QCI

Day 2: Intro to Linear Algebra

Complex Numbers

$$i^2 = -1$$
$$i = \sqrt{-1}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} a+d \\ b+e \\ c+f \end{pmatrix}$$

$$c \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c \times x \\ c \times y \\ c \times z \end{pmatrix}$$

Quick Math!

$$(4+3i)\cdot\begin{pmatrix}3\\1\\4\end{pmatrix}=\begin{pmatrix}12+9i\\4+3i\\16+12i\end{pmatrix}$$

Matrices

$$A = \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} \\ a_{1,0} & a_{1,1} & a_{1,2} \\ a_{2,0} & a_{2,1} & a_{2,2} \end{pmatrix}$$

$$A[1][0] = a_{1,0}$$

Representation in Code

```
my_vector = [3, 1, 4, 1, 5, 9, 2]
my_matrix = [
  [6, 5, 3, 5, 8, 9],
  [7, 9, 3, 2, 3, 8],
  [4, 6, 2, 6, 4, 3]
]

print(my_vector[0]) # 3
print(my_matrix[0][1]) # 5
```

Basic Matrix Operations

$$
\begin{pmatrix} c_{0,0} & c_{0,1} & c_{0,2} \\ c_{1,0} & c_{1,1} & c_{1,2} \\ c_{2,0} & c_{2,1} & c_{2,2} \end{pmatrix} + \begin{pmatrix} d_{0,0} & d_{0,1} & d_{0,2} \\ d_{1,0} & d_{1,1} & d_{1,2} \\ d_{2,0} & d_{2,1} & d_{2,2} \end{pmatrix}
$$

$$
= \begin{pmatrix} c_{0,0} + d_{0,0} & c_{0,1} + d_{0,1} & c_{0,1} + d_{0,2} \\ c_{1,0} + d_{1,0} & c_{1,1} + d_{1,1} & c_{1,1} + d_{1,2} \\ c_{2,0} + d_{1,0} & c_{2,1} + d_{2,1} & c_{2,1} + d_{2,2} \end{pmatrix}
$$

$$c \cdot \begin{pmatrix} d_{0,0} & d_{0,1} \\ d_{1,0} & d_{1,1} \\ d_{2,0} & d_{2,1} \end{pmatrix}$$

$$= \begin{pmatrix} c \times d_{0,0} & c \times d_{0,1} \\ c \times d_{1,0} & c \times d_{1,1} \\ c \times d_{1,0} & c \times d_{2,1} \end{pmatrix}$$

Funny Matrix Business

Transpose

"Reflection across the diagonal"

$$A^T[i,j] = A[j,i]$$

$$A = \begin{pmatrix} a_{0,0} & a_{0,1} \\ a_{1,0} & a_{1,1} \end{pmatrix}$$

What is A^T?

$$A = \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} \\ a_{1,0} & a_{1,1} & a_{1,2} \end{pmatrix}$$

What is A^T?

Matrix Multiplication

Special condition: $m = n$

$$A = \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} \\ a_{1,0} & a_{1,1} & a_{1,2} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{0,0} & b_{0,1} \\ b_{1,0} & b_{1,1} \\ b_{2,0} & b_{2,1} \end{pmatrix}$$

What is $\mathbf{AB}$?

(Not So) Quick Math!

$$\begin{pmatrix} 2 & 7 & 1 \\ 8 & 2 & 8 \end{pmatrix} \begin{pmatrix} 6 & 2 \\ 8 & 3 \\ 1 & 8 \end{pmatrix} =$$

Identity Matrix

For every matrix exists an identity $\mathbf{I}$ such that

$$\mathbf{AI} = A$$

$$\begin{pmatrix} 2 & 7 & 1 \\ 8 & 2 & 8 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 7 & 1 \\ 8 & 2 & 8 \end{pmatrix}$$

Tensor Products + Factoring

$$\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} \otimes \begin{pmatrix} b_0 \\ b_1 \end{pmatrix} = \begin{pmatrix} a_0 \begin{pmatrix} b_0 \\ b_1 \end{pmatrix} \\ a_1 \begin{pmatrix} b_0 \\ b_1 \end{pmatrix} \end{pmatrix}$$

Quick Math!

$$\begin{pmatrix} 3 \\ 42 \end{pmatrix} \otimes \begin{pmatrix} 2 \\ 6 \\ 12 \end{pmatrix}$$

Factoring Product States

$$\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} \otimes \begin{pmatrix} b_0 \\ b_1 \end{pmatrix} = \begin{pmatrix} a_0 b_0 \\ a_0 b_1 \\ a_1 b_0 \\ a_1 b_1 \end{pmatrix}$$

More Quick Math!

$$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Factor this into two vectors

Even More Quick Math!

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Factor this into two vectors

A Few Special Vectors

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Bra-Ket Notation

$$\langle q_0| = (0, 1)$$

$$|\psi\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{pmatrix}$$

$$
\begin{aligned}
|\psi\rangle &= \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle \\
&= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{i}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{i}{\sqrt{2}} \end{pmatrix}
\end{aligned}
$$

Intro to Qiskit

References

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