End-to-End Efficient Quantum Thermal and Ground State Preparation Made Simple

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(***Ding***, *Zhan, Preskill, Lin, arxiv:2508.05703*)

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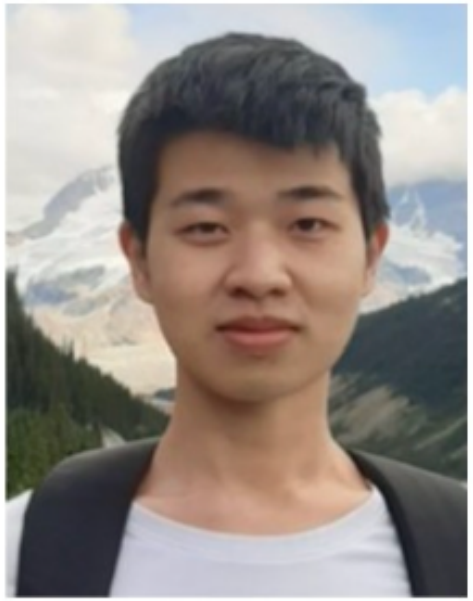
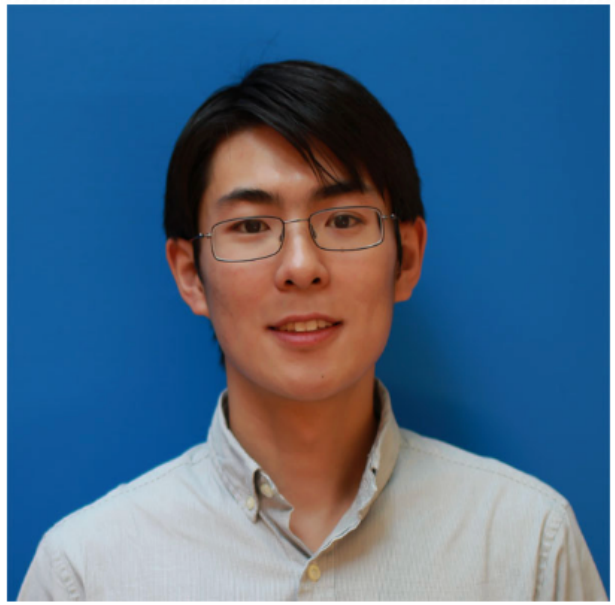
Collaborators:



Yongtao Zhan

Caltech



Lin Lin

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John Preskill

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Outline

- Introduction
- Lindblad dynamics based algorithm
- Weak system-bath interaction based algorithm

Quantum ground and thermal state preparation

Given a Hamiltonian H and inverse temperature β, prepare

$$\sigma_\beta = \frac{\exp(-\beta H)}{\mathrm{Tr}\left(\exp(-\beta H)\right)}.$$

Thermal state

When $\beta \to \infty$, $\sigma_\beta \to \left|\psi_0\right\rangle \left\langle\psi_0\right|$.

Ground state

Quantum ground and thermal state preparation

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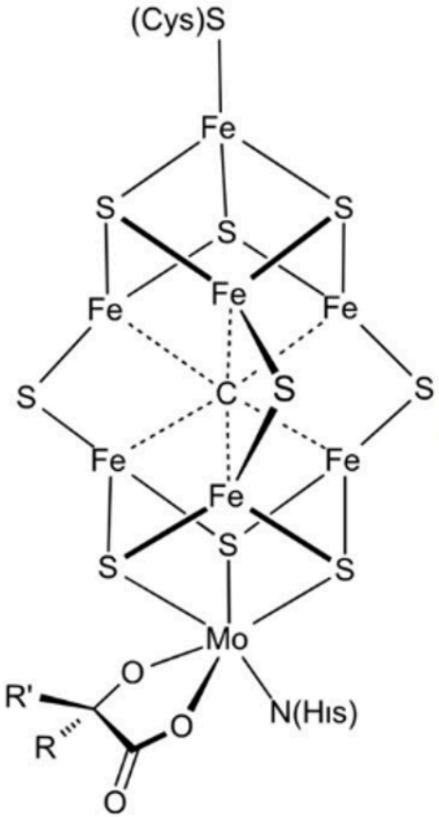
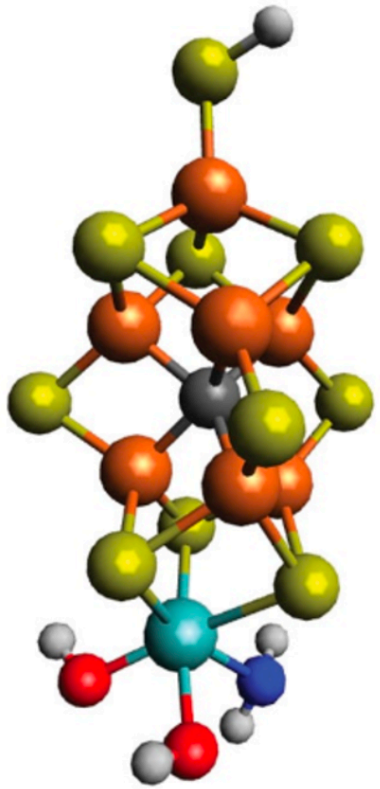
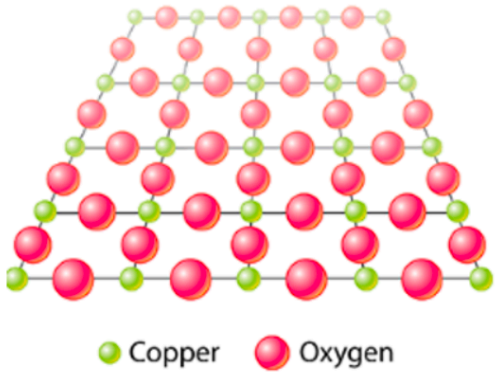
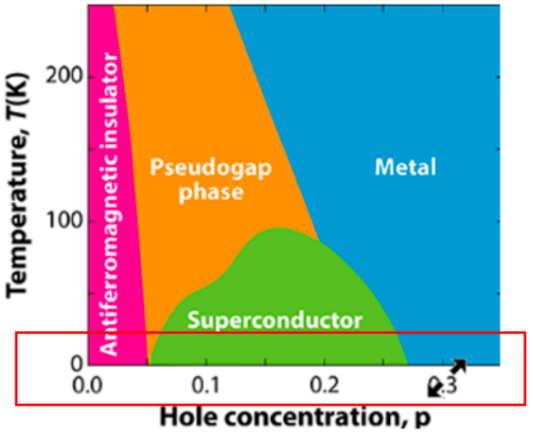
Ground state

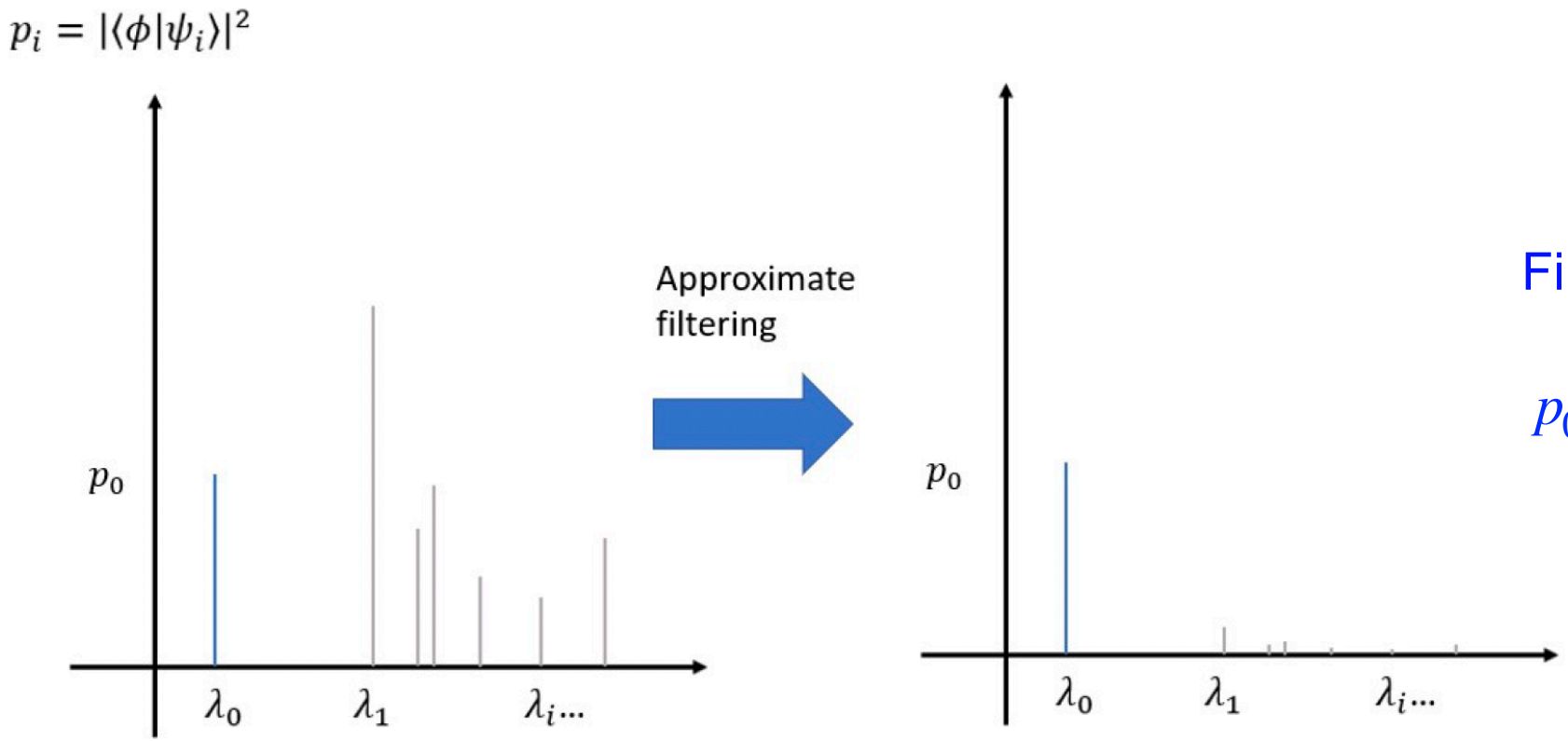








QPE-based ground state preparation algorithm



Filtering method

$$p_0 = \left|\left\langle \phi \middle| \psi_0 \right\rangle\right|^2$$

- Repetition p_0^{-1}
- Does not work when $p_0 = 0$ or $p_0 = \mathcal{O}\left(\exp(-n)\right)$

