Autómatas y Lenguajes formales 2019-2

Ejercicio Semanal 4

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1. Demuestra que el operador de derivada preserva equivalencias, es decir si $\alpha = \beta$, entonces $\partial_a\alpha = \partial_a\beta$.
   Tenemos que $\alpha = \beta \iff \mathcal{L}[\![\alpha]\!] = \mathcal{L}[\![\beta]\!]$, por la defినción de equivalencia en expresiones regulares.
   Por lo que

$$\begin{aligned}
\partial_a\alpha &= \{v | av \in \mathcal{L}[\![\alpha]\!]\}\\
&= \{v | av \in \mathcal{L}[\![\beta]\!]\}\\
&= \partial_a\beta
\end{aligned}$$

2. Calcula la derivada de las expresiones regulares en cada inciso.

a) $\partial bb(a^* \ + \ (a^*ba^*ba^*)^*)$

$$\begin{aligned}
&= \partial b(\partial b(a^* \ + \ (a^*ba^*ba^*)^*))\\
&= \partial b(\partial b(a^*) \ + \ \partial b((a^*ba^*ba^*)^*))\\
&= \partial b(\partial b(a)a^* \ + \ \partial b(a^*ba^*ba^*)((a^*ba^*ba^*)^*))\\
&= \partial b(\emptyset a^* \ + \ (\partial b(a^*)(ba^*ba^*) \ + \ v(a^*)\partial b(ba^*ba^*))((a^*ba^*ba^*)^*))\\
&= \partial b(\emptyset \ + \ (\partial b(a)a^*(ba^*ba^*) \ + \ \epsilon\partial b(ba^*ba^*))((a^*ba^*ba^*)^*))\\
&= \partial b(\emptyset \ + \ (\emptyset a^*(ba^*ba^*) \ + \ \epsilon(\partial b(b)(a^*ba^*) + v(b)\partial b(a^*ba^*)))((a^*ba^*ba^*)^*))\\
&= \partial b(\emptyset \ + \ (\emptyset \ + \ \epsilon(\epsilon(a^*ba^*) + \emptyset\partial b(a^*ba^*)))((a^*ba^*ba^*)^*))\\
&= \partial b((a^*ba^*)((a^*ba^*ba^*)^*))\\
&= \partial b(a^*ba^*)((a^*ba^*ba^*)^*) \ + \ v(a^*ba^*)\partial b((a^*ba^*ba^*)^*)\\
&= (\partial b(a^*)ba^* \ + \ v(a^*)\partial b(ba^*))((a^*ba^*ba^*)^*) \ + \ \emptyset\partial b((a^*ba^*ba^*)^*)\\
&= (\partial b(a)a^*ba^* \ + \ \epsilon(\partial b(b)a^* \ + \ v(b)\partial(a^*))((a^*ba^*ba^*)^*) \ + \ \emptyset\\
&= (\emptyset a^*ba^* \ + \ \epsilon(\epsilon(a^*) \ + \ \emptyset\partial(a^*))((a^*ba^*ba^*)^*) \ + \ \emptyset\\
&= (\emptyset \ + \ \epsilon(a^* \ + \ \emptyset)((a^*ba^*ba^*)^*) \ + \ \emptyset\\
&= a^*(a^*ba^*ba^*)^*
\end{aligned}$$

b) $\partial ab((a^*(baa)^*a^*)^*)$

$$\begin{aligned}
&= \partial b(\partial a((a^*(baa)^*a^*)^*))\\
&= \partial b(\partial a(a^*(baa)^*a^*)(a^*(baa)^*a^*)^*))\\
&= \partial b((\partial a(a^*)((baa)^*a^*) + v(a^*)\partial a((baa)^*a^*))(a^*(baa)^*a^*)^*))\\
&= \partial b((\partial a(a)(a^*)((baa)^*a^*) + \epsilon(\partial a((baa)^*)(a^*) \ + \ v((baa)^*)\partial a(a^*))(a^*(baa)^*a^*)^*))\\
&= \partial b((\epsilon(a^*)((baa)^*a^*) + \epsilon(\partial a(baa)(baa)^*a^* \ + \ \epsilon\partial a(a)(a^*))(a^*(baa)^*a^*)^*))\\
&= \partial b((a^*(baa)^*a^* + (\partial a(b)aa + v(b)\partial(aa))(baa)^*a^* \ + \ \epsilon\epsilon(a^*)))(a^*(baa)^*a^*)^*))\\
&= \partial b((a^*(baa)^*a^* + (\emptyset aa + \emptyset\partial(aa))(baa)^*a^* \ + \ a^*))(a^*(baa)^*a^*)^*))\\
&= \partial b(a^*(baa)^*a^* + a^*)(a^*(baa)^*a^*)^*)\\
&= \partial b(a^*(baa)^*a^* + a^*)(a^*(baa)^*a^*)^*) \ + \ v(a^*(baa)^*a^* + a^*)\partial b((a^*(baa)^*a^*)^*))\\
&= (\partial b(a^*(baa)^*a^*) + \partial b(a^*))(a^*(baa)^*a^*)^*) \ + \ \epsilon\partial b(a^*(baa)^*a^*)((a^*(baa)^*a^*)^*))\\
&= (\partial b(a^*)((baa)^*a^*)+v(a^*)\partial b((baa)^*a^*)+\emptyset)(a^*(baa)^*a^*)^*) + (\partial b(a^*)((baa)^*a^*)+v(a^*)\partial b((baa)^*a^*))((a^*(baa)^*a^*)^*))\\
&= \emptyset((baa)^*a^*)+\epsilon(\partial b((baa)^*)a^*+v((baa)^*)\partial b(a^*))(a^*(baa)^*a^*)^*)+(\emptyset((baa)^*a^*)+\epsilon(\partial b((baa)^*)a^*+v((baa)^*)\partial b(a^*))(a^*(baa)\\
&= (\epsilon(\partial b(baa)((baa)^*))a^* + \epsilon\emptyset)(a^*(baa)^*a^*)^*) + (\epsilon(\partial b(baa)((baa)^*)a^* + \epsilon\emptyset)(a^*(baa)^*a^*)^*)\\
&= ((\partial b(b)aa + v(b)\partial b(aa)((baa)^*))a^*)(a^*(baa)^*a^*)^*) + ((\partial b(b)aa + v(b)\partial b(aa))((baa)^*)a^*)(a^*(baa)^*a^*)^*)\\
&= ((\epsilon aa + \emptyset\partial b(aa)((baa)^*))a^*)(a^*(baa)^*a^*)^*) + ((\epsilon aa + \emptyset\partial b(aa))((baa)^*)a^*)(a^*(baa)^*a^*)^*)\\
&= (aa(baa)^*a^*)(a^*(baa)^*a^*)^*) + (aa(baa)^*)a^*)(a^*(baa)^*a^*)^*)\\
&= (aa(baa)^*a^*)(a^*(baa)^*a^*)^*
\end{aligned}$$

c) $\partial a((aa\ +\ bb)^*)$

$= (\partial a((aa\ +\ bb))(aa\ +\ bb)^*$

$= (\partial a(aa)\ +\ \partial a(bb))(aa\ +\ bb)^*$

$= ((\partial a(a)a\ +\ v(a)\partial a(a)a)\ +\ (\partial a(b)b + v(b)\partial b(b)b))(aa\ +\ bb)^*$

$= ((\epsilon a + \emptyset\epsilon a)\ +\ (\emptyset b + \emptyset\emptyset b))(aa\ +\ bb)^*$

$= (a + \emptyset)(aa\ +\ bb)^*$

$= a(aa\ +\ bb)^*$