QCI

Day 4: Quantum Gates [0]

Quantum Gates

Single-Bit Operators

Identity

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Constant-0

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

Negation

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Constant-1

$$\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$$

Speaker notes

Remember we said we can show changes to state as matrices?

What are the 4 single-bit operators? Guess their matrices.

Show math on Jamboard. Multiply these by I0> (default state)

Which ones are of these valid quantum operations? (identity, negation)

Single-Qubit Operators

Identity

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Negation

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

I Gate

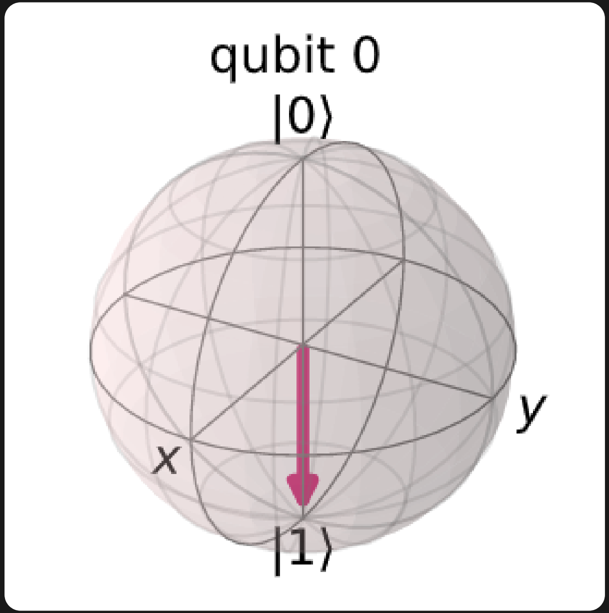
$$f(x) = x$$

$$0 \longrightarrow 0$$
$$1 \longrightarrow 1$$

Identity

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

X Gate



Negation

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Speaker notes

Rotates our state by π radians on the X axis

Show X gate on Qiskit

Y Gate

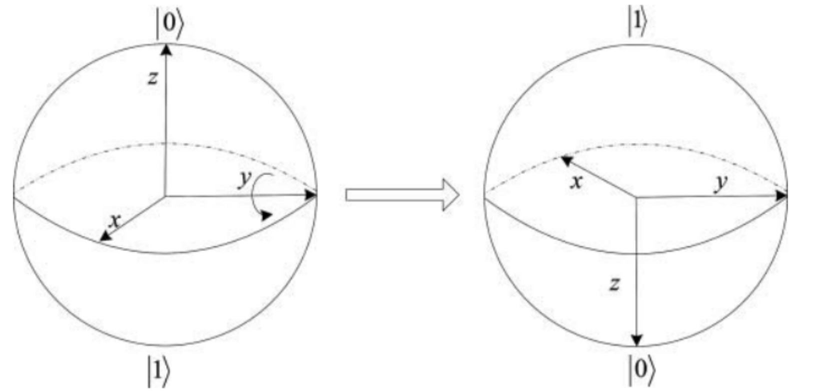


Figure 5.7. A rotation of the Bloch sphere at y.

Negation

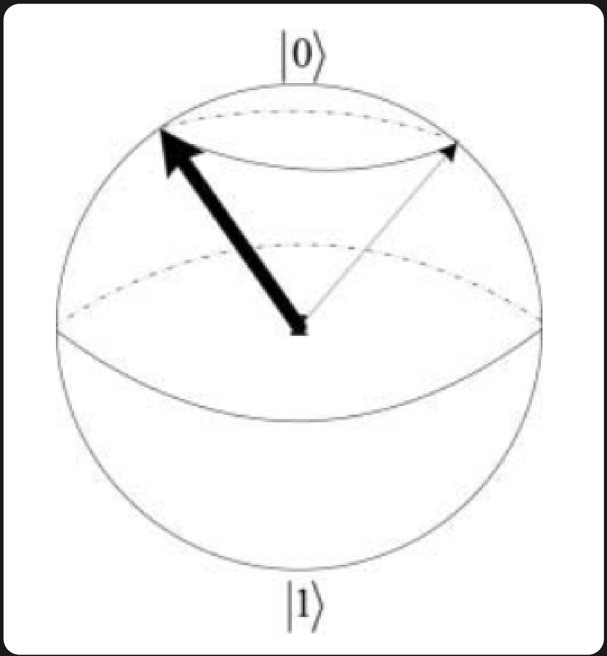
$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Speaker notes

Rotates our state by π radians on the Y axis. Will turn $|0\rangle$ into $|1\rangle$ much like the X gate.

