Autómatas y Lenguajes formales
Ejercicio Semanal 5

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7 de marzo del 2019

1. Para cada lenguajes responda los incisos

   (a) Diseña un AFN (sin $\epsilon - transiciones$) que acepte a L

   (b) Muestra el procesamiento formal de las cadenas $aabbbb$ y $abaab$ usando la función δ^*.

   (c) Transforma M a un AFD mediante la construcción de subconjuntos.

   - $L = (a + b)^*(aaa + bbb)(a + b)^*$

     (a) El autómata sería

$$M = \langle Q, \Sigma, \delta, q_0, F \rangle \; con$$
$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$
$$\Sigma = \{a, b\}$$
$$q_0 = q_0$$
$$F = \{q_5\}$$

Y con función de transición δ

δ	q_0	q_1	q_2	q_3	q_4	q_5
a	$\{q_0, q_1\}$	$\{q_2\}$	$\{q_5\}$	$\varnothing$	$\varnothing$	$\{q_5\}$
b	$\{q_0, q_3\}$	$\varnothing$	$\varnothing$	$\{q_4\}$	$\{q_5\}$	$\{q_5\}$

Con representación gráfica.
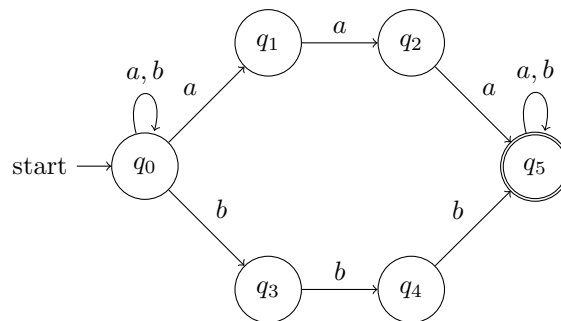

Figure 1: M que reconoce a L

(b) Procesamiento de las cadenas

   − $aabbbb$

$$\delta^*(\{q_0\}, aabbbb) = \delta^*(\delta(q_0, a), abbbb)$$
$$= \delta^*(\{q_0, q_1\}, abbbb) = \delta^*(\delta(q_0, a) \cup \delta(q_1, a), bbbb)$$
$$= \delta^*(\{q_0, q_1, q_2\}, bbbb) = \delta^*(\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b), bbb)$$
$$= \delta^*(\{q_0, q_3\}, bb) = \delta^*(\delta(q_0, b) \cup \delta(q_3, b), bb)$$
$$= \delta^*(\{q_0, q_3, q_4\}, bb) = \delta^*(\delta(q_0, b) \cup \delta(q_3, b) \cup \delta(q_4, b), b)$$
$$= \delta^*(\{q_0, q_3, q_4, q_5\}, b) = \delta^*(\delta(q_0, b) \cup \delta(q_3, b) \cup \delta(q_4, b) \cup \delta(q_5, b), \epsilon)$$
$$= \delta^*(\{q_0, q_3, q_4, q_5\}, \epsilon) = \{q_0, q_3, q_4, q_5\}$$

Luego
$$\delta^*(\{q_0\}, aabbbb) \cap F = \{q_0, q_3, q_4, q_5\} \cap \{q_5\} = \{q_5\} \neq \varnothing$$

Por lo que $aabbbb$ sí es aceptado por el autómata.

   − $abaab$

$$\delta^*(\{q_0\}, abaab) = \delta^*(\delta(q_0, a), baab)$$
$$= \delta^*(\{q_0, q_1\}, baab) = \delta^*(\delta(q_0, b) \cup \delta(q_1, b), aab)$$
$$= \delta^*(\{q_0, q_3\}, aab) = \delta^*(\delta(q_0, a) \cup \delta(q_3, a), ab)$$
$$= \delta^*(\{q_0, q_1\}, ab) = \delta^*(\delta(q_0, a) \cup \delta(q_1, a), b)$$
$$= \delta^*(\{q_0, q_1, q_2\}, b) = \delta^*(\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b), \epsilon)$$
$$= \delta^*(\{q_0, q_3\}, \epsilon) = \{q_0, q_3\}$$

Luego
$$\delta^*(\{q_0\}, abaab) \cap F = \{q_0, q_3\} \cap \{q_5\} = \varnothing$$

Por lo que $abaab$ no es aceptado por el autómata.

