日本語訳『Qiskit Textbook』勉強会
第7章 演習3 ベストなANDゲートの構築

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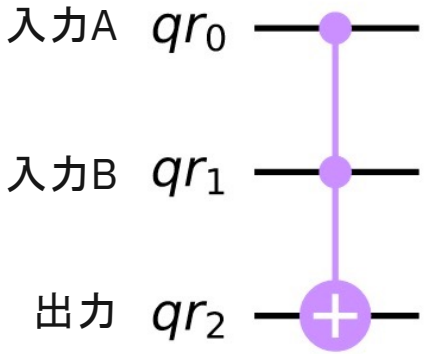
量子コンピューターでのANDゲート

入力A	入力B	出力
0	0	0
0	1	0
1	0	0
1	1	1

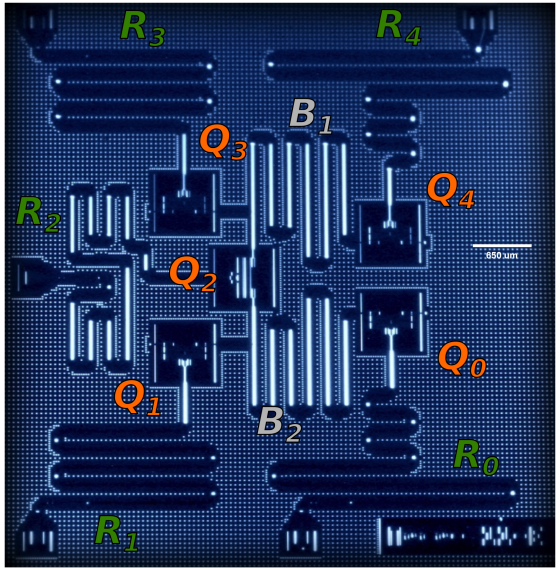


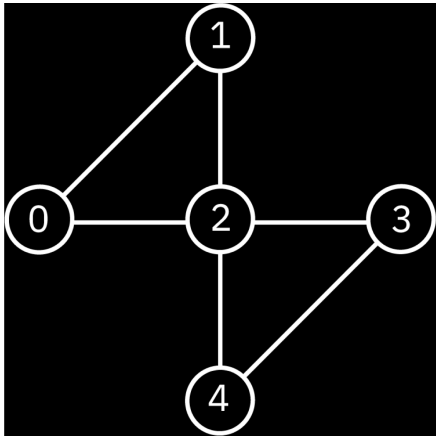

```
q = QuantumRegister(3)
c = ClassicalRegister(1)
qc = QuantumCircuit(q,c)
qc.ccx(q[0], q[1], q[2])
qc.measure(q[2], c[0])
qc.draw(output='mpl')
```

今回は、実デバイスを対象にします。
本物のデバイスを使う場合、接続性(コネクティビティー)の制限とノイズが加わります。

実デバイスでの制約その１：接続性（Connectivity）

ibmq_5_Tenerife (backend_name: ibmqx4)
（Textbookで使われている例ですが今は使われていないデバイスのようです。
ibmq_5_yorktown - ibmqx2と同じ形です。）





https://github.com/Qiskit/ibmq-device-information/tree/master/backends/tenerife/V1

```
coupling_map = [[1, 0], [2, 0], [2, 1], [3, 2], [3, 4], [4, 2]]
```

