

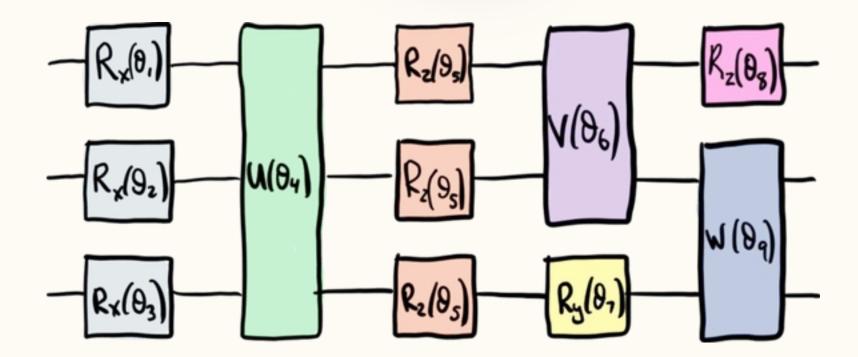
Introduction to Quantum Computing: From Qubits to Circuits

Quantum Computing Club Yachay Tech

By Gabriel Balarezo

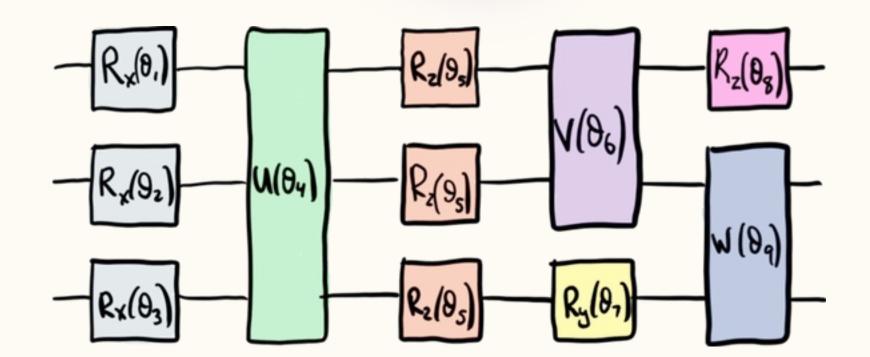


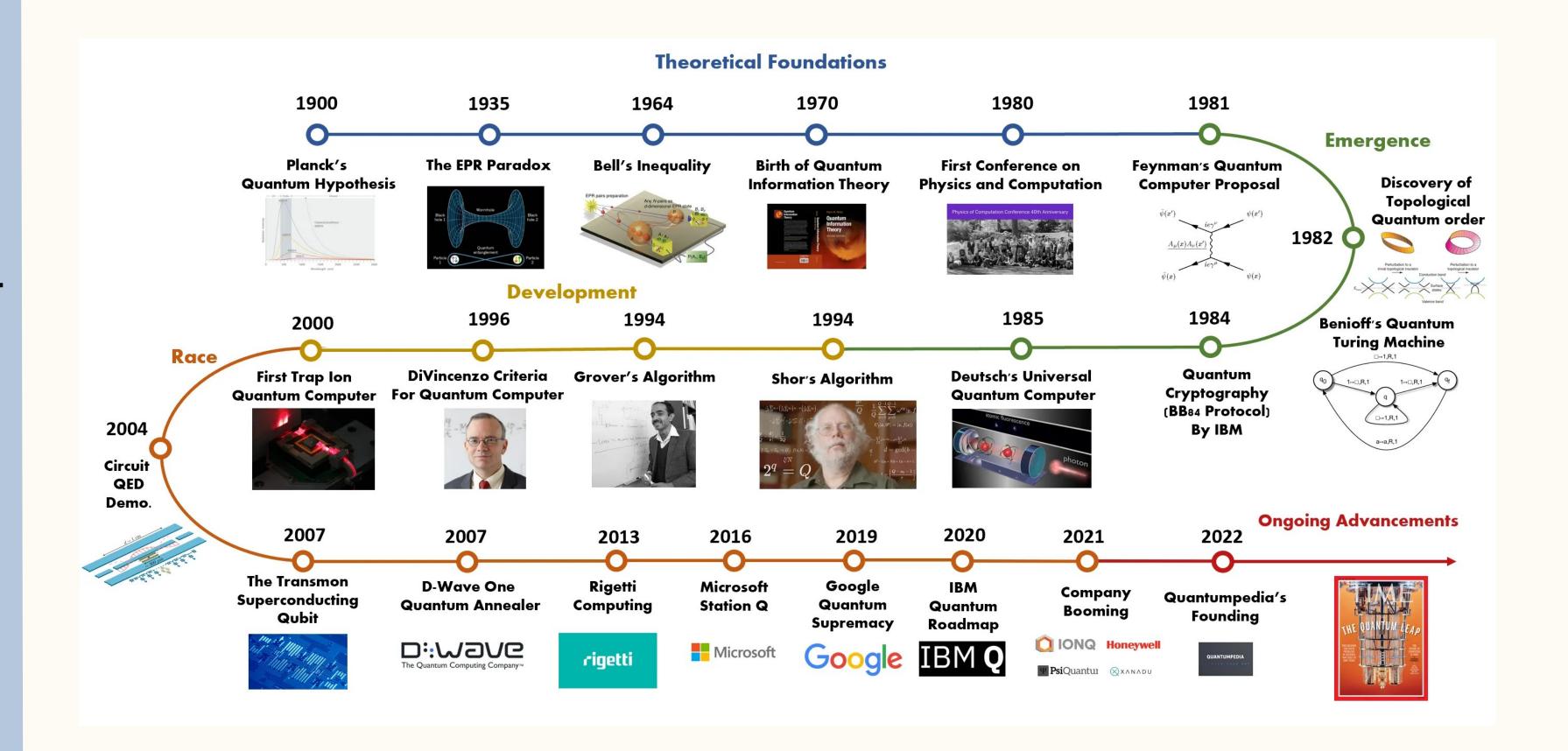
Overview



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- Introduction to Quantum
 Computing
- Quantum Bits and Superposition
- Entanglement
- Quantum Gates and Circuits





Quantum computing uses quantum mechanics to process information in ways classical computers cannot.

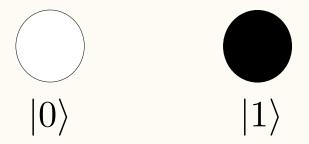
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Classical bits Quantum bits (Qubit)

 $\alpha|0\rangle+\beta|1\rangle$...superposition!!

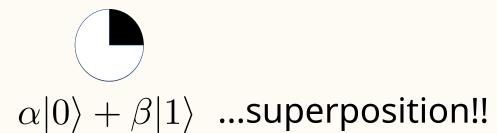
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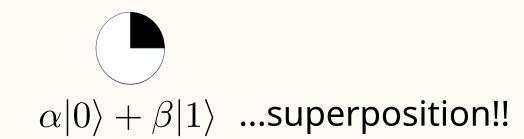
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Basis states

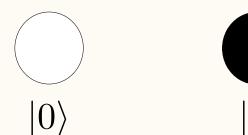
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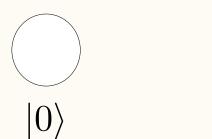
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$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \\ |\alpha|^2 + |\beta|^2 = 1 \\ |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \alpha \text{ and } \beta \text{ can be complex!}$$

