Random Walks and Monte Carlo

WISER Quantum Projects 2025

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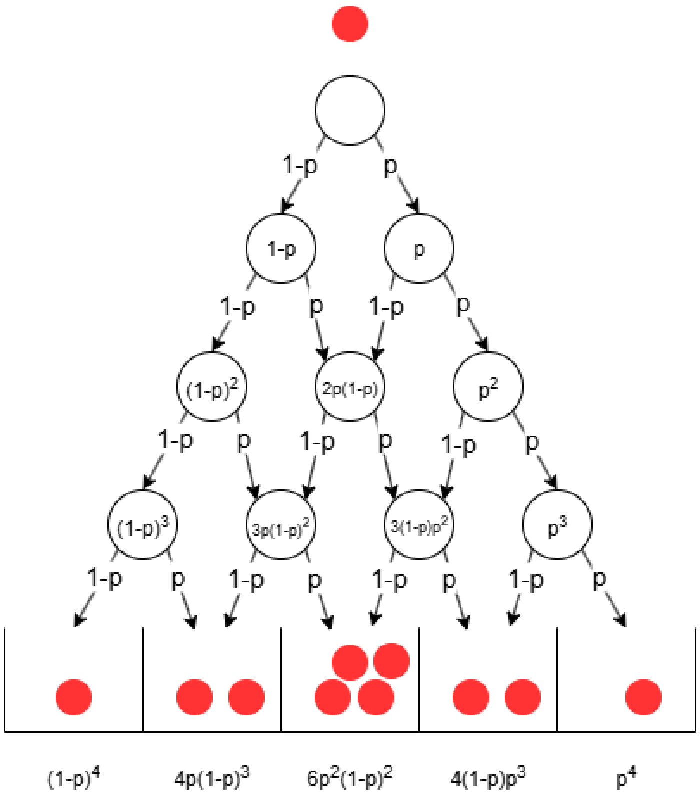
09 August 2025

Project Goals

- State Preparation: Apply the Universal Statistical Simulator method to generate a range of distributions within quantum states.
- Quantum Galton Board Implementation: Construct a quantum analogue of the n-level Galton Board to illustrate symmetric Gaussian distributions.
- Distribution Adjustment: Alter quantum pegs and coin parameters to produce various distributions, including Exponential and Hadamard Random Walk (Bi-Modal).
- Noise-Resilient Optimization: Design a Galton Board-inspired method to load arbitrary distributions and demonstrate outcomes on noisy simulators using the IBM ibm_torino noise model.
- Performance and Depth Analysis: Evaluate circuit accuracy under noisy conditions and examine the relationship between circuit depth and the number of Galton Board levels.
- Real Backend Runs: Execute Resulting Circuits on physical QPUs and observe the results

