Autómatas y Lenguajes formales
Ejercicio Semanal 5

Sandra del Mar Soto Corderi
Edgar Quiroz Castañeda

7 de marzo del 2019

1. Para cada lenguajes responda los incisos
   (a) Diseña un AFN (sin $\epsilon - transiciones$) que acepte a L
   (b) Muestra el procesamiento formal de las cadenas $aabbbb$ y $abaab$ usando la función δ^*.
   (c) Transforma M a un AFD mediante la construcción de subconjuntos.

- $L = (a+b)^*(aaa+bbb)(a+b)^*$
  (a) El autómata sería

$$\begin{aligned} M &= \langle Q, \Sigma, \delta, q_0, F \rangle \; con \\ Q &= \{q_0, q_1, q_2, q_3, q_4, q_5\} \\ \Sigma &= \{a, b\} \\ q_0 &= q_0 \\ F &= \{q_5\} \end{aligned}$$

Y con función de transición δ

δ	q_0	q_1	q_2	q_3	q_4	q_5
a	$\{q_0, q_1\}$	$\{q_2\}$	$\{q_5\}$	$\varnothing$	$\varnothing$	$\{q_5\}$
b	$\{q_0, q_3\}$	$\varnothing$	$\varnothing$	$\{q_4\}$	$\{q_5\}$	$\{q_5\}$

Con representación gráfica.

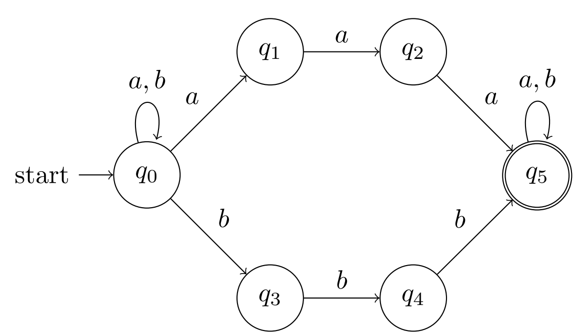

Figure 1: M que reconoce a L

(b) Procesamiento de las cadenas

- *aabbbb*

$$\begin{aligned}\delta^*(\{q_0\}, aabbbb) &= \delta^*(\delta(q_0,a), abbbb)\\ &= \delta^*(\{q_0,q_1\}, abbbb) = \delta^*(\delta(q_0,a)\cup\delta(q_1,a), bbbb)\\ &= \delta^*(\{q_0,q_1,q_2\}, bbbb) = \delta^*(\delta(q_0,b)\cup\delta(q_1,b)\cup\delta(q_2,b), bbb)\\ &= \delta^*(\{q_0,q_3\}, bb) = \delta^*(\delta(q_0,b)\cup\delta(q_3,b), bb)\\ &= \delta^*(\{q_0,q_3,q_4\}, bb) = \delta^*(\delta(q_0,b)\cup\delta(q_3,b)\cup\delta(q_4,b), b)\\ &= \delta^*(\{q_0,q_3,q_4,q_5\}, b) = \delta^*(\delta(q_0,b)\cup\delta(q_3,b)\cup\delta(q_4,b)\cup\delta(q_5,b), \epsilon)\\ &= \delta^*(\{q_0,q_3,q_4,q_5\}, \epsilon) = \{q_0,q_3,q_4,q_5\}\end{aligned}$$

Luego

$$\delta^*(\{q_0\}, aabbbb)\cap F = \{q_0,q_3,q_4,q_5\}\cap\{q_5\} = \{q_5\} \neq \varnothing$$

Por lo que *aabbbb* sí es aceptado por el autómata.

