

QCI

Day 3: Intro to Quantum Mechanics

Intro to QM

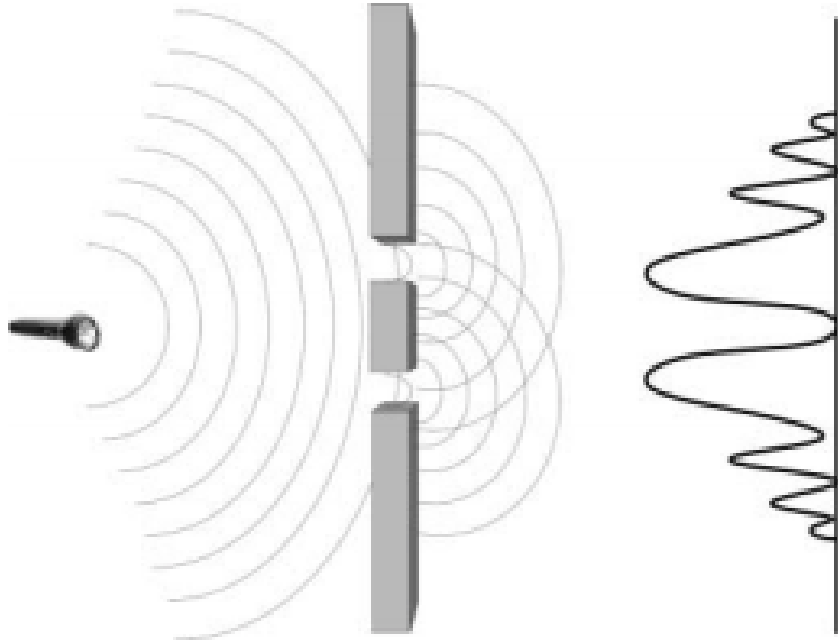


Figure 4.1. Young's double-slit experiment.

Is light a wave or a
particle?

Interference patterns

Send one photon in at a
time

Speaker notes

One photon can go through both slits at the same time and even cancel itself out.

We can do this experiment with small particles—not just light

Pg. 105-106 in Mannucci, Yanofsky

Wave Equation

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle$$

Speaker notes

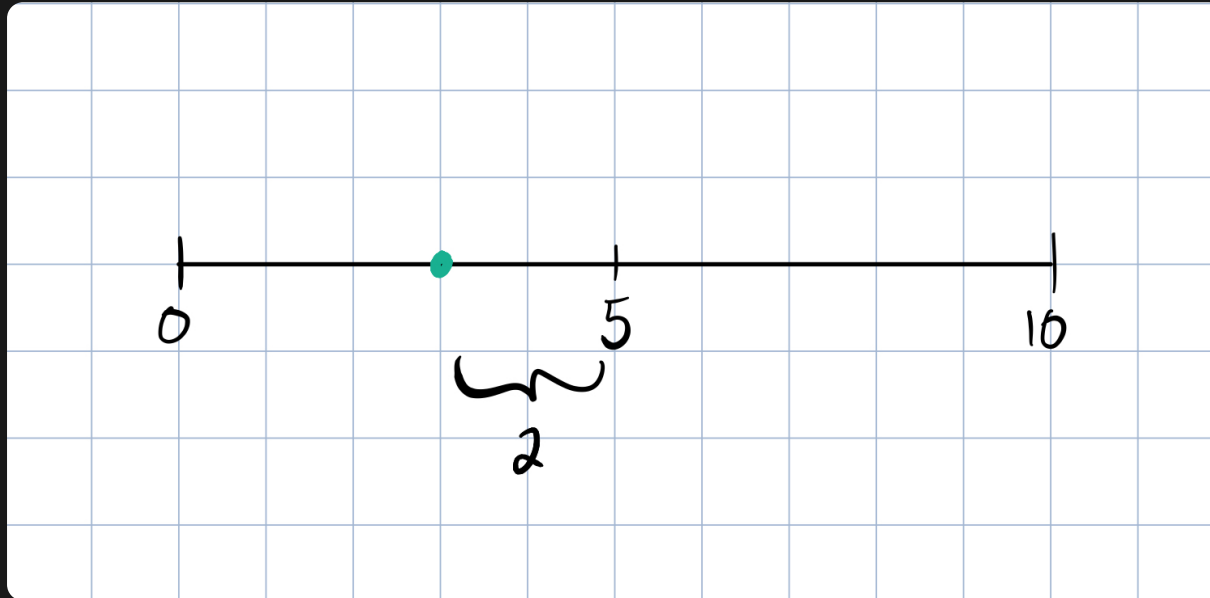
Pattern on the detector and its relation to the wave function