QPE-based thermal state preparation algorithm

$$\sigma_\beta = \frac{\exp(-\beta H)}{Z_\beta}, \quad Z_\beta = \mathrm{Tr}\left(\exp(-\beta H)\right)$$

- Purified thermal state

$$\left|\sigma_\beta\right\rangle = \sqrt{\frac{2^N}{Z_\beta}}\left(I \otimes \exp(-\beta H/2)\right)\left(\frac{1}{\sqrt{2^N}}\sum_{j=1}^{2^N}\left|j\right\rangle \otimes \left|j\right\rangle\right)$$

- Cost: $\mathcal{O}\left(\sqrt{2^N/Z_\beta}\right)$; Efficient only if $2^N/Z_\beta = \mathrm{poly}(N)$

Often means $\beta^{-1} = \mathrm{poly}(N)$

Q1: How to efficiently prepare ground state

from arbitrarily small $p_0 = \left| \left\langle \phi \middle| \psi_0 \right\rangle \right|^2$,

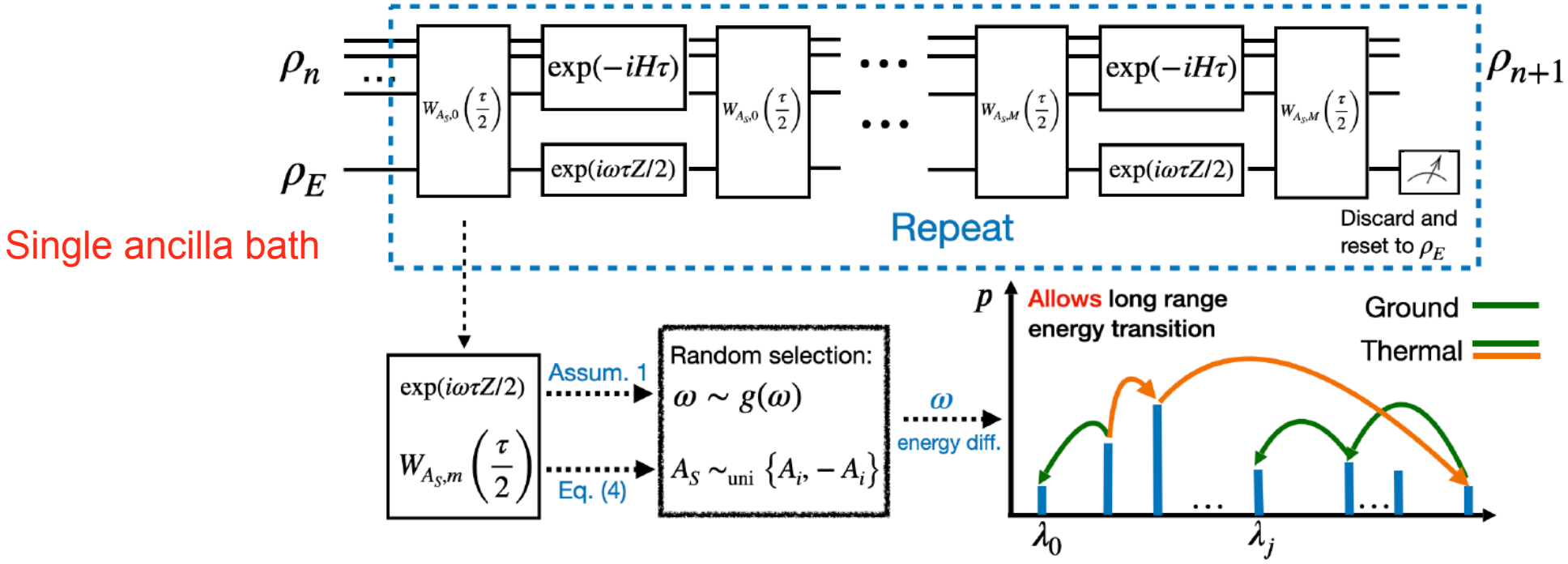
for general physical/chemical Hamiltonian?

Q2: How to efficiently prepare thermal state

at constant or low temperature $\beta = \mathcal{O}(1)$,

for general physical/chemical Hamiltonian?

Our solution



- System-bath interaction:

$$H_{\text{tot}}(t) = H + \left(-\frac{\omega}{2}Z_E\right) + \alpha f(t)\left(A_S \otimes |0\rangle\langle 1| + A_S^\dagger \otimes |1\rangle\langle 0|\right)$$

$$\rho_{n+1} = \Phi(\rho_n) := \mathrm{Tr}_E\left(\exp(-iH_{\text{tot}}T)\rho_n \otimes \rho_E \exp(iH_{\text{tot}}T)\right)$$

$$\rho_n \to \sigma_\beta, \quad n \to \infty$$

Outline

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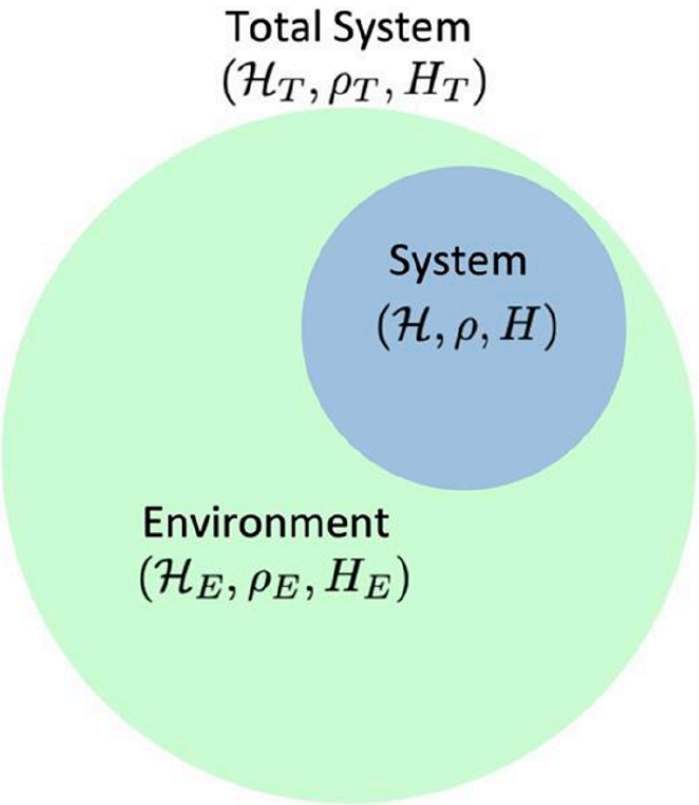
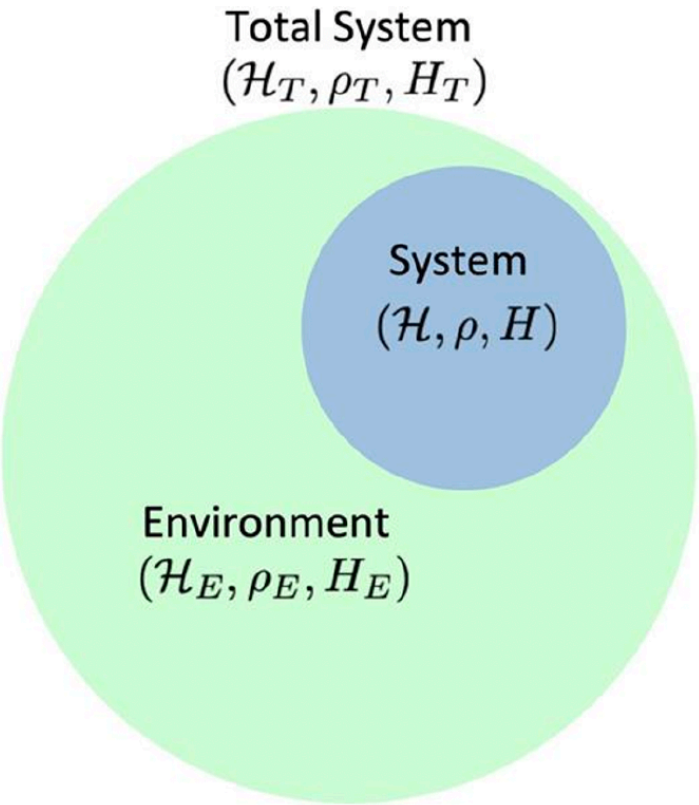
- Lindblad dynamics based algorithm
- Weak system-bath interaction based algorithm

Open quantum system



- $\rho_S(t) = \Phi(\rho_S(0)) := \mathrm{Tr}_E\left(\exp(-iH_T t)\rho_S(0) \otimes \rho_E \exp(iH_T t)\right)$ (interacting with bath+trace out)
- $\rho_S(t) \to \sigma_\beta, \quad t \to \infty$ Nature thermalizes fast.

