# Cavity QED Systems

## What is a Cavity QED System?

Figure 1: Originally, a cavity-QED system was a system of atoms interacting with quantized light in the cavity.

Two-level systems (atoms) are described by **Qubits**, denoted by symbol **S**.

In[ ]:=
```
Let[Qubit, S]
```

In[ ]:=
```
Dimension[S]
```

Out[ ]=

2

Bosonic modes (cavity photons) are described by **Bosons**, denoted by symbol **c**.

In[ ]:=
```
Let[Boson, c]
```

In[ ]:=
```
{Bottom[c], Top[c], Dimension[c]}
```

Out[ ]=

```
{0, 5, 6}
```

# Model: A qubit interacting with single-mode bosons

In[ ]:=
```
Let[Real, Ω]
Hq = (Ω / 2) S[3]
```

Out[ ]=

$$\frac{\Omega\ S^z}{2}$$

In[ ]:=
```
Let[Real, w]
Hc = ω * Dagger[c] ** c
```

Out[ ]=

$$\omega\ c^\dagger c$$

In[ ]:=
```
Let[Real, g]
Hg = g * (S[4] ** c + Dagger[c] ** S[5])
```

Out[ ]=

$$g\ \left(c\ S^+ + c^\dagger\ S^-\right)$$

In[ ]:=
```
HH = Hq + Hc + Hg
```

Out[ ]=

$$\omega\ c^\dagger c + g\ \left(c\ S^+ + c^\dagger\ S^-\right) + \frac{\Omega\ S^z}{2}$$

In[ ]:=
```
mat = Matrix[HH, {c, S}];
mat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
\frac{\Omega}{2} & 0 & 0 & g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \omega + \frac{\Omega}{2} & 0 & 0 & \sqrt{2}\ g & 0 & 0 & 0 & 0 & 0 & 0 \\
g & 0 & 0 & \omega - \frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2\ \omega + \frac{\Omega}{2} & 0 & 0 & \sqrt{3}\ g & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{2}\ g & 0 & 0 & 2\ \omega - \frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3\ \omega + \frac{\Omega}{2} & 0 & 0 & 2\ g & 0 & 0 \\
0 & 0 & 0 & 0 & \sqrt{3}\ g & 0 & 0 & 3\ \omega - \frac{\Omega}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4\ \omega + \frac{\Omega}{2} & 0 & 0 & \sqrt{5}\ g \\
0 & 0 & 0 & 0 & 0 & 0 & 2\ g & 0 & 0 & 4\ \omega - \frac{\Omega}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5\ \omega + \frac{\Omega}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{5}\ g & 0 & 0 & 5\ \omega - \frac{\Omega}{2}
\end{pmatrix}$$

In[ ]:=
```
mat = Matrix[HH, {S, c}];
mat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 & g & 0 & 0 & 0 & 0 \\ 0 & \omega+\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2}\,g & 0 & 0 & 0 \\ 0 & 0 & 2\,\omega+\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3}\,g & 0 & 0 \\ 0 & 0 & 0 & 3\,\omega+\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 2\,g & 0 \\ 0 & 0 & 0 & 0 & 4\,\omega+\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{5}\,g \\ 0 & 0 & 0 & 0 & 0 & 5\,\omega+\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 \\ g & 0 & 0 & 0 & 0 & 0 & 0 & \omega-\frac{\Omega}{2} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{2}\,g & 0 & 0 & 0 & 0 & 0 & 0 & 2\,\omega-\frac{\Omega}{2} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{3}\,g & 0 & 0 & 0 & 0 & 0 & 0 & 3\,\omega-\frac{\Omega}{2} & 0 & 0 \\ 0 & 0 & 0 & 2\,g & 0 & 0 & 0 & 0 & 0 & 0 & 4\,\omega-\frac{\Omega}{2} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{5}\,g & 0 & 0 & 0 & 0 & 0 & 0 & 5\,\omega-\frac{\Omega}{2} \end{pmatrix}$$

