

# Autómatas y Lenguajes formales

## Ejercicio Semanal 6

Sandra del Mar Soto Corderi  
Edgar Quiroz Castañeda

14 de marzo del 2019

1. Para cada  $ANF_\epsilon$ , resuelve los siguientes incisos.

(a) Calcula la  $\epsilon$ -cerradura de cada estado.

(b) Elimina las  $\epsilon$ -transiciones obteniendo un AFN, mostrando el proceso de cálculo de las nuevas transiciones.

1. Autómata 1

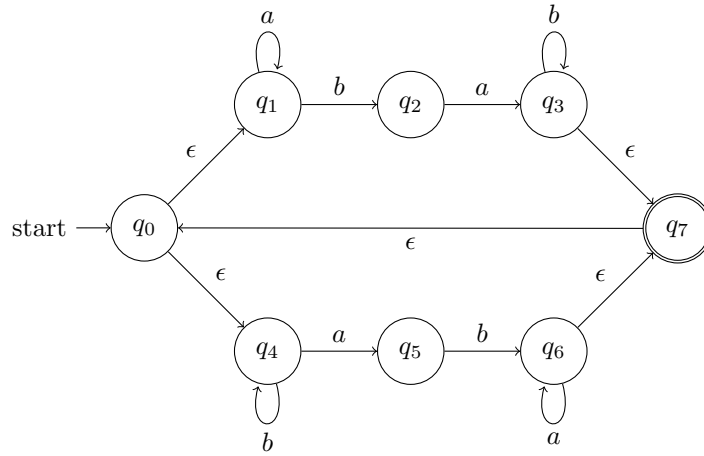


Figure 1: El autómata M

La función de transición  $\delta$  definida con la siguiente tabla:

$q$	$\delta(q, a)$	$\delta(q, b)$	$\delta(q, \epsilon)$
0	$\emptyset$	$\emptyset$	$\{1, 4\}$
1	$\{1\}$	$\{2\}$	$\emptyset$
2	$\{3\}$	$\emptyset$	$\emptyset$
3	$\emptyset$	$\{3\}$	$\{7\}$
4	$\{5\}$	$\{4\}$	$\emptyset$
5	$\emptyset$	$\{6\}$	$\emptyset$
6	$\{6\}$	$\emptyset$	$\{7\}$
7	$\emptyset$	$\emptyset$	$\{0\}$

(a) Vamos a calcular la  $Cl_\epsilon(q)$  para todo  $q \in Q$

$$\begin{aligned}
Cl_\epsilon(q_0) &= \{q_0, q_1, q_4\} \\
Cl_\epsilon(q_1) &= \{q_1\} \\
Cl_\epsilon(q_2) &= \{q_2\} \\
Cl_\epsilon(q_3) &= \{q_3, q_7, q_0, q_1, q_4\} \\
Cl_\epsilon(q_4) &= \{q_4\} \\
Cl_\epsilon(q_5) &= \{q_5\} \\
Cl_\epsilon(q_6) &= \{q_6, q_7, q_0, q_4, q_1\} \\
Cl_\epsilon(q_7) &= \{q_7, q_0, q_4, q_1\}
\end{aligned}$$

(b) Ya con la  $\epsilon$ -cerradura de cada estado podemos definir  $\delta^*$  para los símbolos del alfabeto:

$$\begin{aligned}
\delta^*(q_0, a) &= \\
Cl_\epsilon(\cup_{p \in \delta^*(q_0, \epsilon)} \delta(p, a)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_0)} \delta(p, a)) &= \\
Cl_\epsilon(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_4, a)) &= \\
Cl_\epsilon(\emptyset \cup \{q_1\} \cup \{q_5\}) &= \\
Cl_\epsilon(\{q_1, q_5\}) &= \\
\{q_1, q_5\}
\end{aligned}$$

$$\begin{aligned}
\delta^*(q_0, b) &= \\
Cl_\epsilon(\cup_{p \in \delta^*(q_0, \epsilon)} \delta(p, b)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_0)} \delta(p, b)) &= \\
Cl_\epsilon(\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_4, b)) &= \\
Cl_\epsilon(\emptyset \cup \{q_2\} \cup \{q_4\}) &= \\
Cl_\epsilon(\{q_2, q_4\}) &= \\
\{q_2, q_4\}
\end{aligned}$$

