IMPERIAL COLLEGE LONDON

Mechanical Engineering Department

ME2 Dynamics

Vibration Experiment Report

Rishi Kumra
01211064
5d
13th January 2018

1. Introduction

Short section describing the purpose of this experiment, the test setup and equipment as well as the measurement procedure - A brief explanation of how the two transducers you have used work and the importance of using two transducers in the experiment.

The aim of this experiment is to study and understand the vibration phenomena of resonant forced vibrations and free vibration decay. The test setup (figure 1) includes a uniform, rectangular cross-section steel beam which is supported at each end to provide a flexible spring support, and carries a steel block at its midpoint. This beam is then vibrated by a shaker in experiment 1 and by an unmeasured forced in experiment 2. The purpose of this variation is to explore two different methods of determining the natural frequency and damping factor.

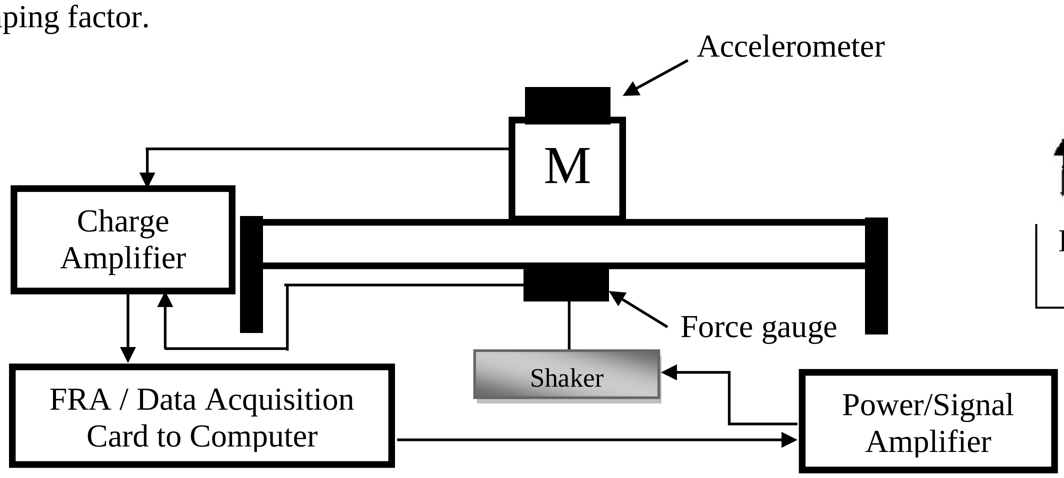

Figure 1.1 – Experimental Setup

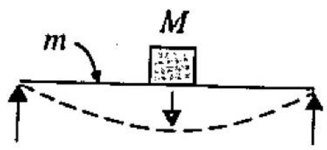

Figure 1.2 – Free body diagram of system

In the forced response experiment, there is a continuous sine excitation from the shaker connected to the beam. We initially varied the frequency of the shaker vibrations across a coarse 1 Hz sweep from 10 Hz to 20 Hz and exported readings of excitation force and acceleration from the computer system. After making observations to the approximate region of resonance, we varied the frequency across a fine 0.1 Hz sweep in the region of observed resonance (15.0 – 17.0 Hz).

In the free response experiment, the shaker was disconnected, and the beam was excited by an unmeasured force (hand striking) which caused the vibration amplitude to decay freely after impact. Sinusoidal decaying time-dependent readings of acceleration were recorded from the computer system, with an initial transient error.

The force gauge in figure 1.1 and figure 2.1 uses the Piezoelectric effect to convert applied strain/mechanical stress in the crystal gauge into electrical charge. This charge from applied strain is converted by a charge amplifier into voltage signals that are then converted from analogue to digital by the Data Acquisition Card and finally displayed as graphs on a computer. Similarly, an accelerometer has a voltage output proportional to acceleration of the mass. Force and acceleration aren’t always in phase with each other, therefore using two transducers allows for independent and valid measurements to be made for each of the values of force and acceleration.

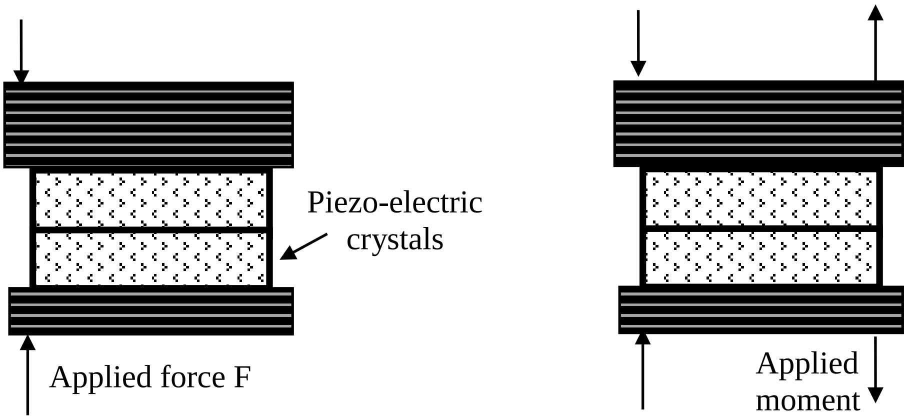

Figure 2.1 – Force Gauge Principle

2. Force and acceleration

Show graphs of the measured force and acceleration as a function of frequency.