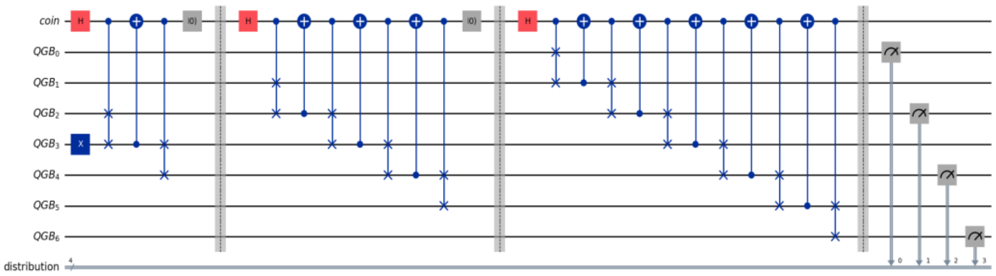
Galton Board as Decision DAG

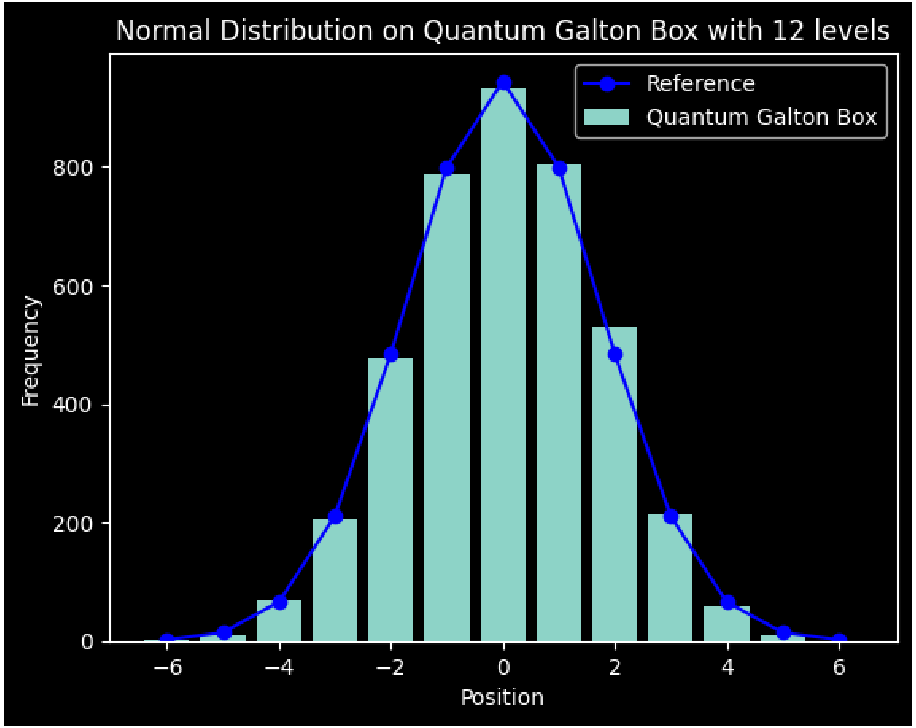
- Galton Board is a decision DAG (Directed Acyclic Graph)
- If p=1/2, a symmetric normal distribution is achieved in the tally bins.
- Modifying the probability (p) will skew the distribution either to the left or right.



n-levels Galton Board: Quantum Circuit

Diagram for n=3:



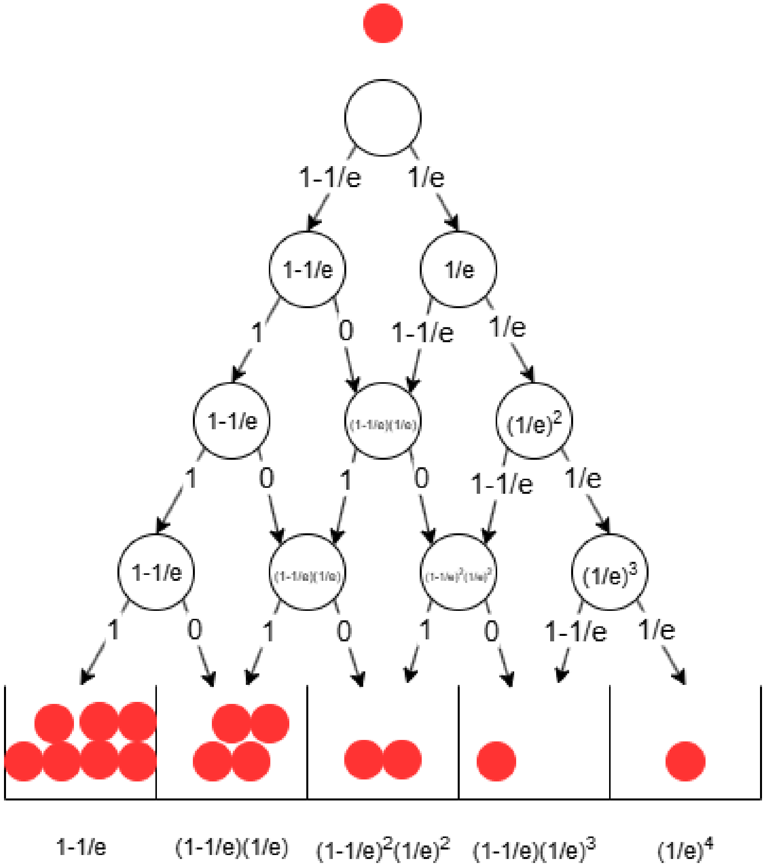


For n levels

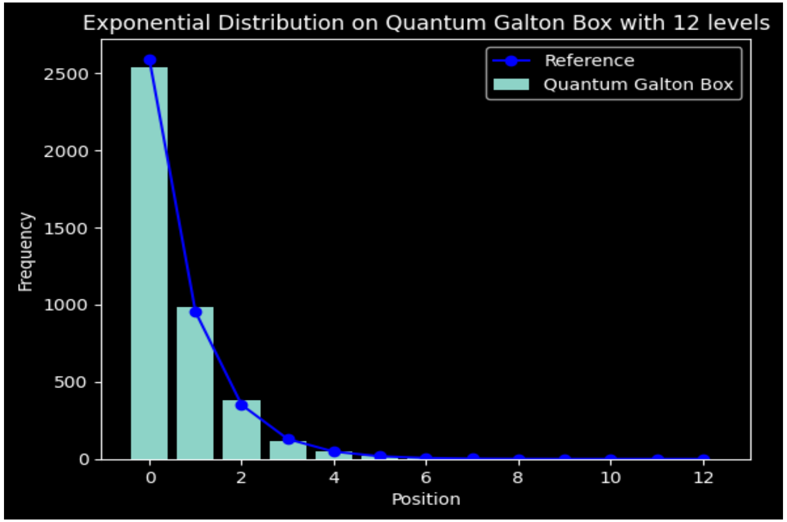
Qubits: $2n + 2$

Depth: $4 + 8 + 12 + \ldots + 3n = 2(n^2 + n) = O(n^2)$

Exponential Distribution

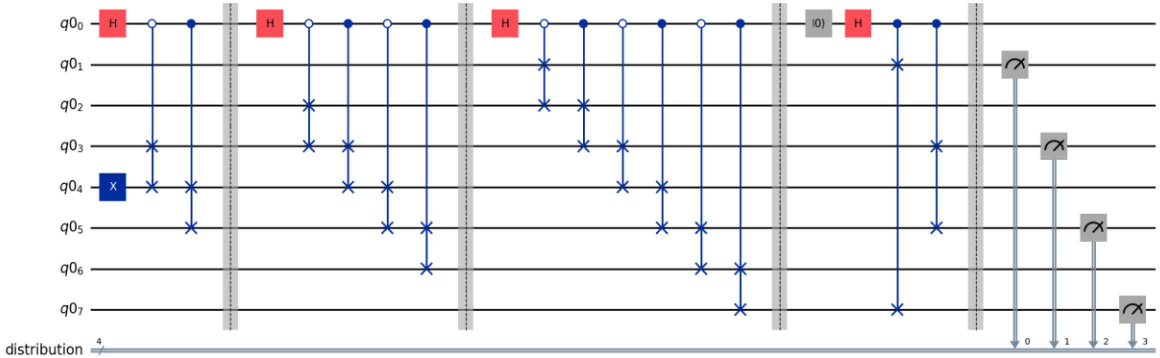


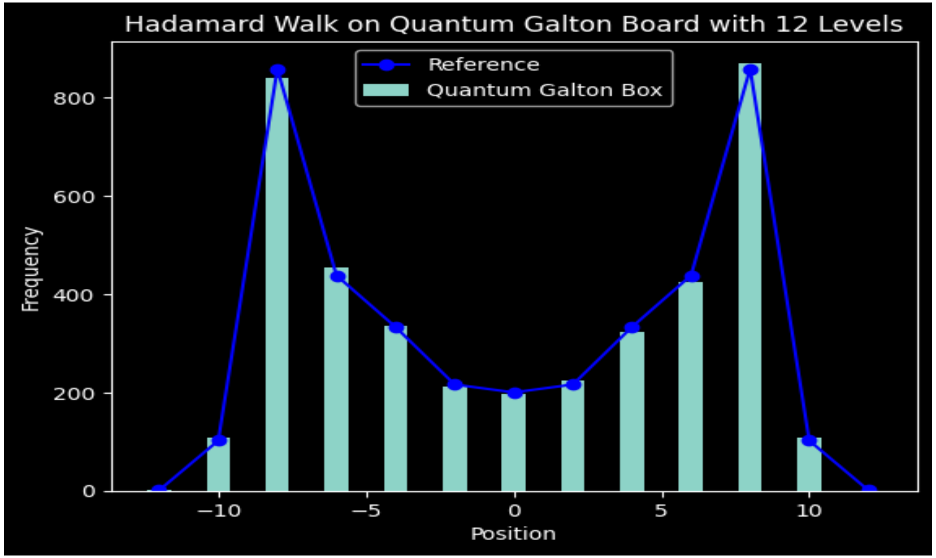
- To achieve a different distribution, probabilities must be adjusted at each level.
- The figure illustrates assigning probabilities only to the rightmost quantum peg, requiring either a rotation of the "coin" qubit by a specific angle or flipping it.
- In the quantum circuit, this involves modifying the quantum coin function using controlled RY rotations to bias the "coin" accordingly.
- The angle of rotation is given by $2arccos(\sqrt{1-\frac{1}{e}})$