$$\begin{aligned}
\delta^*(q_1, a) &= \\
Cl_\epsilon(\cup_{p \in \delta^*(q_1, \epsilon)} \delta(p, a)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_1)} \delta(p, a)) &= \\
Cl_\epsilon(\delta(q_1, a)) &= \\
Cl_\epsilon(\{q_1\}) &= \\
\{q_1\}
\end{aligned}$$

$$\begin{aligned}
\delta^*(q_1, b) &= \\
Cl_\epsilon(\cup_{p \in \delta^*(q_1, \epsilon)} \delta(p, b)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_1)} \delta(p, b)) &= \\
Cl_\epsilon(\delta(q_1, b)) &= \\
Cl_\epsilon(\{q_2\}) &= \\
\{q_2\}
\end{aligned}$$

$$\begin{aligned}
& \delta^*(q_2, a) = \\
& Cl_\epsilon(\cup_{p \in \delta^*(q_2, \epsilon)} \delta(p, a)) = \\
& Cl_\epsilon(\cup_{Cl_\epsilon(q_2)} \delta(p, a)) = \\
& Cl_\epsilon(\delta(q_2, a)) = \\
& Cl_\epsilon(\{q_3\}) = \\
& \{q_3\}
\end{aligned}$$

$$\begin{aligned}
& \delta^*(q_2, b) = \\
& Cl_\epsilon(\cup_{p \in \delta^*(q_2, \epsilon)} \delta(p, b)) = \\
& Cl_\epsilon(\cup_{Cl_\epsilon(q_2)} \delta(p, b)) = \\
& Cl_\epsilon(\delta(q_2, b)) = \\
& Cl_\epsilon(\emptyset) = \\
& \emptyset
\end{aligned}$$

$$\begin{aligned}
& \delta^*(q_3, a) = \\
& Cl_\epsilon(\cup_{p \in \delta^*(q_3, \epsilon)} \delta(p, a)) = \\
& Cl_\epsilon(\cup_{Cl_\epsilon(q_3)} \delta(p, a)) = \\
& Cl_\epsilon(\delta(q_3, a) \cup \delta(q_7, a) \cup \delta(q_0, a) \cup \delta(q_1, a)) = \\
& Cl_\epsilon(\emptyset \cup \emptyset \cup \emptyset \cup \{q_1\}) = \\
& Cl_\epsilon(\{q_1\}) = \\
& \{q_1\}
\end{aligned}$$

$$\begin{aligned}
& \delta^*(q_3, b) = \\
& Cl_\epsilon(\cup_{p \in \delta^*(q_3, \epsilon)} \delta(p, b)) = \\
& Cl_\epsilon(\cup_{Cl_\epsilon(q_3)} \delta(p, b)) = \\
& Cl_\epsilon(\delta(q_3, b) \cup \delta(q_7, b) \cup \delta(q_0, b) \cup \delta(q_1, b)) = \\
& Cl_\epsilon(\emptyset \cup \emptyset \cup \emptyset \cup \{q_2\}) = \\
& Cl_\epsilon(\{q_2\}) = \\
& \{q_2\}
\end{aligned}$$

$$\begin{aligned}
& \delta^*(q_4, a) = \\
& Cl_\epsilon(\cup_{p \in \delta^*(q_4, \epsilon)} \delta(p, a)) = \\
& Cl_\epsilon(\cup_{Cl_\epsilon(q_4)} \delta(p, a)) = \\
& Cl_\epsilon(\delta(q_4, a)) = \\
& Cl_\epsilon(\{q_5\}) = \\
& \{q_5\}
\end{aligned}$$

$$\begin{aligned}
& \delta^*(q_4, b) = \\
& Cl_\epsilon(\cup_{p \in \delta^*(q_4, \epsilon)} \delta(p, b)) = \\
& Cl_\epsilon(\cup_{Cl_\epsilon(q_4)} \delta(p, b)) = \\
& Cl_\epsilon(\delta(q_4, b)) = \\
& Cl_\epsilon(\{q_4\}) = \\
& \{q_4\}
\end{aligned}$$

$$\begin{aligned}
& \delta^*(q_5, a) = \\
& Cl_\epsilon(\cup_{p \in \delta^*(q_5, \epsilon)} \delta(p, a)) = \\
& Cl_\epsilon(\cup_{Cl_\epsilon(q_5)} \delta(p, a)) = \\
& Cl_\epsilon(\delta(q_5, a)) = \\
& Cl_\epsilon(\emptyset) = \emptyset
\end{aligned}$$

