Stokes Theorem as Generalized Abstract Vapid Nonsense

"We'll only use as much category theory as is necessary. [*famous last words*]" –Roman Abramovich[1]

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[1]Attribution: "Higher Gauge Theory: The Fundamentals" (quote is almost certainly misattributed to a Russian oligarch instead of the algebraic geometrist who is likely responsible for it)

Outline

1.1 Manifolds

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- Manifolds are just a continuous set of points
- Moreover, at any given point, they look like a **Euclidean space**

1.2 Euclidean Spaces

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- Euclidean space is the space we inhabit (not the full story; more on this later)

What actually is a manifold then?

- When we say that a manifold "looks like" a Euclidean space, we're actually talking about **homeomorphism**

1.3 General Topology

- Topologists do not understand shapes, so they must deform objects into meaningless blobs to count holes
- Two numbers are equal, but two topological spaces are **homeomorphic** to each other

1.3 General Topology

1.3.1 Definitions

Homeomorphism

A pair of continuous bijective (all $x \in X$ in the domain of f and all $y \in Y$ in the range of f) mappings $f : X \longrightarrow Y$ and $f^{-1} : Y \longrightarrow X$

Homeomorphic equivalence

Two topological objects X and Y are homeomorphic to each other iff there exists a **homeomorphism** between them.

1.4 Formalizing the Manifold

1.4.1 Definitions

Read: more abstract nonsense

Manifold

A k (represents intrinsic dimension) manifold is a continuous set of points X **homeomorphic** to $\mathbb{R}^k$

Embedding

The embedding of a manifold X is n iff $x \in \mathbb{R}^n \ \forall x \in X$

Note: If a k manifold is embedded in n space ($\mathbb{R}^n$), we must have $n > k$ (this is a trivial consequence of an advanced mathematical technique called "visualization")

1.5 Manifolds in the Wild

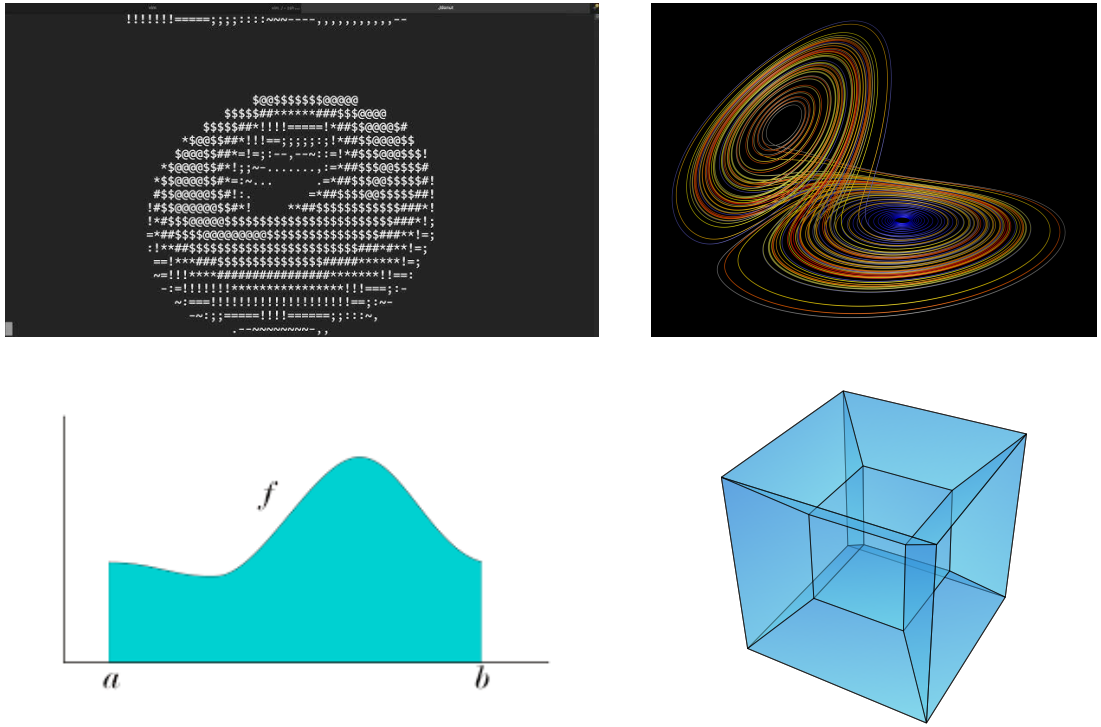


Figure 1: From left to right, top to bottom*: A 2 manifold in 3 space, a 1 manifold in 3 space, a 2 manifold in 2 space, and a 2 manifold in 4 space (projected onto 3 space–the drawing is of a hypercube)*

1.5 Manifolds in the Wild

1.5.1 The Prototypical Example of a Manifold

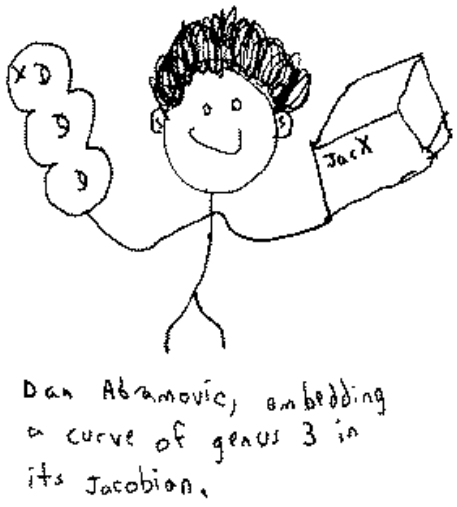
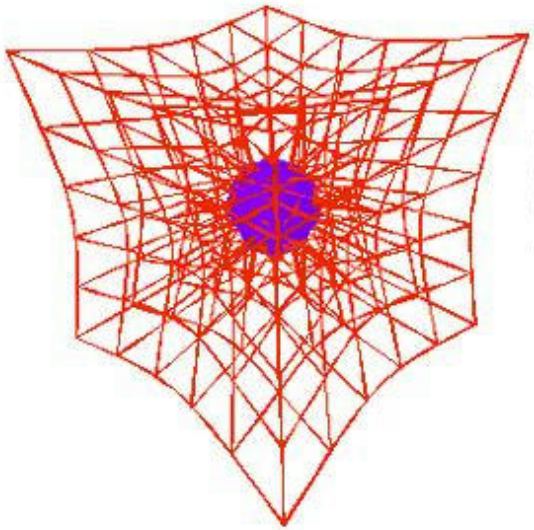


Figure 2: Don't ask me what this is because I am just as clueless as you

1.6 The Universe

We live in a **3 manifold** (possibly embedded in a *higher dimensional* space–and we have no way of knowing)!

Completely irrelevant to understanding Generalized Stokes Theorem, but very cool nevertheless.



This three-dimensional grid gives a better idea of what curved space-time might look like than the two-dimensional analogies do.

1.7 Differential Forms

1.8 Orientability

1.9 Boundaries

1.10 Exterior Derivative

1.11 Generalized Stokes Theorem

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