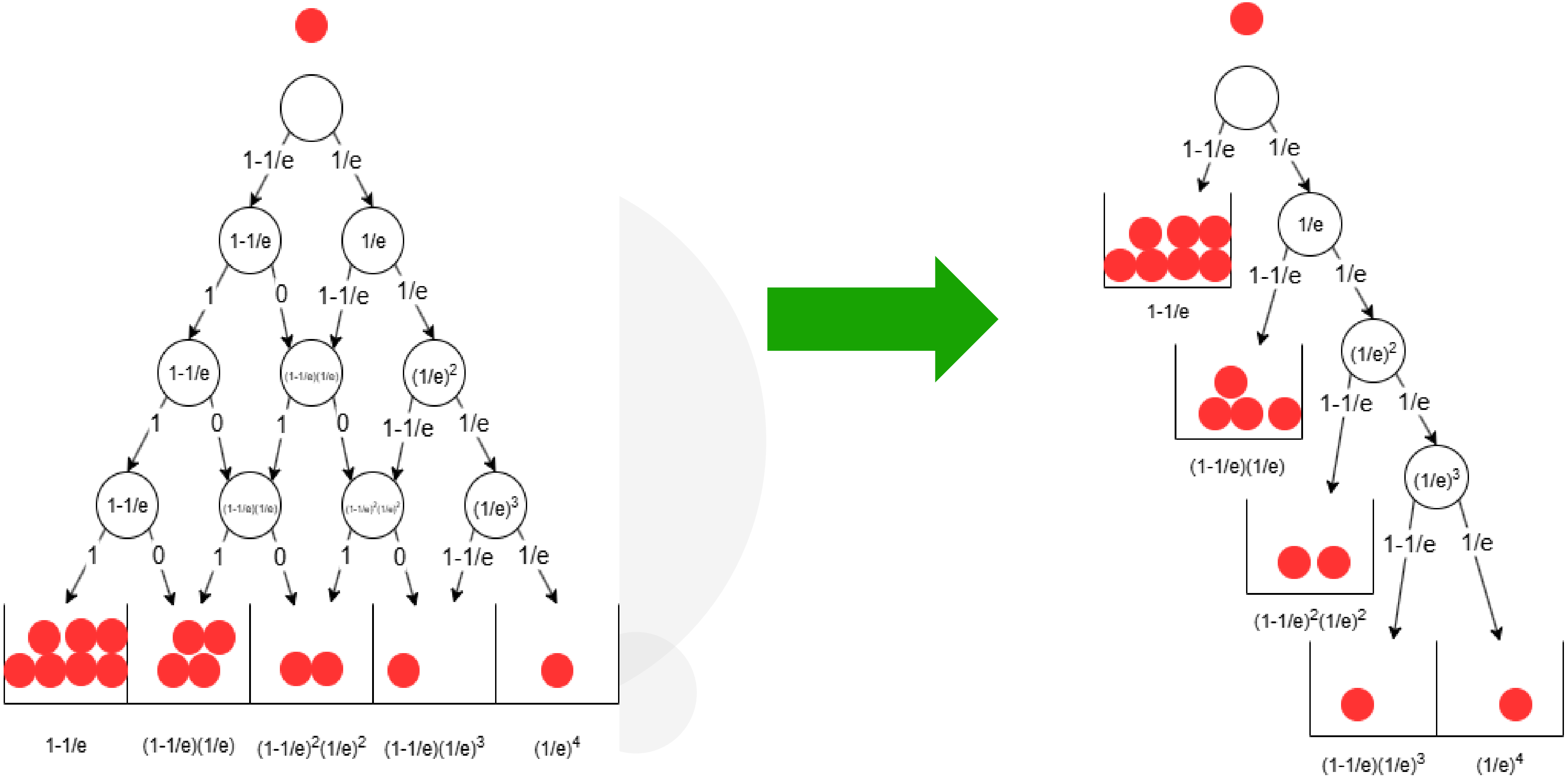
Hadamard Random Walk (bi-modal) Distribution

