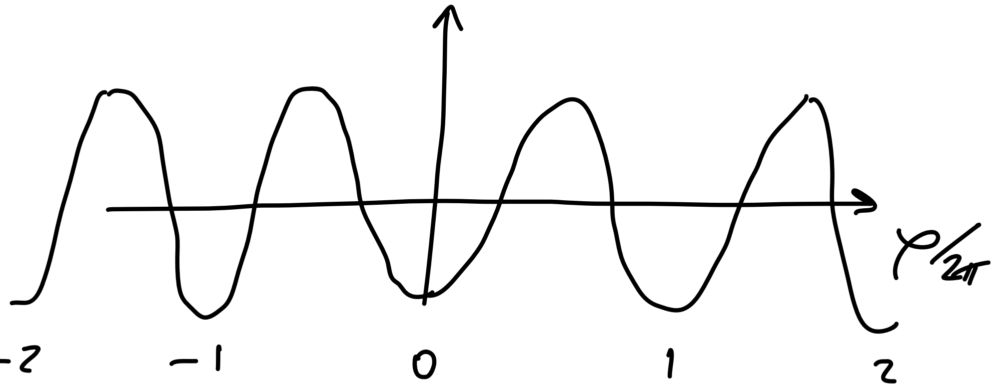
- *idea*: encode qubit in continuous d.o.f.
  → improved robustness against perturbations
  → basis for bosonic codes

- consider cont. d.o.f.

  $[\varphi, q] = i$

  imagine potential:




def. stabilizers:

$$S_\varphi := e^{+i\varphi}$$

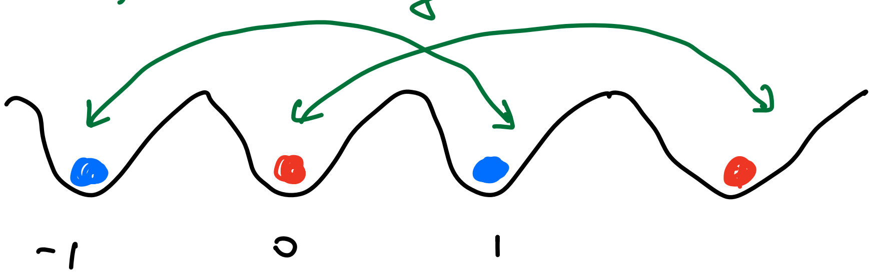
code space: $S_\varphi |\psi_L\rangle = +|\psi_L\rangle \Rightarrow \varphi = 2\pi k$

↳ need another state for qubit:

e.g. consider even vs odd minima

translation by $4\pi$

i.e.




same state $|1\rangle$, $|0\rangle$

⇒ second stabilizer:

$$S_q := e^{i4\pi q}$$

<u>check</u>: $[S_\varphi, S_q] = e^{i\varphi} e^{i4\pi q} - e^{i4\pi q} e^{i\varphi} = 0$:

recall BCH: $e^{iX} e^{iY} = e^{iX + iY - \frac{1}{2}[X,Y] - \frac{i}{12}[X,[X,Y]] + \dots}$

for $[\varphi, q] = i \Rightarrow$ all higher order comm. vanish

$$\Rightarrow e^{i\varphi} e^{i4\pi q} = e^{i(\varphi + 4\pi q)} - \frac{4\pi i}{2} = e^{i(\varphi + 4\pi q)}$$

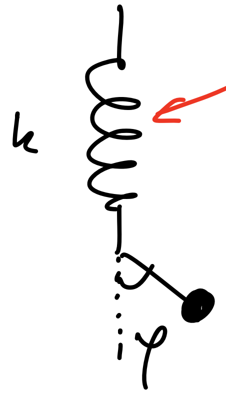
$$e^{i4\pi q} e^{i\varphi} = e^{i(4\pi q + \varphi)} + \frac{4\pi i}{2} = e^{i(\varphi + 4\pi q)}$$

$\uparrow$ $4\pi$ is smallest possible #

intuition: torsion pendulum (cavendish experiment)

spring angle can tell winding!

