QCI

Day 5: Quantum Gates [1]

State Vectors

Does this satisfy conservation of probability?

$$|\psi\rangle = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$$

Exercise: Qiskit Statevectors

Speaker notes

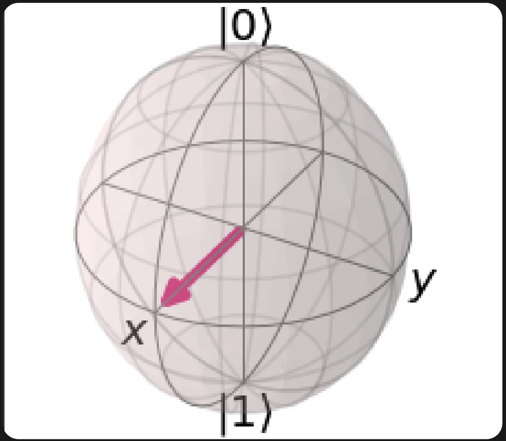
Show custom state and bloch sphere plots

Superposition & Phase

Speaker notes

Information from this section comes from Qiskit Textbook. Single Qubit Gates.

Hadamard Gate



$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H|0\rangle = |+\rangle$$

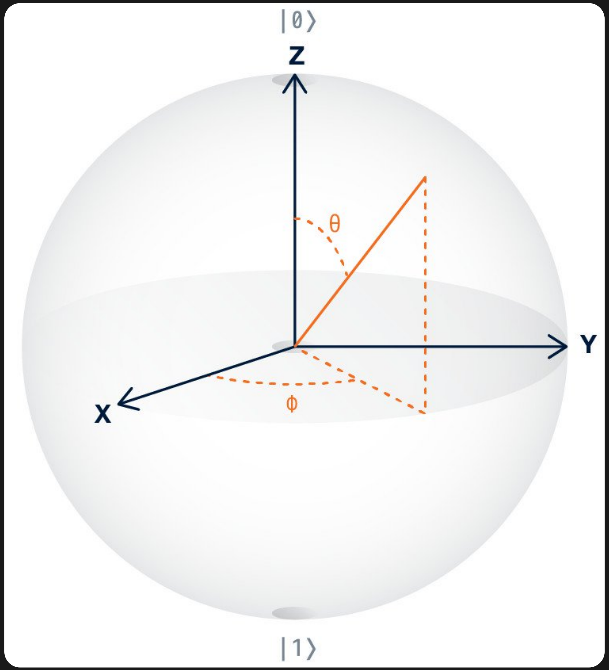
$$H|1\rangle = |-\rangle$$

Speaker notes

Apply H to I0> and I1>. What is the difference? Show math on iPad

The + and - have to do with the sign of the phase (we'll see this on the bloch sphere)

R_ϕ Gate



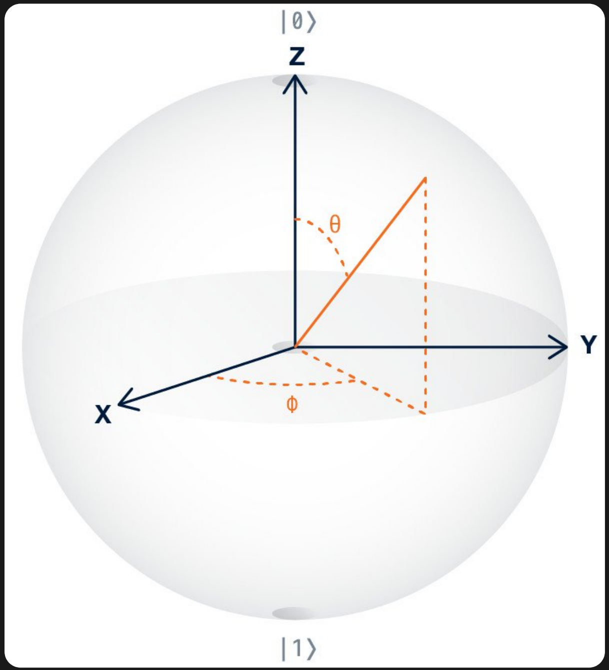
$$R_\phi = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix}$$

Speaker notes

Performs a rotation by ϕ radians on the Z axis.

What gate does this resemble?

$S/S\dagger$ ($\sqrt{Z}/\sqrt{Z}^\dagger$) Gates



$$\sqrt{Z} = S = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{2}} \end{bmatrix}$$

$$\sqrt{Z}^\dagger = S^\dagger = \begin{bmatrix} 1 & 0 \\ 0 & e^{-\frac{i\pi}{2}} \end{bmatrix}$$

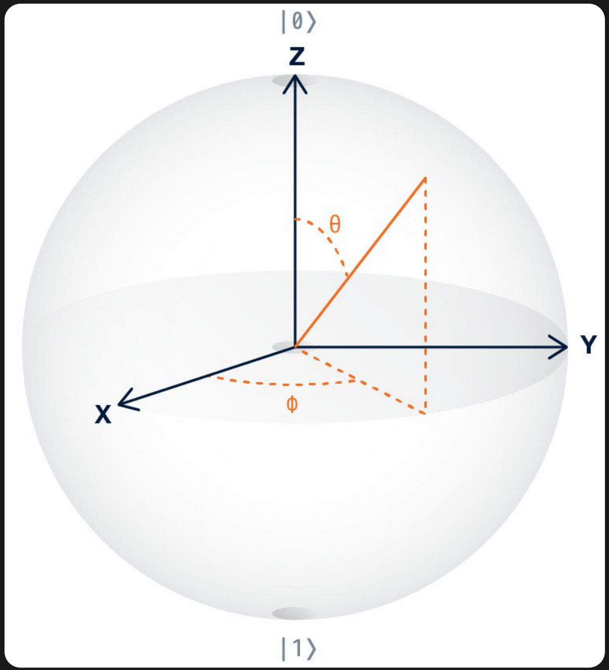
Speaker notes

Special case of the R_ϕ gate with $\phi = \frac{\pi}{2}$

Why is this called $sqrtZ$? (applying this gate twice results in Z)

Are they their own inverses? (No. Show math on Jamboard)

$T/T^\dagger$ ($\sqrt[4]{Z}/\sqrt[4]{Z}^\dagger$) Gates



$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{bmatrix}$$

$$T^\dagger = \begin{bmatrix} 1 & 0 \\ 0 & e^{-\frac{i\pi}{4}} \end{bmatrix}$$

Exercise: Qiskit Superposition and Phase

Entanglement

Review: Entangled Statevectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

How might we entangle two qubits?

Speaker notes

Bell State: H + CNOT (Show in Qiskit, show histogram and bloch sphere)

Qiskit: Intro to Phase Kickback

Speaker notes

Put a $|0\rangle$ into superposition with H, it has positive phase.

Put a $|1\rangle$ into superposition with H, it has negative phase.

Show matrix multiplication with CNOT (from $|+\rangle$ to $|-\rangle$) to see what will happen

Phase Kickback: Using a controlled gate to modify the control qubit

Speaker notes

Wrap a CNOT in H gates -- shows kickback

References

- Qiskit Textbook. Single Qubit Gates
- Qiskit Textbook. Phase Kickback
- Qiskit Textbook. Multiple Qubits and Entangled States