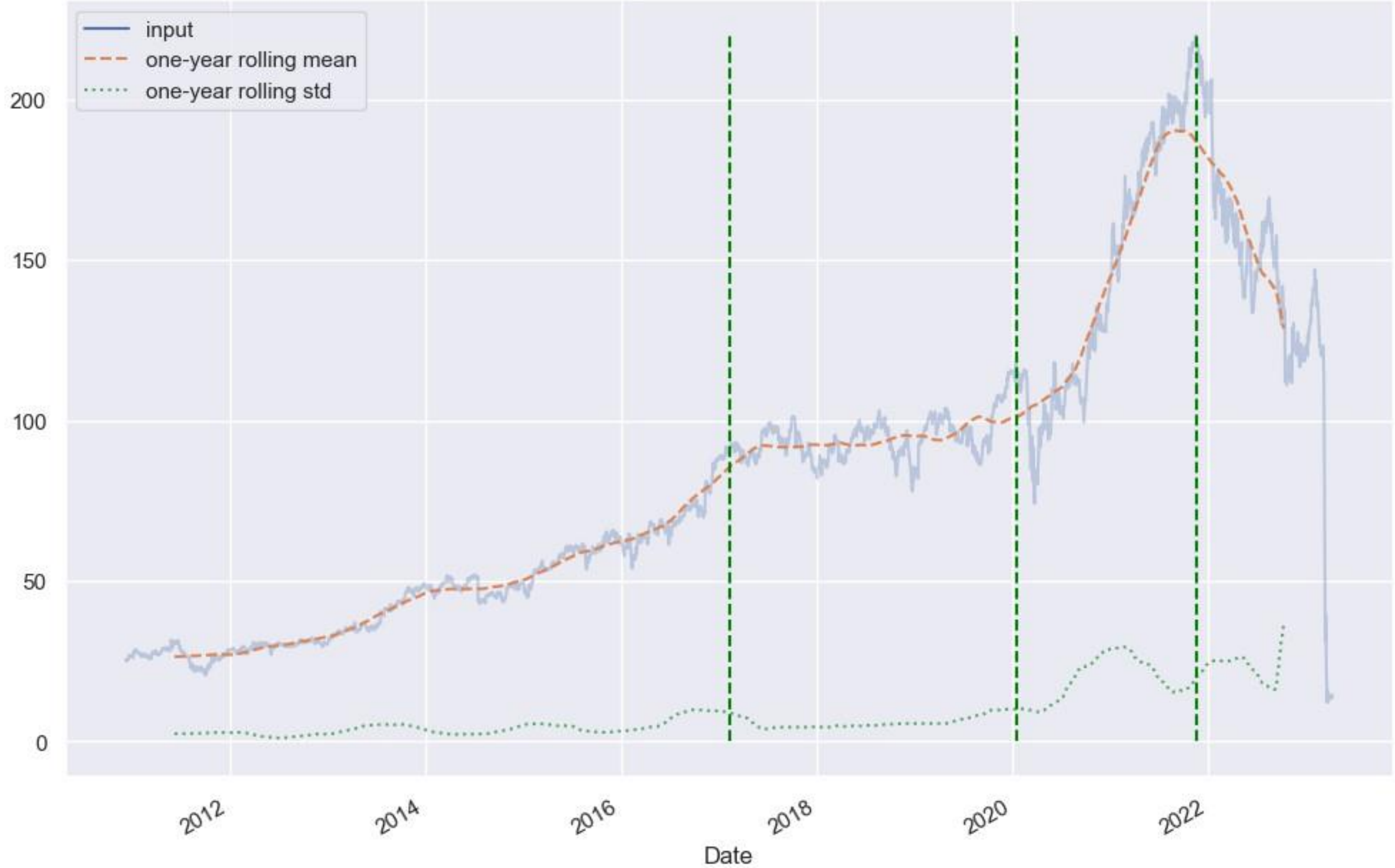
The Log-Periodic Power Law (LPPL) Model for Stock Market Crash