For each frequency value in the course sweep and fine sweep tests, the data collected included force and acceleration data as a function of time. For each of these frequencies (10-20Hz), a maximum force and maximum acceleration value was calculated. These values were then plotted against frequency to obtain graphs of force and acceleration as a function of frequency.

Figure 3.1 gives a sense of how each maximum value was determined and evaluated. Figure 3.2 is the result of plotting all the maximum forces and accelerations against frequency.

s	N	Maximum N
0	-0.13399	3.0666122
0.001	0.103254	
0.002	0.331868	
0.003	0.538915	
0.004	0.750275	
0.005	0.974576	
0.006	1.164369	
0.007	1.38867	
0.008	1.58709	
0.009	1.768256	
0.01	1.932168	
0.011	2.121961	
0.012	2.259992	

Force 12Hz

Figure 3.1 – Time-force data at a specific frequency of 12 Hz. The maximum value of force across all times is evaluated.

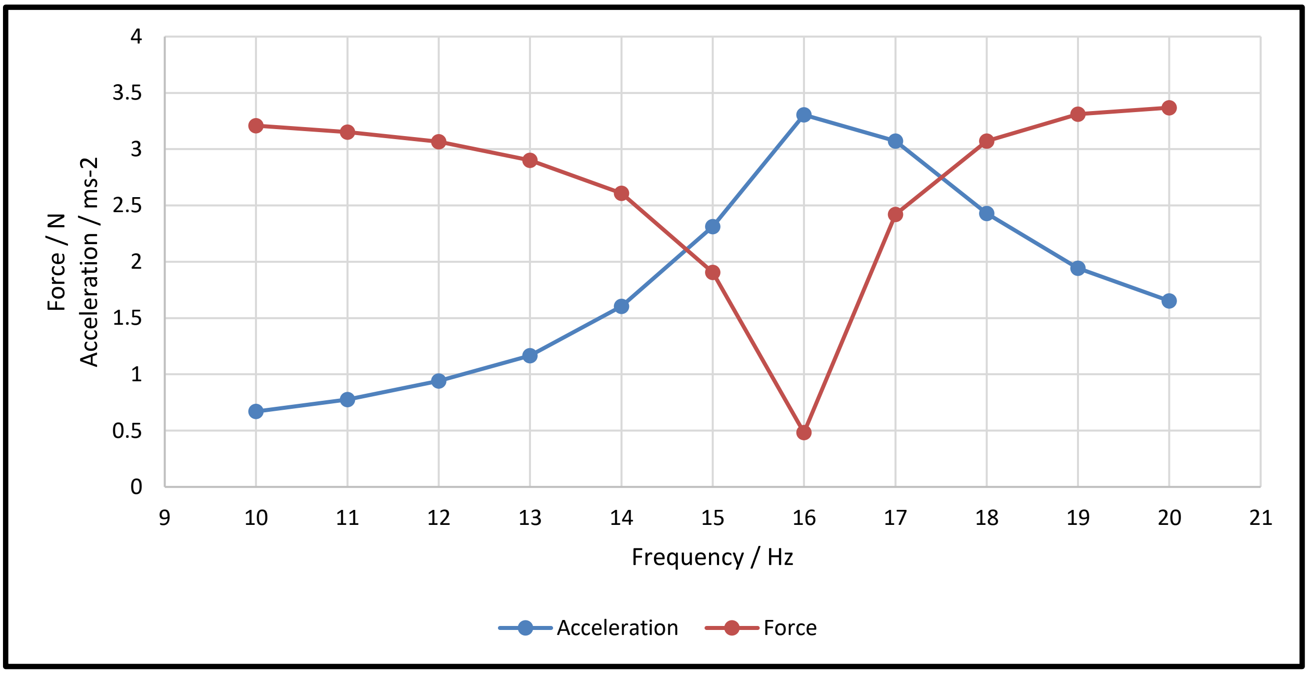

Figure 3.2 – Coarse sweep graph showing maximum values of force and acceleration as a function of frequency

From figure 3.2, it was noted that the maximum value of acceleration and minimum value of force occurred around the range of 15-17 Hz. This was taken as the **resonance range** and a fine sweep with a 0.1 Hz interval was taken across this range. The results were plotted with maximum force and acceleration again as a function of frequency (figure 4). These graphs are important as they will be the basis graphs for further calculations and derivations for the FRF magnitude and phase values in sections 3 and 4.