Open quantum system



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- $\rho_S(t) \to \sigma_\beta, \quad t \to \infty$ Nature thermalizes fast.

Q: How to design an algorithm working as a fridge?

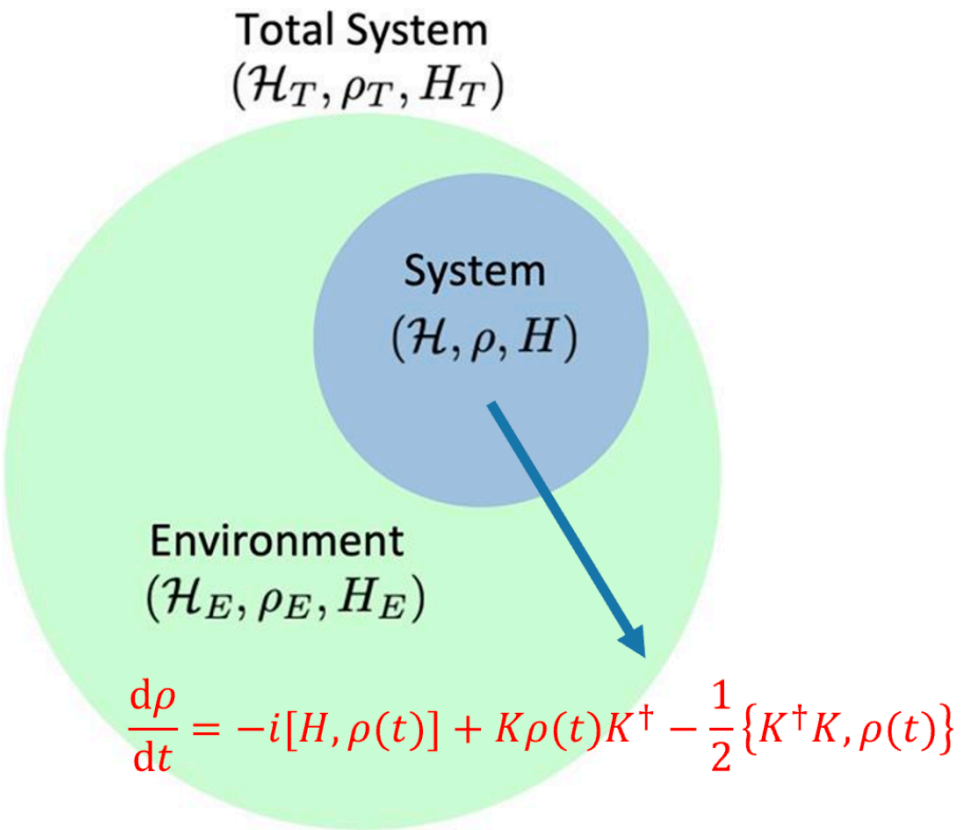
Lindblad dynamics

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \mathcal{L}[\rho] = \underbrace{-i[G,\rho]}_{\text{coherent}} + \sum_a \underbrace{K_a \rho K_a^\dagger}_{\text{transition}} - \frac{1}{2}\underbrace{\left\{K_a^\dagger K_a, \rho\right\}}_{\text{dissipation}}$$

System-bath
interaction system

Born-Markov-Secular
approximation

Lindblad dynamics



(Lindblad, CMP 1976) (Gorini-Kossakowski-Sudarshan, JMP 1976)

Lindblad dynamics

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System-bath interaction system

~~Born-Markov Secular approximation~~

Lindblad dynamics

As an algorithm tool

Article | Open access | Published: 15 October 2025

Efficient quantum thermal simulation

Chi-Fang Chen, Michael Kastoryano, Fernando G. S. L. Brandão & András Gilyén

Nature **646**, 561–566 (2025) | Cite this article

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Single-ancilla ground state preparation via Lindbladians Z Ding, CF Chen, L Lin Physical Review Research 6 (3), 033147	74	2024	Ground state
Simulating open quantum systems using Hamiltonian simulations Z Ding, X Li, L Lin PRX quantum 5 (2), 020332	70	2024	Simulation
Efficient quantum Gibbs samplers with Kubo--Martin--Schwinger detailed balance condition Z Ding, B Li, L Lin Communications in Mathematical Physics 406, 67	57	2025	Thermal state
Rapid quantum ground state preparation via dissipative dynamics Y Zhan, Z Ding, J Huhn, J Gray, J Preskill, GK Chan, L Lin arXiv preprint arXiv:2503.15827	21	2025	Ground state
End-to-end efficient quantum thermal and ground state preparation made simple Z Ding, Y Zhan, J Preskill, L Lin arXiv preprint arXiv:2508.05703	10	2025	Thermal/Ground state

Lindblad dynamics based algorithm

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \mathcal{L}[\rho] = \underbrace{-i[G,\rho]}_{\text{coherent}} + \sum_a \underbrace{K_a \rho K_a^\dagger}_{\text{transition}} - \frac{1}{2}\underbrace{\left\{K_a^\dagger K_a, \rho\right\}}_{\text{dissipation}}$$

- Choose G, K_a properly so that

$$\rho(t) \to \sigma_\beta, \quad t \to \infty, \quad \forall \rho(0)$$

Q1: How to choose G, K?

Q2: What is the simulation cost and convergence rate?

Lindblad dynamics based algorithm

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Q1: How to choose G, K?

Ground state case

- Coherent term: $G = H$ or $G = 0$

- Jump operator: $K_a = \int_{-\infty}^{\infty} f(t)\exp(-iHt)A_a\exp(iHt)\mathrm{d}t$ $\quad \hat{f}(\omega) = 0, \quad \omega \geq 0$

$$\mathscr{L}(\sigma_\infty) = 0$$

Ground state is a fixed point

(***Ding***, *Lin, Chen, PRR, 2024*)

Example:

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = K\rho(t)K^\dagger - \frac{1}{2}\left\{K^\dagger K, \rho(t)\right\} = \mathcal{L}[\rho(t)].$$

1-qubit system: $H = -Z = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

- Coupling operator $A = X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.
- There exists[1] a filter f s.t. $K = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\quad K_a = \int_{-\infty}^{\infty} f(t)\exp(-iHt)A_a\exp(iHt)\mathrm{d}t$
- Coherent term $G = 0$.

Direct calculation:

$$\sigma = |0\rangle\langle 0| \quad \Rightarrow \quad \mathcal{L}[\sigma] = 0.$$

(***Ding***, *Lin, Chen, PRR, 2024*)

Example:

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Starting from $\rho(0) = |1\rangle\langle 1|$ with $p_0 = \mathrm{Tr}[\rho(0)\,|0\rangle\langle 0|] = 0$.

$$\rho(t) = \begin{bmatrix} 1 - e^{-t} & 0 \\ 0 & e^{-t} \end{bmatrix} \to |0\rangle\langle 0| \ , \quad t \to \infty \, .$$

- Converge to the ground state exponentially fast (from any $\rho(0)$)!
- Trace-preserving $\Rightarrow$ no post-selection.

*(**Ding**, Lin, Chen, PRR, 2024)*

Thermal state: (Ding, Li, Lin, CMP, 2024)

Lindblad dynamics based algorithm

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Q1: How to choose G, K?

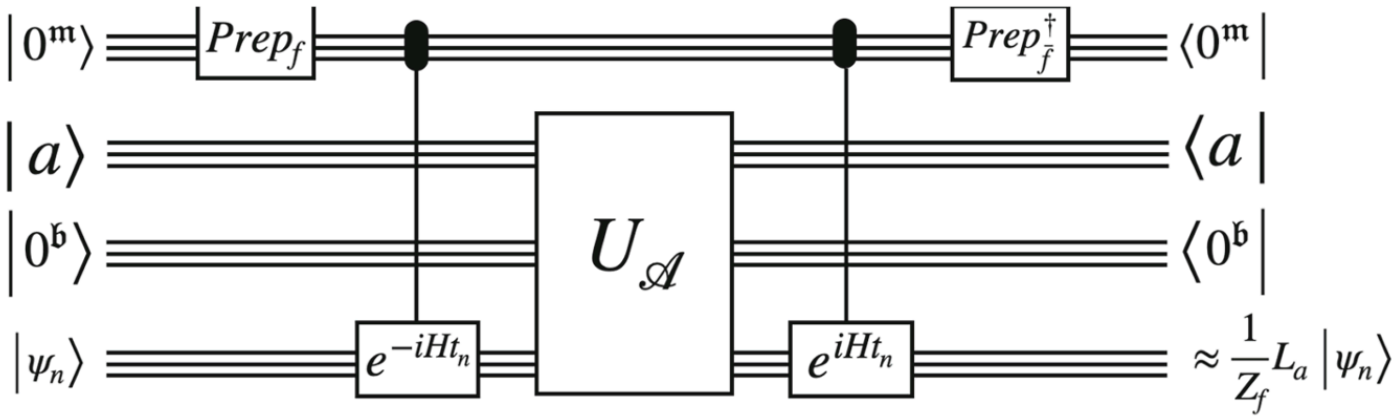
$$K_a = \int_{-\infty}^{\infty} f(t)\exp(-iHt)A_a \exp(iHt)\mathrm{d}t$$

Q2: What is the simulation cost and convergence rate?

