Visualizing Neuron Density Fields in Higher Dimensions

Supplementary Material for Riemannian SKA Neural Fields

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Abstract

This document presents visualizations of neuron density fields generated using Simplex noise across 3D, 4D, and 5D spaces. These fields serve as the computational substrate for the Riemannian SKA Neural Fields framework, where spatially varying neuron density $\rho(\mathbf{r})$ determines local computational capacity and influences the geometry of the information manifold through the metric tensor $g_{ij}(\mathbf{r})$.

1 Neuron Density in Continuous Neural Fields

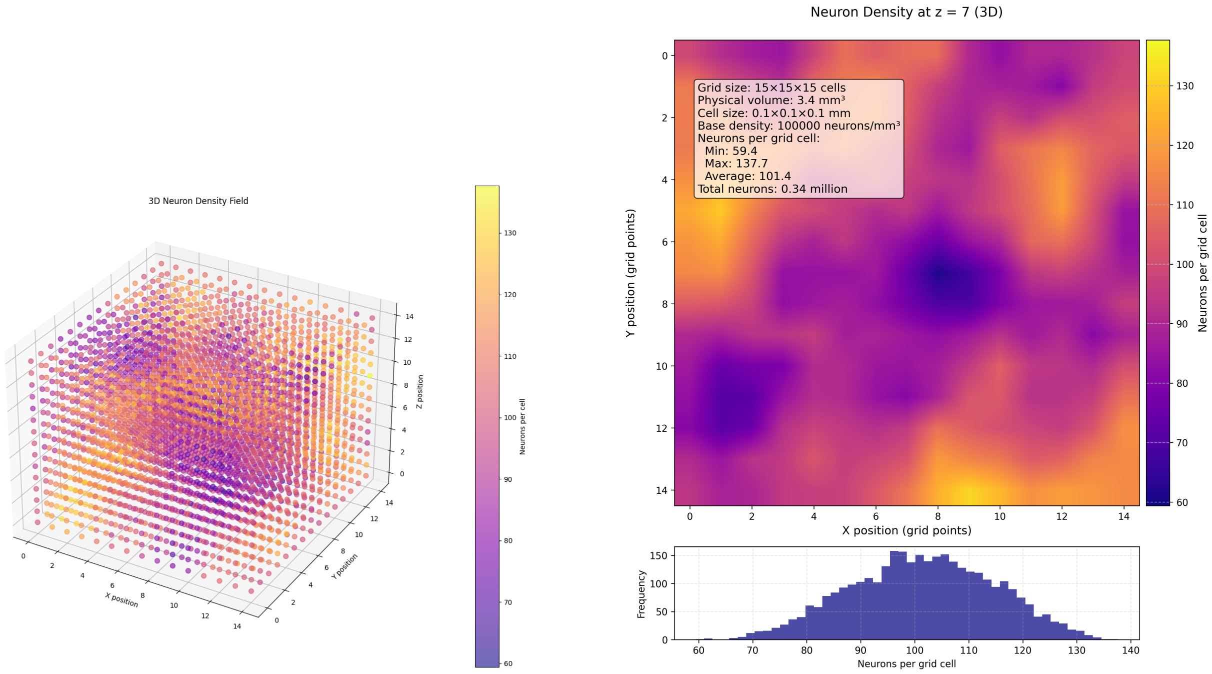
In the Riemannian SKA Neural Fields framework, we model neural tissue as a continuous medium with spatially varying neuron density $\rho(\mathbf{r})$. Following biological observations of primate cortex density variations (30,000–180,000 neurons/mm^3), we generate realistic density fields using Simplex noise:

$$\rho(\mathbf{r}) = \rho_0 \cdot (1 + \alpha \cdot \mathcal{N}(\mathbf{r})) \tag{1}$$

where ρ_0 is the base density, $\alpha \in [0, 1]$ controls variation amplitude, and $\mathcal{N}(\mathbf{r})$ is multi-octave Simplex noise providing smooth, biologically plausible spatial variations.