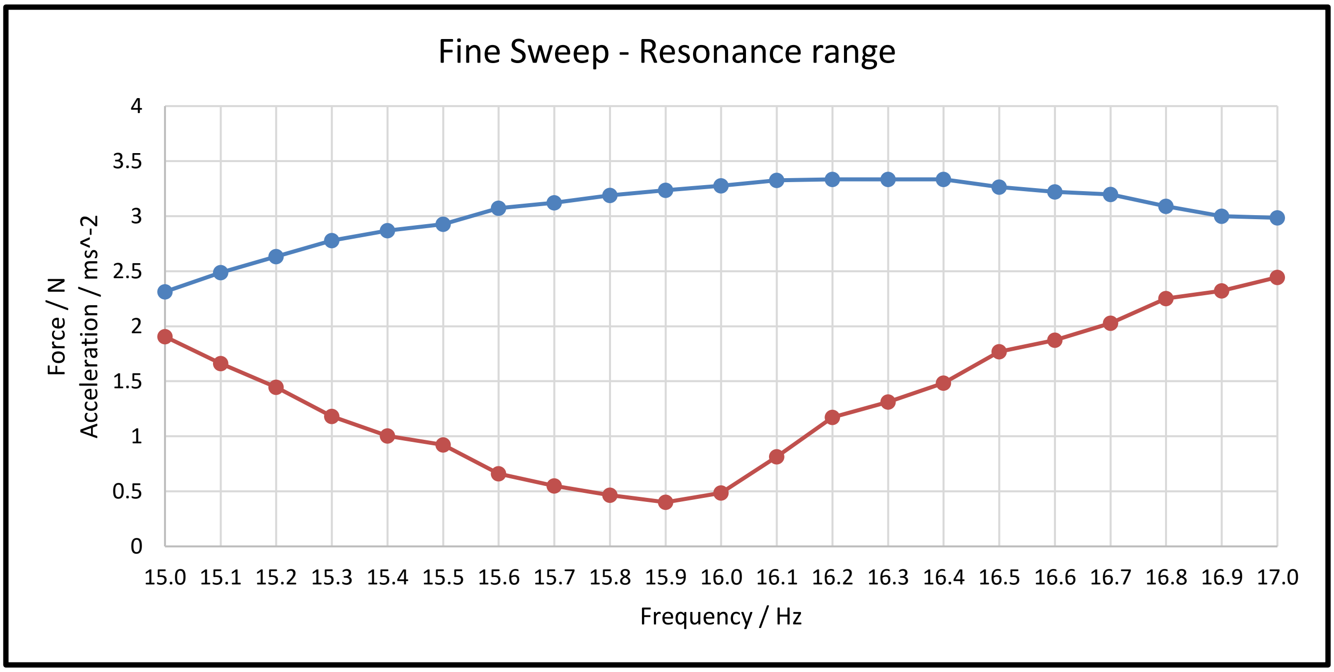

Figure 4 – Fine sweep graph near the resonance frequency showing maximum values of force and acceleration as a function of frequency

3. Frequency response functions

Show graphs of the measured frequency response graphs, describe them, and show how the damping and natural frequency is extracted from them.

Describe the phase angle between acceleration and force below and above the natural frequency and its evolution.

The magnitude of the frequency response function is defined as:

$$FRF_{mag}(\omega) = \frac{max(Displacement)}{max(Force)} \quad (1)$$

ie the ratio of the maximum displacement to maximum force experienced by the system, as a function of the excitation frequency.

However, it can be shown that displacement is proportional to and in phase with the acceleration. Therefore, FRF can be redefined as a ratio of maximum acceleration to maximum force:

$$FRF_{mag}(\omega) = \frac{max(Acceleration)}{max(Force)} \quad (2)$$

Therefore by applying equation 2 to the maximum values of acceleration and force for course and fine sweep frequencies, we obtain the frequency response function magnitude graphs below.