Efficient simulation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \mathcal{L}[\rho] = \underbrace{-i[G,\rho]}_{\text{coherent}} + \sum_a \underbrace{K_a \rho K_a^\dagger}_{\text{transition}} - \frac{1}{2}\underbrace{\left\{K_a^\dagger K_a, \rho\right\}}_{\text{dissipation}}$$

- Provide oracles: $\exp(-iH\tau)$ and block encodings $\left\{\mathcal{U}_{A_a}\right\}_a$



Block encodings
$G,\ \left\{K_a\right\}_a$

(***Ding***, *Li, Lin, CMP, 2024*)

Efficient simulation

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- Provide oracles: $\exp(-iH\tau)$ and block encodings $\left\{\mathcal{U}_{A_a}\right\}_a$

→ Block encodings G, $\left\{K_a\right\}_a$

→ Complexity: $\rho(T)$ requires $\mathcal{O}(T\,\mathrm{polylog}(1/\epsilon))$ queries to $\exp(-iH\tau)$ and $\left\{\mathcal{U}_{A_a}\right\}_a$.

[1] (Cleve-Wang, ICALP 2017)(Li-Wang, ICALP 2023) (**Ding**, X. Li, L. Lin, PRX Quantum 2024)

Efficient simulation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \mathcal{L}[\rho] = \underbrace{-i[G,\rho]}_{\text{coherent}} + \sum_a \underbrace{K_a \rho K_a^\dagger}_{\text{transition}} - \frac{1}{2}\underbrace{\left\{K_a^\dagger K_a, \rho\right\}}_{\text{dissipation}}$$

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Q2.5: How large is T ?

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- T is upper bounded by mixing time

- How long it takes for the system to evolve close enough to steady state, regardless of initial state.

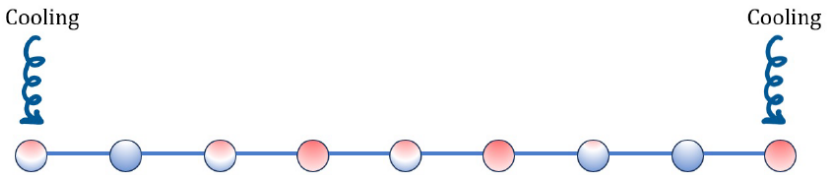
$$t_{\text{mix}} = \inf_{t} \sup_{\rho_0} \left\{ \left\| \exp(\mathscr{L}t)\rho_0 - \sigma_\beta \right\|_{Tr} < \epsilon \right\}$$

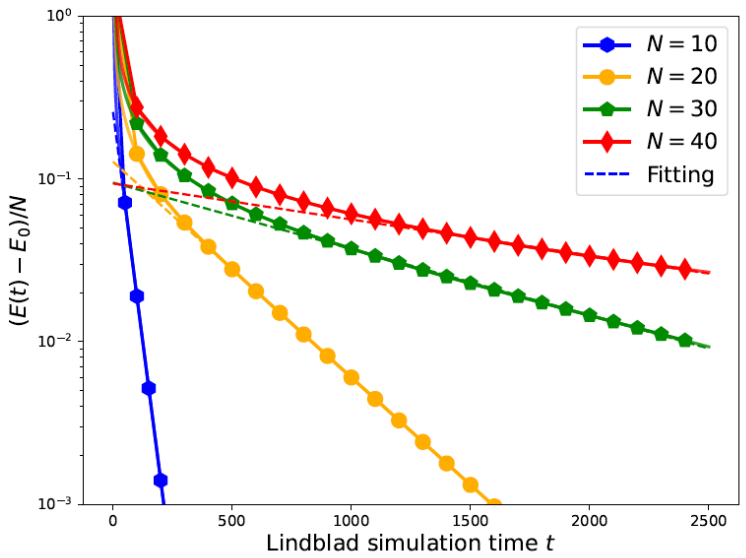
- Characterizing the mixing time of Lindblad dynamics is hard.

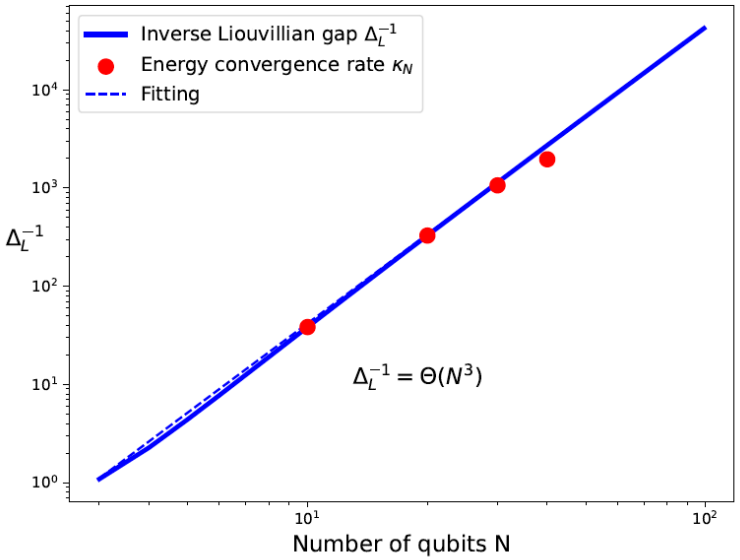
(Rouze, Franca, Alhamba, 2403.12691;2411.04885) (Tong, Zhan, 2501.0043) (Smid et al, 2501.01412) (Zhan, **Ding** et al, 2503.15827) (Bergamaschi, Chen, 2510.08533) ….

TFIM with boundary dissipation: $O(N^3)$ mixing time

$$H = -g\sum_{i=1}^{N} Z_i - J\sum_{i=1}^{N-1} X_i X_{i+1}, \quad \mathcal{A} = \{X_1, Y_1, X_N, Y_N\}$$



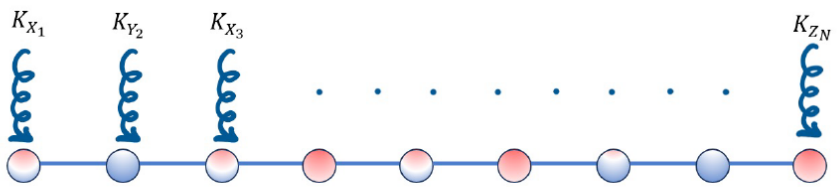


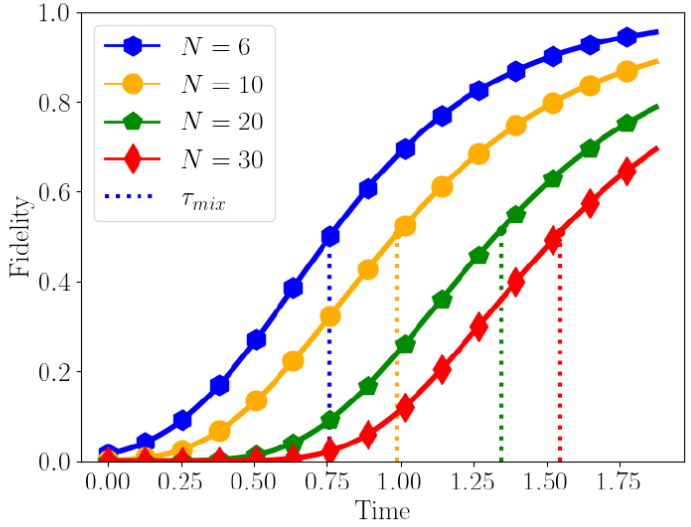


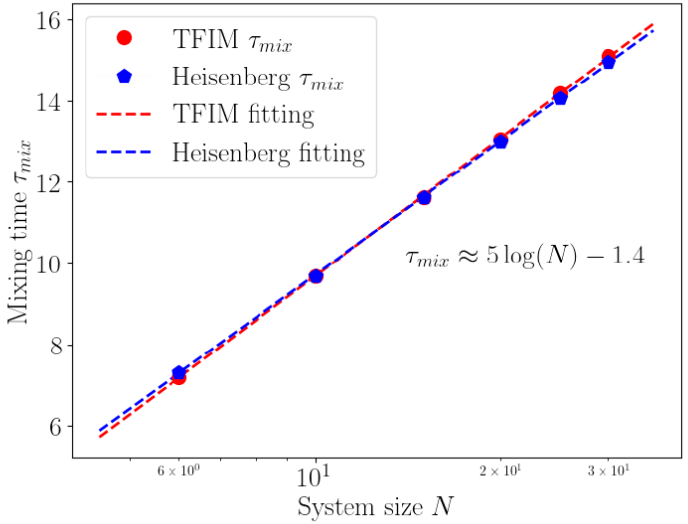
(Zhan, **Ding** et al, 2503.15827)