Image from Mannucci, Yanofsky pg. 163

Z Gate



Phase Flip

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Speaker notes

Rotates our state by π radians on the Z axis. Doesn't change probabilities (how high/low the vector points), but it does change the phase

Image from Mannucci, Yanofsky pg. 162

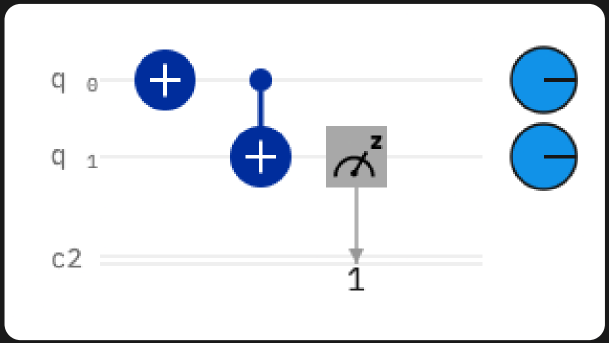
Qiskit Demo: Hands-On With Pauli Gates

Speaker notes

Show in Qiskit and then IBM Quantum Composer

Multi-Qubit Gates

CNOT Gate



XOR

Flips target if control is 1

Speaker notes

Functions like the XOR operator and requires 2 qubits.

What special matrix does this look similar to? (Identity)

If the control qubit is 1, then the target qubit flips. Show truth table and compare to XOR on Jamboard.

Explain circuit diagram

Controlled-U Gate

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{pmatrix}$$

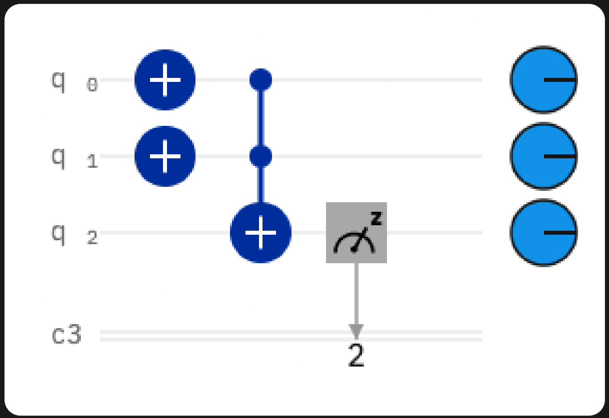
Speaker notes

Here's something cool:

We can replace a/b/c/d with complex numbers that represent a single-qubit operator and use this to perform controlled operations (not just the NOT operator)

Mannucci, Yanofsky pg. 165-166

Toffoli Gate (CCNOT)



Matrix similar to CNOT

Flips target if both control qubits are 1

Speaker notes

The matrix is big, and we're not really going to worry about it, but it's similar to CNOT but it's an 8x8 matrix.

If both control qubits are 1, then the target qubit flips

With the Toffoli gate, we can basically create any logical gate– and it's reversible! We can chain Toffoli gates together to have 3 control qubits and keep going forever

Side note (may not mention in class): In theory, we can make a computer that uses no energy

Mannucci, Yanofsky pg. 154-155

Project: Quantum Adding Machine

Speaker notes

"Truth table" for binary addition Reference

References

- Yanofsky, Mannucci. Quantum Computing for Computer Scientists
- YouTube. Quantum Computing for Computer Scientists
- YouTube. Quantum Gates
- Qiskit Textbook. Single Qubit Gates
- Qiskit Textbook. The Atoms of Computation