$$\begin{aligned}
& \delta^*(q_5, b) = \\
& Cl_\epsilon(\cup_{p \in \delta^*(q_5, \epsilon)} \delta(p, b)) = \\
& Cl_\epsilon(\cup_{Cl_\epsilon(q_5)} \delta(p, b)) = \\
& Cl_\epsilon(\delta(q_5, b)) = \\
& Cl_\epsilon(\{q_6\}) = \\
& \{q_6\}
\end{aligned}$$

$$\begin{aligned}
& \delta^*(q_6, a) = \\
& Cl_\epsilon(\cup_{p \in \delta^*(q_6, \epsilon)} \delta(p, a)) = \\
& Cl_\epsilon(\cup_{Cl_\epsilon(q_6)} \delta(p, a)) = \\
& Cl_\epsilon(\delta(q_6, a) \cup \delta(q_7, a) \cup \delta(q_0, a) \cup \delta(q_1, a)) = \\
& Cl_\epsilon(\{q_6\} \cup \emptyset \cup \emptyset \cup \{q_1\}) = \\
& Cl_\epsilon(\{q_6, q_1\}) = \\
& \{q_6, q_7, q_0, q_1\}
\end{aligned}$$

$$\begin{aligned}
& \delta^*(q_6, b) = \\
& Cl_\epsilon(\cup_{p \in \delta^*(q_6, \epsilon)} \delta(p, b)) = \\
& Cl_\epsilon(\cup_{Cl_\epsilon(q_6)} \delta(p, b)) = \\
& Cl_\epsilon(\delta(q_6, b) \cup \delta(q_7, b) \cup \delta(q_0, b) \cup \delta(q_1, b)) = \\
& Cl_\epsilon(\emptyset \cup \emptyset \cup \emptyset \cup \{q_2\}) = \\
& Cl_\epsilon(\{q_2\}) = \\
& \{q_2\}
\end{aligned}$$

$$\begin{aligned}
& \delta^*(q_7, a) = \\
& Cl_\epsilon(\cup_{p \in \delta^*(q_7, \epsilon)} \delta(p, a)) = \\
& Cl_\epsilon(\cup_{Cl_\epsilon(q_7)} \delta(p, a)) = \\
& Cl_\epsilon(\delta(q_7, a) \cup \delta(q_0, a) \cup \delta(q_1, a)) = \\
& Cl_\epsilon(\emptyset \cup \emptyset \cup \{q_1\}) = \\
& Cl_\epsilon(\{q_1\}) = \\
& \{q_1\}
\end{aligned}$$

$$\begin{aligned}
& \delta^*(q_7, b) = \\
& Cl_\epsilon(\cup_{p \in \delta^*(q_7, \epsilon)} \delta(p, b)) = \\
& Cl_\epsilon(\cup_{Cl_\epsilon(q_7)} \delta(p, b)) = \\
& Cl_\epsilon(\delta(q_7, b) \cup \delta(q_0, b) \cup \delta(q_1, b)) = \\
& Cl_\epsilon(\emptyset \cup \emptyset \cup \{q_2\}) = \\
& Cl_\epsilon(\{q_2\}) = \\
& \{q_2\}
\end{aligned}$$

Vamos a construir un AFN  $M_n$  eliminando las  $\epsilon$ -transiciones.

$M_N = \langle Q, \Sigma, \delta_N, q_0, F_n \rangle$ , donde:

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$$

$$\Sigma = \{a, b\}$$

$\delta_N(q, \sigma) = \delta^*(q, \sigma)$ , así que se define con la siguiente tabla:

q	$\delta_N(q, a)$	$\delta_N(q, b)$
0	{1, 5}	{2, 4}
1	{1}	{2}
2	{3}	$\emptyset$
3	{1}	{2}
4	{5}	{4}
5	$\emptyset$	{6}
6	{0, 1, 6, 7}	{2}
7	{1}	{2}

$F_N = F$  pues  $Cl_\epsilon(q_0) \cap F = \emptyset$ .

