total marks = 10 x 7 = 70
↳ #Questions

QUESTION=paper, MARKS=1.00

Instructions: Q1 (not graded)

- This test contains 7 questions, each worth 10 points (excluding this one, i.e. Q1, which is meant only for instructions). Attempt them all.
- Number the questions in the same way as they appear on SAFE.
- Please stop writing at 10:30 am sharp and scan and upload your answers on SAFE as well as moodle.
- Please have your webcam on right through until you have submitted your answers on SAFE as well as moodle.
- **Avoid writing lengthy answers. Be brief for all answers.**
- This is a closed book, closed notes, closed internet exam. No calculators are allowed (or required).

==

QUESTION=paper, MARKS=1.00 Q2

We know that the restricted isometry constant (RIC) of order *s* of a *m* x *n* sensing matrix **A** (where *m* is much less than is *n*), denoted as δ_s , is the smallest constant for which the following is always true for all *s*-sparse vectors **x.** Recall that an *s*-sparse vector has *s* or less non-zero elements.

$$(1-\delta_s)\|\boldsymbol{x}\|^2 \leq \|\boldsymbol{A}\boldsymbol{x}\|^2 \leq (1+\delta_s)\|\boldsymbol{x}\|^2.$$

Computing the order-*s* RIC for any sensing matrix is a very expensive process as it requires enumeration of subsets (denoted as B) of the *n* column indices of **A** where each subset will have up to *s* elements. This will lead us to generate submatrices $\boldsymbol{A_B}$ of size *m* x |B|, where each submatrix contains columns at only those indices which are in the set B. This is followed by computation of the minimum and maximum eigenvalues of the matrix $\boldsymbol{A}_B^T\boldsymbol{A_B}$. A curious student chooses a total of (say) 2000 such subsets at random and approximates the *s*-order RIC as follows:

$$\kappa_s := \max_B \max(1-\lambda_{min}(\boldsymbol{A}_B^T\boldsymbol{A_B}), \lambda_{max}(\boldsymbol{A}_B^T\boldsymbol{A_B})-1).$$

In the equation above, B denotes a subset in the set of all 2000 subsets generated by the student. Assume 2000 is much less than C(*n*,*s*), i.e. the number of subsets of size *s* that can be created out of *n* elements. Which of the following inequalities is always true? Give a thorough justification. (No marks without justification):

1) $\kappa_s \leq \delta_s$; 2) $\kappa_s \geq \delta_s$; 3) $\kappa_s = \delta_s$ 4) no precise equality/inequality can be ascertained. [10 points]

==

QUESTION=paper, MARKS=1.00 Q3

Explain what is meant by the global rotation/reflection ambiguity in tomography under unknown angles. Why does this ambiguity arise? [6+4=10 points]

==

QUESTION=paper, MARKS=1.00

Q4

Q4

We say that f is a bandlimited signal if there exists a finite b such that the Fourier transform of f at any frequency u for which $|u| >= b$, is zero in value. This notion of bandlimited-ness extends to signals of any dimension. For example, in 2D, we will have f = (u,v), and a 2D signal is bandlimited if its Fourier transform is zero for all frequencies for which either $|u| >= b$ or $|v| >= b$.

If f is a bandlimited 2D signal, can we say that its Radon projection in any given direction theta is also bandlimited? Justify. Recall that the Radon projection of f is defined as follows:

$$R_f(\rho, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\delta(x \cos\theta + y \sin\theta - \rho)dxdy.$$

No marks without proper justification. [10 points]

==

QUESTION=paper, MARKS=1.00 Q5

Briefly describe the image formation model in the multi-frame CASSI camera in the following manner:

Write an equation or a set of equations which relate the measurement made by the multi-frame CASSI camera with the underlying hyperspectral image. Define the meaning of each term and its dimensions carefully.

Explain very briefly how the different hardware components of the multi-frame CASSI camera effectively implement the equation. [10 points]

==

QUESTION=paper, MARKS=1.00 Q6

Designing a sensing matrix **A** (in compressed sensing) by optimizing the coherence criterion discourages any one column of **A** from being too similar to any other column of **A**. For this, the criterion is based on minimizing the maximum absolute value (or in some variants, the average of the squares of the values) of normalized dot products between pairs of columns. Instead consider the following: Suppose I want to design **A** such that no column of **A** can be accurately represented as the linear combination of any two other columns of **A** (in the sense of squared error). Write a mathematical expression for this criterion, which we will term the `double coherence'. Simplify the expression so that it is entirely represented in terms of column vectors or elements of **A**. For sensing matrix design, should the double coherence be maximized or minimized? Why? [4+4+2=10 points]

==

QUESTION=paper, MARKS=1.00 Q7

Consider compressive measurements of the form $\boldsymbol{y} = \boldsymbol{\Phi x} + \boldsymbol{\eta}$ where **y** is a vector of m compressive measurements, $\boldsymbol{\Phi}$ is a m x n sensing matrix ($m < n$) , **x** is a sparse signal with n elements and $\boldsymbol{\eta}$ is a noise vector with m elements. Consider the problem P1 (also called Basis Pursuit Denoising), which is given as:

$$min\|\boldsymbol{x}\|_1 \text{ such that } \|\boldsymbol{y} - \boldsymbol{\Phi x}\|_2 \leq \epsilon.$$

Q7

Here \epsilon is a parameter that is chosen in a manner that it upper bounds the magnitude of the noise vector η.

If the elements of η were drawn independently from a uniform distribution taking values between -*a* and *a* where *a* is known, explain how you would select \epsilon and the reasoning behind your choice.

If the elements of η were drawn independently from a Gaussian distribution with mean zero and known standard deviation \sigma, explain how you would select \epsilon and the reasoning behind your choice. [5+5=10 points]

==

QUESTION=paper, MARKS=1.00 Q8

Suppose there are *n* subjects being tested by Dorfman pooling and only $k < n$ out of these are infected. In the first round, assume that the *n* subjects are divided into groups of size *g* each. For simplicity, assume *n*/*g* is an integer. Now additionally suppose that the cost of a pooled test (regardless of the number of samples in the pool) costs *q* times that of an individual test. Derive a formula for the average number of tests required to be performed in Dorfman pooling. What is the optimal group size in the worst case in terms of total cost? [5+5=10 points]

==