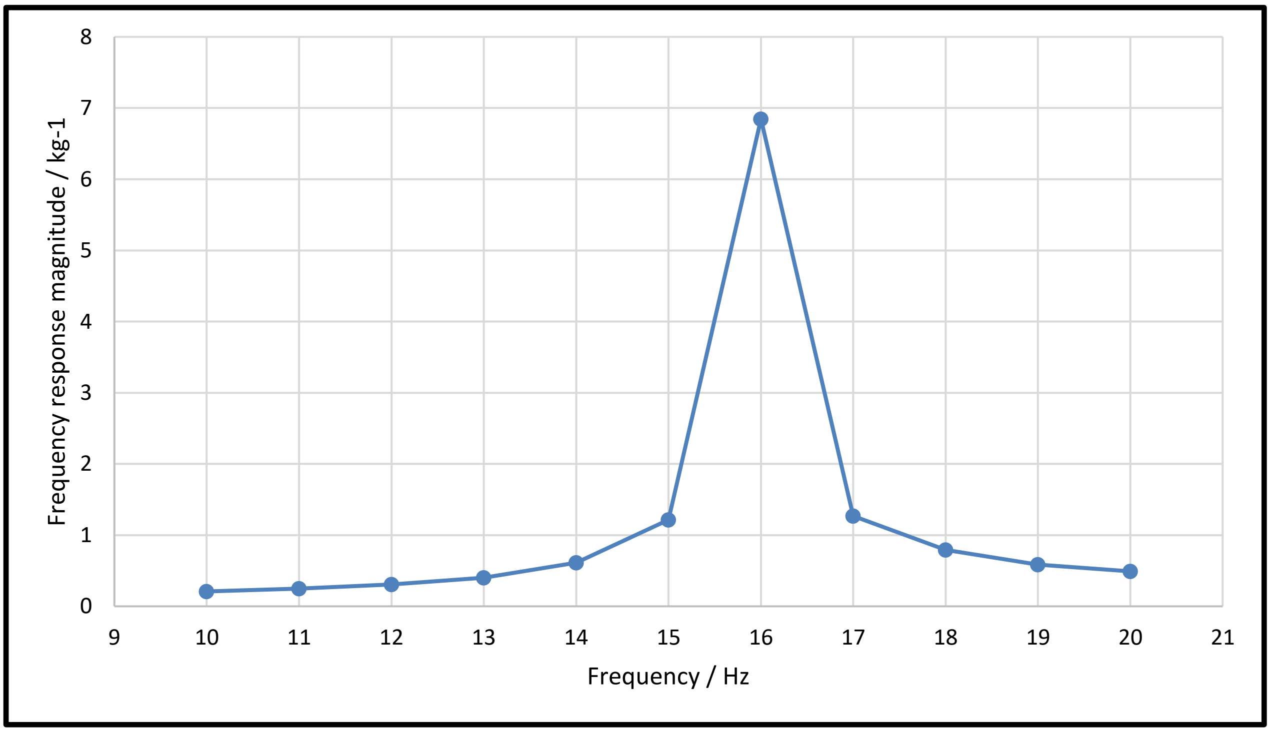

Figure 5 – Coarse sweep graph showing FRF magnitude vs frequency

From the course sweep graph, an approximate region of maximum magnitude can be isolated around 16 Hz. The damping frequency is the exact value at which the magnitude is a maximum and therefore is approximated using the 0.1 Hz fine sweep between 15.0 Hz and 17.0 Hz.

