Autómatas y Lenguajes formales
Ejercicio Semanal 6

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14 de marzo del 2019

1. Para cada ANF_{ϵ}, resuelve los siguientes incisos.
   (a) Calcula la ϵ-cerradura de cada estado.
   (b) Elimina las ϵ-transiciones obteniendo un AFN, mostrando el proceso de cálculo de las nuevas transiciones.

1. Autómata 1

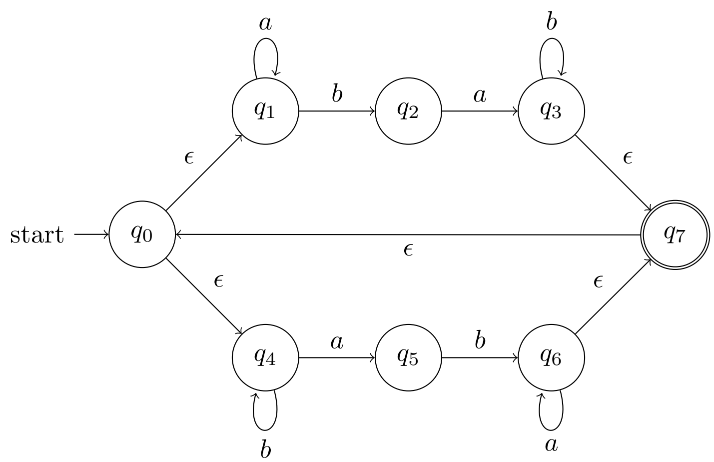


Figure 1: El autómata M

La función de transición δ definida con la siguiente tabla:

q	$\delta(q,a)$	$\delta(q,b)$	$\delta(q,\epsilon)$
0	$\varnothing$	$\varnothing$	$\{1,4\}$
1	$\{1\}$	$\{2\}$	$\varnothing$
2	$\{3\}$	$\varnothing$	$\varnothing$
3	$\varnothing$	$\{3\}$	$\{7\}$
4	$\{5\}$	$\{4\}$	$\varnothing$
5	$\varnothing$	$\{6\}$	$\varnothing$
6	$\{6\}$	$\varnothing$	$\{7\}$
7	$\varnothing$	$\varnothing$	$\{0\}$

(a) Vamos a calcular la $Cl_{\epsilon}(q)$ para todo $q \in Q$

$$
\begin{aligned}
Cl_\epsilon(q_0) &= \{q_0, q_1, q_4\} \\
Cl_\epsilon(q_1) &= \{q_1\} \\
Cl_\epsilon(q_2) &= \{q_2\} \\
Cl_\epsilon(q_3) &= \{q_3, q_7, q_0, q_1, q_4\} \\
Cl_\epsilon(q_4) &= \{q_4\} \\
Cl_\epsilon(q_5) &= \{q_5\} \\
Cl_\epsilon(q_6) &= \{q_6, q_7, q_0, q_4, q_1\} \\
Cl_\epsilon(q_7) &= \{q_7, q_0, q_4, q_1\}
\end{aligned}
$$

(b) Ya con la ϵ-cerradura de cada estado podemos definir δ^* para los símbolos del alfabeto:

