Autómatas y Lenguajes formales
Ejercicio Semanal 6

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1. Para cada ANF_ϵ, resuelve los siguientes incisos.
   (a) Calcula la ϵ-cerradura de cada estado.
   (b) Elimina las ϵ-transiciones obteniendo un AFN, mostrando el proceso de cálculo de las nuevas transiciones.

1. Autómata 1

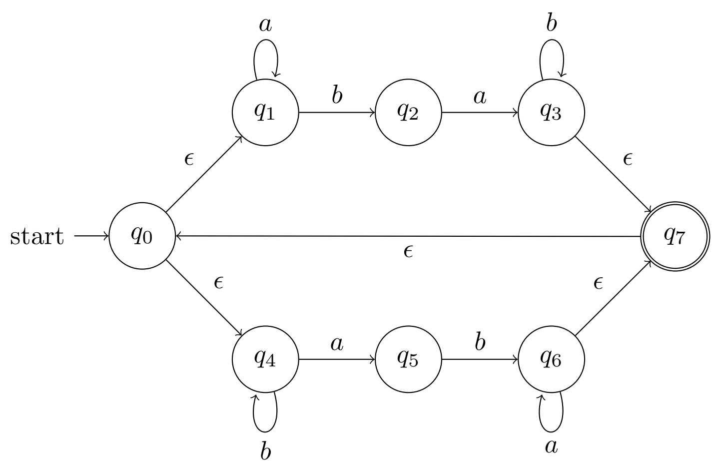


Figure 1: El autómata M

La función de transición δ definida con la siguiente tabla:

q	$\delta(q,a)$	$\delta(q,b)$	$\delta(q,\epsilon)$
0	$\varnothing$	$\varnothing$	$\{1,4\}$
1	$\{1\}$	$\{2\}$	$\varnothing$
2	$\{3\}$	$\varnothing$	$\varnothing$
3	$\varnothing$	$\{3\}$	$\{7\}$
4	$\{5\}$	$\{4\}$	$\varnothing$
5	$\varnothing$	$\{6\}$	$\varnothing$
6	$\{6\}$	$\varnothing$	$\{7\}$
7	$\varnothing$	$\varnothing$	$\{0\}$

(a) Vamos a calcular la $Cl_\epsilon(q)$ para todo $q \in Q$

$$
\begin{aligned}
Cl_\epsilon(q_0) &= \{q_0, q_1, q_4\} \\
Cl_\epsilon(q_1) &= \{q_1\} \\
Cl_\epsilon(q_2) &= \{q_2\} \\
Cl_\epsilon(q_3) &= \{q_3, q_7, q_0, q_1, q_4\} \\
Cl_\epsilon(q_4) &= \{q_4\} \\
Cl_\epsilon(q_5) &= \{q_5\} \\
Cl_\epsilon(q_6) &= \{q_6, q_7, q_0, q_4, q_1\} \\
Cl_\epsilon(q_7) &= \{q_7, q_0, q_4, q_1\}
\end{aligned}
$$

(b) Ya con la ϵ-cerradura de cada estado podemos definir δ^* para los símbolos del alfabeto:

$$
\begin{aligned}
\delta^*(q_0, a) &= Cl_\epsilon(\cup_{p\in\delta^*(q_0,\epsilon)}\delta(p, a)) &&= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_0)}\delta(p, a)) &= Cl_\epsilon(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_4, a)) &&= \\
Cl_\epsilon(\varnothing \cup \{q_1\} \cup \{q_5\}) &= Cl_\epsilon(\{q_1, q_5\}) &&= \\
\{q_1, q_5\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_0, b) &= Cl_\epsilon(\cup_{p\in\delta^*(q_0,\epsilon)}\delta(p, b)) &&= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_0)}\delta(p, b)) &= Cl_\epsilon(\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_4, b)) &&= \\
Cl_\epsilon(\varnothing \cup \{q_2\} \cup \{q_4\}) &= Cl_\epsilon(\{q_2, q_4\}) &&= \\
\{q_2, q_4\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_1, a) &= Cl_\epsilon(\cup_{p\in\delta^*(q_1,\epsilon)}\delta(p, a)) &&= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_1)}\delta(p, a)) &= Cl_\epsilon(\delta(q_1, a)) &&= \\
Cl_\epsilon(\{q_1\}) &= \{q_1\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_1, b) &= Cl_\epsilon(\cup_{p\in\delta^*(q_1,\epsilon)}\delta(p, b)) &&= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_1)}\delta(p, b)) &= Cl_\epsilon(\delta(q_1, b)) &&= \\
Cl_\epsilon(\{q_2\}) &= \{q_2\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_2, a) &= Cl_\epsilon(\cup_{p\in\delta^*(q_2,\epsilon)}\delta(p, a)) &&= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_2)}\delta(p, a)) &= Cl_\epsilon(\delta(q_2, a)) &&= \\
Cl_\epsilon(\{q_3\}) &= \{q_3\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_2, b) &= Cl_\epsilon(\cup_{p\in\delta^*(q_2,\epsilon)}\delta(p, b)) &&= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_2)}\delta(p, b)) &= Cl_\epsilon(\delta(q_2, b)) &&= \\
Cl_\epsilon(\varnothing) &= \varnothing
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_3, a) &= Cl_\epsilon(\cup_{p\in\delta^*(q_3,\epsilon)}\delta(p, a)) &&= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_3)}\delta(p, a)) &= Cl_\epsilon(\delta(q_3, a) \cup \delta(q_7, a) \cup \delta(q_0, a) \cup \delta(q_1, a)) &&= \\
Cl_\epsilon(\varnothing \cup \varnothing \cup \varnothing \cup \{q_1\}) &= Cl_\epsilon(\{q_1\}) &&= \\
\{q_1\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_3, b) &= Cl_\epsilon(\cup_{p\in\delta^*(q_3,\epsilon)}\delta(p,b)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_3)}\delta(p,b)) &= Cl_\epsilon(\delta(q_3,b)\cup\delta(q_7,b)\cup\delta(q_0,b)\cup\delta(q_1,b)) &= \\
Cl_\epsilon(\varnothing\cup\varnothing\cup\varnothing\cup\{q_2\}) &= Cl_\epsilon(\{q_2\}) &= \\
\{q_2\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_4, a) &= Cl_\epsilon(\cup_{p\in\delta^*(q_4,\epsilon)}\delta(p,a)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_4)}\delta(p,a)) &= Cl_\epsilon(\delta(q_4,a)) &= \\
Cl_\epsilon(\{q_5\}) &= \{q_5\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_4, b) &= Cl_\epsilon(\cup_{p\in\delta^*(q_4,\epsilon)}\delta(p,b)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_4)}\delta(p,b)) &= Cl_\epsilon(\delta(q_4,b)) &= \\
Cl_\epsilon(\{q_4\}) &= \{q_4\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_5, a) &= Cl_\epsilon(\cup_{p\in\delta^*(q_5,\epsilon)}\delta(p,a)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_5)}\delta(p,a)) &= Cl_\epsilon(\delta(q_5,a)) &= \\
Cl_\epsilon(\varnothing) &= \varnothing
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_5, b) &= Cl_\epsilon(\cup_{p\in\delta^*(q_5,\epsilon)}\delta(p,b)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_5)}\delta(p,b)) &= Cl_\epsilon(\delta(q_5,b)) &= \\
Cl_\epsilon(\{q_6\}) &= \{q_6\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_6, a) &= Cl_\epsilon(\cup_{p\in\delta^*(q_6,\epsilon)}\delta(p,a)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_6)}\delta(p,a)) &= Cl_\epsilon(\delta(q_6,a)\cup\delta(q_7,a)\cup\delta(q_0,a)\cup\delta(q_1,a)) &= \\
Cl_\epsilon(\{q_6\}\cup\varnothing\cup\varnothing\cup\{q_1\}) &= Cl_\epsilon(\{q_6,q_1\}) &= \\
\{q_6,q_7,q_0,q_1\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_6, b) &= Cl_\epsilon(\cup_{p\in\delta^*(q_6,\epsilon)}\delta(p,b)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_6)}\delta(p,b)) &= Cl_\epsilon(\delta(q_6,b)\cup\delta(q_7,b)\cup\delta(q_0,b)\cup\delta(q_1,b)) &= \\
Cl_\epsilon(\varnothing\cup\varnothing\cup\varnothing\cup\{q_2\}) &= Cl_\epsilon(\{q_2\}) &= \\
\{q_2\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_7, a) &= Cl_\epsilon(\cup_{p\in\delta^*(q_7,\epsilon)}\delta(p,a)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_7)}\delta(p,a)) &= Cl_\epsilon(\delta(q_7,a)\cup\delta(q_0,a)\cup\delta(q_1,a)) &= \\
Cl_\epsilon(\varnothing\cup\varnothing\cup\{q_1\}) &= Cl_\epsilon(\{q_1\}) &= \\
\{q_1\}
\end{aligned}
$$

$$
\begin{aligned}
\delta^*(q_7, b) &= Cl_\epsilon(\cup_{p\in\delta^*(q_7,\epsilon)}\delta(p,b)) &= \\
Cl_\epsilon(\cup_{Cl_\epsilon(q_7)}\delta(p,b)) &= Cl_\epsilon(\delta(q_7,b)\cup\delta(q_0,b)\cup\delta(q_1,b)) &= \\
Cl_\epsilon(\varnothing\cup\varnothing\cup\{q_2\}) &= Cl_\epsilon(\{q_2\}) &= \\
\{q_2\}
\end{aligned}
$$

Vamos a construir un AFN M_n eliminando las ϵ-transiciones.