- The walk must be conducted in superposition to maintain the interference effect
- Alternate the control value for each CSWAP gate to simulate stepping in superposition.
- Refrain from resetting the coin state to preserve interference.
- Execute a final swap after all levels have been applied to ensure a symmetric distribution.





Symmetric DAG => Optimal Decision Tree



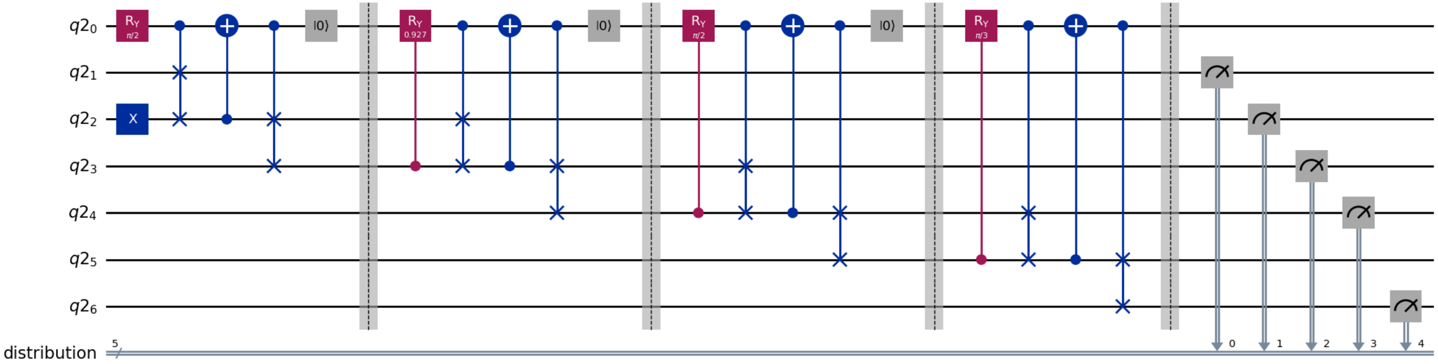
The thinking was inspired by observing the Exponential Distribution decision DAG

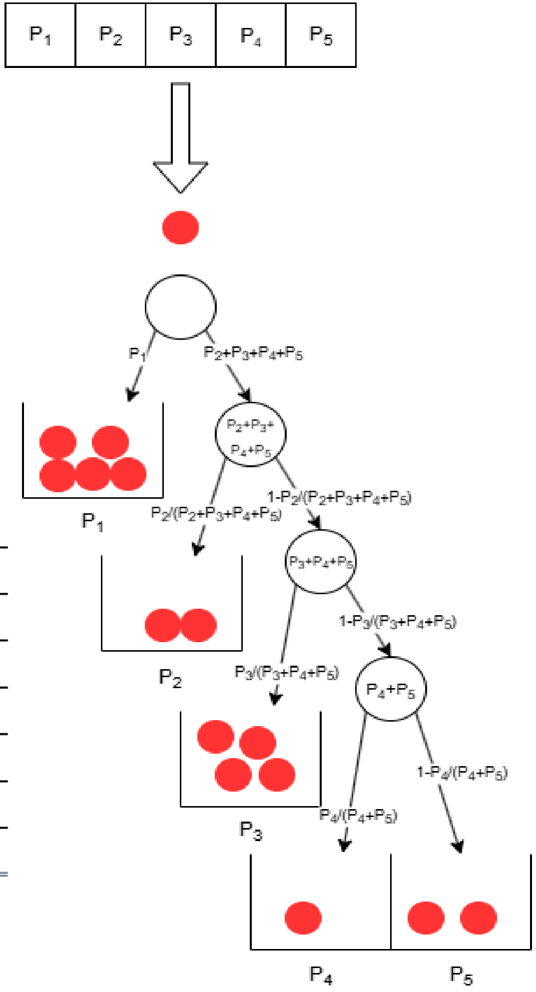
Optimal Decision Tree for an Arbitrary Distribution

