Cavity QED Systems

What is a Cavity QED System?

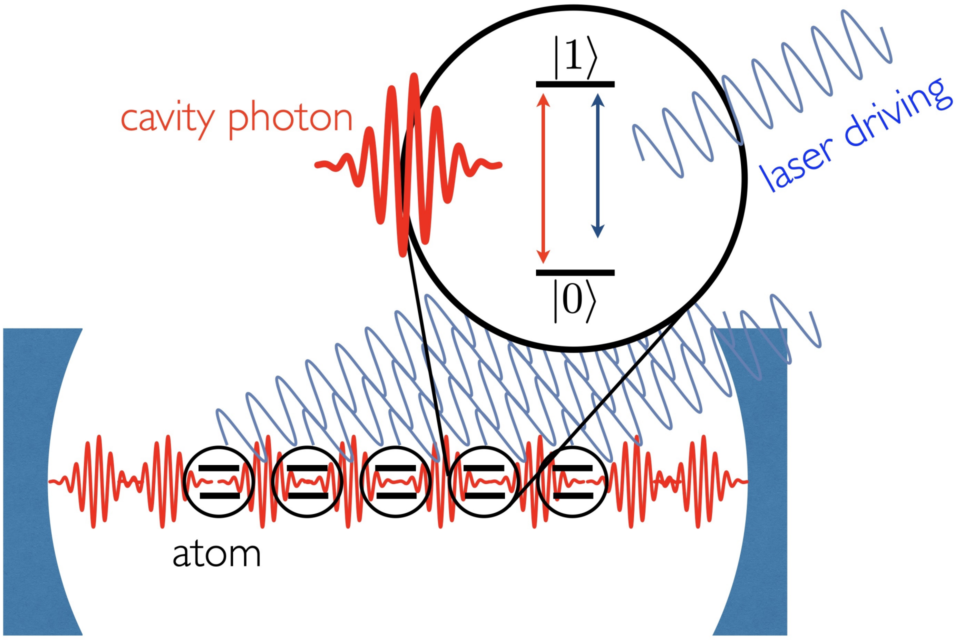

Figure 1: Originally, a cavity-QED system was a system of atoms interacting with quantized light in the cavity.

Two-level systems (atoms) are described by **Qubits**, denoted by symbol **S**.

In[◦]:=
```
Let[Qubit, S]
```

In[◦]:=
```
Dimension[S]
```

Out[◦]=
```
2
```

Bosonic modes (cavity photons) are described by **Bosons**, denoted by symbol **c**.

In[◦]:=
```
Let[Boson, c]
```

In[◦]:=
```
{Bottom[c], Top[c], Dimension[c]}
```

Out[◦]=
```
{0, 5, 6}
```

Model: A qubit interacting with single-mode bosons

In[ ]:=
```
Let[Real, Ω]
Hq = (Ω / 2) S[3]
```

Out[ ]=

$$\frac{\Omega\ S^z}{2}$$

In[ ]:=
```
Let[Real, w]
Hc = ω * Dagger[c] ** c
```

Out[ ]=

$$\omega\ c^{\dagger}c$$

In[ ]:=
```
Let[Real, g]
Hg = g * (S[4] ** c + Dagger[c] ** S[5])
```

Out[ ]=

$$g\ \left(c\ S^{+} + c^{\dagger}\ S^{-}\right)$$

In[ ]:=
```
HH = Hq + Hc + Hg
```

Out[ ]=

$$\omega\ c^{\dagger}c + g\ \left(c\ S^{+} + c^{\dagger}\ S^{-}\right) + \frac{\Omega\ S^z}{2}$$

In[ ]:=
```
mat = Matrix[HH, {c, S}];
mat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
\frac{\Omega}{2} & 0 & 0 & g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \omega+\frac{\Omega}{2} & 0 & 0 & \sqrt{2}\ g & 0 & 0 & 0 & 0 & 0 & 0 \\
g & 0 & 0 & \omega-\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2\ \omega+\frac{\Omega}{2} & 0 & 0 & \sqrt{3}\ g & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{2}\ g & 0 & 0 & 2\ \omega-\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3\ \omega+\frac{\Omega}{2} & 0 & 0 & 2\ g & 0 & 0 \\
0 & 0 & 0 & 0 & \sqrt{3}\ g & 0 & 0 & 3\ \omega-\frac{\Omega}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4\ \omega+\frac{\Omega}{2} & 0 & 0 & \sqrt{5}\ g \\
0 & 0 & 0 & 0 & 0 & 0 & 2\ g & 0 & 0 & 4\ \omega-\frac{\Omega}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5\ \omega+\frac{\Omega}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{5}\ g & 0 & 0 & 5\ \omega-\frac{\Omega}{2}
\end{pmatrix}$$

In[•]:=
```
mat = Matrix[HH, {S, c}];
mat // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 & g & 0 & 0 & 0 & 0 \\
0 & \omega+\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2}\,g & 0 & 0 & 0 \\
0 & 0 & 2\,\omega+\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3}\,g & 0 & 0 \\
0 & 0 & 0 & 3\,\omega+\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 2\,g & 0 \\
0 & 0 & 0 & 0 & 4\,\omega+\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{5}\,g \\
0 & 0 & 0 & 0 & 0 & 5\,\omega+\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{\Omega}{2} & 0 & 0 & 0 & 0 & 0 \\
g & 0 & 0 & 0 & 0 & 0 & 0 & \omega-\frac{\Omega}{2} & 0 & 0 & 0 & 0 \\
0 & \sqrt{2}\,g & 0 & 0 & 0 & 0 & 0 & 0 & 2\,\omega-\frac{\Omega}{2} & 0 & 0 & 0 \\
0 & 0 & \sqrt{3}\,g & 0 & 0 & 0 & 0 & 0 & 0 & 3\,\omega-\frac{\Omega}{2} & 0 & 0 \\
0 & 0 & 0 & 2\,g & 0 & 0 & 0 & 0 & 0 & 0 & 4\,\omega-\frac{\Omega}{2} & 0 \\
0 & 0 & 0 & 0 & \sqrt{5}\,g & 0 & 0 & 0 & 0 & 0 & 0 & 5\,\omega-\frac{\Omega}{2}
\end{pmatrix}$$