$$
\begin{aligned}
\delta^*(q_0, a) &= \\
Cl_\epsilon(\cup_{p \in \delta^*(q_0,\epsilon)} \delta(p, a)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_0)} \delta(p, a)) &= \\
Cl_\epsilon(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_4, a)) &= \\
Cl_\epsilon(\varnothing \cup \{q_1\} \cup \{q_5\}) &= \\
Cl_\epsilon(\{q_1, q_5\}) &= \\
\{q_1, q_5\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_0, b) &= \\
Cl_\epsilon(\cup_{p \in \delta^*(q_0,\epsilon)} \delta(p, b)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_0)} \delta(p, b)) &= \\
Cl_\epsilon(\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_4, b)) &= \\
Cl_\epsilon(\varnothing \cup \{q_2\} \cup \{q_4\}) &= \\
Cl_\epsilon(\{q_2, q_4\}) &= \\
\{q_2, q_4\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_1, a) &= \\
Cl_\epsilon(\cup_{p \in \delta^*(q_1,\epsilon)} \delta(p, a)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_1)} \delta(p, a)) &= \\
Cl_\epsilon(\delta(q_1, a)) &= \\
Cl_\epsilon(\{q_1\}) &= \\
\{q_1\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_1, b) &= \\
Cl_\epsilon(\cup_{p \in \delta^*(q_1,\epsilon)} \delta(p, b)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_1)} \delta(p, b)) &= \\
Cl_\epsilon(\delta(q_1, b)) &= \\
Cl_\epsilon(\{q_2\}) &= \\
\{q_2\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_2, a) = \\
Cl_\epsilon(\cup_{p \in \delta^*(q_2, \epsilon)} \delta(p, a)) = \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_2)} \delta(p, a)) = \\
Cl_\epsilon(\delta(q_2, a)) = \\
Cl_\epsilon(\{q_3\}) = \\
\{q_3\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_2, b) = \\
Cl_\epsilon(\cup_{p \in \delta^*(q_2, \epsilon)} \delta(p, b)) = \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_2)} \delta(p, b)) = \\
Cl_\epsilon(\delta(q_2, b)) = \\
Cl_\epsilon(\varnothing) = \\
\varnothing
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_3, a) = \\
Cl_\epsilon(\cup_{p \in \delta^*(q_3, \epsilon)} \delta(p, a)) = \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_3)} \delta(p, a)) = \\
Cl_\epsilon(\delta(q_3, a) \cup \delta(q_7, a) \cup \delta(q_0, a) \cup \delta(q_1, a)) = \\
Cl_\epsilon(\varnothing \cup \varnothing \cup \varnothing \cup \{q_1\}) = \\
Cl_\epsilon(\{q_1\}) = \\
\{q_1\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_3, b) = \\
Cl_\epsilon(\cup_{p \in \delta^*(q_3, \epsilon)} \delta(p, b)) = \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_3)} \delta(p, b)) = \\
Cl_\epsilon(\delta(q_3, b) \cup \delta(q_7, b) \cup \delta(q_0, b) \cup \delta(q_1, b)) = \\
Cl_\epsilon(\varnothing \cup \varnothing \cup \varnothing \cup \{q_2\}) = \\
Cl_\epsilon(\{q_2\}) = \\