- Given the vector of n+1 probabilities, we can build an optimized coin function

- Quantum circuit will assure the ball reaches each tally bin with the desired probability

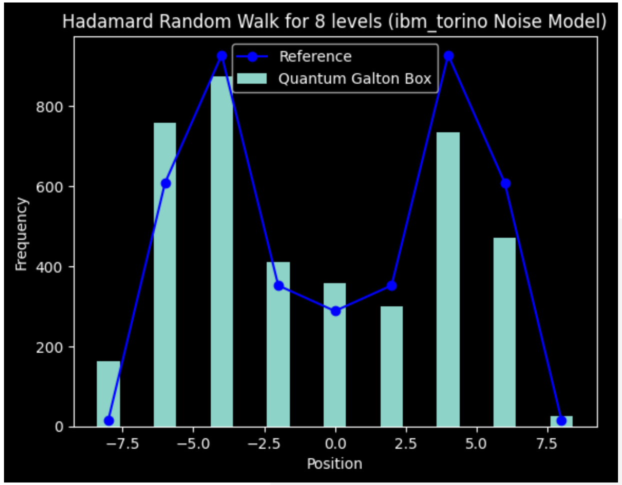
- Rotation angle for level `i` is given by: $\theta_i = 2arccos(\sqrt{1 - \frac{p_i}{\sum_{j=i}^{n+1} p_j}})$