Yan Zeng, 5/23/2023

Outline

- The challenge: forecast the timing and magnitude of the stock price crash of First Republic Bank (ticker: FRC)
- The methodology: Econometrics (Kepler) vs. Econophysics (Newton)
- The Log-Periodic Power Law (LPPL) model in econophysics for stock market crash modeling
  - Success stories: past and current
  - The model formula and its theoretical grounding
  - A roadmap to productionize the LPPL model in a commercial environment

The challenge: FRC stock price peaked on 11/16/2021 and then crashed. *How to forecast it?*



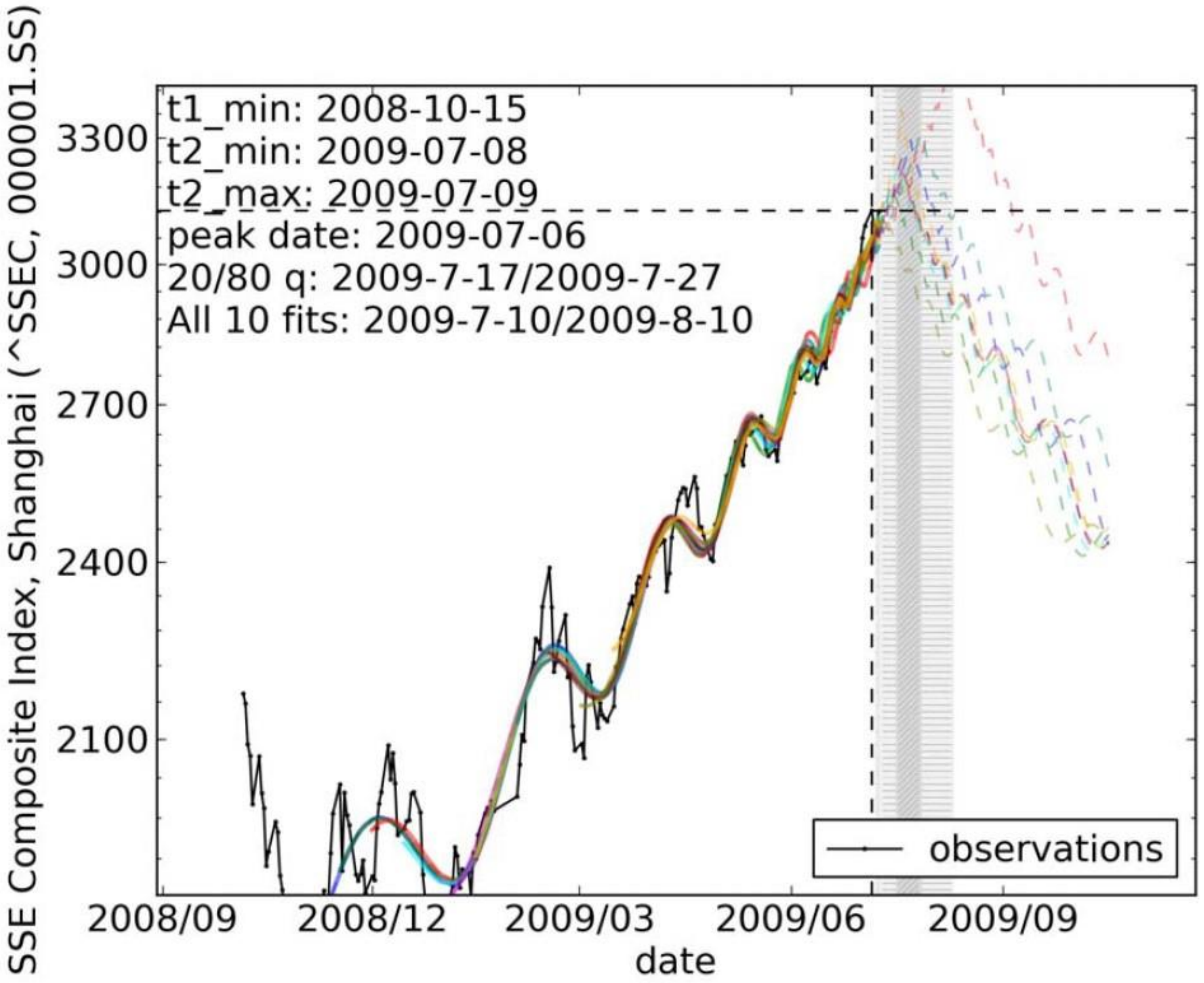
Knowing any planetary trajectory is an ellipse, how do we know why? *Choice of modeling methodology*