Don't worry, you won't really see this equation later. Understanding it is beyond the scope of our course, but this wave function determines our quantum state.

Copenhagen interpretation of QM - wave function describes probabilities

Quantum States

The Number Line Analogy



$$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}$$

Infinite possible positions spaced distance dx apart

Speaker notes

Imagine a quantum particle's state as a position on this line. Represent its state in a vector

For those of you who are familiar with calculus, you'll recognize this as a differential (infinitely small) distance

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Speaker notes

Remember these? We said we'd use them.

These vectors represent a quantum state (a qubit, a quantum bit) which can have either 0 or 1. Like bits form classical computers, qubits form quantum computers. The 0 or 1 can be represented by the spin of a particle like an electron (explain spin) and can be manipulated by precise microwave pulses

These are simpler than the more complex number line we saw earlier, and thankfully we won't be dealing too much with state vectors of higher dimensions

Superposition

$$|\psi\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{pmatrix}$$

Speaker notes

If this vector represents a state, what does this even mean?

Our vector ψ is in a **superposition** of 0 and 1. It is both 0 and 1 at the same time

Draw an analogy to how a photon can go through 2 slits at once

Our amplitudes can be complex numbers, as they are like the amplitudes in a wave and can cancel each other out

Observer Effect

Observing a system changes its state

Causes a superposition to collapse

We cannot see a quantum state in superposition

Speaker notes

Blind person trying to locate a ball with a stick analogy

Probabilities

$$|\psi\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{pmatrix}$$

$$p(|x\rangle) = |\langle x|\psi\rangle|^2$$

Speaker notes

What information can we gain from knowing the superposition state of a quantum system?

We square the inner product (a form of the dot product) to find probabilities. We can also just find the square the absolute value of the amplitude

Show math on Jamboard

From Qiskit Textbook: Representing Qubit States and Manuucci, Yanofsky p. 96

$$p(|\psi\rangle) = 1$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\therefore \sqrt{|\alpha|^2 + |\beta|^2} = 1$$

$$|\alpha|^2 + |\beta|^2 = 1$$

Speaker notes

Probability must be conserved. α and β are the amplitudes

Show math on Jamboard. Sum of amplitudes squared must add to 1

Remember where else we saw the Pythagorean theorem? Vectors! The norm of the vector is 1, so we can plot this on a unit circle (actually, we need a unit sphere -- we'll get to this in a second')

We need complex numbers as when we square the complex amplitudes a and b , sometimes these probabilities cancel themselves out. This is like the interference in the double slit experiment

From Qiskit Textbook: Representing Qubit States

$$|\alpha|^2 + |\beta|^2 = 1$$

$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle$$

ϕ is the phase

Speaker notes

We'll skip the math, but we can express a qubit state in terms of angles (like we did for complex numbers)