\{q_2\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_4, a) = \\
Cl_\epsilon(\cup_{p \in \delta^*(q_4, \epsilon)} \delta(p, a)) = \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_4)} \delta(p, a)) = \\
Cl_\epsilon(\delta(q_4, a)) = \\
Cl_\epsilon(\{q_5\}) = \\
\{q_5\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_4, b) = \\
Cl_\epsilon(\cup_{p \in \delta^*(q_4, \epsilon)} \delta(p, b)) = \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_4)} \delta(p, b)) = \\
Cl_\epsilon(\delta(q_4, b)) = \\
Cl_\epsilon(\{q_4\}) = \\
\{q_4\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_5, a) = \\
Cl_\epsilon(\cup_{p \in \delta^*(q_5,\epsilon)} \delta(p, a)) = \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_5)} \delta(p, a)) = \\
Cl_\epsilon(\delta(q_5, a)) = \\
Cl_\epsilon(\varnothing) = \varnothing
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_5, b) = \\
Cl_\epsilon(\cup_{p \in \delta^*(q_5,\epsilon)} \delta(p, b)) = \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_5)} \delta(p, b)) = \\
Cl_\epsilon(\delta(q_5, b)) = \\
Cl_\epsilon(\{q_6\}) = \\
\{q_6\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_6, a) = \\
Cl_\epsilon(\cup_{p \in \delta^*(q_6,\epsilon)} \delta(p, a)) = \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_6)} \delta(p, a)) = \\
Cl_\epsilon(\delta(q_6, a) \cup \delta(q_7, a) \cup \delta(q_0, a) \cup \delta(q_1, a)) = \\
Cl_\epsilon(\{q_6\} \cup \varnothing \cup \varnothing \cup \{q_1\}) = \\
Cl_\epsilon(\{q_6, q_1\}) = \\
\{q_6, q_7, q_0, q_1\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_6, b) = \\
Cl_\epsilon(\cup_{p \in \delta^*(q_6,\epsilon)} \delta(p, b)) = \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_6)} \delta(p, b)) = \\
Cl_\epsilon(\delta(q_6, b) \cup \delta(q_7, b) \cup \delta(q_0, b) \cup \delta(q_1, b)) = \\
Cl_\epsilon(\varnothing \cup \varnothing \cup \varnothing \cup \{q_2\}) = \\
Cl_\epsilon(\{q_2\}) = \\
\{q_2\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_7, a) = \\
Cl_\epsilon(\cup_{p \in \delta^*(q_7,\epsilon)} \delta(p, a)) = \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_7)} \delta(p, a)) = \\
Cl_\epsilon(\delta(q_7, a) \cup \delta(q_0, a) \cup \delta(q_1, a)) = \\
Cl_\epsilon(\varnothing \cup \varnothing \cup \{q_1\}) = \\
Cl_\epsilon(\{q_1\}) = \\
\{q_1\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_7, b) = \\
Cl_\epsilon(\cup_{p \in \delta^*(q_7,\epsilon)} \delta(p, b)) = \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_7)} \delta(p, b)) = \\
Cl_\epsilon(\delta(q_7, b) \cup \delta(q_0, b) \cup \delta(q_1, b)) = \\
Cl_\epsilon(\varnothing \cup \varnothing \cup \{q_2\}) = \\
Cl_\epsilon(\{q_2\}) = \\
\{q_2\}
\end{aligned}
$$

