^{n(\mathbf{r})}$ denote the *knowledge tensor* and $\mathbf{D}(\mathbf{r}) \in \mathbb{R}^{n(\mathbf{r})}$ the *decision probability tensor* at that location. The **local entropy density** is defined as:

$$h(\mathbf{r}) \;=\; -\frac{1}{\ln 2}\,\mathbf{z}(\mathbf{r}) \cdot \Delta\mathbf{D}(\mathbf{r}) \tag{1}$$

where $\Delta\mathbf{D}(\mathbf{r}) = \mathbf{D}(\mathbf{r}, t+\Delta t) - \mathbf{D}(\mathbf{r}, t)$ is the change in decision probability over a learning step, and the dot product sums over all $n(\mathbf{r})$ components.

Implicit Encoding of Neuron Density

A crucial observation is that neuron density $\rho(\mathbf{r})$ is *already encoded* in the entropy field through tensor dimensionality. The local neuron count is $n(\mathbf{r}) = \rho(\mathbf{r}) \cdot dV$. Since both $\mathbf{z}(\mathbf{r})$ and $\Delta\mathbf{D}(\mathbf{r})$ have dimension $n(\mathbf{r})$, the dot product sums over exactly $n(\mathbf{r})$ terms:

$$h(\mathbf{r}) = -\frac{1}{\ln 2}\sum_{i=1}^{n(\mathbf{r})} z_i(\mathbf{r}) \cdot \Delta D_i(\mathbf{r})$$

Regions with higher neuron density contribute *more terms* to the sum. For fixed average alignment between $\mathbf{z}$ and $\Delta\mathbf{D}$, denser regions produce proportionally larger entropy magnitudes—no explicit density term required.

Universality: From Layered Networks to Neural Fields

A remarkable property of this entropy definition is its *universality.* The identical equation governs discrete layered networks, and continuous neural fields in *any* spatial dimension:

$$h^{(l)} = -\frac{1}{\ln 2}\,\mathbf{z}^{(l)} \cdot \Delta\mathbf{D}^{(l)} \quad \longleftrightarrow \quad h(\mathbf{r}) = -\frac{1}{\ln 2}\,\mathbf{z}(\mathbf{r}) \cdot \Delta\mathbf{D}(\mathbf{r})$$

where $\mathbf{r} \in \mathbb{R}^D$ for $D = 3, 4, 5, \ldots$ These are genuine spatial dimensions: $D = 3$ is physical 3D space, $D = 4$ embeds the field in 4D space, and so on. The mathematical form is *unchanged*—the entropy depends only on the dot product, which is independent of the ambient spatial dimension D.

Consequences and the Entropy Field

This implicit coupling means: (1) high-density regions have greater capacity to reduce entropy, becoming natural hubs for knowledge accumulation; (2) the entropy remains a scalar despite varying tensor dimensions; (3) the system self-regulates computational load according to local capacity. Because $h(\mathbf{r})$ is defined everywhere, it forms a scalar field that evolves as learning progresses, with $\nabla h(\mathbf{r})$ guiding information flow—density effects naturally incorporated through tensor dimensionality.