- Finance and economics theories can tell “stories”, but are short of providing model-based quantitative characterization of stock market crashes
- “Extreme value theory + self-exciting models (+ ARMA-GARCH)” may offer some hope, but the data-fitting exercise is hard given the high frequency and extreme magnitude of stock market crashes
- The bottom-up approach of physics is the most desirable methodology, if a model can be built based on first principles

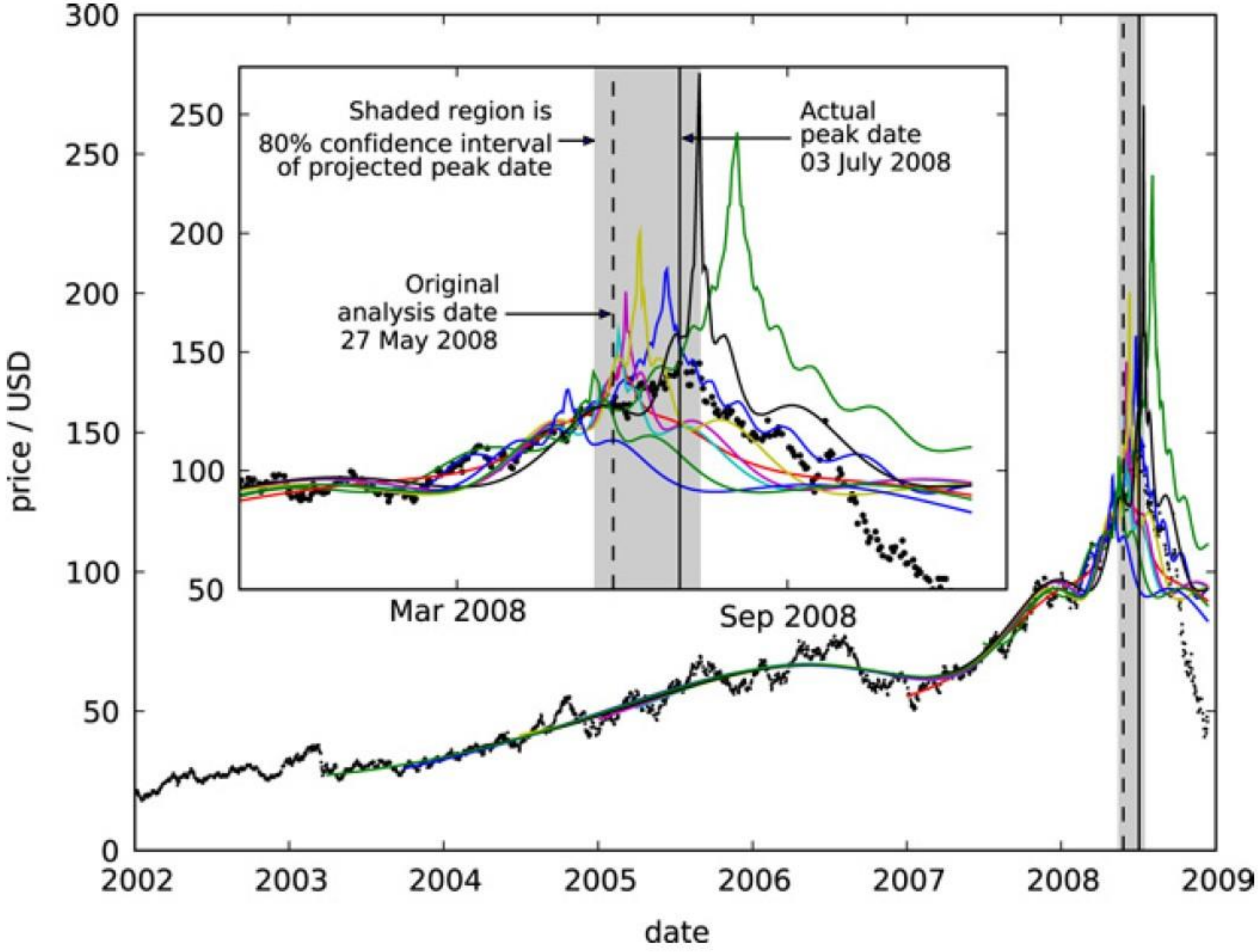
The first principles for the LPPL model

- Herding behavior: Market participants imitate each other so that asset prices are driven up away from their fundamental values