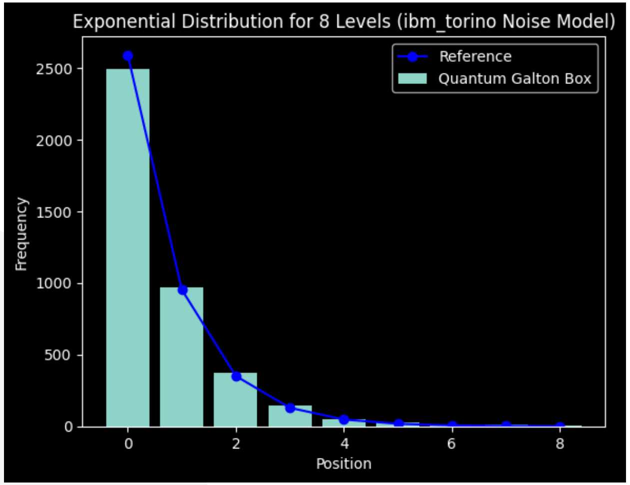


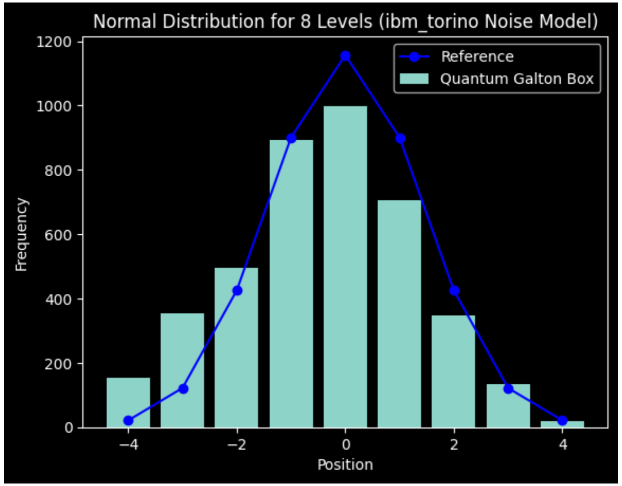


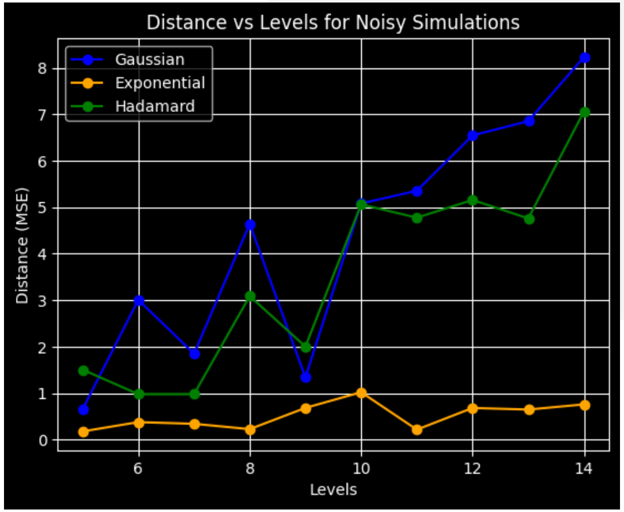
Qubits: n + 3
Depth: 4n = O(n)

Noisy Simulations



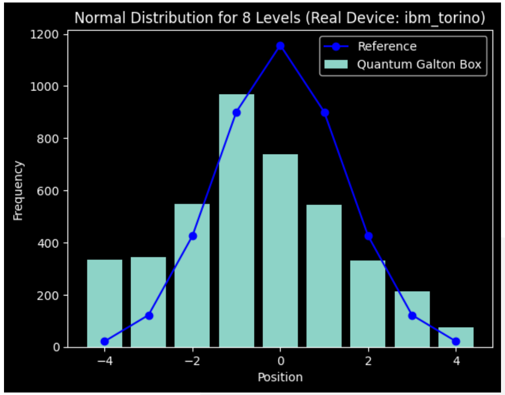


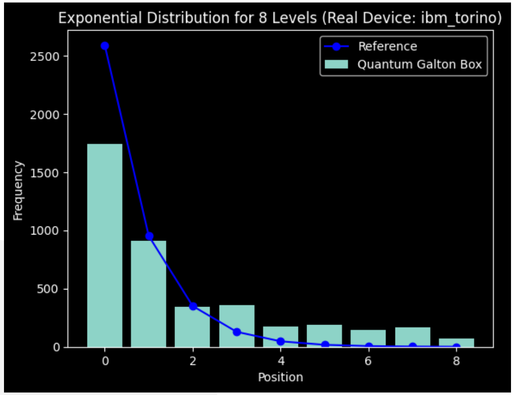


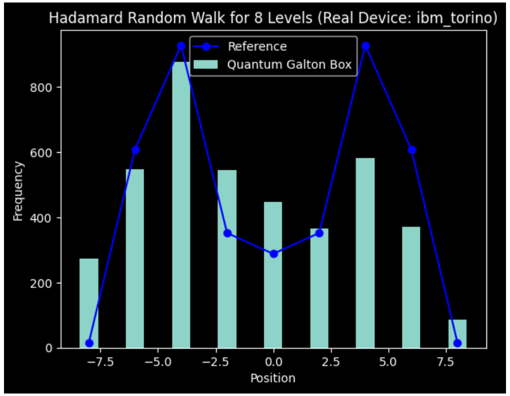


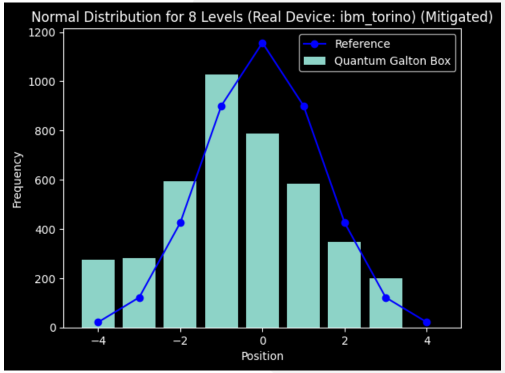
- The exponential distribution demonstrated the greatest improvement in accuracy following the Galton Board optimizations.
- By decreasing the number of required qubits from 2n+2 to n+3 and markedly reducing the circuit depth, the results remained sufficiently close to the reference values for all distributions despite the presence of noise.
- The visual similarity of the distributions is preserved even when noise is introduced.