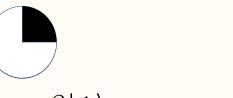
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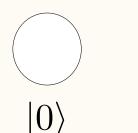
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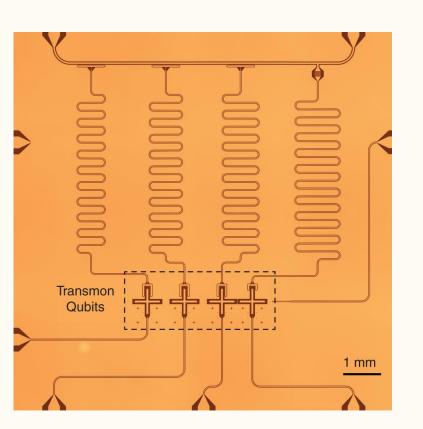
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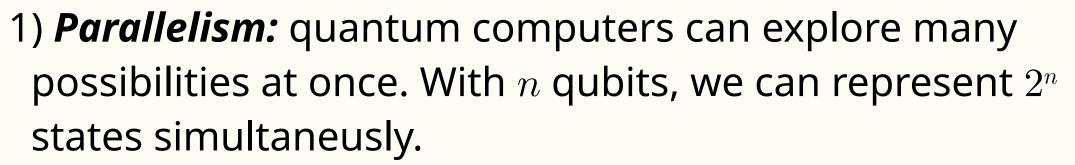
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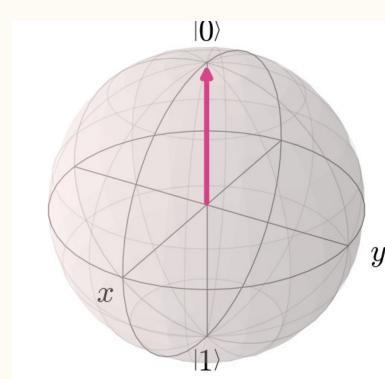
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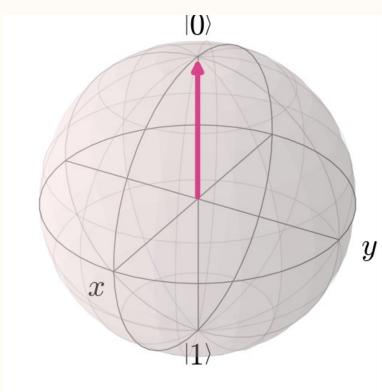
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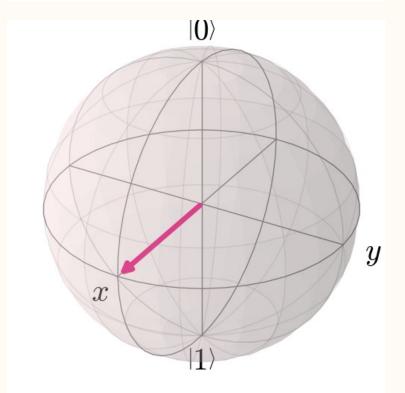
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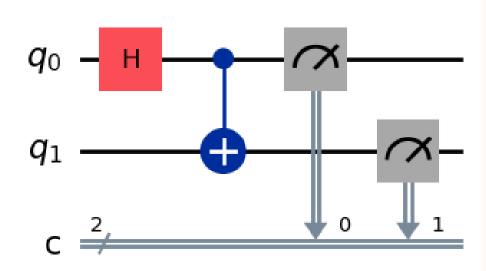
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Bell states

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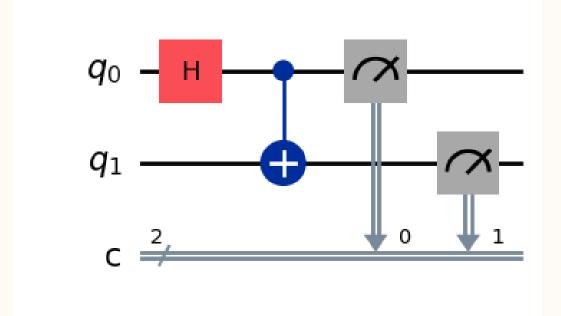


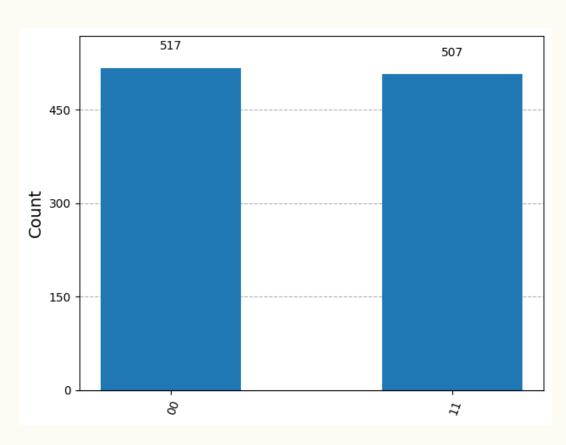
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Symbol	Truth Table		
	А	В	Q
A —	0	0	0
8 Q	0	1	0
2-input AND Gate	1	0	0
	1	1	1
Boolean Expression Q = A.B	Read as A AND B gives Q		ives Q

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2-input NAND Gate	1	0	1
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Boolean Expression Q = $\overline{A.B}$	Read as A AND B gives NOT-Q		es NOT-Q

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Quantum Gates! Represented by unitary matrices $\mathcal{U}^{\dagger}U=1$



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$$\frac{1}{\sqrt{2}}egin{bmatrix} 1 & 1 \ 1 & -1 \end{bmatrix}$$

$$\text{Hadamard Gate} \quad \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \mathbf{q} - \mathbf{H} - \quad \Longrightarrow \quad \frac{H \left| 0 \right\rangle = \frac{1}{\sqrt{2}} (\left| 0 \right\rangle + \left| 1 \right\rangle)}{H \left| 1 \right\rangle = \frac{1}{\sqrt{2}} (\left| 0 \right\rangle - \left| 1 \right\rangle)}$$



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Pauli X Gate

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$$X|0\rangle = |1$$

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$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

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$$\Longrightarrow$$

$$\Rightarrow \quad rac{Z_{\parallel }}{Z_{\parallel }}$$

$$Z|1\rangle = -|1\rangle$$





S Gate

$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

$$egin{aligned} \mathbf{q} - \mathbf{s} - & \Longrightarrow & egin{aligned} Z \ket{0} &= \ket{0} \ Z \ket{1} &= -\ket{1} \end{aligned}$$



S Gate

$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

$$\implies \frac{Z |0\rangle - |0\rangle}{Z |1\rangle = -|1\rangle}$$

Multi-qubit gates!