Rapid mixing of 1D local Hamiltonians: $O(\log N)$ mixing time

$$H = -J\sum_{i=1}^{N-1} X_i X_{i+1} - \xi \sum_{i=1}^{N-1} (Y_i Y_{i+1} + Z_i Z_{i+1}) - g\sum_{i=1}^{N} Z_i, \quad \mathcal{A} = \{X_i, Y_i, Z_i\}_{i=1}^{N}$$







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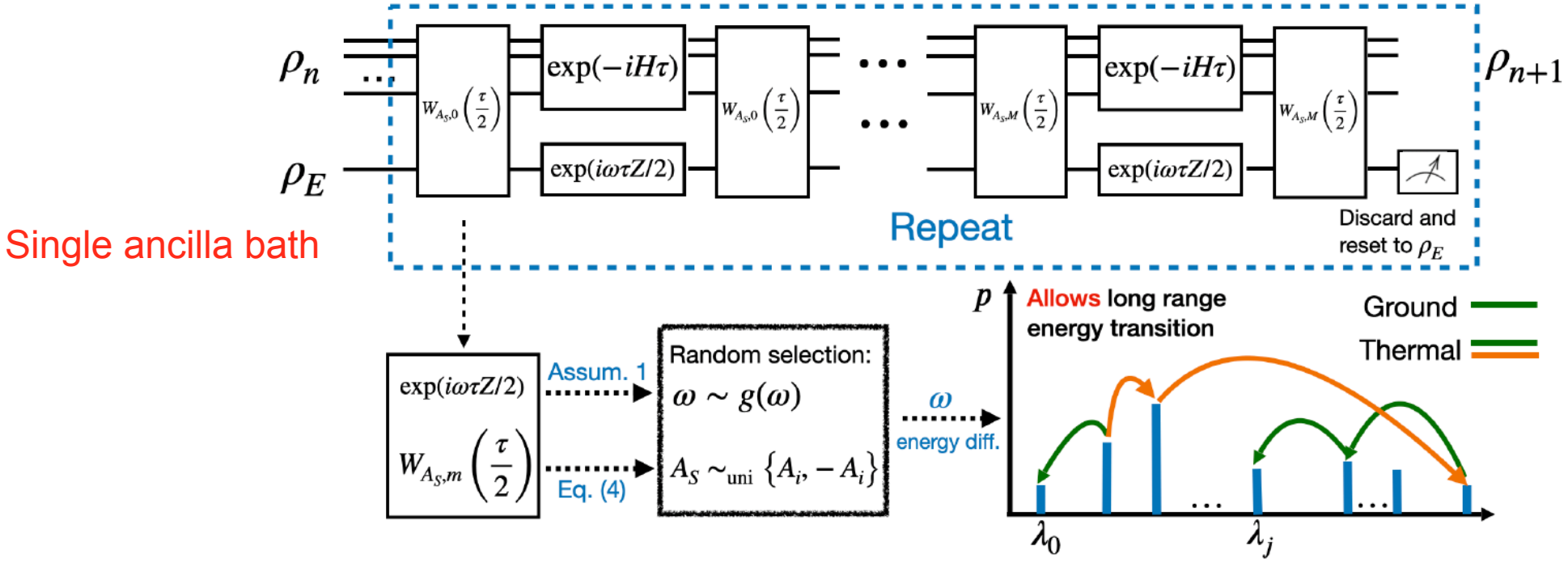
Our solution



- System-bath interaction:

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$$\rho_{n+1} = \Phi(\rho_n) := \mathrm{Tr}\left(\exp(-iH_{\text{tot}}T)\rho_n \otimes \rho_E \exp(iH_{\text{tot}}T)\right)$$

$$\rho_n \to \sigma_\beta, \quad n \to \infty$$

Early fault-tolerant (EFT) quantum algorithm

Limited useful quantum resources:

- Selective application of quantum error correction to components most susceptible to noise
- Hybrid architectures combining analog and digital elements[1]
- Non fault-tolerant error mitigation and suppression strategies[2]

Need significantly simplified algorithm for near-term and practical deployment[3]

No universally accepted definition of an early fault-tolerant quantum computer. See recent discussions: (Katabarwa et al, *Early fault-tolerant quantum computing*, PRX Quantum 2024)

[1] Partially fault-tolerant quantum computer: (Toshio et al, PRX 2025)

[2] Quantum error mitigation: (Cai et al, Rev. Mod. Phys. 2023)

[3] Simplicity is a guiding principle; ultimately algorithms need to be compiled to gate level.

Early fault-tolerant (EFT) quantum algorithm

Algorithms	Properties			Remarks
	Fixed-point error bound	Mixing time guarantee	Early-fault tolerant Implementation	
Lindblad dynamics based thermal state preparation [3, 4, 34]	✓	✓	×	Controlled Hamiltonian simulation; Complex logic gates
Discrete dynamics simulating Metropolis-type sampling [19, 41]	✓	?	×	Controlled Hamiltonian simulation; Complex logic gates
Lindblad dynamics based ground state preparation [8, 9]	✓	✓	?	Time-reversed Hamiltonian simulation
Hahn *et al* [18]	?	?	✓	Variational compilation
Weakly-coupled system bath interaction				
Hagan *et al* [16]	✓	?	?	Haar-random system-bath coupling Exponential simulation time
Hahn *et al* [17]	✓	?	✓	Only allow small energy transitions
Langbehn *et al* [24]	?	?	✓	Rotating wave approximation
Lloyd *et al* [27]	?	?	✓	Quasiparticle approximation
Scandi *et al* [38]	✓	?	?	Gaussian bath coupling
Shtanko *et al* [39]	✓	?	?	ETH hypothesis
This work	✓	✓	✓	Large energy transitions; Can prepare ground state

*(**Ding**, Zhan, Preskill, Lin, arxiv:2508.05703)*

First work with mixing time guarantee for physical Hamiltonians in weakly-coupled system bath interaction framework.

Lindblad dynamics

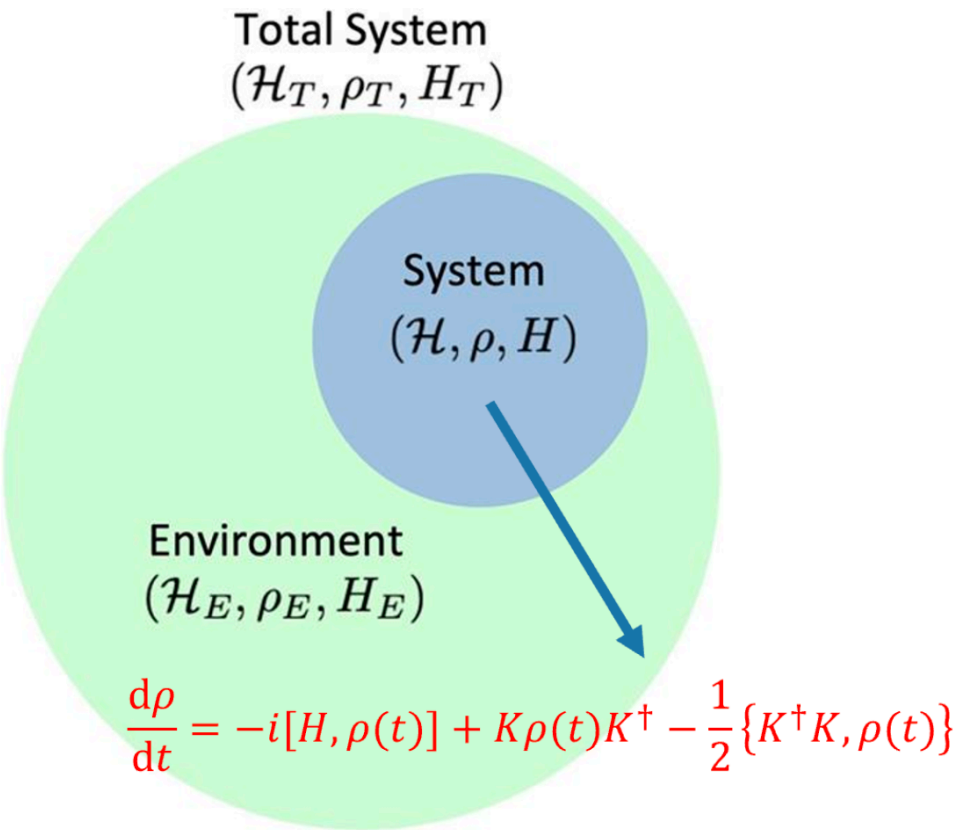
$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \mathcal{L}[\rho] = \underbrace{-i[G,\rho]}_{\text{coherent}} + \sum_a \underbrace{K_a \rho K_a^\dagger}_{\text{transition}} - \frac{1}{2}\underbrace{\left\{K_a^\dagger K_a, \rho\right\}}_{\text{dissipation}}$$