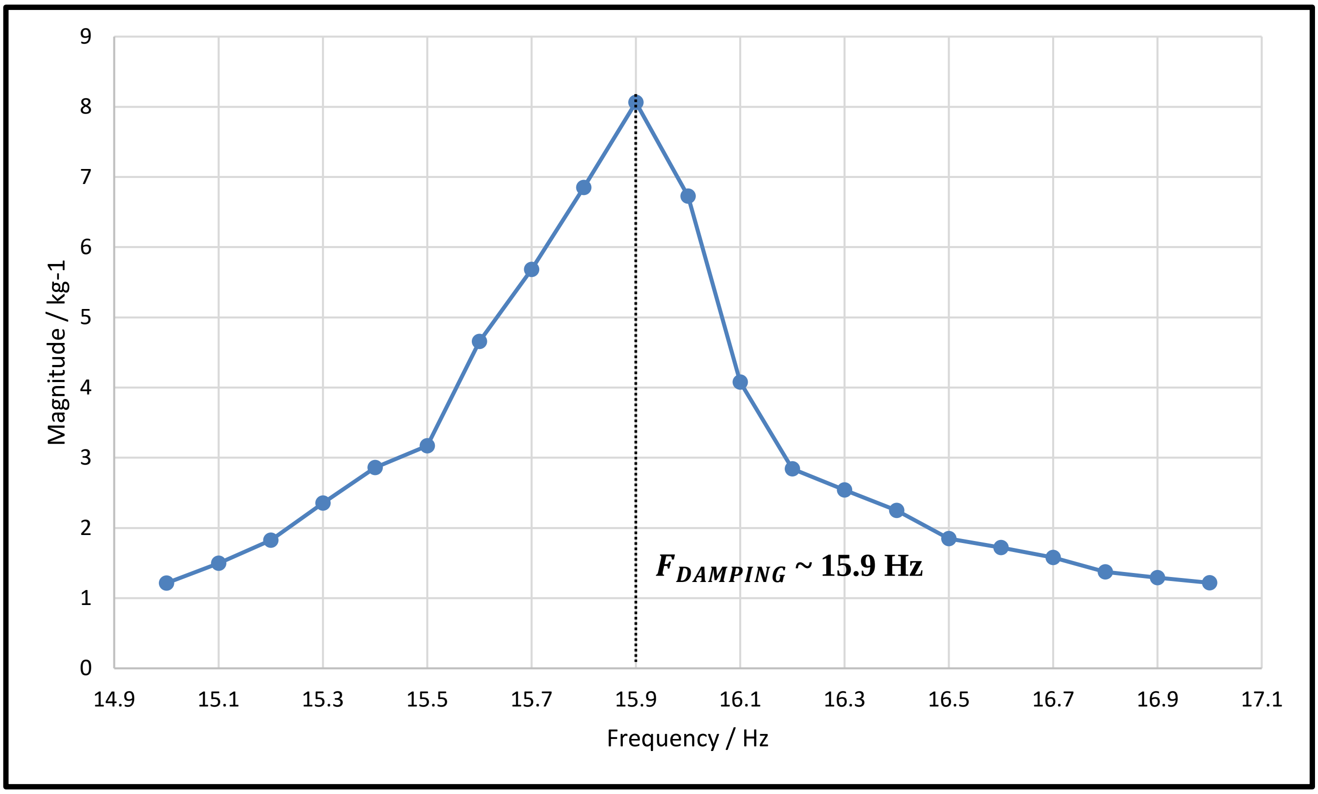

Figure 6 – Fine sweep graph showing FRF magnitude vs frequency

By observing the fine sweep graph (figure 6), it is clear that the maximum value of the FRF magnitude is approximately at a frequency of 15.9 Hz. This gives a value for the damping frequency of the system, however in order to find the critical damping ratio, the FRF magnitude plot must first be converted into a modulus-frequency plot, where:

$$Modulus\ = 20\ln(FRF_{mag}) \tag{3}$$

The units of the modulus are in dB and can be used directly to calculate the critical damping ratio. Using equation 3, the values for modulus were obtained in the fine sweep region and plotted against frequency in figure 7 below.

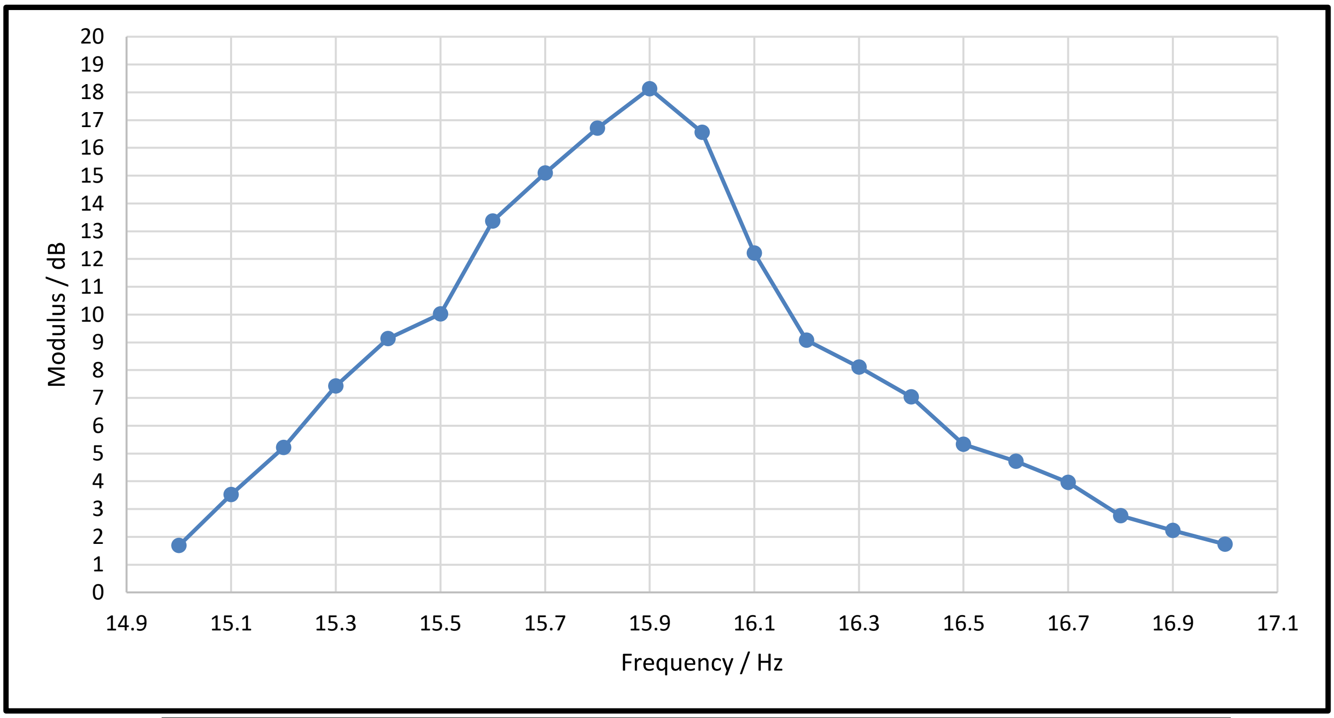

Figure 7 – Fine sweep graph showing modulus vs frequency

Given the graph in figure 7, we can approximate values to help us calculate the damping ratio. We firstly need the half-power values and then use them to find the lower and upper half-power frequencies:

$$Half - power = \left|\frac{modulus}{\sqrt{2}}\right| = \left|\frac{18.13}{\sqrt{2}}\right| = 12.82\ dB \tag{4}$$

The damping ratio is calculated by using the following equation:

$$\zeta = \frac{\omega_2 - \omega_1}{\omega_d} = \frac{16.1 - 15.6}{15.9} = 0.03145 \tag{5}$$

The natural frequency, only slightly larger due to light damping, is given by:

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{15.9}{0.99} = 16.06\ Hz \tag{6}$$

Limitations in the precision of the force gauge and accelerometers means that such a tiny difference between natural and damping frequencies cannot be reported with great reliability. The phase difference between the force and acceleration was also measured across each of the frequencies. Each phase value was then plotted on a FRF phase diagram against frequency for both the coarse and fine sweep (figures 8 and 9 respectively).