(c) El equivalente determinista M^d de M sería

$$M^d = \langle Q^d, \Sigma, \delta^d, p_0, F^d \rangle \text{ con}$$
$$Q^d = \mathcal{P}(Q)$$
$$\Sigma = \{a, b\}$$
$$p_0 = \{q_0\}$$
$$F^d = \{S \in Q^d | S \cap F \neq \varnothing\}$$

En cuanto a δ^d, se construye a continuación usando subconjuntos.

$$\delta^d(\{q_0\}, a) = \{q_0, q_1\}$$
$$\delta^d(\{q_0\}, b) = \{q_0, q_3\}$$
$$\delta^d(\{q_0, q_1\}, a) = \{q_0, q_1, q_2\}$$
$$\delta^d(\{q_0, q_1\}, b) = \{q_0, q_3\}$$
$$\delta^d(\{q_0, q_3\}, a) = \{q_0, q_1\}$$
$$\delta^d(\{q_0, q_3\}, b) = \{q_0, q_3, q_4\}$$
$$\delta^d(\{q_0, q_1, q_2\}, a) = \{q_0, q_1, q_2, q_5\}$$
$$\delta^d(\{q_0, q_1, q_2\}, b) = \{q_0, q_3\}$$
$$\delta^d(\{q_0, q_3, q_4\}, a) = \{q_0, q_1\}$$
$$\delta^d(\{q_0, q_3, q_4\}, b) = \{q_0, q_3, q_4, q_5\}$$
$$\delta^d(\{q_0, q_1, q_2, q_5\}, a) = \{q_0, q_1, q_2, q_5\}$$
$$\delta^d(\{q_0, q_1, q_2, q_5\}, b) = \{q_0, q_3, q_5\}$$
$$\delta^d(\{q_0, q_3, q_4, q_5\}, a) = \{q_0, q_1, q_5\}$$
$$\delta^d(\{q_0, q_3, q_4, q_5\}, b) = \{q_0, q_3, q_4, q_5\}$$
$$\delta^d(\{q_0, q_3, q_5\}, a) = \{q_0, q_3, q_5\}$$
$$\delta^d(\{q_0, q_3, q_5\}, b) = \{q_0, q_3, q_4, q_5\}$$
$$\delta^d(\{q_0, q_1, q_5\}, a) = \{q_0, q_1, q_2, q_5\}$$
$$\delta^d(\{q_0, q_1, q_5\}, b) = \{q_0, q_3, q_5\}$$

Luego, renombrando los estados

$$p_0 = \{q_0\}, p_1 = \{q_0, q_1\}, p_2 = \{q_0, q_3\}, p_3 = \{q_0, q_1, q_2\}, p_4 = \{q_0, q_3, q_4\},$$
$$p_5 = \{q_0, q_1, q_2, q_5\}, p_6 = \{q_0, q_3, q_4, q_5\}, p_7 = \{q_0, q_3, q_5\}, p_8 = \{q_0, q_1, q_5\}$$

Se puede describir δ^d de la siguiente manera

δ^d	p_0	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
a	p_1	p_3	p_1	p_5	p_1	p_5	p_8	p_7	p_5
b	p_2	p_2	p_4	p_2	p_6	p_7	p_6	p_6	p_8

Con representación gráfica.
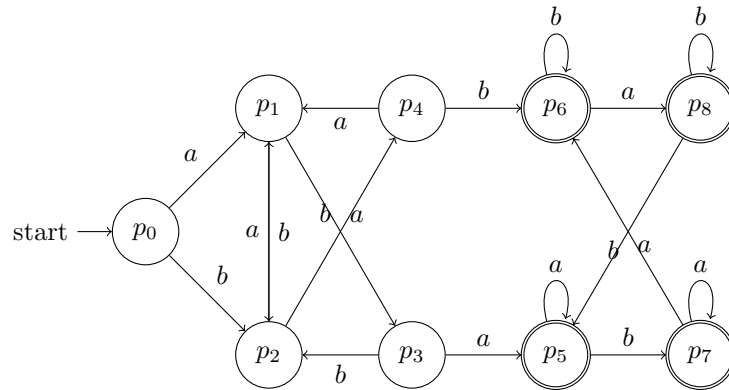

Figure 2: M^d que reconoce a L

- $L = \{a^n b^m | n + m \text{ es par}\}$

  (a) El autómata sería

$$M = \langle Q, \Sigma, \delta, q_0, F \rangle \; con$$
$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$$
$$\Sigma = \{a, b\}$$
$$q_0 = q_0$$
$$F = \{q_0, q_5\}$$

Y con función de transición δ

δ	q_0	q_1	q_2	q_3	q_4	q_5	q_6
a	$\{q_1, q_3\}$	$\{q_0\}$	$\varnothing$	$\{q_4\}$	$\{q_3\}$	$\varnothing$	$\varnothing$
b	$\{q_2\}$	$\varnothing$	$\{q_0\}$	$\{q_5\}$	$\varnothing$	$\{q_6\}$	$\{q_5\}$