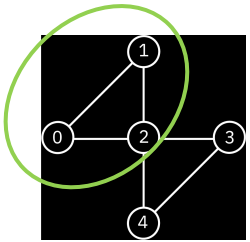
実デバイスでの制約その2：ノイズ

ibmq_5_Tenerife (backend_name: ibmqx4) のノイズを今回はシミュレートして使います。

```
noise_dict = {'errors': [{'type': 'qerror', 'operations': ['u2'], \
                          'instructions': [[{'name': 'x', 'qubits': [0]}], \
                                           [{'name': 'y', 'qubits': [0]}], \
                                           [{'name': 'z', 'qubits': [0]}], \
                                           [{'name': 'id', 'qubits': [0]}]], \
                          'probabilities': [0.0004721766167523067, 0.0004721766167523067, \
                                            0.0004721766167523067, 0.9985834701497431], \
                          'gate_qubits': [[0]]}, \
                         {'type': 'qerror', 'operations': ['u2'], \
                          'instructions': [[{'name': 'x', 'qubits': [0]}], \
                                           [{'name': 'y', 'qubits': [0]}], \
                                           [{'name': 'z', 'qubits': [0]}], \
                                           [{'name': 'id', 'qubits': [0]}]], \
                          'probabilities': [0.0005151090708174488, 0.0005151090708174488, \
                                            0.0005151090708174488, 0.9984546727875476], \
                          'gate_qubits': [[1]]}, \
                         {'type': 'qerror', 'operations': ['u2'], \
                          'instructions': [[{'name': 'x', 'qubits': [0]}], \
                                           [{'name': 'y', 'qubits': [0]}], \
                                           [{'name': 'z', 'qubits': [0]}], \
                                           [{'name': 'id', 'qubits': [0]}]], \
                          'probabilities': [0.0005151090708174488, 0.0005151090708174488, \
                                            0.0005151090708174488, 0.9984546727875476], \
                          'gate_qubits': [[2]]}, \
                         {'type': 'qerror', 'operations': ['u2'], \
                          'instructions': [[{'name': 'x', 'qubits': [0]}], \
                                           [{'name': 'y', 'qubits': [0]}], \
                                           [{'name': 'z', 'qubits': [0]}], \
                                           [{'name': 'id', 'qubits': [0]}]], \
                          'probabilities': [0.000901556048412383, 0.000901556048412383, \
                                            0.000901556048412383, 0.9972953318547628], \
                          'gate_qubits': [[3]]}, {'type': 'qerror', 'operations': ['u2'], \
                         'instructions': [[{'name': 'x', 'qubits': [0]}], \
                                          ⋮
```

ibmq_5_TenerifeでCCXを使ったANDの計算

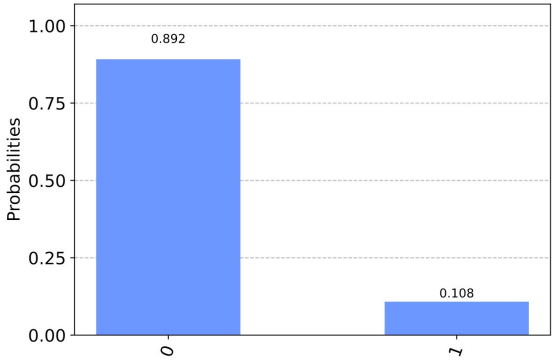
入力1:量子ビット番号0、入力2:量子ビット番号1、出力:量子ビット番号2を使った場合、



0 x 0 の計算：89.2%の確率で成功

```
result = AND('0','0')
print( result )
plot_histogram( result )
```

```
{'0': 8917, '1': 1083}
```



0x0, 0x1, 1x0, 1x1の計算では最も悪い確率で89.4%

```
worst = 1
for input1 in ['0','1']:
    for input2 in ['0','1']:
        print('\nProbability of correct answer for inputs',input1,input2)
        prob = AND(input1,input2, q_1=0,q_2=1,q_out=2)\
        [str(int( input1=='1' and input2=='1' ))]/10000
        print( prob )
        worst = min(worst,prob)
print('\nThe lowest of these probabilities was',worst)
```

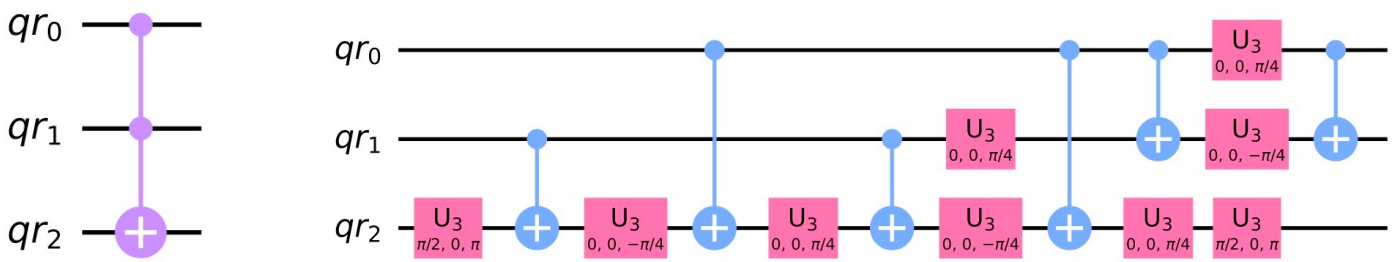
```
Probability of correct answer for inputs 0 0
0.9019

Probability of correct answer for inputs 0 1
0.9028

Probability of correct answer for inputs 1 0
0.9067

Probability of correct answer for inputs 1 1
0.8943

The lowest of these probabilities was 0.8943
```