From Qiskit Textbook: Representing Qubit States

Understand Phase

Comes from the wave-like nature of quantum

Does not affect probabilities

Can be used to cancel things out

Speaker notes

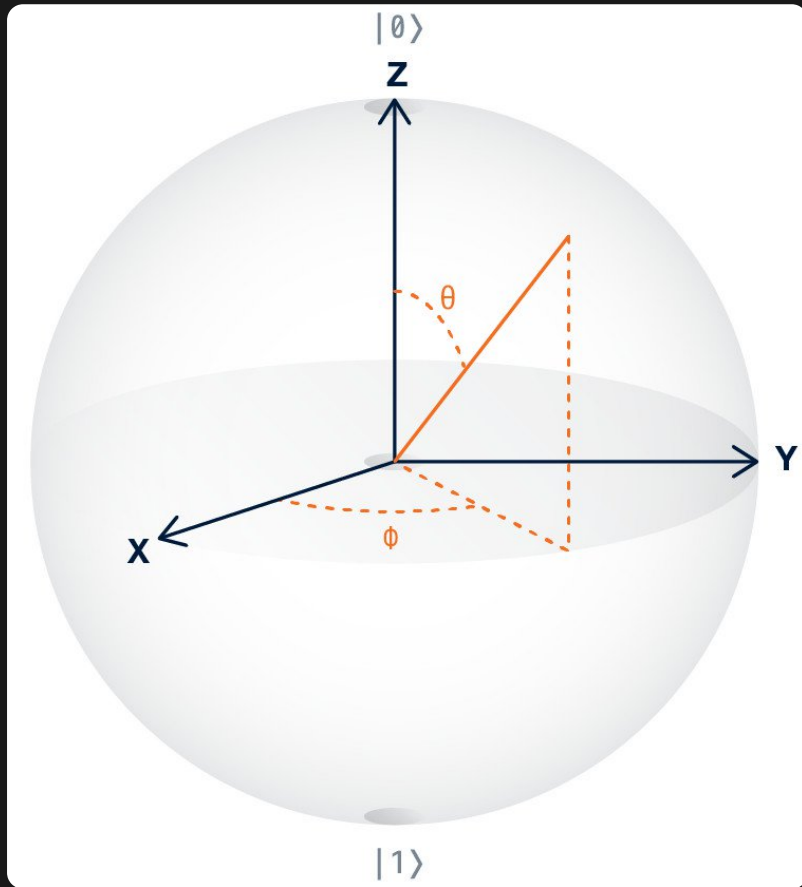
Phase shift analogy in trigonometry. Show math on Jamboard/Desmos

We can have the exact same quantum state but shifted in phase (same amplitude, different phase). We need to know phase as well to understand qubit states

Quantum logical gates can use phase differences to add/cancel things

[Show interference on this article](#)

Bloch Sphere and Phase



Phase on y -axis

Probabilities depend only
on θ

Speaker notes

The probabilities only depend on the one-ness or zero-ness of the Bloch vector

The phase is just the $e^{i\phi}$ term, and it's a complex number. The complex number is why we have a sphere and not a circle

Mannucci, Yanofsky pg. 160-162

Qiskit Exercise

Speaker notes

Only do if time permits

Work through the statevectors and superposition notebook in my principles folder

Show Bloch Sphere and use [code examples](#) as necessary

Multi-Qubit States

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} \otimes \begin{pmatrix} b_0 \\ b_1 \end{pmatrix} = \begin{pmatrix} a_0 b_0 \\ a_0 b_1 \\ a_1 b_0 \\ a_1 b_1 \end{pmatrix}$$

Speaker notes

We use the tensor product to describe two quantum states together

Show math on Jamboard

a_0b_0 shows the amplitude for both states being 0

Recall that the 0th element tells us the probability of the state being a 0. The 1st element tells us the probability of 0

Entangled States

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Cannot be factored!

Speaker notes

Recall that we cannot factor entangled states

The states only make sense **together**

When one state collapses, the other will collapse instantaneously (even if the two quantum systems are very far apart)

Show math on Jamboard

Intro to Dynamics

Speaker notes

How do things change?