In[•]:=
```
Basis[{c, S}]
Basis[{S, c}]
```

Out[•]=

$$\{|0_c 0_S\rangle, |0_c 1_S\rangle, |1_c 0_S\rangle, |1_c 1_S\rangle, |2_c 0_S\rangle, |2_c 1_S\rangle, |3_c 0_S\rangle, |3_c 1_S\rangle, |4_c 0_S\rangle, |4_c 1_S\rangle, |5_c 0_S\rangle, |5_c 1_S\rangle\}$$

Out[•]=

$$\{|0_c 0_S\rangle, |1_c 0_S\rangle, |2_c 0_S\rangle, |3_c 0_S\rangle, |4_c 0_S\rangle, |5_c 0_S\rangle, |0_c 1_S\rangle, |1_c 1_S\rangle, |2_c 1_S\rangle, |3_c 1_S\rangle, |4_c 1_S\rangle, |5_c 1_S\rangle\}$$

In[•]:=
```
ProperValues[HH]
```

Out[•]=

$$\Big\{-\frac{\Omega}{2}, \frac{1}{2}(10\,\omega+\Omega), \frac{1}{2}\left(\omega-\sqrt{4\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right), \frac{1}{2}\left(\omega+\sqrt{4\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right),$$
$$\frac{1}{2}\left(3\,\omega-\sqrt{8\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right), \frac{1}{2}\left(3\,\omega+\sqrt{8\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right),$$
$$\frac{1}{2}\left(5\,\omega-\sqrt{12\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right), \frac{1}{2}\left(5\,\omega+\sqrt{12\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right),$$
$$\frac{1}{2}\left(7\,\omega-\sqrt{16\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right), \frac{1}{2}\left(7\,\omega+\sqrt{16\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right),$$
$$\frac{1}{2}\left(9\,\omega-\sqrt{20\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right), \frac{1}{2}\left(9\,\omega+\sqrt{20\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right)\Big\}$$

In[∘]:= `ProperStates[HH]`

Out[∘]=

$$\left\{\left|0_c 1_S\right\rangle,\ \left|5_c 0_S\right\rangle,\ -\frac{\left(\omega-\Omega+\sqrt{4\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right)\ \left|0_c 0_S\right\rangle}{2\,g}+\left|1_c 1_S\right\rangle,\right.$$

$$-\frac{\left(\omega-\Omega-\sqrt{4\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right)\ \left|0_c 0_S\right\rangle}{2\,g}+\left|1_c 1_S\right\rangle,$$

$$-\frac{\left(\omega-\Omega+\sqrt{8\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right)\ \left|1_c 0_S\right\rangle}{2\,\sqrt{2}\,g}+\left|2_c 1_S\right\rangle,$$

$$\frac{\left(-\omega+\Omega+\sqrt{8\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right)\ \left|1_c 0_S\right\rangle}{2\,\sqrt{2}\,g}+\left|2_c 1_S\right\rangle,$$

$$-\frac{\left(\omega-\Omega+\sqrt{12\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right)\ \left|2_c 0_S\right\rangle}{2\,\sqrt{3}\,g}+\left|3_c 1_S\right\rangle,$$

$$\frac{\left(-\omega+\Omega+\sqrt{12\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right)\ \left|2_c 0_S\right\rangle}{2\,\sqrt{3}\,g}+\left|3_c 1_S\right\rangle,$$

$$-\frac{\left(\omega-\Omega+\sqrt{16\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right)\ \left|3_c 0_S\right\rangle}{4\,g}+\left|4_c 1_S\right\rangle,$$

$$-\frac{\left(\omega-\Omega-\sqrt{16\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right)\ \left|3_c 0_S\right\rangle}{4\,g}+\left|4_c 1_S\right\rangle,$$

$$-\frac{\left(\omega-\Omega+\sqrt{20\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right)\ \left|4_c 0_S\right\rangle}{2\,\sqrt{5}\,g}+\left|5_c 1_S\right\rangle,$$

$$\left.\frac{\left(-\omega+\Omega+\sqrt{20\,g^2+\omega^2-2\,\omega\,\Omega+\Omega^2}\right)\ \left|4_c 0_S\right\rangle}{2\,\sqrt{5}\,g}+\left|5_c 1_S\right\rangle\right\}$$