- Rational expectation: super-exponential growth of asset price appears since market participants need it to compensate for the increasing risk of a crash, until the moment of phase transition
- Critical phenomena of a complex system: “the road to crash” is bumpy, full of oscillations. E.g. earthquakes, avalanches, etc.
- Mean field theory for macroscopic modeling
- “Ising model → hierarchical diamond lattice” for microscopic modeling

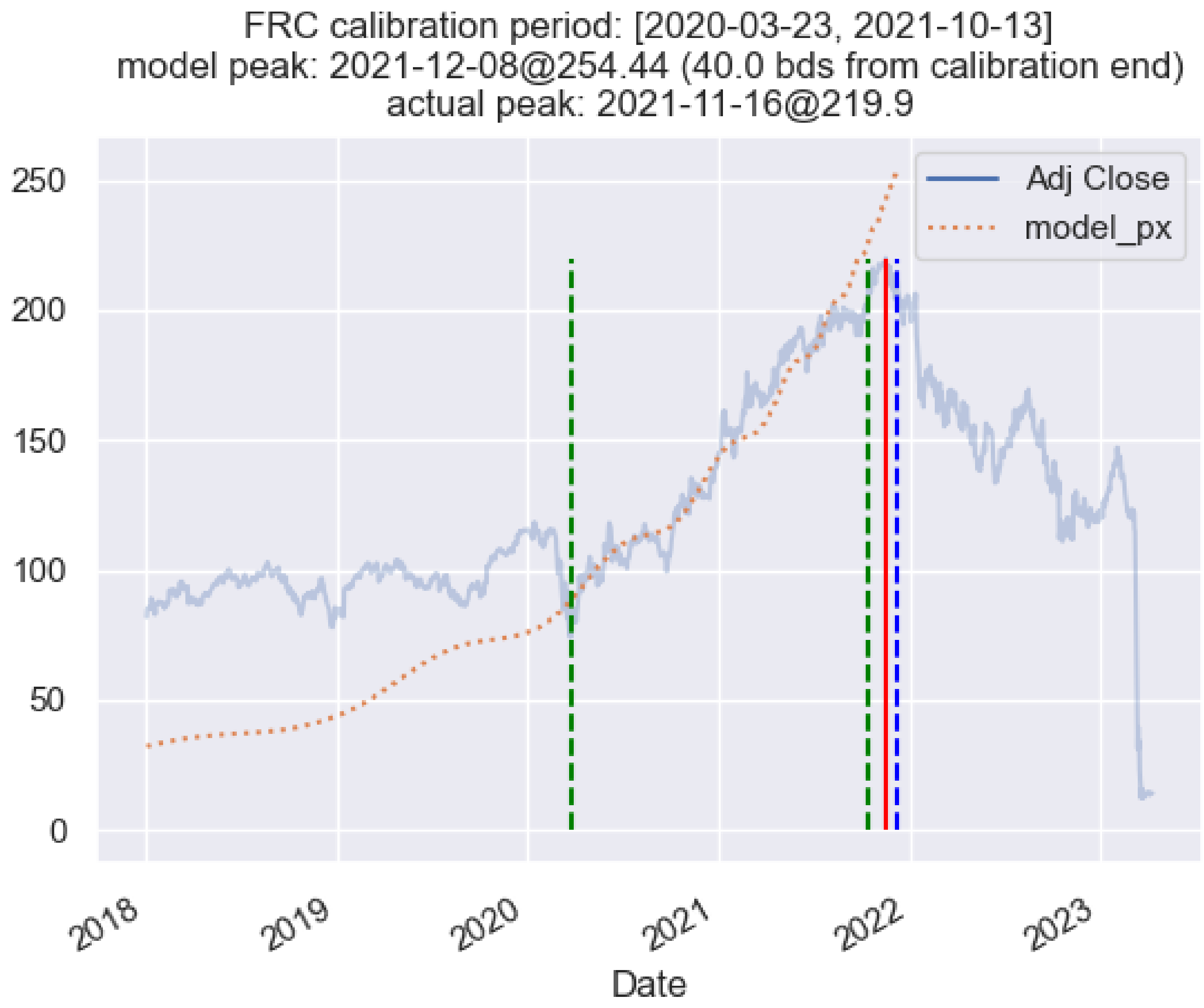
Success stories: The Chinese equity bubble to burst on 7/17-7/27, 2009 (20%/80% quantile confidence interval)