In[ ]:=
```
Basis[{c, S}]
Basis[{S, c}]
```

Out[ ]=

$$\{|0_c 0_S\rangle, |0_c 1_S\rangle, |1_c 0_S\rangle, |1_c 1_S\rangle, |2_c 0_S\rangle, |2_c 1_S\rangle, |3_c 0_S\rangle, |3_c 1_S\rangle, |4_c 0_S\rangle, |4_c 1_S\rangle, |5_c 0_S\rangle, |5_c 1_S\rangle\}$$

Out[ ]=

$$\{|0_c 0_S\rangle, |1_c 0_S\rangle, |2_c 0_S\rangle, |3_c 0_S\rangle, |4_c 0_S\rangle, |5_c 0_S\rangle, |0_c 1_S\rangle, |1_c 1_S\rangle, |2_c 1_S\rangle, |3_c 1_S\rangle, |4_c 1_S\rangle, |5_c 1_S\rangle\}$$

In[ ]:=
```
ProperValues[HH]
```

Out[ ]=

$$\left\{-\frac{\Omega}{2}, \frac{1}{2}(10\,\omega+\Omega), \frac{1}{2}\left(\omega-\sqrt{4\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right), \frac{1}{2}\left(\omega+\sqrt{4\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right), \frac{1}{2}\left(3\,\omega-\sqrt{8\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right), \frac{1}{2}\left(3\,\omega+\sqrt{8\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right), \frac{1}{2}\left(5\,\omega-\sqrt{12\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right), \frac{1}{2}\left(5\,\omega+\sqrt{12\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right), \frac{1}{2}\left(7\,\omega-\sqrt{16\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right), \frac{1}{2}\left(7\,\omega+\sqrt{16\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right), \frac{1}{2}\left(9\,\omega-\sqrt{20\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right), \frac{1}{2}\left(9\,\omega+\sqrt{20\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right)\right\}$$

In[ ]:=
```
ProperStates[HH]
```

Out[ ]=

$$\left\{ \left|0_c 1_s\right\rangle, \left|5_c 0_s\right\rangle, -\frac{\left(\omega - \Omega + \sqrt{4 g^2 + \omega^2 - 2 \omega \Omega + \Omega^2}\right) \left|0_c 0_s\right\rangle}{2 g} + \left|1_c 1_s\right\rangle, \right.$$

$$-\frac{\left(\omega - \Omega - \sqrt{4 g^2 + \omega^2 - 2 \omega \Omega + \Omega^2}\right) \left|0_c 0_s\right\rangle}{2 g} + \left|1_c 1_s\right\rangle,$$

$$-\frac{\left(\omega - \Omega + \sqrt{8 g^2 + \omega^2 - 2 \omega \Omega + \Omega^2}\right) \left|1_c 0_s\right\rangle}{2 \sqrt{2}\, g} + \left|2_c 1_s\right\rangle,$$

$$\frac{\left(-\omega + \Omega + \sqrt{8 g^2 + \omega^2 - 2 \omega \Omega + \Omega^2}\right) \left|1_c 0_s\right\rangle}{2 \sqrt{2}\, g} + \left|2_c 1_s\right\rangle,$$

$$-\frac{\left(\omega - \Omega + \sqrt{12 g^2 + \omega^2 - 2 \omega \Omega + \Omega^2}\right) \left|2_c 0_s\right\rangle}{2 \sqrt{3}\, g} + \left|3_c 1_s\right\rangle,$$

$$\frac{\left(-\omega + \Omega + \sqrt{12 g^2 + \omega^2 - 2 \omega \Omega + \Omega^2}\right) \left|2_c 0_s\right\rangle}{2 \sqrt{3}\, g} + \left|3_c 1_s\right\rangle,$$

$$-\frac{\left(\omega - \Omega + \sqrt{16 g^2 + \omega^2 - 2 \omega \Omega + \Omega^2}\right) \left|3_c 0_s\right\rangle}{4 g} + \left|4_c 1_s\right\rangle,$$