$\rightarrow$ decompactifies $\varphi$

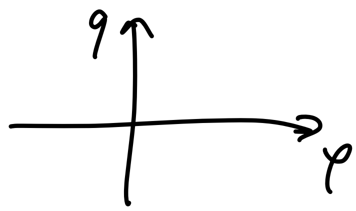



pendulum bob: position def. mod $2\pi$

can observe bob position (not spring angle)

$\Rightarrow$ all observables $f(\varphi)$: periodic w/ $2\pi$

phase space:

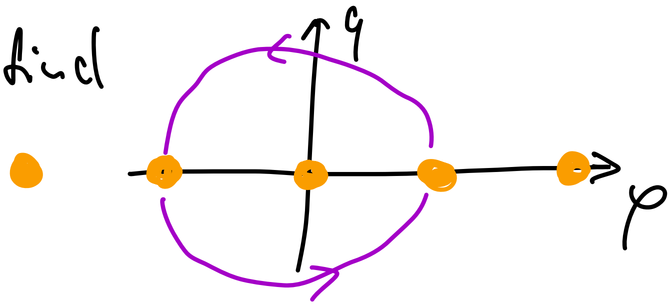



harmonic osc. [ignoring man. inertia & torque, etc.]

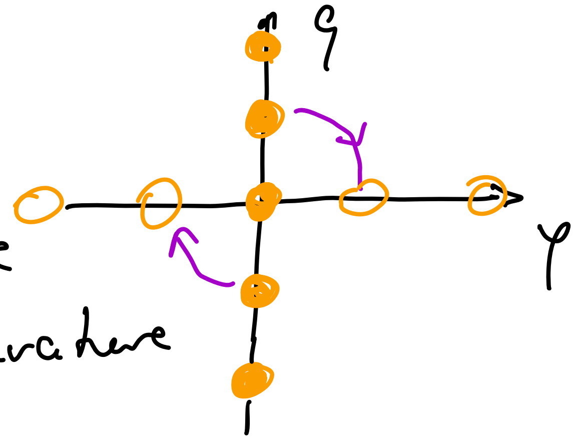
$$\begin{cases} \dot{\varphi} = \frac{1}{m} q \\ \dot{q} = -k\varphi \end{cases} \qquad \omega = \sqrt{k/m} \;\Rightarrow\; \begin{aligned} \varphi(t) &= \sin \omega t \\ q(t) &= m\omega \cos \omega t \end{aligned}$$

- phase variable:
- at times $t_n = n\frac{T}{2}$ we find the same state

$\Rightarrow$ "stabilization" of $\varphi$



- momentum variable:




- wait $t = T/4$

→ momentum quadrature turns into phase quadrature

- then measure phase

want: measurement of momentum from meas. of phase

but: $q(t) = \underbrace{m\omega}\, \varphi(t - T/4)$

↑ amplitude mismatch, want to set to unity

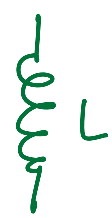
but: stabilizer: extra $4\pi$

$$\Rightarrow 4\pi\, q(t) = 4\pi m\omega\, \varphi(t - T/4) \overset{!}{=} \varphi(t - T/4)$$

$$\Rightarrow 4\pi m\omega \overset{!}{=} 1$$

⇒ measurements only in $\varphi$ can stabilize periodic structure in $\varphi$ & $q$

- relation to LC circuits:




inductor:

voltage: $V = L \frac{dI}{dt}$

↗ inductance

Josephson relation for phase $\varphi$ of SC:

$$\frac{d\varphi}{dt} = \frac{2e}{\hbar} V(t) = \frac{2eL}{\hbar} \frac{dI}{dt} \quad (*)$$

$$\Rightarrow \varphi = \frac{2eL}{\hbar} I = \frac{2eL}{\hbar} \frac{dq}{dt}$$ ← charge