S Gate

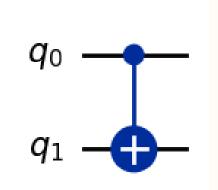
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Multi-qubit gates!

CNOT Gate

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



Flips the state of

q1 if q0's state is

1. Entanglement!!



S Gate

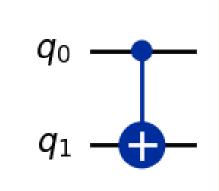
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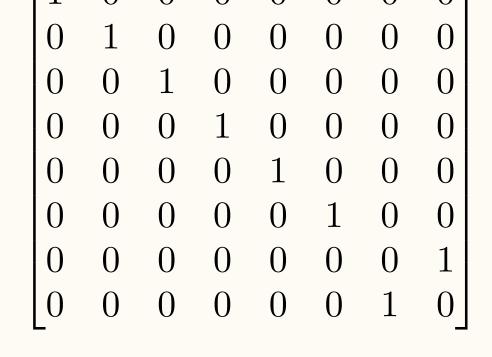
CNOT Gate

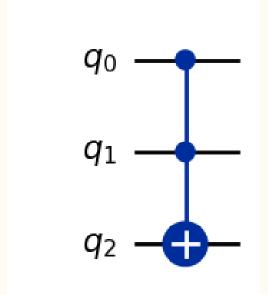
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



Flips the state of ⇒ q1 if q0's state is 1. Entanglement!!

Toffoli Gate





Flips q2 only and if

only q0 and q1 are

in 1 state!

Universal gate!



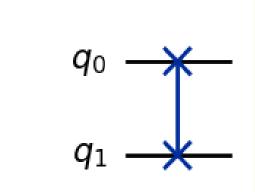
Multi-qubit gates!



Multi-qubit gates!

SWAP Gate

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Swaps q0 and q2 states.

Sequence of building blocks that carry out elementary operations, called gates!

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Key components

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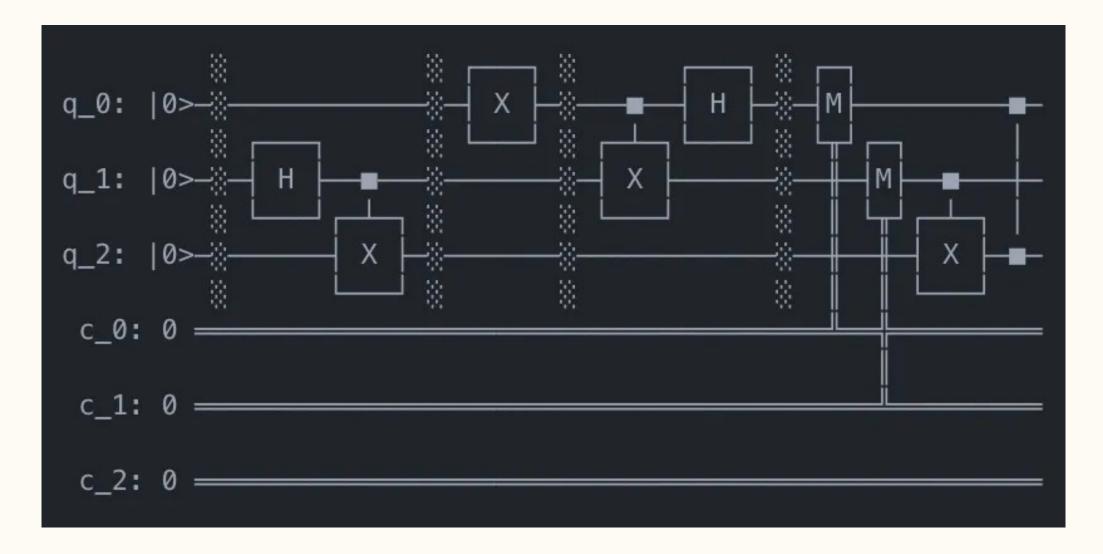
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Coding time!!!

