Autómatas y Lenguajes formales Ejercicio Semanal 6

Sandra del Mar Soto Corderi Edgar Quiroz Castañeda

14 de marzo del 2019

- 1. Para cada $ANF_{\epsilon},$ resuelve los siguientes incisos.
 - (a) Calcula la ϵ -cerradura de cada estado.
 - (b) Elimina las ϵ -transiciones obteniendo un AFN, mostrando el proceso de cálculo de las nuevas transiciones.

1. Autómata 1

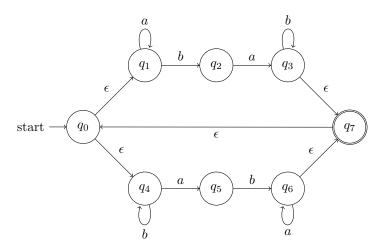


Figure 1: El autómata M

La función de transición δ definida con la siguiente tabla:

q	$\delta(q,a)$	$\delta(q,b)$	$\delta(q,\epsilon)$
0	Ø	Ø	$\{1,4\}$
1	{1}	{2}	Ø
2	{3}	Ø	Ø
3	Ø	{3}	{7}
4	$\{5\}$	{4}	Ø
5	Ø	{6}	Ø
6	$\{6\}$	Ø	{7}
7	Ø	Ø	{0}

(a) Vamos a calcular la $Cl_{\epsilon}(q)$ para todo $q \in Q$

$$Cl_{\epsilon}(q_0) = \{q_0, q_1, q_4\}$$

$$Cl_{\epsilon}(q_1) = \{q_1\}$$

$$Cl_{\epsilon}(q_2) = \{q_2\}$$

$$Cl_{\epsilon}(q_3) = \{q_3, q_7, q_0, q_1, q_4\}$$

$$Cl_{\epsilon}(q_4) = \{q_4\}$$

$$Cl_{\epsilon}(q_5) = \{q_5\}$$

$$Cl_{\epsilon}(q_6) = \{q_6, q_7, q_0, q_4, q_1\}$$

$$Cl_{\epsilon}(q_7) = \{q_7, q_0, q_4, q_1\}$$

(b) Ya con la ϵ -cerradura de cada estado podemos definir δ^* para los símbolos del alfabeto:

$$\delta^*(q_0, a) =$$

$$Cl_{\epsilon}(\cup_{p \in \delta^*(q_0, \epsilon)} \delta(p, a)) =$$

$$Cl_{\epsilon}(\cup_{Cl_{\epsilon}(q_0)} \delta(p, a)) =$$

$$Cl_{\epsilon}(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_4, a)) =$$

$$Cl_{\epsilon}(\emptyset \cup \{q_1\} \cup \{q_5\}) =$$

$$Cl_{\epsilon}(\{q_1, q_5\}) =$$

$$\{q_1, q_5\}$$

$$\delta^*(q_0, b) =$$

$$Cl_{\epsilon}(\cup_{p \in \delta^*(q_0, \epsilon)} \delta(p, b)) =$$

$$Cl_{\epsilon}(\cup_{Cl_{\epsilon}(q_0)} \delta(p, b)) =$$

$$Cl_{\epsilon}(\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_4, b)) =$$