System-bath
interaction system

Born-Markov-Secular
approximation

Lindblad dynamics

As an algorithm tool

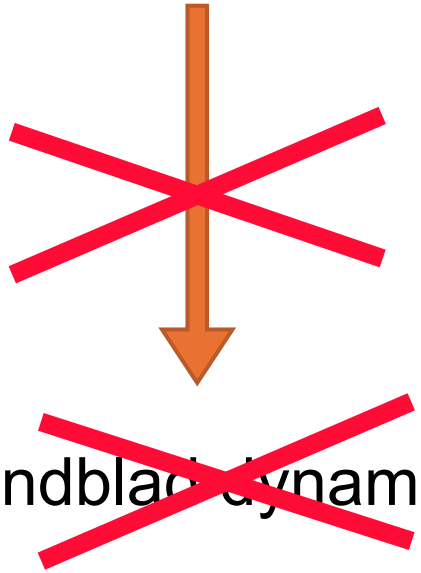


(Lindblad, CMP 1976) (Gorini-Kossakowski-Sudarshan, JMP 1976)

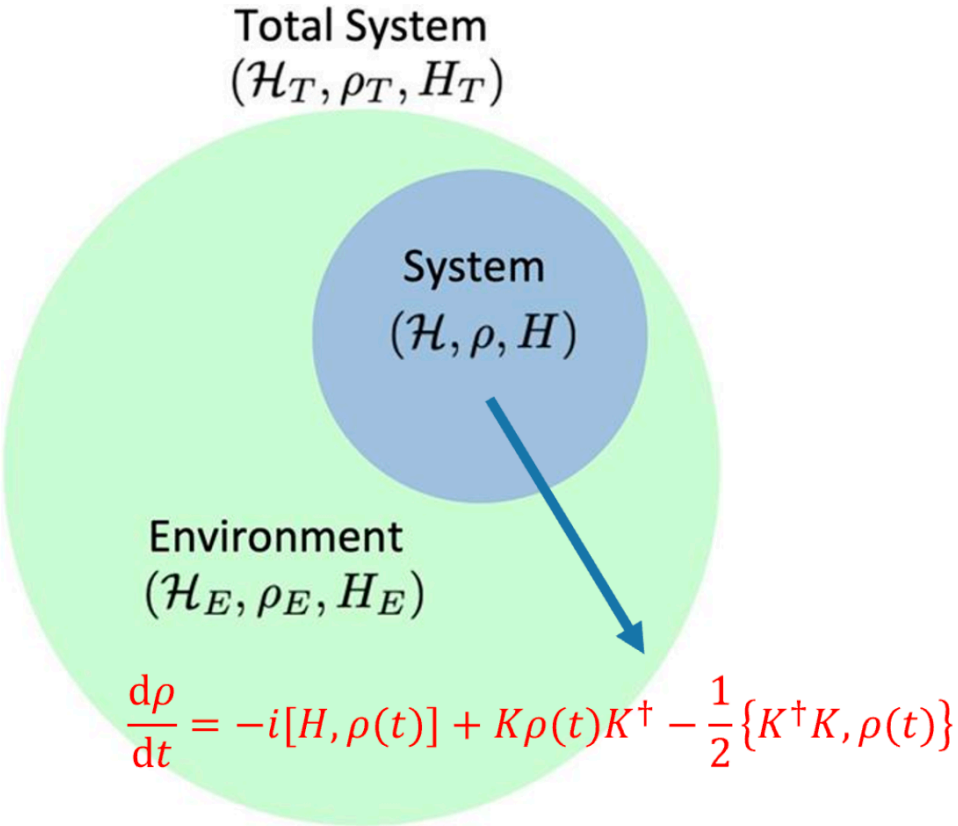
Interaction system: an algorithm

$$\rho_S(t) = \Phi(\rho_S(0)) := \mathrm{Tr}\left(\exp(-iH_T t)\rho_S(0) \otimes \rho_E \exp(iH_T t)\right)$$

System-bath
interaction system as an algorithm tool



Lindblad dynamics



(Lindblad, CMP 1976) (Gorini-Kossakowski-Sudarshan, JMP 1976)

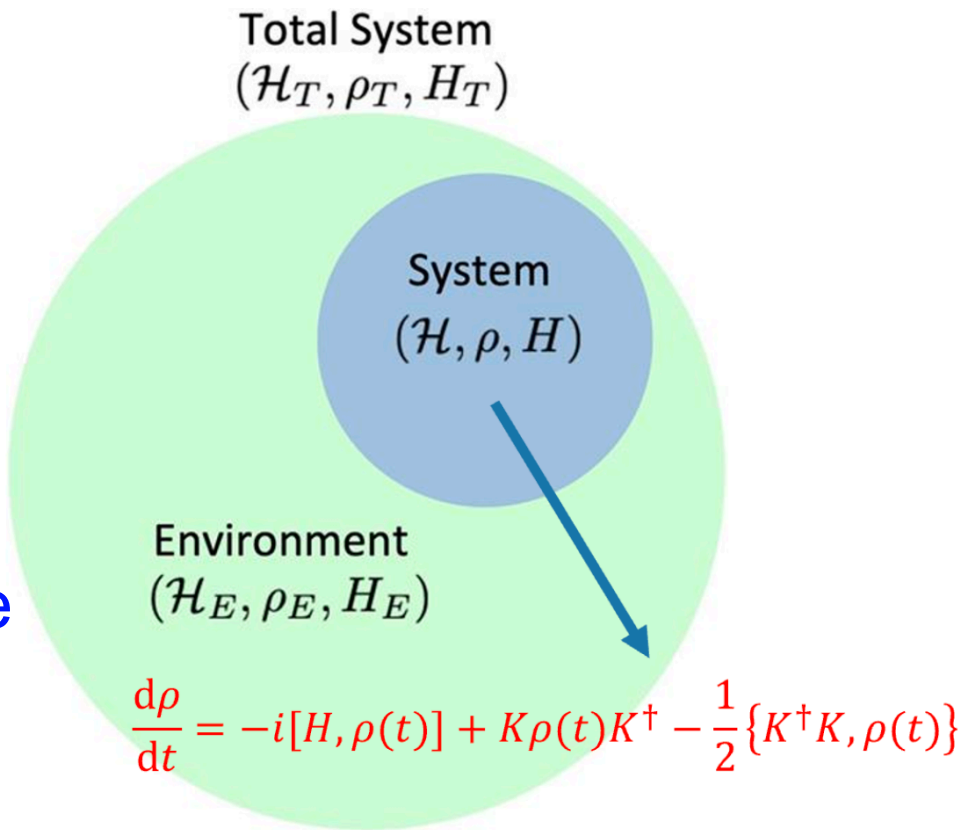
Interaction system: an algorithm

$$\rho_S(t) = \Phi(\rho_S(0)) := \mathrm{Tr}\left(\exp(-iH_T t)\rho_S(0) \otimes \rho_E \exp(iH_T t)\right)$$

System-bath
interaction system as an algorithm tool

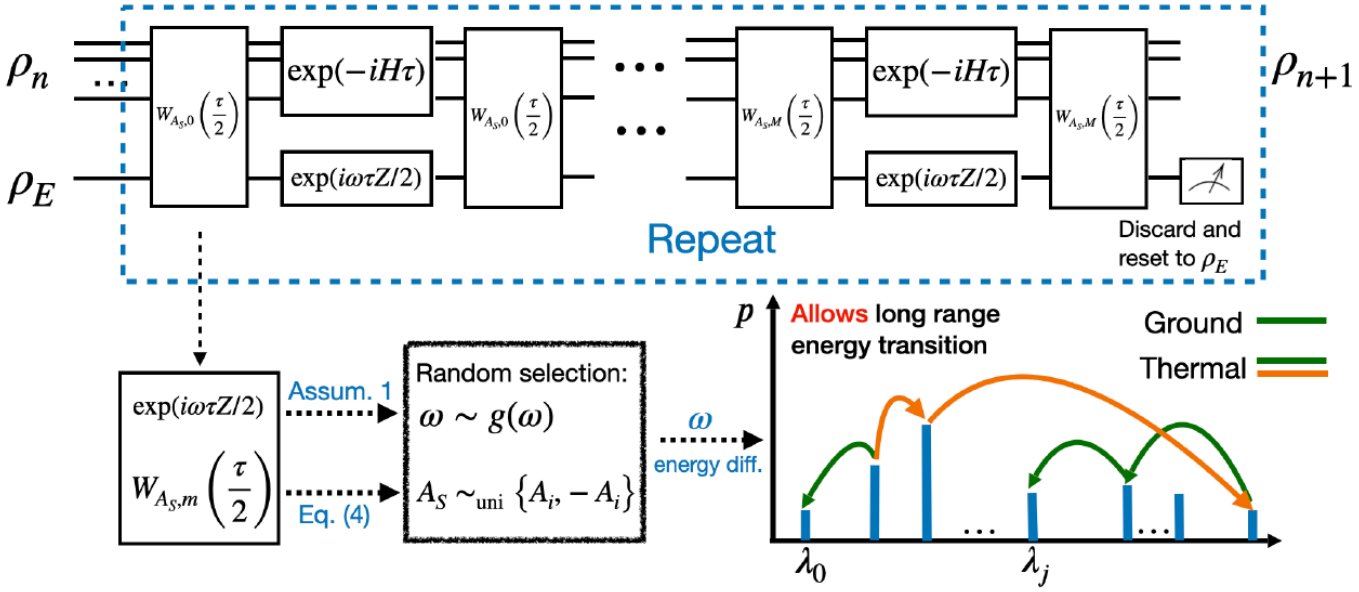
Q1: How to set up the interaction system?