The neuron count per grid cell directly determines the dimensionality of local knowledge and decision tensors, creating a heterogeneous computational substrate where geodesic learning paths naturally emerge.

2 3D Neuron Density Field



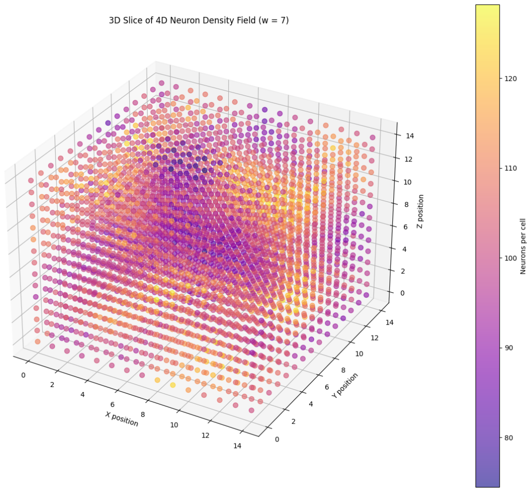
(a) Scattered grid approximation of the continuous 3D density field. Each point represents a computational unit with color encoding local neuron count.

(b) Cross-sectional slice at $z = 7$ with histogram showing the distribution of neurons per cell across the volume.

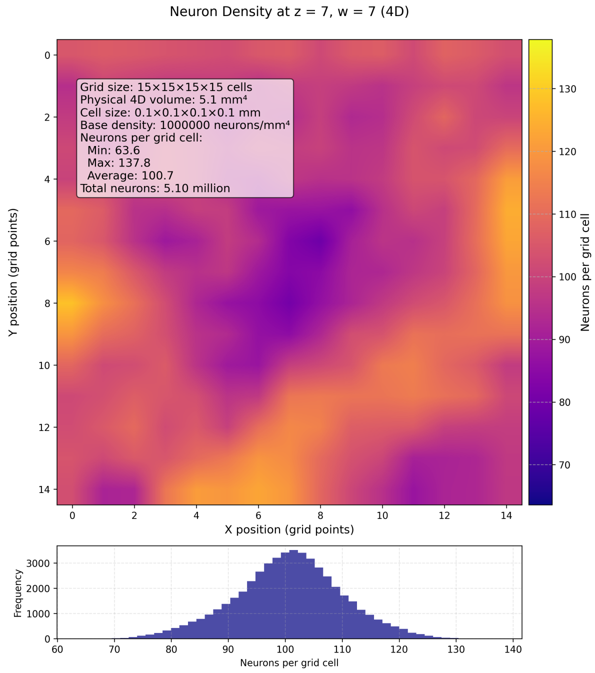
Figure 1: **3D Neuron Density Field** ($15 \times 15 \times 15$ grid, 0.34 million neurons). The density gradient $\nabla\rho$ contributes to the Riemannian metric tensor, influencing geodesic learning paths.

3 4D Neuron Density Field

Higher-dimensional neural fields extend the spatial framework to 4D and 5D spaces, demonstrating the dimensional scalability of the SKA approach while maintaining the same entropy-driven learning principles.



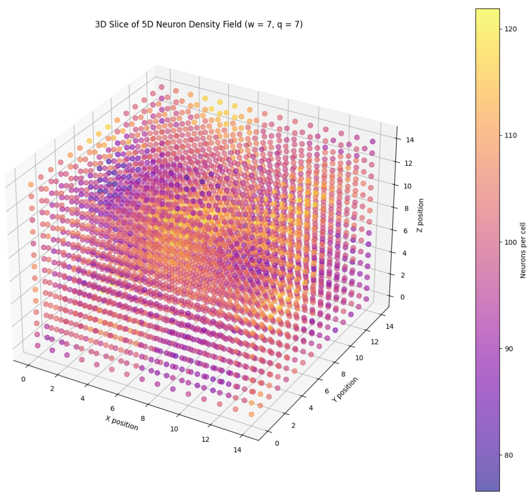
(a) 3D slice through the 4D field at $w = 7$, revealing the density structure in the (x, y, z) subspace.