$$Cl_{\epsilon}(\emptyset \cup \{q_2\} \cup \{q_4\}) =$$

$$Cl_{\epsilon}(\{q_2, q_4\}) =$$

$$\{q_2, q_4\}$$

$$\delta^*(q_1, a) =$$

$$Cl_{\epsilon}(\bigcup_{p \in \delta^*(q_1, \epsilon)} \delta(p, a)) =$$

$$Cl_{\epsilon}(\bigcup_{Cl_{\epsilon}(q_1)} \delta(p, a)) =$$

$$Cl_{\epsilon}(\delta(q_1, a)) =$$

$$Cl_{\epsilon}(\{q_1\}) =$$

$$\{q_1\}$$

$$\delta^*(q_1, b) =$$

$$Cl_{\epsilon}(\bigcup_{p \in \delta^*(q_1, \epsilon)} \delta(p, b)) =$$

$$Cl_{\epsilon}(\bigcup_{Cl_{\epsilon}(q_1)} \delta(p, b)) =$$

$$Cl_{\epsilon}(\delta(q_1, b)) =$$

$$Cl_{\epsilon}(\{q_2\}) =$$

$$\{q_2\}$$

$$\delta^*(q_2, a) =$$

$$Cl_{\epsilon}(\cup_{p \in \delta^*(q_2, \epsilon)} \delta(p, a)) =$$

$$Cl_{\epsilon}(\cup_{Cl_{\epsilon}(q_2)} \delta(p, a)) =$$

$$Cl_{\epsilon}(\delta(q_2, a)) =$$

$$Cl_{\epsilon}(\{q_3\}) =$$

$$\{q_3\}$$

$$\delta^*(q_2, b) =$$

$$Cl_{\epsilon}(\cup_{p \in \delta^*(q_2, \epsilon)} \delta(p, b)) =$$

$$Cl_{\epsilon}(\cup_{cl_{\epsilon}(q_2)} \delta(p, b)) =$$

$$Cl_{\epsilon}(\delta(q_2, b)) =$$

$$Cl_{\epsilon}(\delta(q_2, b)) =$$

$$Cl_{\epsilon}(\delta(q_2, b)) =$$

$$Cl_{\epsilon}(\delta(q_3, a)) =$$

$$Cl_{\epsilon}((\cup_{cl_{\epsilon}(q_3)} \delta(p, b)) =$$

$$Cl_{\epsilon}((\cup_{cl_{\epsilon}(q_3)} \delta(p, a)) =$$

$$Cl_{\epsilon}((\cup_{cl_{\epsilon}(q_3)} \delta(p, a)) =$$

$$Cl_{\epsilon}((\cup_{cl_{\epsilon}(q_3)} \delta(p, a)) =$$

$$\delta^*(q_4, a) =$$

$$Cl_{\epsilon}(\bigcup_{p \in \delta^*(q_4, \epsilon)} \delta(p, a)) =$$

$$Cl_{\epsilon}(\bigcup_{Cl_{\epsilon}(q_4)} \delta(p, a)) =$$

$$Cl_{\epsilon}(\delta(q_4, a)) =$$

$$Cl_{\epsilon}(\{q_5\}) =$$

$$\{q_5\}$$

$$\delta^*(q_4, b) =$$

$$Cl_{\epsilon}(\cup_{p \in \delta^*(q_4, \epsilon)} \delta(p, b)) =$$

$$Cl_{\epsilon}(\cup_{cl_{\epsilon}(q_4)} \delta(p, b)) =$$

$$Cl_{\epsilon}(\delta(q_4, b)) =$$

$$Cl_{\epsilon}(\{q_4\}) =$$

$$\{q_4\}$$