$M_N =< Q, \Sigma, \delta_N, q_0, F_n >$, donde:

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$
$\Sigma = \{a, b\}$
$\delta_N(q, \sigma) = \delta^*(q, \sigma)$, así que se define con la siguiente tabla:

q	$\delta_N(q,a)$	$\delta_N(q,b)$
0	$\{1,5\}$	$\{2,4\}$
1	$\{1\}$	$\{2\}$
2	$\{3\}$	$\varnothing$
3	$\{1\}$	$\{2\}$
4	$\{5\}$	$\{4\}$
5	$\varnothing$	$\{6\}$
6	$\{0,1,6,7\}$	$\{2\}$
7	$\{1\}$	$\{2\}$

$F_N = F$ pues $Cl_\epsilon(q_0) \cap F = \varnothing$.
Y visto de forma gráfica, AFN es:

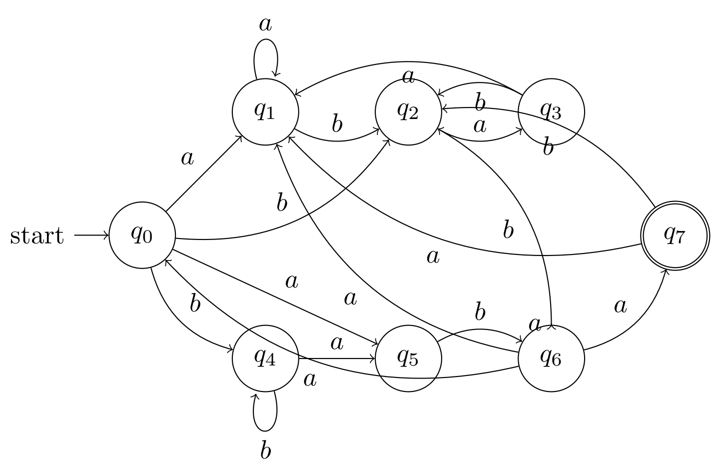
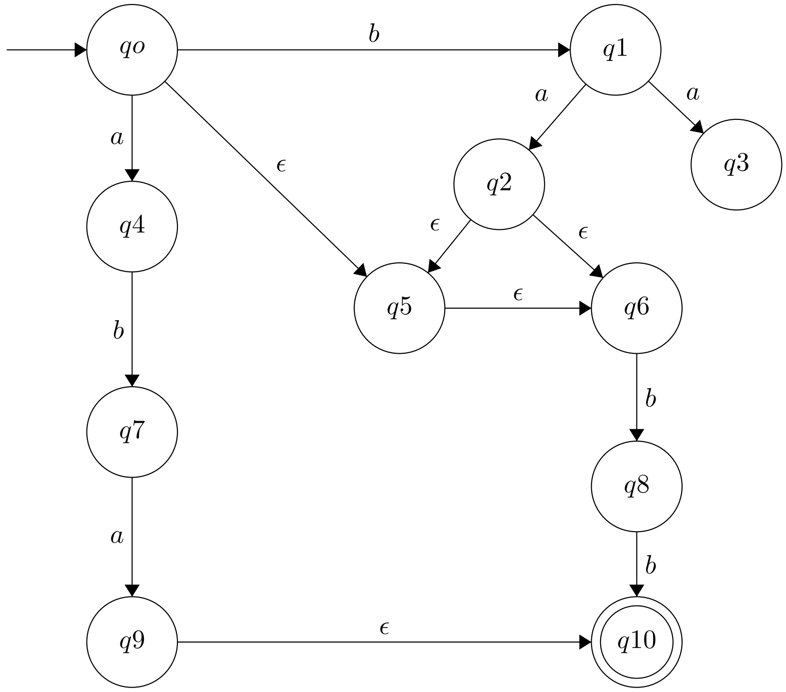


Figure 2: El autómata M_N

2. Autómata 2



qo
b
$q1$
a
a
a
ϵ
$q2$
$q3$
$q4$
ϵ
ϵ
$q5$
ϵ
$q6$
b
b
$q7$
$q8$
a
b
$q9$
ϵ
$q10$