Y visto de forma gráfica, AFN es:

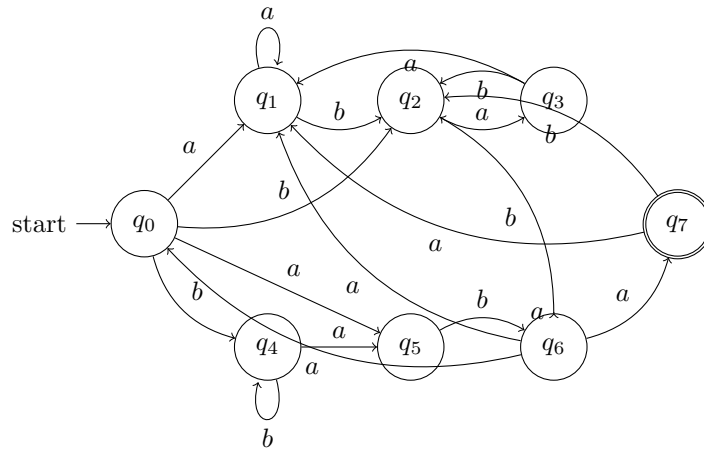
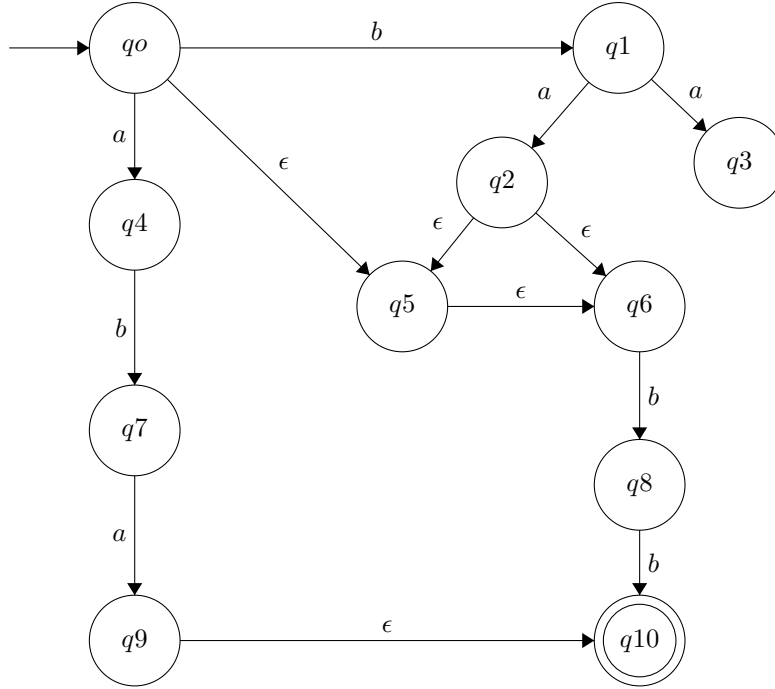


Figure 2: El autómata  $M_N$

## 2. Autómata 2



(a) Calcula la  $\epsilon$ -cerradura de cada estado

$$\begin{aligned}
 Cl_\epsilon(q_0) &= \{q_0, q_5, q_6\} & Cl_\epsilon(q_1) &= \{q_1\} \\
 Cl_\epsilon(q_2) &= \{q_2, q_5, q_6\} & Cl_\epsilon(q_3) &= \{q_3\} \\
 Cl_\epsilon(q_4) &= \{q_4\} & Cl_\epsilon(q_5) &= \{q_5, q_6\} \\
 Cl_\epsilon(q_6) &= \{q_6\} & Cl_\epsilon(q_7) &= \{q_7\} \\
 Cl_\epsilon(q_8) &= \{q_8\} & Cl_\epsilon(q_9) &= \{q_9, q_{10}\} \\
 Cl_\epsilon(q_{10}) &= \{q_{10}\}
 \end{aligned}$$

(b) Elimina las  $\epsilon$ -transiciones obteniendo un AFN, mostrando el proceso de cálculo de las nuevas transiciones.

Sea  $M_\epsilon = \langle Q_\epsilon, \Sigma_\epsilon, \delta_\epsilon, q_{0\epsilon}, F_\epsilon \rangle$  el atómata de la figura.

El nuevo automata sería  $M = \langle Q, \Sigma, \delta, q_0, F \rangle$  dado por

$$Q = Q_\epsilon, \Sigma = \Sigma_\epsilon, q_0 = q_{0\epsilon}, F = F_\epsilon$$

Pues  $F_\epsilon \cap Cl_\epsilon(q_0) = \emptyset$ .