$$\delta^*(q_5,a) = \\ Cl_{\epsilon}(\cup_{p \in \delta^*(q_5,\epsilon)}\delta(p,a)) = \\ Cl_{\epsilon}(\cup_{Cl_{\epsilon}(q_5)}\delta(p,a)) = \\ Cl_{\epsilon}(\delta(q_5,a)) = \\ Cl_{\epsilon}(\delta(q_5,a)) = \\ Cl_{\epsilon}(\emptyset) = \emptyset \\ \\ \delta^*(q_5,b) = \\ Cl_{\epsilon}(\cup_{p \in \delta^*(q_5,\epsilon)}\delta(p,b)) = \\ Cl_{\epsilon}(\cup_{Cl_{\epsilon}(q_5)}\delta(p,b)) = \\ Cl_{\epsilon}(\delta(q_5,b)) = \\ Cl_{\epsilon}(\delta(q_5,b)) = \\ Cl_{\epsilon}(\{q_6\}) = \\ \{q_6\} \\ \\ Cl_{\epsilon}(\{q_6\}) = \\ \{q_6\} \\ \\ Cl_{\epsilon}(\{q_6,a) \cup \delta(q_7,a) \cup \delta(q_0,a) \cup \delta(q_1,a)) = \\ Cl_{\epsilon}(\{q_6,q_1\}) = \\ Cl_{\epsilon}(\{q_6,q_1\}) = \\ Cl_{\epsilon}(\{q_6,q_7,q_0,q_1\}) \\ \\ \delta^*(q_6,b) = \\ Cl_{\epsilon}(\{q_6,b) \cup \delta(q_7,b) \cup \delta(q_0,b) \cup \delta(q_1,b)) = \\ Cl_{\epsilon}(\{q_6\}) \cup \delta(q_7,b) \cup \delta(q_0,b) \cup \delta(q_1,b) = \\ Cl_{\epsilon}(\{q_2\}) = \\ Cl_{\epsilon}(\{q_1\}) = \\ Cl_{\epsilon}(\{q_2\}) = \\ Cl_{\epsilon}(\{q_$$

4

 $\{q_2\}$

Vamos a construir un AFN M_n eliminando las $\epsilon\text{-transiciones}.$

 $M_N = \langle Q, \Sigma, \delta_N, q_0, F_n \rangle$, donde:

$$\begin{split} Q &= \{q_0,q_1,q_2,q_3,q_4,q_5,q_6,q_7\} \\ \Sigma &= \{a,b\} \\ \delta_N(q,\sigma) &= \delta^*(q,\sigma), \text{ así que se define con la siguiente tabla:} \end{split}$$

q	$\delta_N(q,a)$	$\delta_N(q,b)$
0	$\{1, 5\}$	$\{2,4\}$
1	{1}	{2}
2	{3}	Ø
3	{1}	{2}
4	{5}	{4}
5	Ø	{6}
6	$\{0, 1, 6, 7\}$	{2}
7	{1}	{2}

 $F_N = F$ pues $Cl_{\epsilon}(q_0) \cap F = \emptyset$. Y visto de forma gráfica, AFN es:

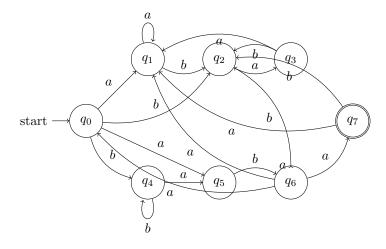


Figure 2: El autómata M_N

2. Autómata 2

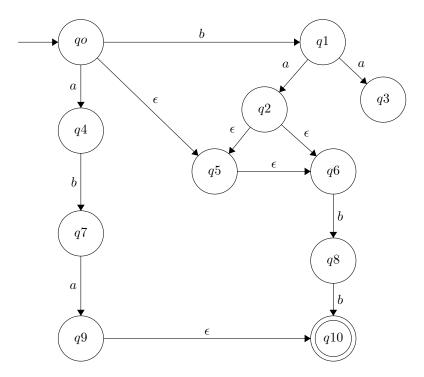


Figure 3: Autómata M

La función de transición δ definida con la siguiente tabla:

q	$\delta_N(q,a)$	$\delta_N(q,b)$	$\delta_N(q,\epsilon)$
0	{4}	{1}	$\{5,6\}$
1	$\{2, 3\}$	Ø	Ø
2	Ø	Ø	$\{5, 6\}$
3	Ø	Ø	Ø
4	Ø	{7}	Ø
5	Ø	Ø	{6}
6	Ø	{8}	Ø
7	{9}	Ø	Ø
8	Ø	{10}	Ø
9	Ø	Ø	{10}
10	Ø	Ø	Ø