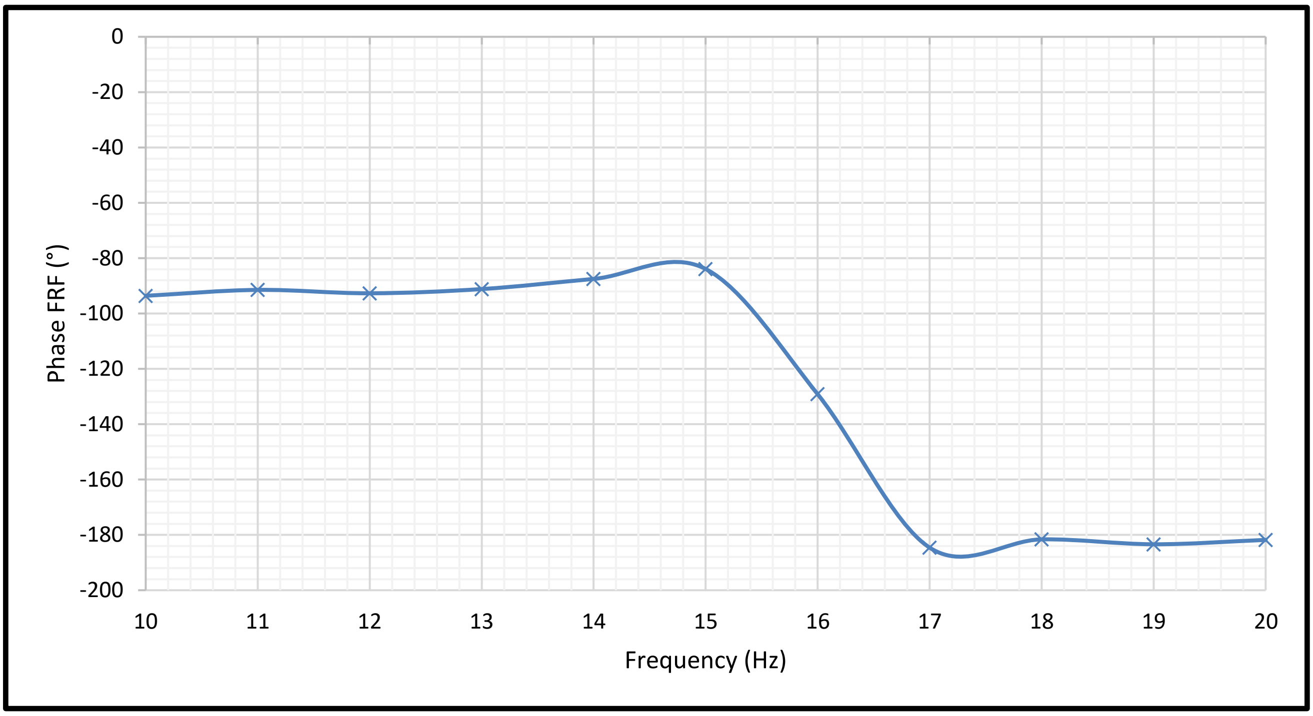

Figure 8 – Coarse sweep graph showing FRF phase vs frequency

The results were slightly erratic however a trend could be observed:

i. Below the natural frequency, the phase angle between acceleration and force was constant around -90 degrees.
ii. As the forced frequency approached the natural frequency, the steepest gradient/fall in phase angle was observed, approximately -125 degrees at natural frequency.
iii. Above the natural frequency, the phase angle continues to drop and then remains constant around -180 degrees.