Vamos a construir un AFN M_n eliminando las ϵ-transiciones.

$M_N = < Q, \Sigma, \delta_N, q_0, F_n >$, donde:

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$
$\Sigma = \{a, b\}$
$\delta_N(q, \sigma) = \delta^*(q, \sigma)$, así que se define con la siguiente tabla:

q	$\delta_N(q,a)$	$\delta_N(q,b)$
0	$\{1,5\}$	$\{2,4\}$
1	$\{1\}$	$\{2\}$
2	$\{3\}$	$\varnothing$
3	$\{1\}$	$\{2\}$
4	$\{5\}$	$\{4\}$
5	$\varnothing$	$\{6\}$
6	$\{0,1,6,7\}$	$\{2\}$
7	$\{1\}$	$\{2\}$

$F_N = F$ pues $Cl_\epsilon(q_0) \cap F = \varnothing$.
Y visto de forma gráfica, AFN es:

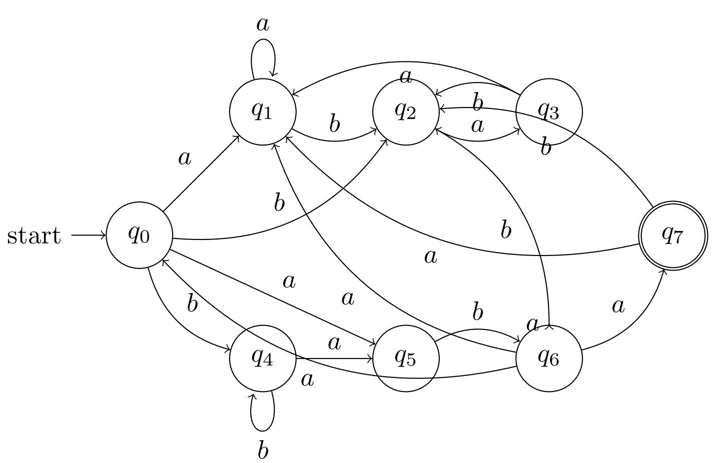
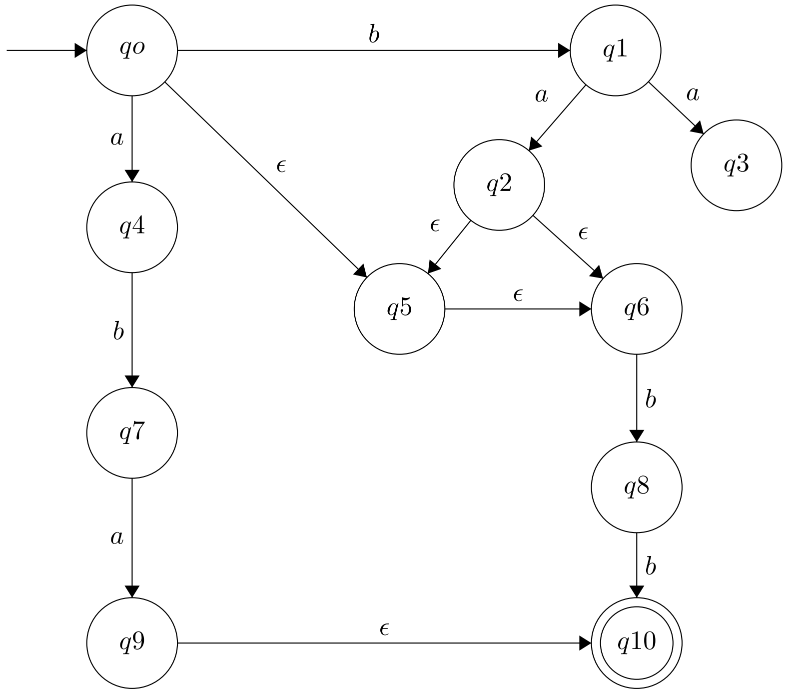


Figure 2: El autómata M_N

2. Autómata 2



(a) Calcula la ϵ-cerradura de cada estado

$$
\begin{aligned}
Cl_\epsilon(q_0) &= \{q_0, q_5, q_6\} & Cl_\epsilon(q_1) &= \{q_1\} \\
Cl_\epsilon(q_2) &= \{q_2, q_5, q_6\} & Cl_\epsilon(q_3) &= \{q_3\} \\
Cl_\epsilon(q_4) &= \{q_4\} & Cl_\epsilon(q_5) &= \{q_5, q_6\} \\
Cl_\epsilon(q_6) &= \{q_6\} & Cl_\epsilon(q_7) &= \{q_7\} \\
Cl_\epsilon(q_8) &= \{q_8\} & Cl_\epsilon(q_9) &= \{q_9, q_{10}\} \\
Cl_\epsilon(q_{10}) &= \{q_{10}\}
\end{aligned}
$$

(b) Elimina las ϵ-transiciones obteniendo un AFN, mostrando el proceso de cálculo de las nuevas transiciones.
Sea $M_\epsilon = \langle Q_\epsilon, \Sigma_\epsilon, \delta_\epsilon, q_{0\epsilon}, F_\epsilon \rangle$ el atómata de la figura.
El nuevo automata sería $M = \langle Q, \Sigma, \delta, q_0, F \rangle$ dado por

$$Q = Q_\epsilon,\ \Sigma = \Sigma_\epsilon,\ q_0 = q_{0\epsilon},\ F = F_\epsilon$$

Pues $F_\epsilon \cap Cl_\epsilon(q_0) = \varnothing$.
En cuento a la δ, esta se construye de la siguiente manera

$$
\begin{aligned}
\delta(q_0, a) &= \delta_\epsilon^*(q_0, a) \\
&= Cl_\epsilon\Big(\bigcup_{p \in \delta_\epsilon(q_0, \epsilon)} \delta_\epsilon(p, a)\Big) \\
&= Cl_\epsilon(\delta_\epsilon(q_0, a) \cup \delta_\epsilon(q_5, a) \cup \delta_\epsilon(q_6, a)) \\
&= Cl_\epsilon(\{q_4\} \cup \varnothing \cup \varnothing) \\
&= \{q_4\}
\end{aligned}
$$