(a) Calcula la ϵ -cerradura de cada estado

$$Cl_{\epsilon}(q_0) = \{q_0, q_5, q_6\}$$

$$Cl_{\epsilon}(q_1) = \{q_1\}$$

$$Cl_{\epsilon}(q_2) = \{q_2, q_5, q_6\}$$

$$Cl_{\epsilon}(q_3) = \{q_3\}$$

$$Cl_{\epsilon}(q_4) = \{q_4\}$$

$$Cl_{\epsilon}(q_6) = \{q_6\}$$

$$Cl_{\epsilon}(q_8) = \{q_8\}$$

$$Cl_{\epsilon}(q_9) = \{q_9, q_{10}\}$$

$$Cl_{\epsilon}(q_{10}) = \{q_{10}\}$$

(b) Elimina las ϵ -transiciones obteniendo un AFN, mostrando el proceso de cálculo de las nuevas transiciones. Sea $M_{\epsilon} = \langle Q_{\epsilon}, \Sigma_{\epsilon}, \delta_{\epsilon}, q_{0\epsilon}, F_{\epsilon} \rangle$ el autómata de la figura. El nuevo autómata sería $M = \langle Q, \Sigma, \delta, q_0, F \rangle$ dado por

$$Q = Q_{\epsilon}, \ \Sigma = \Sigma_{\epsilon}, \ q_0 = q_{0\epsilon}, \ F = F_{\epsilon}$$

Pues $F_{\epsilon} \cap Cl_{\epsilon}(q_0) = \emptyset$.

En cuento a la δ , esta se construye de la siguiente manera

$$\begin{split} \delta(q_0, a) &= \delta_{\epsilon}^*(q_0, a) \\ &= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_0, \epsilon)} \delta_{\epsilon}(p, a)) \\ &= Cl_{\epsilon}(\delta_{\epsilon}(q_0, a) \cup \delta_{\epsilon}(q_5, a) \cup \delta_{\epsilon}(q_6, a)) \\ &= Cl_{\epsilon}(\{q_4\} \cup \varnothing \cup \varnothing) \\ &= \{q_4\} \end{split}$$

$$\begin{split} \delta(q_0,b) &= \delta_{\epsilon}^*(q_0,b) \\ &= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_0,\epsilon)} \delta_{\epsilon}(p,b)) \\ &= Cl_{\epsilon}(\delta_{\epsilon}(q_0,b) \cup \delta_{\epsilon}(q_5,b) \cup \delta_{\epsilon}(q_6,b)) \\ &= Cl_{\epsilon}(\{q_1\} \cup \varnothing \cup \{q_8\}) \\ &= \{q_1,q_8\} \end{split}$$

$$\delta(q_1, a) = \delta_{\epsilon}^*(q_1, a)$$

$$= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_1, \epsilon)} \delta_{\epsilon}(p, a))$$

$$= Cl_{\epsilon}(\delta_{\epsilon}(q_1, a))$$

$$= Cl_{\epsilon}(\{q_2, q_3\})$$

$$= \{q_2, q_3, q_5, q_6\}$$

$$\delta(q_1, b) = \delta_{\epsilon}^*(q_1, b)$$

$$= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_1, \epsilon)} \delta_{\epsilon}(p, b))$$

$$= Cl_{\epsilon}(\delta_{\epsilon}(q_1, b))$$

$$= Cl_{\epsilon}(\varnothing)$$

$$= \varnothing$$

$$\begin{split} \delta(q_2, a) &= \delta_{\epsilon}^*(q_2, a) \\ &= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_2, \epsilon)} \delta_{\epsilon}(p, a)) \\ &= Cl_{\epsilon}(\delta_{\epsilon}(q_2, a) \cup \delta_{\epsilon}(q_5, a) \cup \delta_{\epsilon}(q_6, a)) \\ &= Cl_{\epsilon}(\varnothing \cup \varnothing \cup \varnothing) \\ &= \varnothing \end{split}$$