- *abaab*

$$\begin{aligned}\delta^*(\{q_0\}, abaab) &= \delta^*(\delta(q_0,a), baab)\\ &= \delta^*(\{q_0,q_1\}, baab) = \delta^*(\delta(q_0,b)\cup\delta(q_1,b), aab)\\ &= \delta^*(\{q_0,q_3\}, aab) = \delta^*(\delta(q_0,a)\cup\delta(q_3,a), ab)\\ &= \delta^*(\{q_0,q_1\}, ab) = \delta^*(\delta(q_0,a)\cup\delta(q_1,a), b)\\ &= \delta^*(\{q_0,q_1,q_2\}, b) = \delta^*(\delta(q_0,b)\cup\delta(q_1,b)\cup\delta(q_2,b), \epsilon)\\ &= \delta^*(\{q_0,q_3\}, \epsilon) = \{q_0,q_3\}\end{aligned}$$

Luego

$$\delta^*(\{q_0\}, abaab)\cap F = \{q_0,q_3\}\cap\{q_5\} = \varnothing$$

Por lo que *abaab* no es aceptado por el autómata.

(c) El equivalente determinista M^d de M sería

$$\begin{aligned}M^d &= \langle Q^d, \Sigma, \delta^d, p_0, F^d\rangle \ con\\ Q^d &= \mathcal{P}(Q)\\ \Sigma &= \{a,b\}\\ p_0 &= \{q_0\}\\ F^d &= \{S\in Q^d | S\cap F \neq \varnothing\}\end{aligned}$$

En cuanto a δ^d, se construye a continuación usando subconjuntos.

$$
\begin{aligned}
\delta^d(\{q_0\}, a) &= \{q_0, q_1\} \\
\delta^d(\{q_0\}, b) &= \{q_0, q_3\} \\
\delta^d(\{q_0, q_1\}, a) &= \{q_0, q_1, q_2\} \\
\delta^d(\{q_0, q_1\}, b) &= \{q_0, q_3\} \\
\delta^d(\{q_0, q_3\}, a) &= \{q_0, q_1\} \\
\delta^d(\{q_0, q_3\}, b) &= \{q_0, q_3, q_4\} \\
\delta^d(\{q_0, q_1, q_2\}, a) &= \{q_0, q_1, q_2, q_5\} \\
\delta^d(\{q_0, q_1, q_2\}, b) &= \{q_0, q_3\} \\
\delta^d(\{q_0, q_3, q_4\}, a) &= \{q_0, q_1\} \\
\delta^d(\{q_0, q_3, q_4\}, b) &= \{q_0, q_3, q_4, q_5\} \\
\delta^d(\{q_0, q_1, q_2, q_5\}, a) &= \{q_0, q_1, q_2, q_5\} \\
\delta^d(\{q_0, q_1, q_2, q_5\}, b) &= \{q_0, q_3, q_5\} \\
\delta^d(\{q_0, q_3, q_4, q_5\}, a) &= \{q_0, q_1, q_5\} \\
\delta^d(\{q_0, q_3, q_4, q_5\}, b) &= \{q_0, q_3, q_4, q_5\} \\
\delta^d(\{q_0, q_3, q_5\}, a) &= \{q_0, q_3, q_5\} \\
\delta^d(\{q_0, q_3, q_5\}, b) &= \{q_0, q_3, q_4, q_5\} \\
\delta^d(\{q_0, q_1, q_5\}, a) &= \{q_0, q_1, q_2, q_5\} \\
\delta^d(\{q_0, q_1, q_5\}, b) &= \{q_0, q_3, q_5\}
\end{aligned}
$$

Luego, renombrando los estados

$$
\begin{gathered}
p_0 = \{q_0\}, p_1 = \{q_0, q_1\}, p_2 = \{q_0, q_3\}, p_3 = \{q_0, q_1, q_2\}, p_4 = \{q_0, q_3, q_4\}, \\
p_5 = \{q_0, q_1, q_2, q_5\}, p_6 = \{q_0, q_3, q_4, q_5\}, p_7 = \{q_0, q_3, q_5\}, p_8 = \{q_0, q_1, q_5\}
\end{gathered}
$$

Se puede describir δ^d de la siguiente manera

δ^d	p_0	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
a	p_1	p_3	p_1	p_5	p_1	p_5	p_8	p_7	p_5
b	p_2	p_2	p_4	p_2	p_6	p_7	p_6	p_6	p_8

Con representación gráfica.