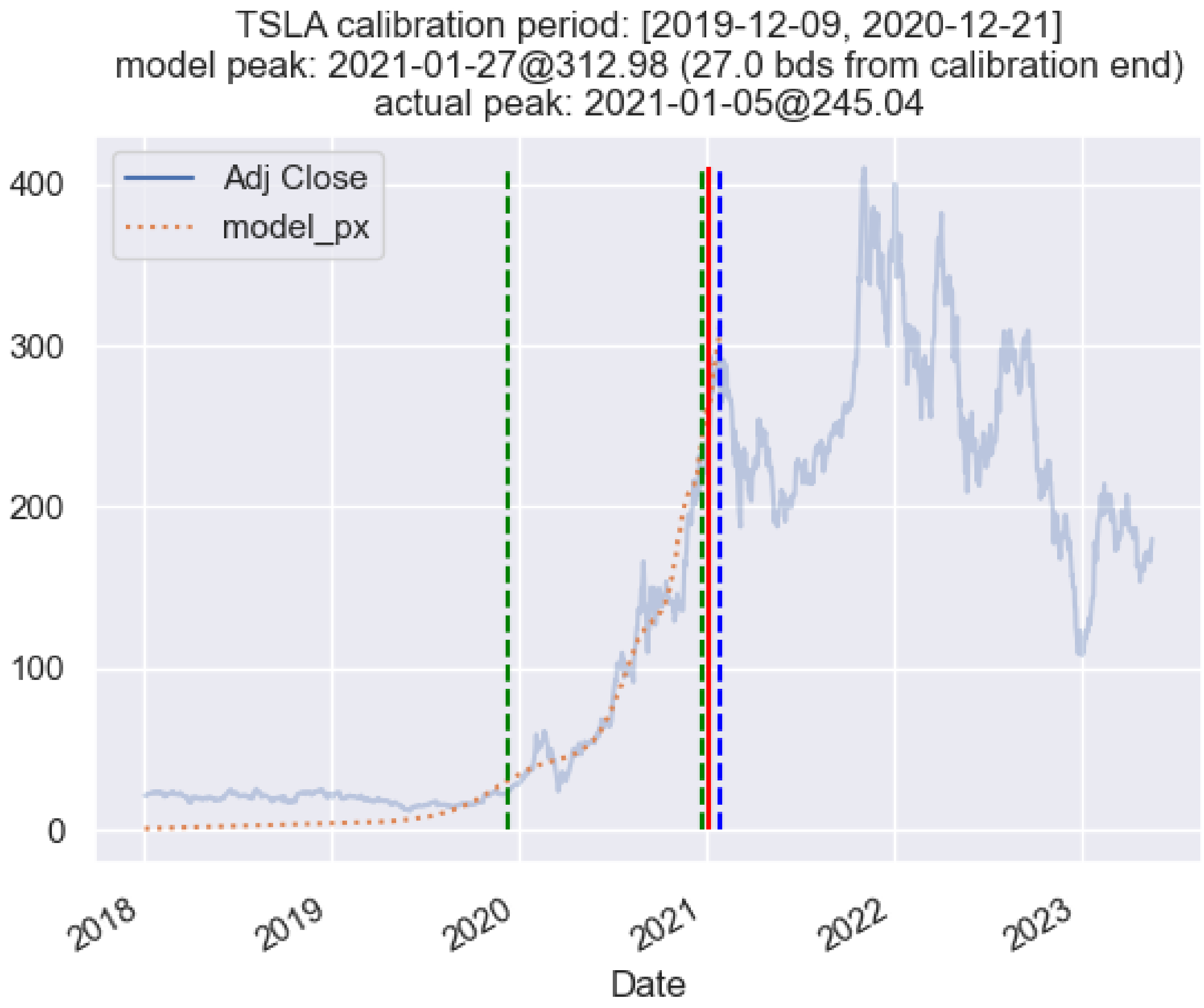
Success stories: The 2006-2008 oil bubble