$$\begin{split} \delta(q_2,b) &= \delta_{\epsilon}^*(q_2,b) \\ &= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_2,\epsilon)} \delta_{\epsilon}(p,b)) \\ &= Cl_{\epsilon}(\delta_{\epsilon}(q_2,b) \cup \delta_{\epsilon}(q_5,b) \cup \delta_{\epsilon}(q_6,b)) \\ &= Cl_{\epsilon}(\varnothing \cup \varnothing \cup \{q_8\}) \\ &= \{q_8\} \end{split}$$

$$\begin{split} \delta(q_3, a) &= \delta_{\epsilon}^*(q_3, a) \\ &= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_3, \epsilon)} \delta_{\epsilon}(p, a)) \\ &= Cl_{\epsilon}(\delta_{\epsilon}(q_3, a)) \\ &= Cl_{\epsilon}(\varnothing) \\ &= \varnothing \end{split}$$

$$\begin{split} \delta(q_3,b) &= \delta_{\epsilon}^*(q_3,b) \\ &= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_3,\epsilon)} \delta_{\epsilon}(p,b)) \\ &= Cl_{\epsilon}(\delta_{\epsilon}(q_3,b)) \\ &= Cl_{\epsilon}(\varnothing) \\ &= \varnothing \end{split}$$

$$\delta(q_4, a) = \delta_{\epsilon}^*(q_4, a)$$

$$= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_4, \epsilon)} \delta_{\epsilon}(p, a))$$

$$= Cl_{\epsilon}(\delta_{\epsilon}(q_4, a))$$

$$= Cl_{\epsilon}(\varnothing)$$

$$= \varnothing$$

$$\delta(q_4, b) = \delta_{\epsilon}^*(q_4, b)$$

$$= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_4, \epsilon)} \delta_{\epsilon}(p, b))$$

$$= Cl_{\epsilon}(\delta_{\epsilon}(q_4, b))$$

$$= Cl_{\epsilon}(\{q_7\})$$

$$= \{q_7\}$$

$$\delta(q_5, a) = \delta_{\epsilon}^*(q_5, a)$$

$$= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_5, \epsilon)} \delta_{\epsilon}(p, a))$$

$$= Cl_{\epsilon}(\delta_{\epsilon}(q_5, a) \cup \delta_{\epsilon}(q_6, a))$$

$$= Cl_{\epsilon}(\varnothing \cup \varnothing)$$

$$= \varnothing$$

$$\begin{split} \delta(q_5,b) &= \delta_{\epsilon}^*(q_5,b) \\ &= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_5,\epsilon)} \delta_{\epsilon}(p,b)) \\ &= Cl_{\epsilon}(\delta_{\epsilon}(q_5,b) \cup \delta_{\epsilon}(q_6,b)) \\ &= Cl_{\epsilon}(\varnothing \cup \{q_8\}) \\ &= \{q_8\} \end{split}$$

$$\begin{split} \delta(q_6, a) &= \delta_{\epsilon}^*(q_6, a) \\ &= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_6, \epsilon)} \delta_{\epsilon}(p, a)) \\ &= Cl_{\epsilon}(\delta_{\epsilon}(q_6, a)) \\ &= Cl_{\epsilon}(\varnothing) \\ &= \varnothing \end{split}$$

$$\begin{split} \delta(q_6,b) &= \delta_{\epsilon}^*(q_6,b) \\ &= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_6,\epsilon)} \delta_{\epsilon}(p,b)) \\ &= Cl_{\epsilon}(\delta_{\epsilon}(q_6,b)) \\ &= Cl_{\epsilon}(\{q_8\}) \\ &= \{q_8\} \end{split}$$

$$\delta(q_7, a) = \delta_{\epsilon}^*(q_7, a)$$

$$= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_7, \epsilon)} \delta_{\epsilon}(p, a))$$

$$= Cl_{\epsilon}(\delta_{\epsilon}(q_7, a))$$

$$= Cl_{\epsilon}(\{q_9\})$$

$$= \{q_9, q_{10}\}$$

$$\delta(q_7, b) = \delta_{\epsilon}^*(q_7, b)$$

$$= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_7, \epsilon)} \delta_{\epsilon}(p, b))$$