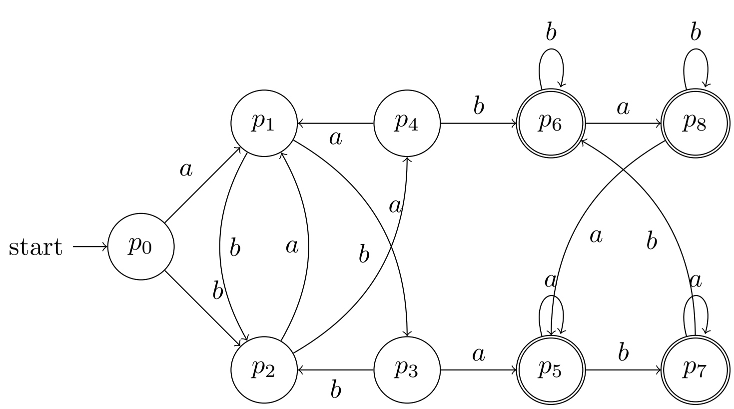

Figure 2: M^d que reconoce a L

- $L = \{a^n b^m | n + m \text{ } es \text{ } par\}$
  (a) El autómata sería

$$\begin{aligned} M &= \langle Q, \Sigma, \delta, q_0, F\rangle \text{ } con \\ Q &= \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\} \\ \Sigma &= \{a, b\} \\ q_0 &= q_0 \\ F &= \{q_0, q_5\} \end{aligned}$$

Y con función de transición δ

δ	q_0	q_1	q_2	q_3	q_4	q_5	q_6
a	$\{q_1, q_3\}$	$\{q_0\}$	$\varnothing$	$\{q_4\}$	$\{q_3\}$	$\varnothing$	$\varnothing$
b	$\{q_2\}$	$\varnothing$	$\{q_0\}$	$\{q_5\}$	$\varnothing$	$\{q_6\}$	$\{q_5\}$

Con representación gráfica.

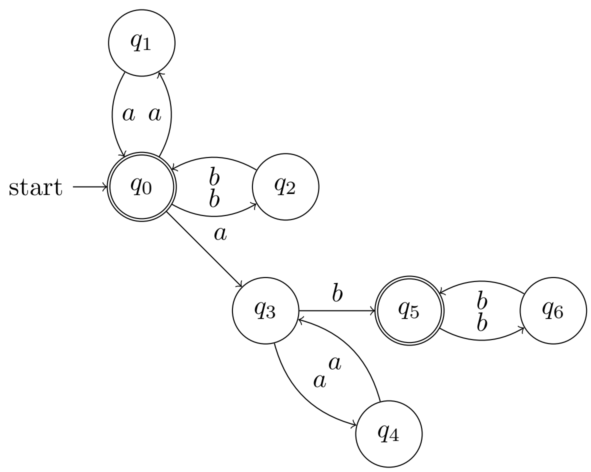

Figure 3: M que reconoce a L

(b) Procesamiento de las cadenas

- $aabbbb$

$$\begin{aligned} \delta^*(\{q_0\}, aabbbb) &= \delta^*(\{q_1, q_3\}, abbbb) = \delta^*(\delta(q_1, a) \cup \delta(q_3, a), bbbb) \\ &= \delta^*(\{q_0, q_4\}, bbbb) = \delta^*(\delta(q_0, b) \cup \delta(q_4, b), bbb) \\ &= \delta^*(\{q_2\}, bbb) = \delta^*(\delta(q_2, b), bb) \\ &= \delta^*(\{q_0\}, bb) = \delta^*(\delta(q_0, b), b) \\ &= \delta^*(\{q_2\}, b) = \delta^*(\delta(q_2, b), \epsilon) \\ &= \delta^*(\{q_0\}, \epsilon) = \{q_0\} \end{aligned}$$

Luego

$$\delta^*(\{q_0\}, aabbbb) \cap F = \{q_0\} \cap \{q_0, q_5\} = \{q_0\} \neq \varnothing$$

Por lo que $aabbbb$ sí es aceptado por el autómata.