Model: A qudit interacting with single-mode bosons

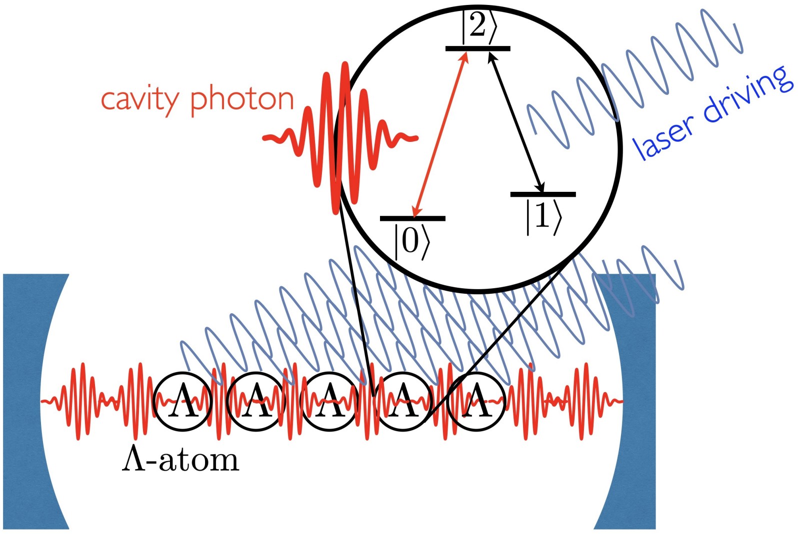

Figure 2: A system of (artificial) atoms with a Λ-level structure and interacting with bosonic modes.

Three-level systems (atoms) are described by **`Qudits`**, denoted by symbol **`A`**.

In[ ]:=
```
Let[Qudit, A]
```

In[ ]:=
```
Dimension[A]
```

Out[ ]=
```
3
```

Bosonic modes (cavity photons) are described by **`Bosons`**, denoted by symbol **`c`**.

In[ ]:=
```
Let[Boson, c]
```

In[ ]:=
```
{Bottom[c], Top[c], Dimension[c]}
```

Out[ ]=
```
{0, 5, 6}
```

Atoms are driven by an external classical laser (Rabi driving).

In[ ]:=
```
Let[Real, Ω]
HR = Ω * (A[1 → 2] + A[2 → 1])
```

Out[ ]=

$\Omega\,\left(\left(|2\rangle\langle 1|\right) + \left(|1\rangle\langle 2|\right)\right)$

In[ ]:=
```
Let[Real, w]
Hc = ω * Dagger[c] ** c
```

Out[ ]=

$\omega\; c^{\dagger} c$

Coupling between the atom and boson.

In[ ]:=
```
Let[Real, g]
Hg = g * (A[2 → 0] ** Dagger[c] + c ** A[0 → 2])
```
Out[ ]=

$$g \left(c\left(|2\rangle\langle 0|\right) + c^{\dagger}\left(|0\rangle\langle 2|\right)\right)$$

In[ ]:=
```
HH = HR + Hc + Hg
```
Out[ ]=

$$\Omega\left(\left(|2\rangle\langle 1|\right) + \left(|1\rangle\langle 2|\right)\right) + \omega\, c^{\dagger}c + g\left(c\left(|2\rangle\langle 0|\right) + c^{\dagger}\left(|0\rangle\langle 2|\right)\right)$$

In[ ]:=
```
mat = Matrix[HH, {c, A}];
mat // MatrixForm
```
Out[ ]//MatrixForm=

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&\Omega&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&\Omega&0&g&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&g&\omega&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&\omega&\Omega&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&\Omega&\omega&\sqrt{2}\,g&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&\sqrt{2}\,g&2\,\omega&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&2\,\omega&\Omega&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&\Omega&2\,\omega&\sqrt{3}\,g&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&\sqrt{3}\,g&3\,\omega&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&3\,\omega&\Omega&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&\Omega&3\,\omega&2\,g&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&2\,g&4\,\omega&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&4\,\omega&\Omega&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&\Omega&4\,\omega&\sqrt{5}\,g&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&\sqrt{5}\,g&5\,\omega&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&5\,\omega&\Omega\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\Omega&5\,\omega
\end{pmatrix}$$

Summary

Functions

- **Qubit, Dimension**
- **Boson, Bottom, Top, Dimension**
- **Matrix, ExpressionFor**
- **ProperValues, ProperStates, ProperSystem**

Related Links

- Mahn-Soo Choi, Advanced Quantum Technologies 12, 2000085 (2020), “Exotic Quantum States of Circuit Quantum Electrodynamics in the Ultra-Strong Coupling Regime.”
- C. Dongni, S. Luo, Y.-D. Wang, S. Chesi, and Mahn-Soo Choi, Physical Review A 105, 022627 (2022), “Geometric manipulation of a decoherence-free subspace in atomic ensembles.”