$$= Cl_{\epsilon}(\delta_{\epsilon}(q_7, b))$$

$$= Cl_{\epsilon}(\varnothing)$$

$$= \varnothing)$$

$$\delta(q_8, a) = \delta_{\epsilon}^*(q_8, a)$$

$$= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_8, \epsilon)} \delta_{\epsilon}(p, a))$$

$$= Cl_{\epsilon}(\delta_{\epsilon}(q_8, a))$$

$$= Cl_{\epsilon}(\varnothing)$$

$$= \varnothing$$

$$\delta(q_8, b) = \delta_{\epsilon}^*(q_8, b)$$

$$= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_8, \epsilon)} \delta_{\epsilon}(p, b))$$

$$= Cl_{\epsilon}(\delta_{\epsilon}(q_8, b))$$

$$= Cl_{\epsilon}(\{q_{10}\})$$

$$= \{q_{10}\}$$

$$\begin{split} \delta(q_9, a) &= \delta_{\epsilon}^*(q_9, a) \\ &= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_9, \epsilon)} \delta_{\epsilon}(p, a)) \\ &= Cl_{\epsilon}(\delta_{\epsilon}(q_9, a) \cup \delta_{\epsilon}(q_{10}, a)) \\ &= Cl_{\epsilon}(\varnothing \cup \varnothing) \\ &= \varnothing \end{split}$$

$$\begin{split} \delta(q_9,b) &= \delta_{\epsilon}^*(q_9,b) \\ &= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_9,\epsilon)} \delta_{\epsilon}(p,b)) \\ &= Cl_{\epsilon}(\delta_{\epsilon}(q_9,b) \cup \delta_{\epsilon}(q_{10},b)) \\ &= Cl_{\epsilon}(\varnothing \cup \varnothing) \\ &= \varnothing \end{split}$$

$$\delta(q_{10}, a) = \delta_{\epsilon}^{*}(q_{10}, a)$$

$$= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_{10}, \epsilon)} \delta_{\epsilon}(p, a))$$

$$= Cl_{\epsilon}(\delta_{\epsilon}(q_{10}, a))$$

$$= Cl_{\epsilon}(\varnothing)$$

$$= \varnothing$$

$$\begin{split} \delta(q_{10},b) &= \delta_{\epsilon}^*(q_{10},b) \\ &= Cl_{\epsilon}(\bigcup_{p \in \delta_{\epsilon}(q_{10},\epsilon)} \delta_{\epsilon}(p,b)) \\ &= Cl_{\epsilon}(\delta_{\epsilon}(q_{10},b)) \\ &= Cl_{\epsilon}(\varnothing) \\ &= \varnothing \end{split}$$

Podemos resumirlo en la siguiente tabla:

q	$\delta_N(q,a)$	$\delta_N(q,b)$
0	$\{4\}$	$\{1, 8\}$
1	$\{2, 3, 5, 6\}$	Ø
2	Ø	{8}
3	Ø	Ø
4	Ø	{7}
5	Ø	{8}
6	Ø	{8}
7	{9, 10}	Ø
8	Ø	{10}
9	Ø	Ø
10	Ø	Ø

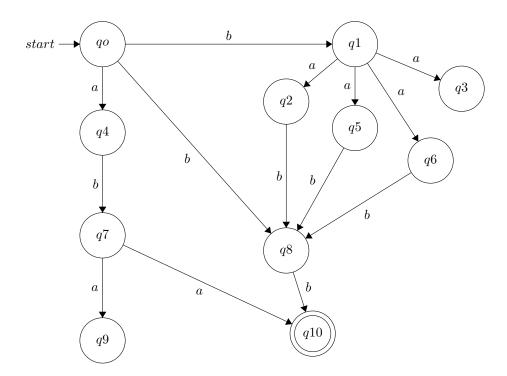


Figure 4: El autómata ${\cal M}_N$