

Structured Knowledge Accumulation: A Dirac Notation Formalism for Market Regime Modeling

Detecting Quantum Coherence in Financial Markets Through Real-Time Entropy Analysis

Abstract

This white paper introduces a quantum-inspired mathematical formalism for modeling financial market regimes using **Dirac notation** within the Structured Knowledge Accumulation (SKA) framework. The market state is represented as a superposition vector in a complex Hilbert space, and its evolution is governed by a density operator encoding both regime probabilities and structural transitions. Each complex amplitude carries an entropy phase, linking probabilistic transitions to information dynamics. The use of operator algebra reveals that regime transitions are not discrete events but emergent features of the SKA state geometry. This formulation provides a theoretical foundation for real-time analytics, entropy-based decision systems, and market structure decoding, with practical implementations across SQL querying, live dashboards, and agent-based forecasting.

Introduction

Traditional models of financial markets rely on discretized time series, deterministic indicators, or Markovian assumptions to represent regime changes. These approaches often overlook the **underlying information structure** and fail to capture the continuity and coherence inherent in real-time transitions.

The **Structured Knowledge Accumulation (SKA)** framework redefines this paradigm by treating the market as a **dynamical information system**, where regime states exist in a superposed form and evolve through entropy-driven learning. Inspired by quantum mechanics, this formalism introduces Dirac notation to express regime states, transition amplitudes, and entropy phases in a unified mathematical structure.

By modeling the market state as a vector in a complex Hilbert space, and transitions as projections within this space, we obtain a **density operator** that simultaneously encodes probabilities and structural couplings. This allows for a natural integration of **real-time learning**, **entropy flow**, and **non-Markovian memory effects**—opening the door to a fundamentally new class of market models.

This document lays the foundation for this approach, defining each component of the model and providing the necessary tools for implementation in analytics pipelines, signal processing, and algorithmic decision systems.

1. State Space Definition

Let \mathcal{H} be a **3-dimensional complex Hilbert space** with orthonormal basis:

$$\mathcal{B} = \{|0\rangle, |1\rangle, |2\rangle\}$$

Interpretation: Basis vectors represent mutually exclusive, exhaustive regimes.

2. Market State Vector

At any time t , the market is in a superposition of basis regimes:

$$|\Psi(t)\rangle = \sum_{i=0}^2 \Psi_i(t) |i\rangle, \quad \Psi_i(t) \in \mathbb{C}$$

$$|\Psi(t)\rangle = \Psi_0(t)|0\rangle + \Psi_1(t)|1\rangle + \Psi_2(t)|2\rangle$$

- $|0\rangle, |1\rangle, |2\rangle$: Basis vectors for Neutral, Bull, Bear
- $\Psi_i(t) = A_i(t)e^{iH_i(t)} \in \mathbb{C}$: Complex amplitude encoding probability and entropy

In the Structured Knowledge Accumulation (SKA) framework, the market is not confined to a single regime (e.g., bull, bear, or neutral) at any moment. Instead, it exists in a **superposition** of all possible regimes, with complex amplitudes $\Psi_i(t)$ encoding both probability and entropy.

Traditional models assume the market flips from one regime to another—an abrupt, externally triggered event. But SKA reveals a continuous evolution through overlapping informational states.

Classical Assumption:

- Market is always *in* one regime.
- Transitions are discrete.
- Probabilities are assigned *after* classification.

SKA View:

- Market is always *between* regimes.
- Transitions emerge from entropy geometry.
- Probabilities arise from the evolving wavefunction $|\Psi(t)\rangle$.

Implications:

- **Why classical models fail:** they discretize what is fundamentally smooth and entangled.
- **Why transitions seem random:** because we're projecting a superposed state onto a single outcome.
- **Why micro-patterns exist at the tick level:** because phase coherence in $\Psi(t)$ encodes deep structural information—**even when the price appears flat.**

SKA does not model the *outcome* of the market. It models the *informational field* from which outcomes emerge.

Normalization:

For any state $|\Psi\rangle \in \mathcal{H}$, $\langle\Psi|\Psi\rangle = 1$.

$$\langle\Psi(t)|\Psi(t)\rangle = |\Psi_0|^2 + |\Psi_1|^2 + |\Psi_2|^2 = 1$$

3. Density Operator

The **density operator** is the outer product of the state vector with itself:

$$\hat{\rho}(t) = |\Psi(t)\rangle\langle\Psi(t)|$$

This object captures both **probabilities** and **structural relationships** (coherences) between regimes.

Matrix Elements:

$$\langle i | \hat{\rho}(t) | j \rangle = \Psi_i^*(t)\Psi_j(t)$$

This yields a 3x3 Hermitian matrix:

$$\hat{\rho}(t) = \begin{pmatrix} |\Psi_0|^2 & \Psi_0^*\Psi_1 & \Psi_0^*\Psi_2 \\ \Psi_1^*\Psi_0 & |\Psi_1|^2 & \Psi_1^*\Psi_2 \\ \Psi_2^*\Psi_0 & \Psi_2^*\Psi_1 & |\Psi_2|^2 \end{pmatrix}$$

Interpretation:

- **Diagonal terms:** $\rho_{ii} = |\Psi_i|^2$ = probability of being in regime i
- **Off-diagonal terms:** $\rho_{ij} = \Psi_i^*\Psi_j$ = coherence between regimes i and j , interpreted as transition structure

4. Structural Transition Operator

A transition from regime $j \rightarrow i$ is represented by:

$$\hat{T}_{i \leftarrow j} = |i\rangle\langle j|$$

This operator transforms basis state $|j\rangle$ into $|i\rangle$.