En cuento a la  $\delta$ , esta se construye de la siguiente manera

$$\begin{aligned}
 \delta(q_0, a) &= \delta_\epsilon^*(q_0, a) \\
 &= Cl_\epsilon\left(\bigcup_{p \in \delta_\epsilon(q_0, a)} \delta_\epsilon(p, a)\right) \\
 &= Cl_\epsilon(\delta_\epsilon(q_0, a) \cup \delta_\epsilon(q_5, a) \cup \delta_\epsilon(q_6, a)) \\
 &= Cl_\epsilon(\{q_4\} \cup \emptyset \cup \emptyset) \\
 &= \{q_4\}
 \end{aligned}$$

$$\begin{aligned}
 \delta(q_0, b) &= \delta_\epsilon^*(q_0, b) \\
 &= Cl_\epsilon\left(\bigcup_{p \in \delta_\epsilon(q_0, b)} \delta_\epsilon(p, b)\right) \\
 &= Cl_\epsilon(\delta_\epsilon(q_0, b) \cup \delta_\epsilon(q_5, b) \cup \delta_\epsilon(q_6, b)) \\
 &= Cl_\epsilon(\{q_1\} \cup \emptyset \cup \{q_8\}) \\
 &= \{q_1, q_8\}
 \end{aligned}$$

$$\begin{aligned}
\delta(q_1, a) &= \delta_\epsilon^*(q_1, a) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_1, \epsilon)} \delta_\epsilon(p, a)) \\
&= Cl_\epsilon(\delta_\epsilon(q_1, a)) \\
&= Cl_\epsilon(\{q_2, q_3\}) \\
&= \{q_2, q_3, q_5, q_6\}
\end{aligned}$$

$$\begin{aligned}
\delta(q_1, b) &= \delta_\epsilon^*(q_1, b) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_1, \epsilon)} \delta_\epsilon(p, b)) \\
&= Cl_\epsilon(\delta_\epsilon(q_1, b)) \\
&= Cl_\epsilon(\emptyset) \\
&= \emptyset
\end{aligned}$$

$$\begin{aligned}
\delta(q_2, a) &= \delta_\epsilon^*(q_2, a) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_2, \epsilon)} \delta_\epsilon(p, a)) \\
&= Cl_\epsilon(\delta_\epsilon(q_2, a) \cup \delta_\epsilon(q_5, a) \cup \delta_\epsilon(q_6, a)) \\
&= Cl_\epsilon(\emptyset \cup \emptyset \cup \emptyset) \\
&= \emptyset
\end{aligned}$$

$$\begin{aligned}
\delta(q_2, b) &= \delta_\epsilon^*(q_2, b) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_2, \epsilon)} \delta_\epsilon(p, b)) \\
&= Cl_\epsilon(\delta_\epsilon(q_2, b) \cup \delta_\epsilon(q_5, b) \cup \delta_\epsilon(q_6, b)) \\
&= Cl_\epsilon(\emptyset \cup \emptyset \cup \{q_8\}) \\
&= \{q_8\}
\end{aligned}$$

$$\begin{aligned}
\delta(q_3, a) &= \delta_\epsilon^*(q_3, a) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_3, \epsilon)} \delta_\epsilon(p, a)) \\
&= Cl_\epsilon(\delta_\epsilon(q_3, a)) \\
&= Cl_\epsilon(\emptyset) \\
&= \emptyset
\end{aligned}$$

$$\begin{aligned}
\delta(q_3, b) &= \delta_\epsilon^*(q_3, b) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_3, \epsilon)} \delta_\epsilon(p, b)) \\
&= Cl_\epsilon(\delta_\epsilon(q_3, b)) \\
&= Cl_\epsilon(\emptyset) \\
&= \emptyset
\end{aligned}$$

$$\begin{aligned}
\delta(q_4, a) &= \delta_\epsilon^*(q_4, a) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_4, \epsilon)} \delta_\epsilon(p, a)) \\
&= Cl_\epsilon(\delta_\epsilon(q_4, a)) \\
&= Cl_\epsilon(\emptyset) \\
&= \emptyset
\end{aligned}$$

$$\begin{aligned}
\delta(q_4, b) &= \delta_\epsilon^*(q_4, b) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_4, \epsilon)} \delta_\epsilon(p, b)) \\
&= Cl_\epsilon(\delta_\epsilon(q_4, b)) \\
&= Cl_\epsilon(\{q_7\}) \\
&= \{q_7\}
\end{aligned}$$

$$\begin{aligned}
\delta(q_5, a) &= \delta_\epsilon^*(q_5, a) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_5, \epsilon)} \delta_\epsilon(p, a)) \\
&= Cl_\epsilon(\delta_\epsilon(q_5, a) \cup \delta_\epsilon(q_6, a)) \\
&= Cl_\epsilon(\emptyset \cup \emptyset) \\
&= \emptyset
\end{aligned}$$