Reversible Operations

Given a function and an output, you can find the input

Quantum operators are their own inverses

Speaker notes

Mannucci, Yanofsky p. 91-92 and 151-152

If there's time, go over pg. 151-152 in Mannucci, Yanofsky

Quantum Computing for Computer Scientists (8:45)

Questions

Is $f(x) = 3x$ reversible? How about $f(x) = x^2$?

Which of our single-bit operations are reversible?

Is negation a valid quantum operator?

Speaker notes

Quantum Computing for Computer Scientists (8:45)

Quick Math!

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} |0\rangle$$

$$= |1\rangle$$

Speaker notes

We represent changes to quantum states with matrices

Changes are simulated by matrix multiplication, and they must be reversible on a quantum computer

Show math (multiplication and reverse) on Jamboard

Teaser: There are 4 single bit operations. What does this remind us of? What's special about this matrix? (It's the inverse of the identity, it's the bit flip gate as a matrix)

Quantum Computing: An Overview

Qubits: Quantum Bits

Obeys quantum principles

Special quantum logic gates

Represented by superconducting chips

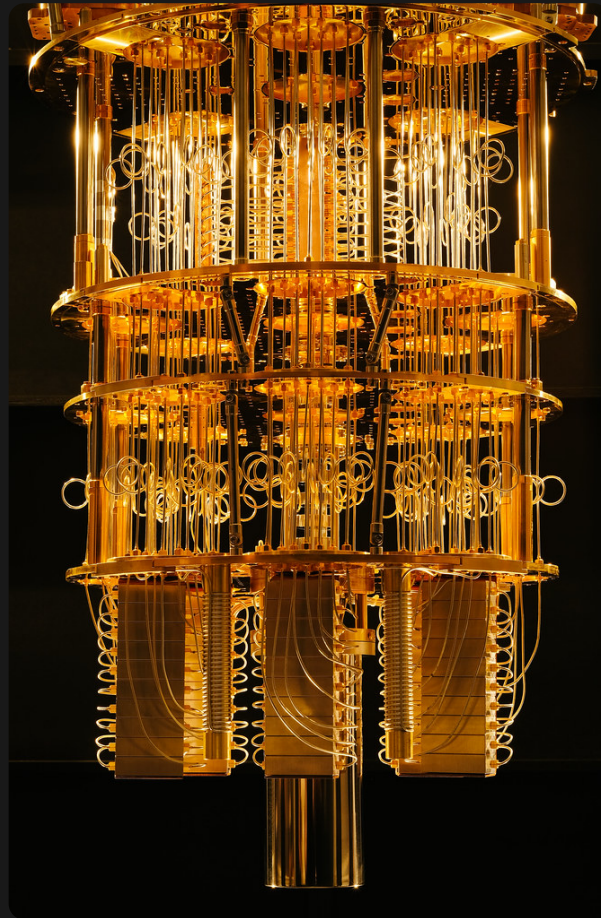
Manipulated by microwave pulses

Speaker notes

Like a bit is a unit of information, so is a qubit. We can implement it in different ways, such as the spin of a particle. Electrons have a property of spin up/down (they have angular momentum). Explain superposition with spin.

Mannucci, Yanofsky p. 141

The Quantum Computer



Speaker notes

Most of the infrastructure goes to cooling – near absolute zero!

Discuss the quantum error correction

Why Quantum?

Superposition

Entanglement

Applications of QC

Simulations

Computer security

Computer science

Speaker notes

We can simulate biological systems much faster than a classical machine

Shor's Algorithm (which we'll get into later)

Grover's Algorithm (search a list in $O(\sqrt{n})$ time)

It's not just faster – quantum computers do tasks that classical computers could never do

References

- Yanofsky, Mannucci. Quantum Computing for Computer Scientists
- [Wikipedia. Schrodinger Equation](#)
- [Qiskit Textbook. Representing Qubit States](#)
- [Quantum Computing for Computer Scientists. Youtube](#)
- [Towards Data Science. The Qubit Phase](#)