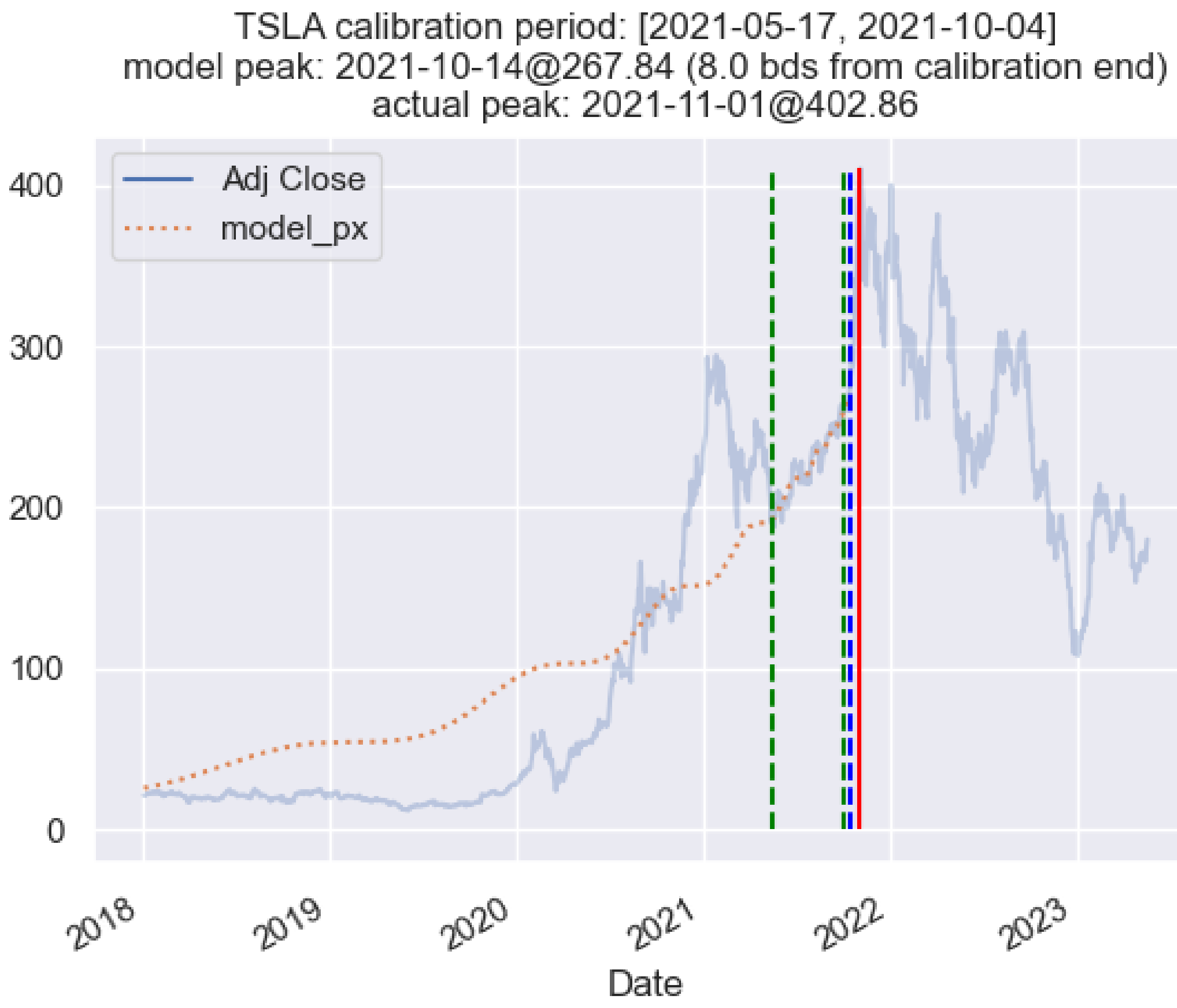
Success stories: FRC stock price crash



Success stories: TSLA stock price crashes



Success stories: TSLA stock price crashes (contd.)



The crash forecasting for FRC and TSLA stock prices could be more accurate

- The model fitting is a non-convex, highly non-linear optimization problem that have many local mimima
- The numerical algorithms need to be customized to work; metaheuristics may be employed (simulated annealing, genetic algorithm etc.)
- Given the limited time for this project, the best possible calibration algorithms were not implemented
- Results for FRC and TSLA, in terms of forecasting accuracy, are really “civilian grade GPS”, not “military grade GPS”

The model formula and its intuition

- $\log(p_t) = A + B(t_c - t)^\beta + C(t_c - t)^\beta \cos(\omega \log(t_c - t) + \varphi)$
- t_c is the critical time (when bubble bursts), $A = \log(p_{t_c})$ is the log price at the critical time
- $B < 0$, $C \in (-1, 1)$, $\beta \in [0.1, 0.9]$, $\omega \in [6, 15]$, $\varphi \in [0, 2\pi]$
- To see the super-exponential growth and the oscillatory behavior, transform the formula to