$$\begin{aligned}
\delta(q_5, b) &= \delta_\epsilon^*(q_5, b) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_5, \epsilon)} \delta_\epsilon(p, b)) \\
&= Cl_\epsilon(\delta_\epsilon(q_5, b) \cup \delta_\epsilon(q_6, b)) \\
&= Cl_\epsilon(\emptyset \cup \{q_8\}) \\
&= \{q_8\}
\end{aligned}$$

$$\begin{aligned}
\delta(q_6, a) &= \delta_\epsilon^*(q_6, a) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_6, \epsilon)} \delta_\epsilon(p, a)) \\
&= Cl_\epsilon(\delta_\epsilon(q_6, a)) \\
&= Cl_\epsilon(\emptyset) \\
&= \emptyset
\end{aligned}$$

$$\begin{aligned}
\delta(q_6, b) &= \delta_\epsilon^*(q_6, b) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_6, \epsilon)} \delta_\epsilon(p, b)) \\
&= Cl_\epsilon(\delta_\epsilon(q_6, b)) \\
&= Cl_\epsilon(\{q_8\}) \\
&= \{q_8\}
\end{aligned}$$



$$\begin{aligned}
\delta(q_7, a) &= \delta_\epsilon^*(q_7, a) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_7, \epsilon)} \delta_\epsilon(p, a)) \\
&= Cl_\epsilon(\delta_\epsilon(q_7, a)) \\
&= Cl_\epsilon(\{q_9\}) \\
&= \{q_9, q_{10}\}
\end{aligned}$$

$$\begin{aligned}
\delta(q_7, b) &= \delta_\epsilon^*(q_7, b) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_7, \epsilon)} \delta_\epsilon(p, b)) \\
&= Cl_\epsilon(\delta_\epsilon(q_7, b)) \\
&= Cl_\epsilon(\emptyset) \\
&= \emptyset
\end{aligned}$$

$$\begin{aligned}
\delta(q_8, a) &= \delta_\epsilon^*(q_8, a) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_8, \epsilon)} \delta_\epsilon(p, a)) \\
&= Cl_\epsilon(\delta_\epsilon(q_8, a)) \\
&= Cl_\epsilon(\emptyset) \\
&= \emptyset
\end{aligned}$$

$$\begin{aligned}
\delta(q_8, b) &= \delta_\epsilon^*(q_8, b) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_8, \epsilon)} \delta_\epsilon(p, b)) \\
&= Cl_\epsilon(\delta_\epsilon(q_8, b)) \\
&= Cl_\epsilon(\{q_{10}\}) \\
&= \{q_{10}\}
\end{aligned}$$

$$\begin{aligned}
\delta(q_9, a) &= \delta_\epsilon^*(q_9, a) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_9, \epsilon)} \delta_\epsilon(p, a)) \\
&= Cl_\epsilon(\delta_\epsilon(q_9, a) \cup \delta_\epsilon(q_{10}, a)) \\
&= Cl_\epsilon(\emptyset \cup \emptyset) \\
&= \emptyset
\end{aligned}$$

$$\begin{aligned}
\delta(q_9, b) &= \delta_\epsilon^*(q_9, b) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_9, \epsilon)} \delta_\epsilon(p, b)) \\
&= Cl_\epsilon(\delta_\epsilon(q_9, b) \cup \delta_\epsilon(q_{10}, b)) \\
&= Cl_\epsilon(\emptyset \cup \emptyset) \\
&= \emptyset
\end{aligned}$$

$$\begin{aligned}
\delta(q_{10}, a) &= \delta_\epsilon^*(q_{10}, a) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_{10}, \epsilon)} \delta_\epsilon(p, a)) \\
&= Cl_\epsilon(\delta_\epsilon(q_{10}, a)) \\
&= Cl_\epsilon(\emptyset) \\
&= \emptyset
\end{aligned}$$

$$\begin{aligned}
\delta(q_{10}, b) &= \delta_\epsilon^*(q_{10}, b) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_{10}, \epsilon)} \delta_\epsilon(p, b)) \\
&= Cl_\epsilon(\delta_\epsilon(q_{10}, b)) \\
&= Cl_\epsilon(\emptyset) \\
&= \emptyset
\end{aligned}$$