Q2: What is the simulation cost and convergence rate?



(Lindblad, CMP 1976) (Gorini-Kossakowski-Sudarshan, JMP 1976)

Interaction system: an algorithm



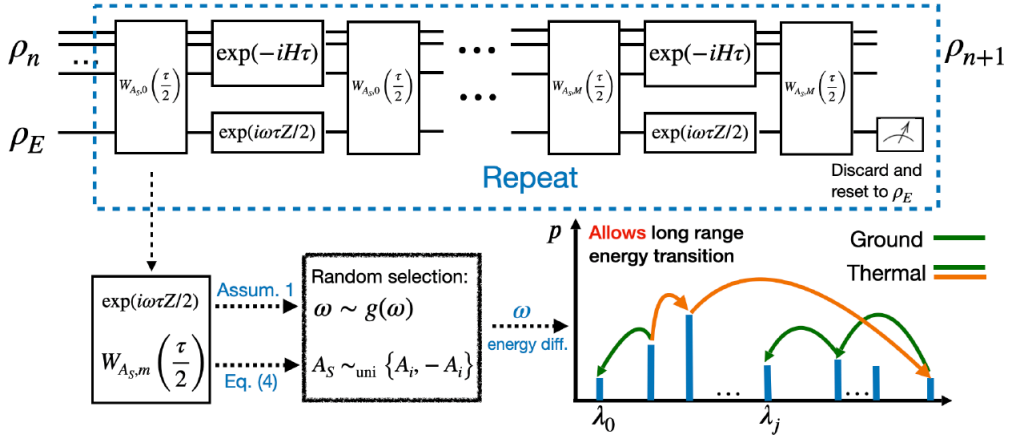
- System-bath interaction: $\rho_{n+1} = \Phi(\rho_n) := \mathrm{Tr}_E\left(\exp(-iH_{\text{tot}}T)\rho_n \otimes \rho_E \exp(iH_{\text{tot}}T)\right)$
- Time-dependent Hamiltonian: $H_{\text{tot}}(t) = H + \left(-\frac{\omega}{2}Z_E\right) + \alpha f(t)\left(A_S \otimes |0\rangle\langle 1| + A_S^\dagger \otimes |1\rangle\langle 0|\right)$
- Bath starts with inverse temperature β: $\rho_E \propto \exp(\omega\beta/2Z_E)$

Single ancilla bath

A_S: free to choose

Choice of parameters

- $H_{\text{tot}}(t) = H + \left(-\frac{\omega}{2} Z_E\right) + \alpha f(t)\left(A_S \otimes |0\rangle\langle 1| + A_S^\dagger \otimes |1\rangle\langle 0|\right)$



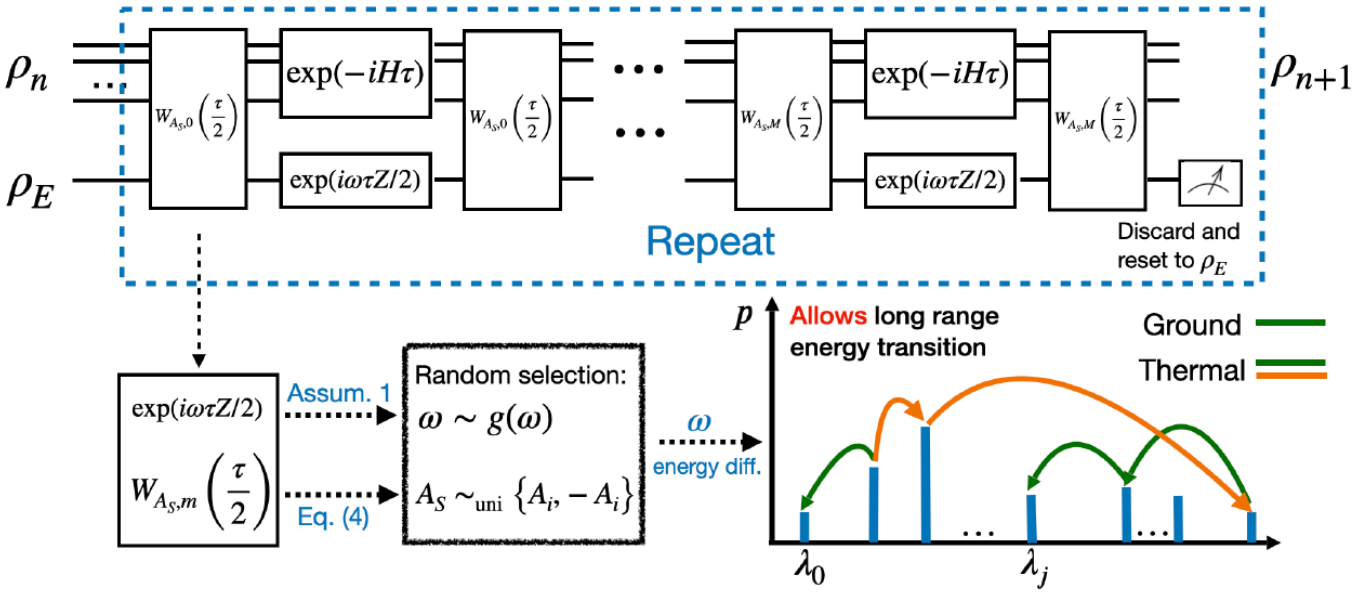
- $\rho_E \propto \exp(\omega\beta/2Z_E)$

- Gaussian interaction function:

$$f(t) = \frac{1}{(2\pi)^{1/4}\sigma^{1/2}} \exp\left(-\frac{t^2}{4\sigma^2}\right), \quad \sigma \gg 1$$

- ω uniformly chosen from $[0, 2\|H\|]$. (Distribution is flexible)

- A_S uniformly chosen from a proper set. (Ex. Single Pauli set, Single creation/annihilation set)



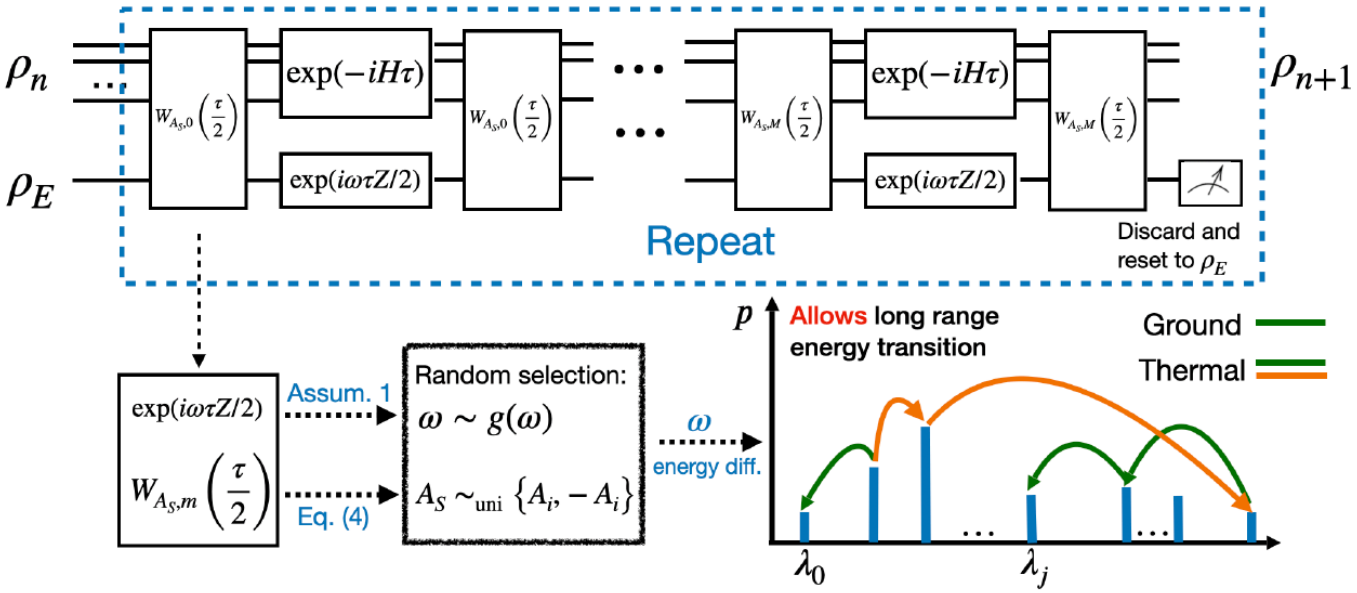
- System-bath interaction: $\rho_{n+1} = \Phi(\rho_n) := \mathrm{Tr}_E\left(\exp(-iH_{\text{tot}}T)\rho_n \otimes \rho_E \exp(iH_{\text{tot}}T)\right)$

Simulation is equivalent to Hamiltonian simulation + trace out

We use second order Trotter for Hamiltonian simulation

Complexity: $\mathcal{O}(T^{3/2}/\epsilon^{1/2})$

Single ancilla;
No need to construct jump operator;
Only involve forward Hamiltonian evolution



