An Introduction to Voting

The basics of social choice theory

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[1]Toquos theory was invented here

Outline

1.1 An Introduction

- Have some description of preferences among some choices/candidates (**alternatives**) for each individual
- Have many individuals (these form a **society**)
- Want to combine individual preferences to find the **socially optimal choice**

1.2 Plurality Voting

- We ask each voter for their **favorite choice**
- **Count** up the votes for each choice
- Winner is the choice with the **most votes**

1.3 Some Issues

- If many candidates with similar positions run, they **split the vote**
- Winning candidate is not necessarily supported by a **majority**
- Leads to a **two-party system** (Duverger's Law)

Outline

2.1 Ranked-Choice Voting Systems

- Instead of only asking for a first choice, we ask for a **ranking of all choices**
- How we sum up the rankings depends on what **voting system** we use
- **Instant-runoff voting** (IRV)
  - If a choice has a **majority of first-place votes**, it wins
  - If not, the choice with the **fewest first-place votes is eliminated**, and the process repeats

2.1 Ranked-Choice Voting Systems

- **Borda count**
  - Assign each choice a **score equal to its ranking**
    - Here, first place ranking gets points = number of candidates
    - Last place gets points = 1
  - **Sum** up all the scores, and choose the candidate with the **highest score**

2.2 Arrow's Impossibility Theorem

- It turns out it is not possible to have a voting system that satisfies all of the following conditions:
  - **Universal domain**: A winner is always selected, regardless of voter preferences
  - **Non-dictatorship**: A single voter does not determine the outcome of the election
  - **Unanimity**: A unanimous first place choice should always win
  - **Independence of irrelevant alternatives** (IIA): Adding a new candidate that doesn't win shouldn't change the result of the election

2.3 Condorcet winners

- If we **relax universal domain**, we can choose a winner satisfying non-dictatorship, unanimity, and IIA
  - But only in certain cases!
- Find the candidate that **beats every other candidate** in a pairwise comparison, if one exists
  - Candidate ranked higher on more ballots wins
- Doesn't exist when we have **Condorcet cycles**

Outline

3.1 The Spatial Voting Model

- We model the things voters care about (their preferences) as **points** in space
- We model candidates' positions as points in space also
- Voters prefer the **candidates closest to them** (they support candidates with beliefs closest to their own)
- Voters rank candidates in order of distance; the so-called **Euclidean model**

3.2 Things to observe

- Play with different geometries in the simulation (drag candidate points around)
- Use different voter distributions
- Why does plurality cause a two-party system?
  - Hint: what does vote splitting look like geometrically?
- What are the differences between plurality, IRV, and Borda count?
- Can you create a scenario where no Condorcet winner exists?
  - Single-peaked preferences prevent this in one-dimension, but not in higher dimensional spatial voting models