$$
\begin{aligned}
\delta(q_0, b) &= \delta_\epsilon^*(q_0, b) \\
&= Cl_\epsilon\Big(\bigcup_{p \in \delta_\epsilon(q_0, \epsilon)} \delta_\epsilon(p, b)\Big) \\
&= Cl_\epsilon(\delta_\epsilon(q_0, b) \cup \delta_\epsilon(q_5, b) \cup \delta_\epsilon(q_6, b)) \\
&= Cl_\epsilon(\{q_1\} \cup \varnothing \cup \{q_8\}) \\
&= \{q_1, q_8\}
\end{aligned}
$$

$$
\begin{aligned}
\delta(q_1, a) &= \delta_\epsilon^*(q_1, a) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_1, \epsilon)} \delta_\epsilon(p, a)) \\
&= Cl_\epsilon(\delta_\epsilon(q_1, a)) \\
&= Cl_\epsilon(\{q_2, q_3\}) \\
&= \{q_2, q_3, q_5, q_6\}
\end{aligned}
$$

$$
\begin{aligned}
\delta(q_1, b) &= \delta_\epsilon^*(q_1, b) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_1, \epsilon)} \delta_\epsilon(p, b)) \\
&= Cl_\epsilon(\delta_\epsilon(q_1, b)) \\
&= Cl_\epsilon(\varnothing) \\
&= \varnothing
\end{aligned}
$$

$$
\begin{aligned}
\delta(q_2, a) &= \delta_\epsilon^*(q_2, a) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_2, \epsilon)} \delta_\epsilon(p, a)) \\
&= Cl_\epsilon(\delta_\epsilon(q_2, a) \cup \delta_\epsilon(q_5, a) \cup \delta_\epsilon(q_6, a)) \\
&= Cl_\epsilon(\varnothing \cup \varnothing \cup \varnothing) \\
&= \varnothing
\end{aligned}
$$

$$
\begin{aligned}
\delta(q_2, b) &= \delta_\epsilon^*(q_2, b) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_2, \epsilon)} \delta_\epsilon(p, b)) \\
&= Cl_\epsilon(\delta_\epsilon(q_2, b) \cup \delta_\epsilon(q_5, b) \cup \delta_\epsilon(q_6, b)) \\
&= Cl_\epsilon(\varnothing \cup \varnothing \cup \{q_8\}) \\
&= \{q_8\}
\end{aligned}
$$

$$
\begin{aligned}
\delta(q_3, a) &= \delta_\epsilon^*(q_3, a) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_3, \epsilon)} \delta_\epsilon(p, a)) \\
&= Cl_\epsilon(\delta_\epsilon(q_3, a)) \\
&= Cl_\epsilon(\varnothing) \\
&= \varnothing
\end{aligned}
$$

$$
\begin{aligned}
\delta(q_3, b) &= \delta_\epsilon^*(q_3, b) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_3, \epsilon)} \delta_\epsilon(p, b)) \\
&= Cl_\epsilon(\delta_\epsilon(q_3, b)) \\
&= Cl_\epsilon(\varnothing) \\
&= \varnothing
\end{aligned}
$$

$$
\begin{aligned}
\delta(q_4, a) &= \delta_\epsilon^*(q_4, a) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_4, \epsilon)} \delta_\epsilon(p, a)) \\
&= Cl_\epsilon(\delta_\epsilon(q_4, a) \cup \delta_\epsilon(q_5, a) \cup \delta_\epsilon(q_6, a)) \\
&= Cl_\epsilon(\{q_4\} \cup \varnothing \cup \varnothing) \\
&= \{q_4\}
\end{aligned}
$$

$$
\begin{aligned}
\delta(q_4, b) &= \delta_\epsilon^*(q_4, b) \\
&= Cl_\epsilon(\bigcup_{p \in \delta_\epsilon(q_4, \epsilon)} \delta_\epsilon(p, b)) \\
&= Cl_\epsilon(\delta_\epsilon(q_4, b) \cup \delta_\epsilon(q_5, b) \cup \delta_\epsilon(q_6, b)) \\
&= Cl_\epsilon(\{q_1\} \cup \varnothing \cup \{q_8\}) \\
&= \{q_1, q_8\}
\end{aligned}
$$