演習問題：もっとよい結果が得られるANDを作ってください。

ゲート数を減らしてみる

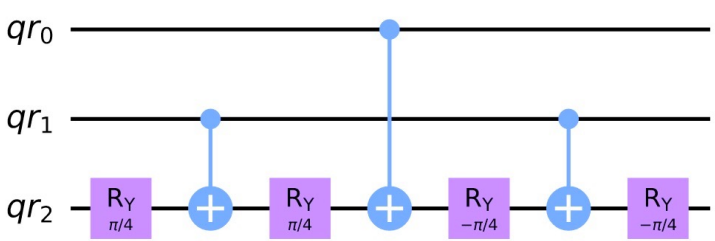
CCXゲートをTranspileしてみるとCNOTが6個、u3ゲート9個

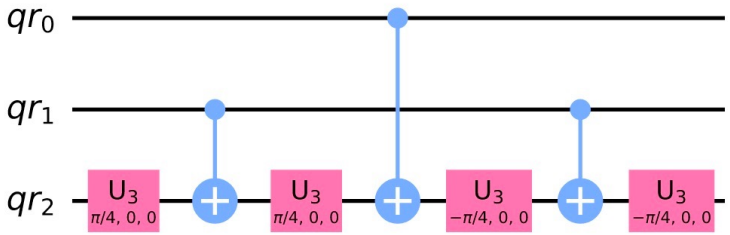
OrderedDict([('u3', 9), ('cx', 6)])



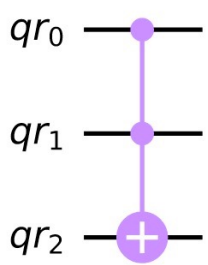
ほぼCCXの別バージョン：CNOTが3個、u3ゲート4個

OrderedDict([('u3', 4), ('cx', 3)])

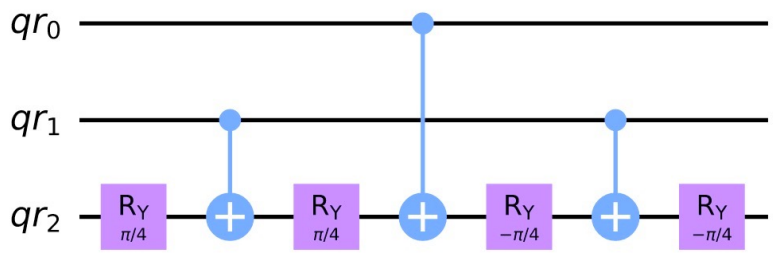




CNOT3個でのCCX



$\cong$

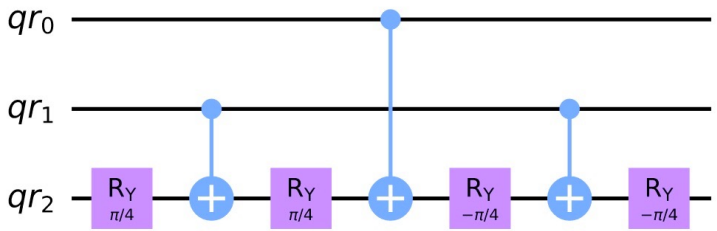


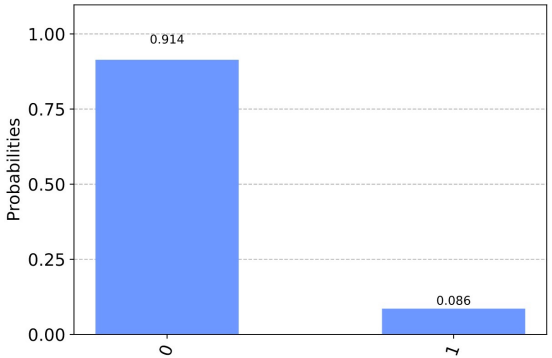
$$
\begin{aligned}
|00\rangle \otimes |\phi\rangle &\mapsto |00\rangle \otimes |\phi\rangle, \\
|01\rangle \otimes |\phi\rangle &\mapsto |01\rangle \otimes |\phi\rangle, \\
|10\rangle \otimes |\phi\rangle &\mapsto |10\rangle \otimes |\phi\rangle, \\
|11\rangle \otimes |\phi\rangle &\mapsto |11\rangle \otimes X|\phi\rangle,
\end{aligned}
$$