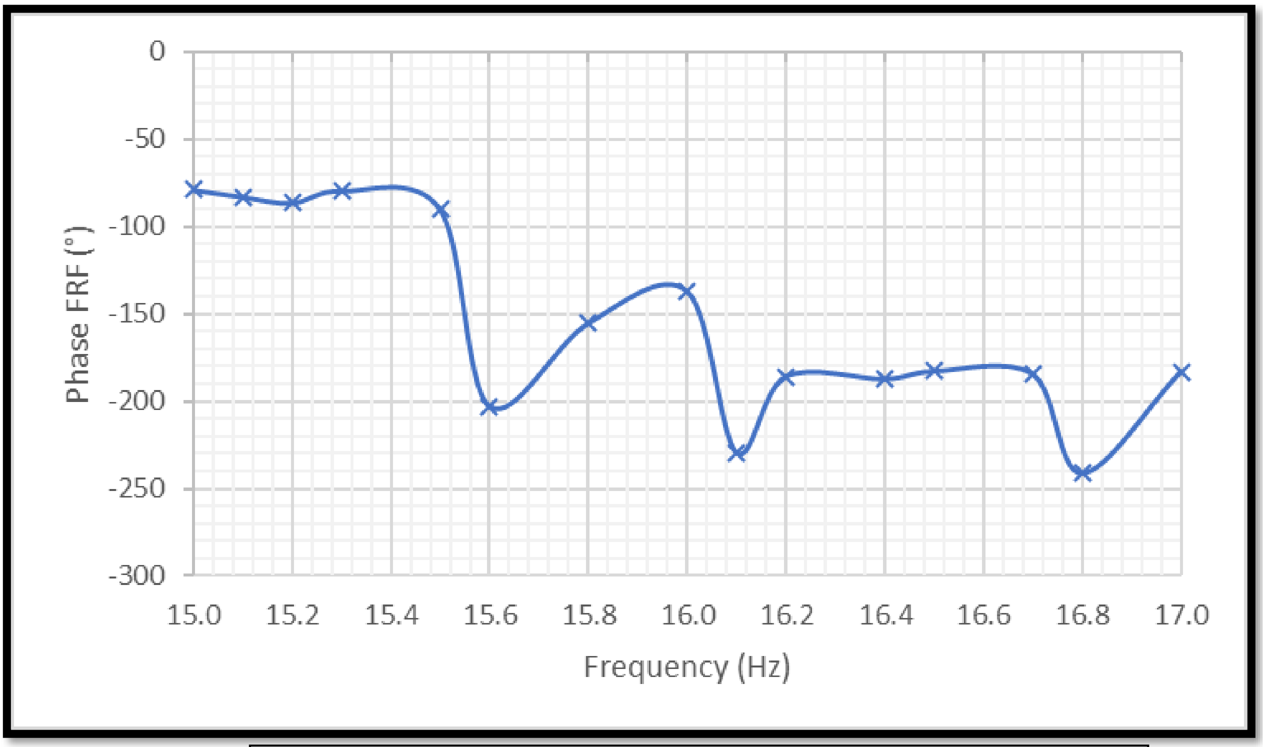

Figure 9 – Fine sweep graph showing FRF phase vs frequency

Vibrations of a system become extremely chaotic when approaching the resonance condition (damping frequency = natural frequency), therefore the fine sweep graph does not follow a linear pattern or trend but is rather erratic due to a greater degree of error in the transducers. This means that this data isn't very reliable other than to confirm the general trend in decreasing phase angle at greater forced frequencies.

4. Time history of free decay after impact

Show a plot of the acceleration time history from the second experiment and how the natural frequency and damping is extracted from it.

Since our measured values are acceleration and not displacement, we need to rearrange our equations to find the general solution in terms of acceleration. Equation 6 shows the derivation of the general solution for free vibration of a damped system (with acceleration instead of displacement):

$$\ddot{x}(t) = Ae^{-\zeta\omega_n t}(C_1 \sin(\omega_n\sqrt{1-\zeta^2 t}) + C_2 \cos(\omega_n\sqrt{1-\zeta^2 t})) \quad (6)$$

where $\ddot{\boldsymbol{x}}$ is time-dependent acceleration, $\boldsymbol{\zeta}$ is the damping ratio, $\boldsymbol{t}$ is the time in seconds, $\boldsymbol{\omega_n}$ is the natural angular frequency.

Figure 10 below shows a graph of acceleration against time, results from our experiment of free decay after an impact from an unmeasured force. After an initial transient phase for approximately the first 0.6 seconds, the classic sinusoidal decaying curve is seen.