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Simulation is equivalent to Hamiltonian simulation + trace out

We use second order Trotter for Hamiltonian simulation

End-to-end efficiency: $\mathcal{O}(T^{3/2}/\epsilon^{1/2}) \times \tau_{\text{mix}}$

Single simulation time and mixing time

Integer mixing time

$$\tau_{\text{mix},\Phi}(\epsilon) = \min\left\{ t \in \mathbb{N} \,\middle|\, \sup_{\rho} \|\Phi^t(\rho) - \rho_{\text{fix}}(\Phi)\|_1 \leq \epsilon \right\}$$

Theoretical guarantee:

Main result 1: Efficiency preparation of the target state under fast mixing

$$\rho_{n+1} = \Phi(\rho_n) := \mathrm{Tr}_E \left(\exp(-iH_{\mathrm{tot}}T)\rho_n \otimes \rho_E \exp(iH_{\mathrm{tot}}T) \right)$$

<u>Theorem 1</u>: Given finite β, to prepare σ_β up to ϵ-trace distance, we need

$$\text{End-to-end cost } = \mathrm{poly}\left(\beta, \tau_{\mathrm{mix}}, 1/\epsilon\right)$$ after choosing $\sigma \gg 1, \quad \alpha \ll 1$

<u>Theorem 2</u>: We assume H has a spectral gap Δ. To prepare σ_∞ up to ϵ-trace distance, we need

$$\text{End-to-end cost } = \mathrm{poly}\left(\Delta^{-1}, \tau_{\mathrm{mix}}, 1/\epsilon\right)$$ after choosing $\sigma \gg 1, \quad \alpha \ll 1$

Theoretical guarantee:

Main result 2: Fast mixing of the channel $\tau_{\mathrm{mix},\Phi}(\epsilon) = \min\left\{ t \in \mathbb{N} \middle| \sup_{\rho} \|\Phi^t(\rho) - \rho_{\mathrm{fix}}(\Phi)\|_1 \leq \epsilon \right\}$

- Rescaled mixing time $t_{\mathrm{mix},\Phi_\alpha}(\epsilon) = \alpha^2 \tau_{\mathrm{mix},\Phi_\alpha}(\epsilon)$.
- Ground state for free fermions

$$t_{\mathrm{mix},\Phi_\alpha}(\epsilon) = \mathcal{O}(N \log(N/\epsilon))$$

- Thermal state for free fermions (constant temperature)

$$t_{\mathrm{mix},\Phi_\alpha}(\epsilon) = \mathcal{O}(N^2 \log(N/\epsilon))$$

- Thermal state for commuting local Hamiltonian (high temperature)

$$t_{\mathrm{mix},\Phi_\alpha}(\epsilon) = \mathcal{O}(N^2 \log(N/\epsilon))$$

Technical notice:

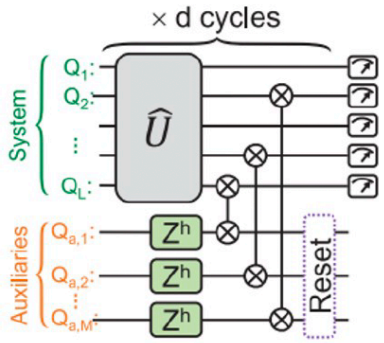
- t_{mix} comes from the analysis:

$$\Phi(\rho) \approx \exp(\mathcal{L}\alpha^2)\rho$$

- t_{mix} is independent of σ, α^{-1} .

Summary:

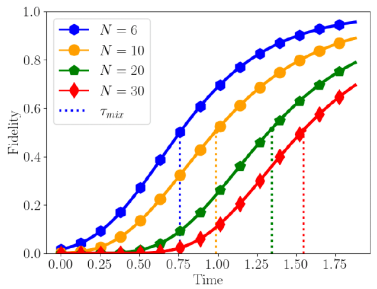
System-bath interaction

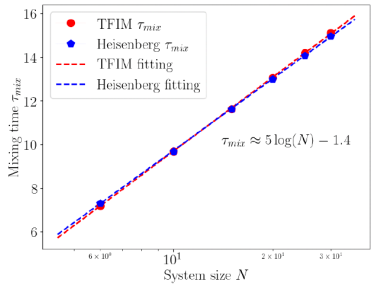


(Mi et al, Science, 2024)

Mixing time analysis
Performance guarantees
Algorithmic design

Born-Markov-Secular approximations
Restricted scope





Lindblad dynamics

$$\partial_t \rho = \mathcal{L}(\rho)$$

Much more to be understood

References:

[1]. **Zhiyan Ding***, Yongtao Zhan*, John Preskill, Lin Lin, End-to-End Efficient Quantum Thermal and Ground State Preparation Made Simple, arXiv/2508.05703

[2]. Yongtao Zhan*, **Zhiyan Ding***, John Preskill, Lin Lin, Rapid quantum ground state preparation via dissipative dynamics, 2503.15827

[3]. **Zhiyan Ding**, Bowen Li, Lin Lin, Efficient Quantum Gibbs Samplers with Kubo–Martin–Schwinger Detailed Balance Condition, Commun. Math. Phys., 2025. (arXiv:2404.05998)

[4]. **Zhiyan Ding**, Xiantao Li, Lin Lin, Simulating Open Quantum Systems Using Hamiltonian Simulations , PRX Quantum, 2024. (arXiv:2311.15533)

[5]. **Zhiyan Ding**, Chi-Fang Chen, Lin Lin, Single-ancilla ground state preparation via Lindbladians, Phys. Rev. Research, 2024. (arXiv:2308.15676)

Thank you!

Lindblad dynamics based algorithm

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \mathcal{L}[\rho] = \underbrace{-i[G,\rho]}_{\text{coherent}} + \sum_a \underbrace{K_a \rho K_a^\dagger}_{\text{transition}} - \frac{1}{2}\underbrace{\left\{K_a^\dagger K_a, \rho\right\}}_{\text{dissipation}}$$

Q1: How to choose G, K?

Ground state case

- Jump operator:

$$K = \int_{-\infty}^{\infty} f(s) e^{iHs} A e^{-iHs}\,\mathrm{d}s = \sum_{i,j\in[N]} \hat{f}(\lambda_i - \lambda_j) \left|\psi_i\right\rangle \left\langle\psi_i\right| A \left|\psi_j\right\rangle \left\langle\psi_j\right|$$

$$= \sum_{i<j} \hat{f}(\lambda_i - \lambda_j) \left|\psi_i\right\rangle \left\langle\psi_i\right| A \left|\psi_j\right\rangle \left\langle\psi_j\right|$$

$$\hat{f}(\omega) = 0, \quad \omega \geq 0$$

(***Ding***, *Lin, Chen, PRR, 2024*)

Lindblad dynamics based algorithm

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \mathcal{L}[\rho] = \underbrace{-i[G,\rho]}_{\text{coherent}} + \sum_a \underbrace{K_a \rho K_a^\dagger}_{\text{transition}} - \frac{1}{2}\underbrace{\left\{K_a^\dagger K_a, \rho\right\}}_{\text{dissipation}}$$

Q1: How to choose G, K?

Thermal state case

- Coherent term: G is defined as a specific function of K_a

- Jump operator: $K_a = \int_{-\infty}^{\infty} f(t)\exp(-iHt)A_a\exp(iHt)\mathrm{d}t$

$$\overline{\hat{f}(\omega)\exp(\beta\omega/4)} = \hat{f}(-\omega)\exp(-\beta\omega/4)$$

$$\mathscr{L}(\sigma_\beta) = 0$$

Thermal state is a fixed point

(***Ding***, *Li, Lin, CMP, 2024*)

Example:

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -i[G, \rho(t)] + K\rho(t)K^\dagger - \frac{1}{2}\left\{K^\dagger K, \rho(t)\right\} = \mathcal{L}[\rho(t)]$$

1-qubit system: $H = -Z = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

- Coupling operator $A = X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.
- There exists[1] a filter f s.t. $K = \begin{bmatrix} 1 & 1 \\ e^{-\beta} & 1 \end{bmatrix}$
- Coherent term $G = \frac{1-e^{-\beta}}{2} Y$.

Direct calculation:

$$\sigma_\beta \propto e^{-\beta H} \quad \Rightarrow \quad \mathcal{L}[\sigma_\beta] = 0.$$

*(**Ding**, Li, Lin, CMP, 2024)*

Example:

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -i[G, \rho(t)] + K\rho(t)K^\dagger - \frac{1}{2}\left\{K^\dagger K, \rho(t)\right\} = \mathcal{L}[\rho(t)]$$

Starting from $\rho(0) = |1\rangle\langle 1|$, define $\gamma = 1 + e^{-2\beta}$

$$\rho(t) = \begin{bmatrix} \frac{e^{2\beta}}{1+e^{2\beta}}\left(1 - e^{-\gamma t}\right) & 0 \\ 0 & \frac{1}{1+e^{2\beta}} + \frac{e^{2\beta}}{1+e^{2\beta}}\, e^{-\gamma t} \end{bmatrix} \to \sigma_\beta\,, \quad t \to \infty\,.$$

- Converge to the thermal state exponentially fast (from any $\rho(0)$)!
- Trace-preserving $\Rightarrow$ no post-selection.

(Ding, Li, Lin, CMP, 2024)