$$
\begin{aligned}
|00\rangle \otimes |\phi\rangle &\mapsto |00\rangle \otimes |\phi\rangle, \\
|01\rangle \otimes |\phi\rangle &\mapsto |01\rangle \otimes |\phi\rangle, \\
|10\rangle \otimes |\phi\rangle &\mapsto |10\rangle \otimes Z|\phi\rangle, \\
|11\rangle \otimes |\phi\rangle &\mapsto |11\rangle \otimes X|\phi\rangle,
\end{aligned}
\quad
\begin{cases}
|100\rangle \to |100\rangle \\
|101\rangle \to -|101\rangle
\end{cases}
$$

CNOT3個の場合

```
def my_AND(input1,input2, q_1=0,q_2=1,q_out=2):
    qc = QuantumCircuit(qr, cr)

        # 量子ビットq1, q2を入力ビットとして準備
    if input1=='1':
        qc.x( qr[ q_1 ] )
    if input2=='1':
        qc.x( qr[ q_2 ] )

    qc.ry(np.pi/4, qr[ q_out ])
    qc.cx(qr[ q_2 ], qr[ q_out ])
    qc.ry(np.pi/4, qr[ q_out ])
    qc.cx(qr[ q_1 ], qr[ q_out ])
    qc.ry(-np.pi/4, qr[ q_out ])
    qc.cx(qr[ q_2 ], qr[ q_out ])
    qc.ry(-np.pi/4, qr[ q_out ])

    qc.measure(qr[ q_out ],cr[0]) # 量子ビット1からの出力を測定

    job = execute(qc, backend, shots=10000, noise_model=noise_model,
                                  coupling_map=coupling_map,
                                  basis_gates=noise_model.basis_gates)
    output = job.result().get_counts()

    return output
```





Probability of correct answer for inputs 0 0
0.9156

Probability of correct answer for inputs 0 1
0.9123

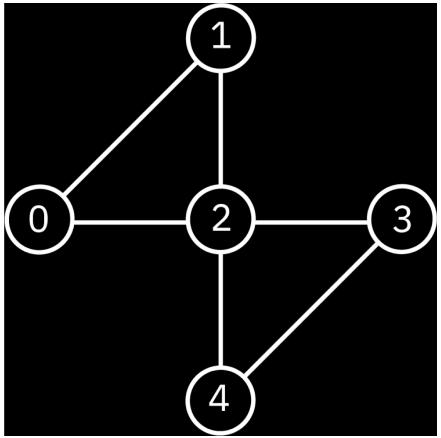
Probability of correct answer for inputs 1 0
0.9127

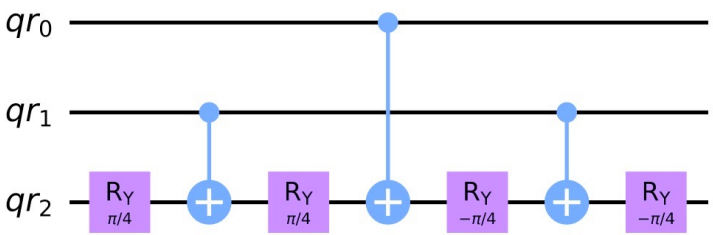
Probability of correct answer for inputs 1 1
0.9063

The lowest of these probabilities was 0.9063

結果、確率が90.06%に上がりました。

量子ビットの組み合わせ（順列）で最も良いものを探してみました





結果：[q0,q1,q2]は、 [1, 0, 2] が最も良さそうでした。

一番良いQubitの配列　[1, 0, 2] 0.9142

もともと89.4%のものを91.4%に改善するAND回路ができました。

Kifumi Numata