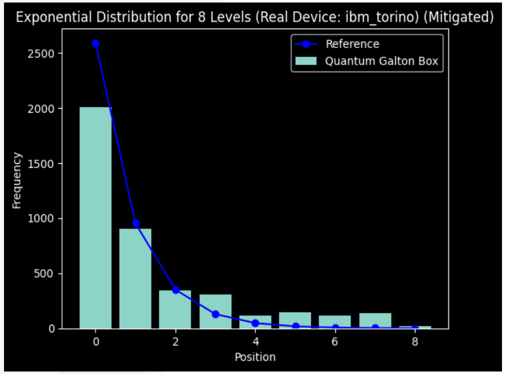
Real Backend Execution

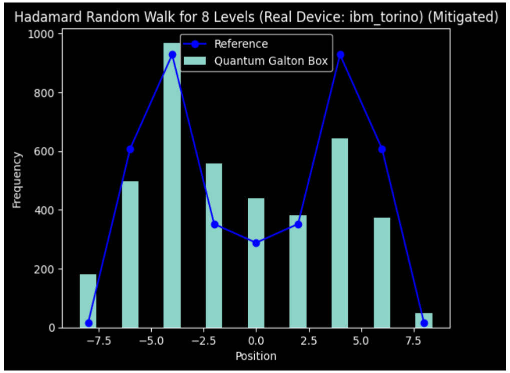






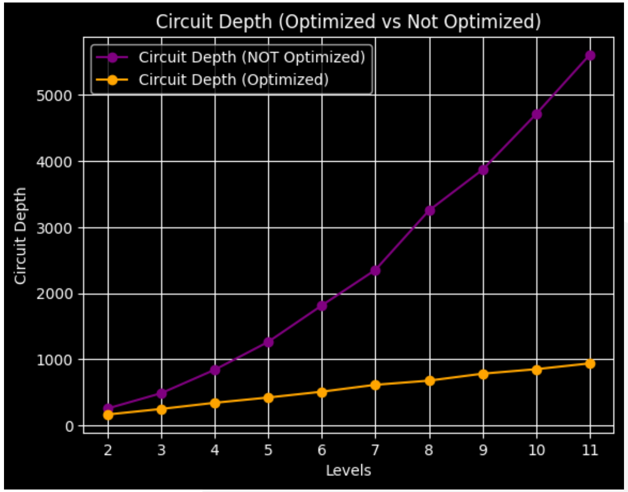


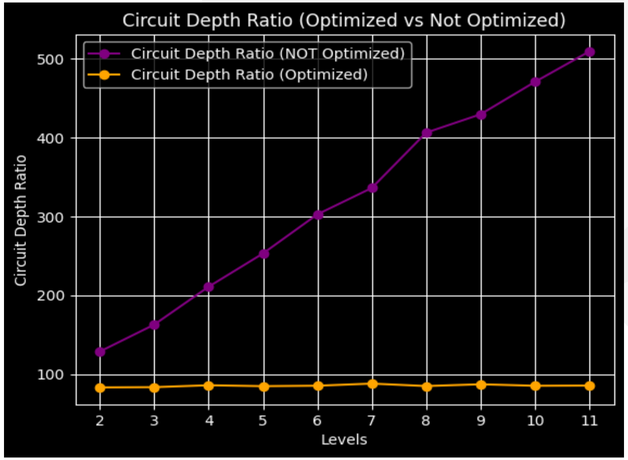




- Despite large distances from the reference, the distribution shapes are identifiable
- Noise reduction using the M3 package had minimal impact
- Overall, the IBM Torino backend performed impressively!!! 🎉 🎉 🎉

Circuit Depth Study





Optimized circuit

Maintaining a constant depth-to-levels ratio results in a linear dependency on levels.

Non-optimized circuits

A linearly increasing ratio signifies a quadratic dependency on levels.

Conclusions

- Demonstrated universal distribution simulator with Hamming Weight 1
- Galton board method shows promise on NISQ devices
- Reduced qubits needed from $2n+2$ to $n+3$
- Improved circuit depth from $O(n^2)$ to $O(n)$ ensuring resource efficiency and noise resilience
- MSE distance stayed below 5 at most levels
- Execution on real backend showcased impressive results