Con representación gráfica.
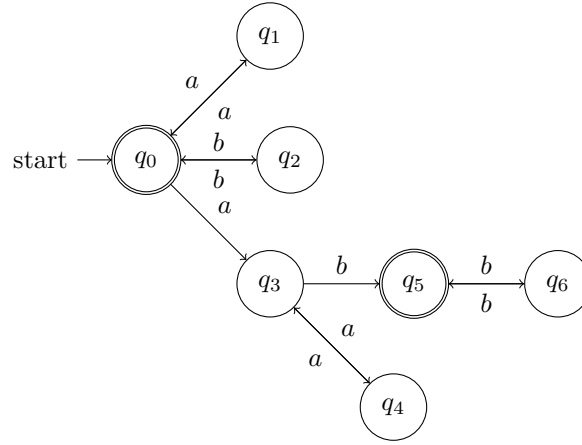

Figure 3: M que reconoce a L

  (b) Procesamiento de las cadenas
    – $aabbbb$

$$\delta^*(\{q_0\}, aabbbb) = \delta^*(\{q_1, q_3\}, abbbb) = \delta^*(\delta(q_1, a) \cup \delta(q_3, a), bbbb)$$
$$= \delta^*(\{q_0, q_4\}, bbbb) = \delta^*(\delta(q_0, b) \cup \delta(q_4, b), bbb)$$
$$= \delta^*(\{q_2\}, bbb) = \delta^*(\delta(q_2, b), bb)$$
$$= \delta^*(\{q_0\}, bb) = \delta^*(\delta(q_0, b), b)$$
$$= \delta^*(\{q_2\}, b) = \delta^*(\delta(q_2, b), \epsilon)$$
$$= \delta^*(\{q_0\}, \epsilon) = \{q_0\}$$

Luego
$$\delta^*(\{q_0\}, aabbbb) \cap F = \{q_0\} \cap \{q_0, q_5\} = \{q_0\} \neq \varnothing$$

Por lo que $aabbbb$ sí es aceptado por el autómata.

    – $abaab$

$$\delta^*(\{q_0\}, abaab) = \delta^*(\delta(q_0, a), baab)$$
$$= \delta^*(\{q_1, q_3\}, baab) = \delta^*(\delta(q_1, b) \cup \delta(q_3, b), aab)$$
$$= \delta^*(\{q_5\}, aab) = \delta^*(\varnothing, ab)$$
$$= \varnothing$$

Luego
$$\delta^*(\{q_0\}, abaab) \cap F = \varnothing \cap \{q_0, q_5\} = \varnothing$$

Por lo que *abaab* no es aceptado por el autómata.

(c) El equivalente determinista M^d de M sería
$$M^d = \langle Q^d, \Sigma, \delta^d, p_0, F^d \rangle \ con$$
$$Q^d = \mathcal{P}(Q)$$
$$\Sigma = \{a, b\}$$
$$p_0 = \{q_0\}$$
$$F^d = \{S \in Q^d | S \cap F \neq \varnothing\}$$

En cuanto a δ^d, se construye a continuación usando subconjuntos.
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$$\delta^d(\{q_1, q_3\}, a) = \{q_0, q_4\}$$
$$\delta^d(\{q_1, q_3\}, b) = \{q_5\}$$
$$\delta^d(\{q_2\}, a) = \varnothing$$
$$\delta^d(\{q_2\}, b) = \{q_0\}$$
$$\delta^d(\{q_0, q_4\}, a) = \{q_1, q_3\}$$
$$\delta^d(\{q_0, q_4\}, b) = \{q_2\}$$
$$\delta^d(\{q_5\}, a) = \varnothing$$
$$\delta^d(\{q_5\}, b) = \{q_6\}$$
$$\delta^d(\{q_6\}, a) = \varnothing$$
$$\delta^d(\{q_6\}, b) = \{q_5\}$$

Luego, renombrando los estados
$$p_0 = \{q_0\}, \ p_1 = \{q_1, q_3\}, \ p_2 = \{q_2\}, \ p_3 = \{q_0, q_4\}, \ p_4 = \{q_5\}, \ p_5 = \{q_6\}$$

Se puede describir δ^d de la siguiente manera

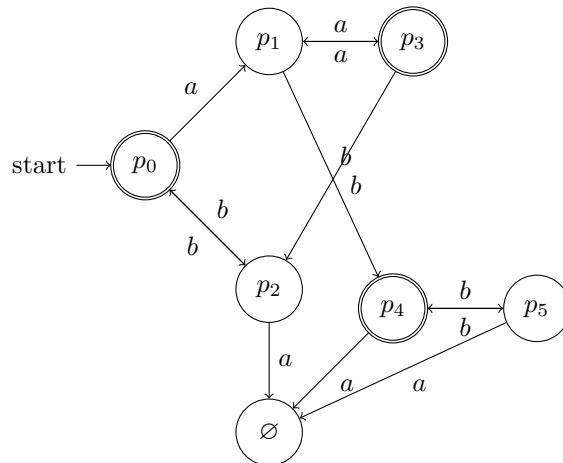| δ^d	p_0	p_1	p_2	p_3	p_4	p_5
a	p_1	p_3	$\varnothing$	p_1	$\varnothing$	$\varnothing$
b	p_2	p_4	p_0	p_2	p_5	p_4

Con representación gráfica.

Figure 4: M^d que reconoce a L