- $abaab$

$$\begin{aligned} \delta^*(\{q_0\}, abaab) &= \delta^*(\delta(q_0, a), baab) \\ &= \delta^*(\{q_1, q_3\}, baab) = \delta^*(\delta(q_1, b) \cup \delta(q_3, b), aab) \\ &= \delta^*(\{q_5\}, aab) = \delta^*(\varnothing, ab) \\ &= \varnothing \end{aligned}$$

Luego

$$\delta^*(\{q_0\}, abaab) \cap F = \varnothing \cap \{q_0, q_5\} = \varnothing$$

Por lo que *abaab* no es aceptado por el autómata.

(c) El equivalente determinista M^d de M sería

$$\begin{aligned}
M^d &= \langle Q^d, \Sigma, \delta^d, p_0, F^d \rangle \ con \\
Q^d &= \mathcal{P}(Q) \\
\Sigma &= \{a, b\} \\
p_0 &= \{q_0\} \\
F^d &= \{S \in Q^d | S \cap F \neq \varnothing\}
\end{aligned}$$

En cuanto a δ^d, se construye a continuación usando subconjuntos.

$$\begin{aligned}
\delta^d(\{q_0\}, a) &= \{q_1, q_3\} \\
\delta^d(\{q_0\}, b) &= \{q_2\} \\
\delta^d(\{q_1, q_3\}, a) &= \{q_0, q_4\} \\
\delta^d(\{q_1, q_3\}, b) &= \{q_5\} \\
\delta^d(\{q_2\}, a) &= \varnothing \\
\delta^d(\{q_2\}, b) &= \{q_0\} \\
\delta^d(\{q_0, q_4\}, a) &= \{q_1, q_3\} \\
\delta^d(\{q_0, q_4\}, b) &= \{q_2\} \\
\delta^d(\{q_5\}, a) &= \varnothing \\
\delta^d(\{q_5\}, b) &= \{q_6\} \\
\delta^d(\{q_6\}, a) &= \varnothing \\
\delta^d(\{q_6\}, b) &= \{q_5\}
\end{aligned}$$

Luego, renombrando los estados

$$p_0 = \{q_0\},\ p_1 = \{q_1, q_3\},\ p_2 = \{q_2\},\ p_3 = \{q_0, q_4\},\ p_4 = \{q_5\},\ p_5 = \{q_6\}$$

Se puede describir δ^d de la siguiente manera

δ^d	p_0	p_1	p_2	p_3	p_4	p_5
a	p_1	p_3	$\varnothing$	p_1	$\varnothing$	$\varnothing$
b	p_2	p_4	p_0	p_2	p_5	p_4

Con representación gráfica.

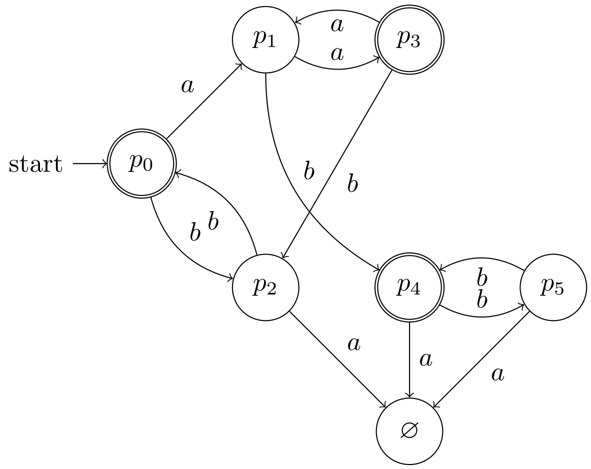

Figure 4: M^d que reconoce a L