Expectation Value:

To extract the amplitude for a structural transition from $j \rightarrow i$:

$$\langle \Psi | \hat{T}_{i \leftarrow j} | \Psi \rangle = \Psi_i^*(t) \Psi_j(t)$$

This equals the (i, j) entry of the density matrix.

Constant Structural Coupling C_{ij}

In practice, the most observable market regularities emerge when structural transition amplitudes become **time-invariant**:

$$C_{ij} = \Psi_i^*(t) \Psi_j(t) = \text{const}$$

This condition implies a stable **directional flow of probability** between regimes i and j .

When C_{ij} remains constant:

- Regime transitions align clearly in real-time analysis.
- Entropy flow across regimes becomes **symmetric** or **balanced**.
- Paired transitions (e.g., Neutral \rightarrow Bull and Bull \rightarrow Neutral) appear as coherent oscillations.

Classical models assume regime transitions are noisy or random. In contrast, SKA shows structural amplitudes C_{ij} can remain highly stable across thousands of trades, enabling precise **real-time alignment** of transitions.

This stability reveals that markets operate within a **coherent informational geometry** rather than a stochastic regime-flip process, explaining why traditional models—treating probabilities as noise instead of geometry—fail to detect these regularities.

5. Entropy Interpretation

Each complex amplitude $\Psi_i(t)$ embeds an **entropy phase**:

$$\Psi_i(t) = A_i(t)e^{iH_i(t)}$$

Here, $A_i(t)$ represents the amplitude, and $H_i(t)$ is the entropy associated with regime i at time t .

The entropy $H_i(t)$ comes from the SKA learning process as it accumulates over the trajectory of market evolution.

We express the entropy as a functional of the Lagrangian:

$$H_i(t) = \frac{1}{\ln 2} \int_0^t \mathcal{L}_i(s) ds$$

where $\mathcal{L}_i(s)$ is the Lagrangian.

This phase represents the **information content** (entropy) of regime i . Therefore, transitions (via $\Psi_i^* \Psi_j$) carry entropy shifts $H_j - H_i$.

6. Transition Operator Action on the State

We consider the global transition operator (density operator):

$$\hat{T} = |\Psi\rangle\langle\Psi|$$

We project this operator acting on the state back onto the basis state $|i\rangle$:

$$\langle i | \hat{T} | \Psi \rangle = \langle i | \Psi \rangle = \sum_j \Psi_j \langle i | j \rangle = \sum_j \Psi_j \delta_{ij} = \Psi_i$$

Here we used the orthonormality of the basis: $\langle i | j \rangle = \delta_{ij}$, the Kronecker delta.

Since $\hat{T} | \Psi \rangle = |\Psi\rangle$, the operator acts as a projector and leaves the state invariant. Therefore:

$$\langle 0 | \hat{T} | \Psi \rangle = \Psi_0$$

$$\langle 1 | \hat{T} | \Psi \rangle = \Psi_1$$

$$\langle 2 | \hat{T} | \Psi \rangle = \Psi_2$$

This shows that transitions are not events but projections. The market doesn't "switch" regimes - our measurement collapses the superposition!

This means:

- Regime changes are observation-dependent
- The act of trading affects the state
- Observer effect is real in markets

The transition operator encodes the full informational identity of the market state and reflects that transitions are emergent properties of the SKA state — not discrete external events.

Upon measurement, the superposed market state $|\Psi\rangle$ collapses to a definite regime $|i\rangle$ with probability $|\Psi_i|^2$, revealing the observed market condition.

7. Summary of Core Equations

Concept	Formula
State vector	$ \Psi\rangle = \sum_i \Psi_i i\rangle$
Density operator	$\hat{\rho} = \Psi\rangle\langle\Psi $
Regime probability	$\rho_{ii} = \ \Psi_i\ ^2$
Structural transition	$\hat{T}_{i\leftarrow j} = i\rangle\langle j $
Transition amplitude	$\langle i \hat{\rho} j \rangle = \Psi_i^* \Psi_j$
Entropy phase	$\arg(\Psi_i) = H_i(t)$

8. Usage

This document serves as the **formal reference** for SKA modules that:

- Visualize transitions in QuestDB or Grafana
- Compute entropy-induced transition probabilities
- Apply the principle of least entropy action

Refer to this file when defining:

- Structural probability matrices
- Real-time SKA signal decoders
- Quantum-inspired simulation of regime evolution

Author Note

This formalism supports a deeper view of the market as an **information-processing system**, not a sequence of discrete events. Transitions are not externally triggered but emerge from the **geometry of knowledge accumulation** encoded in the SKA state.

References

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- [2] Bouarfa Mahi. *Structured Knowledge Accumulation: The Principle of Entropic Least Action in Forward-Only Neural Learning*. arXiv:2504.03214