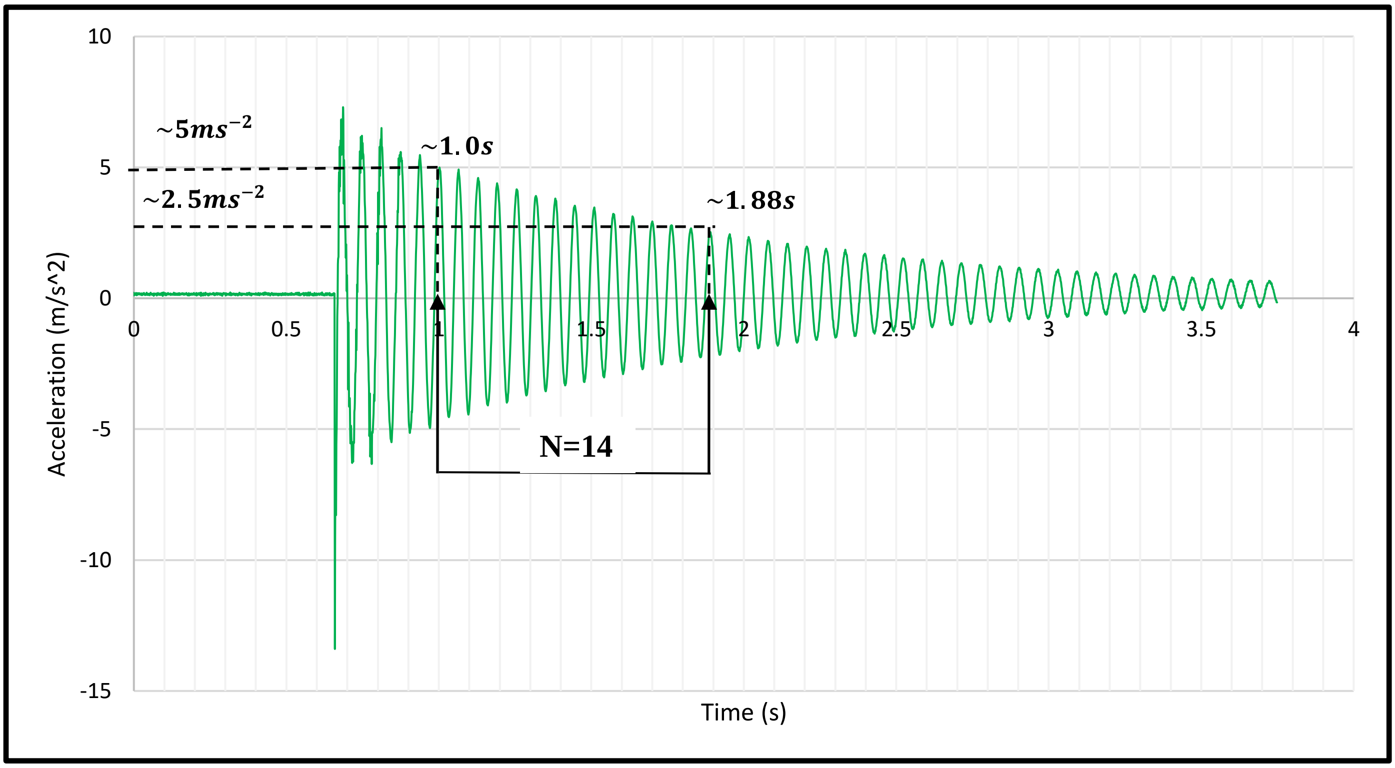

Figure 10 – Exponential decay observed in the acceleration-time graph

Since damping is light, and the critical damping ratio ζ is likely to be less than 0.1, the critical damping ratio can therefore be obtained by firstly evaluating the logarithmic decrement:

$$\delta \approx \frac{1}{N}\ln\left(\frac{\ddot{x}_1}{\ddot{x}_{N+1}}\right) \quad (7)$$

Where $\ddot{\boldsymbol{x}}_{\boldsymbol{1}}$ represents the acceleration evaluated at a reference peak (5 ms^{-2} in figure 10), $\ddot{\boldsymbol{x}}_{\boldsymbol{N+1}}$ represents the acceleration at the *Nth* peak after the reference value.

From equation 7 and the values obtained from figure 10, the critical damping ratio is found to be:

$$\zeta \approx \frac{1}{2\pi N}\ln\left(\frac{\ddot{x}_1}{\ddot{x}_{N+1}}\right) = \frac{1}{2\pi(14)}\ln\left(\frac{5}{2.5}\right) = \mathbf{0.00788} \quad (8)$$

The natural frequency for this decay is calculated by using an equation correlating the time period with the frequency.

$$f_n\sqrt{1-\ \zeta^2} = \frac{1}{T} \tag{9}$$

The time period is determined from data points of the oscillation in figure 10:

$$T = \ \frac{1.88-1.0}{14} = 0.063s \tag{10}$$

Hence frequency is found as:

$$f_n = \frac{1}{T\sqrt{1-\zeta^2}} = \mathbf{15.9\ \mathit{Hz}} \tag{11}$$

5. Table of extracted natural frequencies and damping values and discussion

	Natural Frequency (Hz)	Damping factor
Experiment 1	16.0	0.03145
Experiment 2	15.9	0.00788

Did you expect to obtain exactly the same natural frequency and damping factor values in both experiments? Explain why your values are different (or almost the same)?

From the results, it's evident that the natural frequency obtained was approximately the same however in the case of the damping factor, the results vary by a factor of 4. The reason for this variation is likely to be measurement errors made by the system, since the damping values are extremely small (<0.1). Moreover, the system was experiencing it's greatest vibrations as it approached the resonance condition and this makes it difficult for equipment to measure data reliably.

I would expect the values of the critical damping ratio to be identical in perfect conditions, however differences arose due to experimental errors. The damping factor relied on data to do with phase difference, which required extremely small quantity measurements (order 10^{-1} and lower) and therefore introduced greater amounts of relative error. This error had more of an impact especially since the system was under chaotic conditions near the resonant frequency. In contrast, the natural frequencies were approximately the same value because the relative errors in deriving the natural frequency were less even if the absolute error in measurement was the same (due to a large order of values around 10^{2}).

Therefore, since the system approached the natural frequency, errors were introduced due to the chaotic vibrations of the system in the region of the natural frequency. Moreover, the differences in the values of the damping factor were caused by inaccuracies in small order measurements, such as the phase angle. However, the results for the natural frequency remain similar regardless because of the extremely low values of the critical damping factor. These low values mean that errors in the measurement of the damping factor do not affect the results obtained for natural frequency.