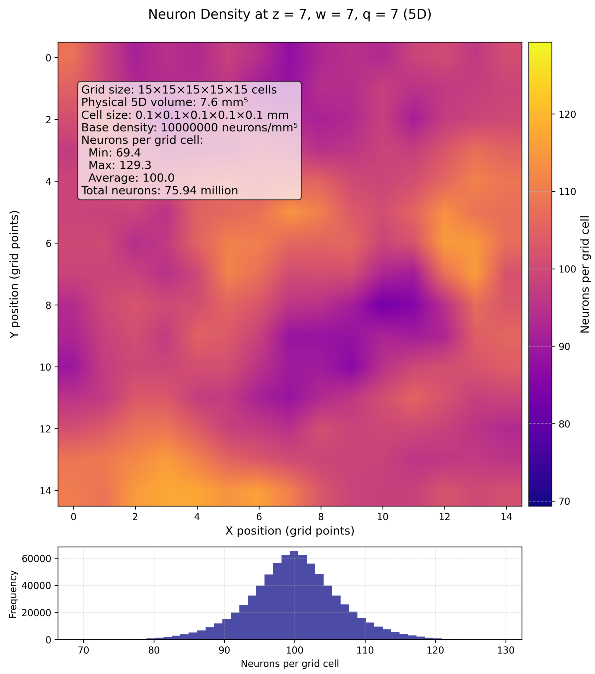
(b) 2D slice at $(z = 7, w = 7)$ showing density variations across 50,625 total cells.

Figure 2: **4D Neuron Density Field** (15^4 cells, 5.10 million neurons). The fourth spatial dimension extends the framework naturally, with geodesics computed on 4-simplex meshes.

4 5D Neuron Density Field



(a) 3D projection of the 5D field at $(w = 7, q = 7)$, showing preserved spatial coherence.



(b) 2D slice with narrowed distribution demonstrating the central limit effect in higher dimensions.

Figure 3: **5D Neuron Density Field** (15^5 cells, 75.94 million neurons). The SKA entropy expression remains dimension-independent, enabling natural extension to arbitrary D.

5 Statistical Properties Across Dimensions

Dim	Grid	Cells	Neurons	Range	Std Dev
3D	15^3	3,375	0.34M	59–138	$\pm$20.1
4D	15^4	50,625	5.10M	64–138	$\pm$14.2
5D	15^5	759,375	75.94M	69–129	$\pm$10.3

Table 1: As dimensionality increases, the density distribution converges toward the mean (≈ 100 neurons/cell) with decreasing variance—a manifestation of the central limit theorem in higher-dimensional sampling.

6 Connection to Riemannian SKA Framework

These density fields directly inform the Riemannian metric tensor:

$$g_{ij}(\mathbf{r}) = \alpha \cdot (\nabla h)_i (\nabla h)_j + \beta \cdot (\nabla \rho)_i (\nabla \rho)_j + \gamma \cdot \delta_{ij} \tag{2}$$

The scattered grid approximation enables:

- **Efficient computation**: Sampling continuous Simplex noise at discrete vertices
- **FEM integration**: Tetrahedral (3D) or D-simplex meshes for gradient recovery
- **Geodesic paths**: Discrete exterior calculus on the resulting simplicial complex
- **Scalability**: Linear complexity increase with additional modulatory fields

The visualizations demonstrate that continuous neural fields can be effectively approximated through sparse sampling while preserving the organic, spatially-varying characteristics essential for biologically plausible learning dynamics.

Full Paper: *Structured Knowledge Accumulation: Geodesic Learning Paths and Architecture Discovery in Riemannian Neural Fields* (TechRxiv, 2026)

Code Repository: https://github.com/quantiota/SKA-Riemannian-Neural-Fields