$$p_{t_c} = p_t \times e^{-\frac{B(t_c - t)}{(t_c - t)^{1-\beta}}} \times e^{-\frac{C(t_c - t)}{(t_c - t)^{1-\beta}} \cos(\omega \log(t_c - t) + \varphi)}$$

A roadmap for productionizing the LPPL model

- Automation of calibration period selection: detection of regime switching (fractional Gaussian process, multi-resolution analysis)
- Numerical optimization for model calibration
  - Generation of initial guesses: grid search, taboo search
  - Optimization algorithms: Nelder-Mead simplex algorithm, Levenberg-Marquardt algorithm, genetic algorithm, etc.
  - Parallelization of the computation
- Statistical analysis
  - Residual validation: unit root test, spectral analysis
  - Residual modeling: Ornstein–Uhlenbeck process, ARCH-GARCH
  - Confidence interval estimation
- Extensions of the original LPPL model

References

- Anders Johansen, Olivier Ledoit, and Didier Sornette. “Crashes as critical points”. *International Journal of Theoretical and Applied Finance*, Vol. 3, No. 2 (2000) 219-255.
- K. Bastiaensen, P. Cauwels, D. Sornette, R. Woodard, and W.-X. Zhou. “The Chinese equity bubble: Ready to burst”. arxiv:0907.1827, July 10, 2009.
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