$$-\frac{\left(\omega - \Omega - \sqrt{16 g^2 + \omega^2 - 2 \omega \Omega + \Omega^2}\right) \left|3_c 0_s\right\rangle}{4 g} + \left|4_c 1_s\right\rangle,$$

$$-\frac{\left(\omega - \Omega + \sqrt{20 g^2 + \omega^2 - 2 \omega \Omega + \Omega^2}\right) \left|4_c 0_s\right\rangle}{2 \sqrt{5}\, g} + \left|5_c 1_s\right\rangle,$$

$$\left. \frac{\left(-\omega + \Omega + \sqrt{20 g^2 + \omega^2 - 2 \omega \Omega + \Omega^2}\right) \left|4_c 0_s\right\rangle}{2 \sqrt{5}\, g} + \left|5_c 1_s\right\rangle \right\}$$

# Model: A qudit interacting with single-mode bosons

Figure 2: A system of (artificial) atoms with a Λ-level structure and interacting with bosonic modes.

Three-level systems (atoms) are described by **Qudits**, denoted by symbol **A**.

In[ ]:=
```
Let[Qudit, A]
```

In[ ]:=
```
Dimension[A]
```

Out[ ]=

3

Bosonic modes (cavity photons) are described by **Bosons**, denoted by symbol **c**.

In[ ]:=
```
Let[Boson, c]
```

In[ ]:=
```
{Bottom[c], Top[c], Dimension[c]}
```

Out[ ]=

{0, 5, 6}

Atoms are driven by an external classical laser (Rabi driving).

In[ ]:=
```
Let[Real, Ω]
HR = Ω * (A[1 → 2] + A[2 → 1])
```

Out[ ]=

$\Omega\ (( |2\rangle \langle 1| ) + ( |1\rangle \langle 2| ))$

In[ ]:=
```
Let[Real, w]
Hc = ω * Dagger[c] ** c
```

Out[ ]=

$\omega\ c^{\dagger} c$

Coupling between the atom and boson.

In[ ]:=
```
Let[Real, g]
Hg = g * (A[2 → 0] ** Dagger[c] + c ** A[0 → 2])
```

Out[ ]=

$$g\left(c\left(|2\rangle\langle 0|\right)+c^{\dagger}\left(|0\rangle\langle 2|\right)\right)$$

In[ ]:=
```
HH = HR + Hc + Hg
```

Out[ ]=

$$\Omega\left(\left(|2\rangle\langle 1|\right)+\left(|1\rangle\langle 2|\right)\right)+\omega\, c^{\dagger}c+g\left(c\left(|2\rangle\langle 0|\right)+c^{\dagger}\left(|0\rangle\langle 2|\right)\right)$$

In[ ]:=
```
mat = Matrix[HH, {c, A}];
mat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \Omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \Omega & 0 & g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & g & \omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \omega & \Omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \Omega & \omega & \sqrt{2}\,g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{2}\,g & 2\,\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\,\omega & \Omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \Omega & 2\,\omega & \sqrt{3}\,g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3}\,g & 3\,\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\,\omega & \Omega & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Omega & 3\,\omega & 2\,g & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\,g & 4\,\omega & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4\,\omega & \Omega & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Omega & 4\,\omega & \sqrt{5}\,g & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{5}\,g & 5\,\omega & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5\,\omega & \Omega \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Omega & 5\,\omega
\end{pmatrix}$$

# Summary

## Functions

- **Qubit, Dimension**
- **Boson, Bottom, Top, Dimension**
- **Matrix, ExpressionFor**
- **ProperValues, ProperStates, ProperSystem**

## Related Links

- Mahn-Soo Choi, Advanced Quantum Technologies 12, 2000085 (2020), “Exotic Quantum States of Circuit Quantum Electrodynamics in the Ultra-Strong Coupling Regime.”
- C. Dongni, S. Luo, Y.-D. Wang, S. Chesi, and Mahn-Soo Choi, Physical Review A 105, 022627 (2022), “Geometric manipulation of a decoherence-free subspace in atomic ensembles.”