

Chebyshev Based Spectral Representations of Neutron-Star Equations of State

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Introduction

- The equation of state describes the relationship between pressure (P) and energy density (ϵ);
- Given an equation of state $\epsilon = \epsilon(P)$, Oppenheimer and Volkoff solved the Einstein equations to get

$$\frac{dm}{dr} = 4\pi r^2 \epsilon \quad (1)$$

$$\frac{dP}{dr} = -(\epsilon + P) \frac{m + 4\pi r^3 P}{r(r - 2m)}; \quad (2)$$

- Neutron star densities surpass lab limits, with no consensus on models, but mass (M) and radius (R) data can help determine their equations of state [1];
- Aim to represent the equations of state numerically with spectral methods to find the best-fit equations of state with M and R data [2];
- Simple power-law bases have been used in previous studies but show slow convergence when adding more spectral parameters for first-order and second-order phase transitions [3];
- Chebyshev polynomials can serve as a faster-converging orthogonal basis, defined by:

$$T_n(x) = \cos(n \arccos(x)) \quad (3)$$

for $n = 0, 1, 2, 3, \dots$

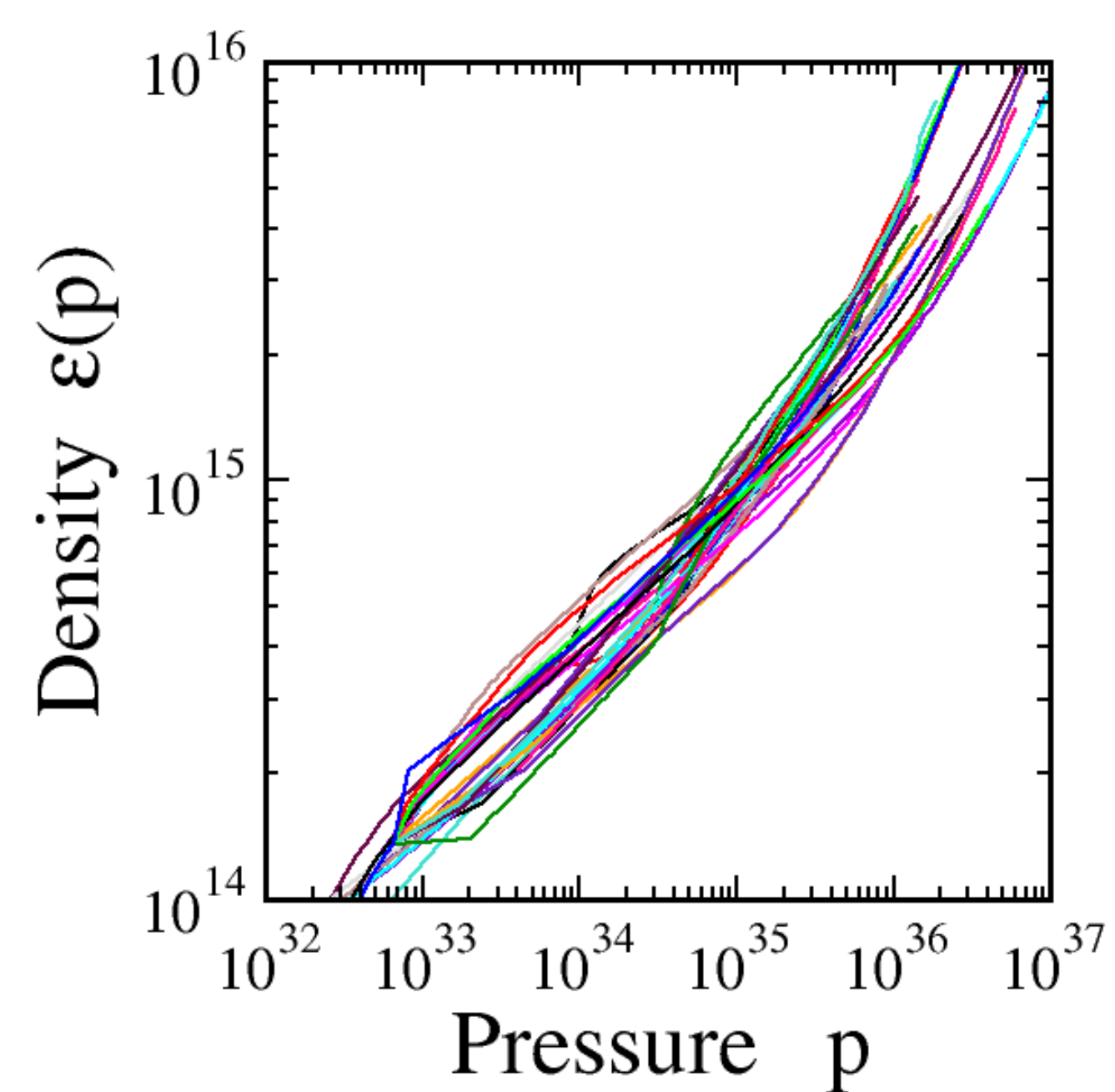


Figure 1: These curves correspond to 34 nuclear-theory models of the neutron stars' equations of state.

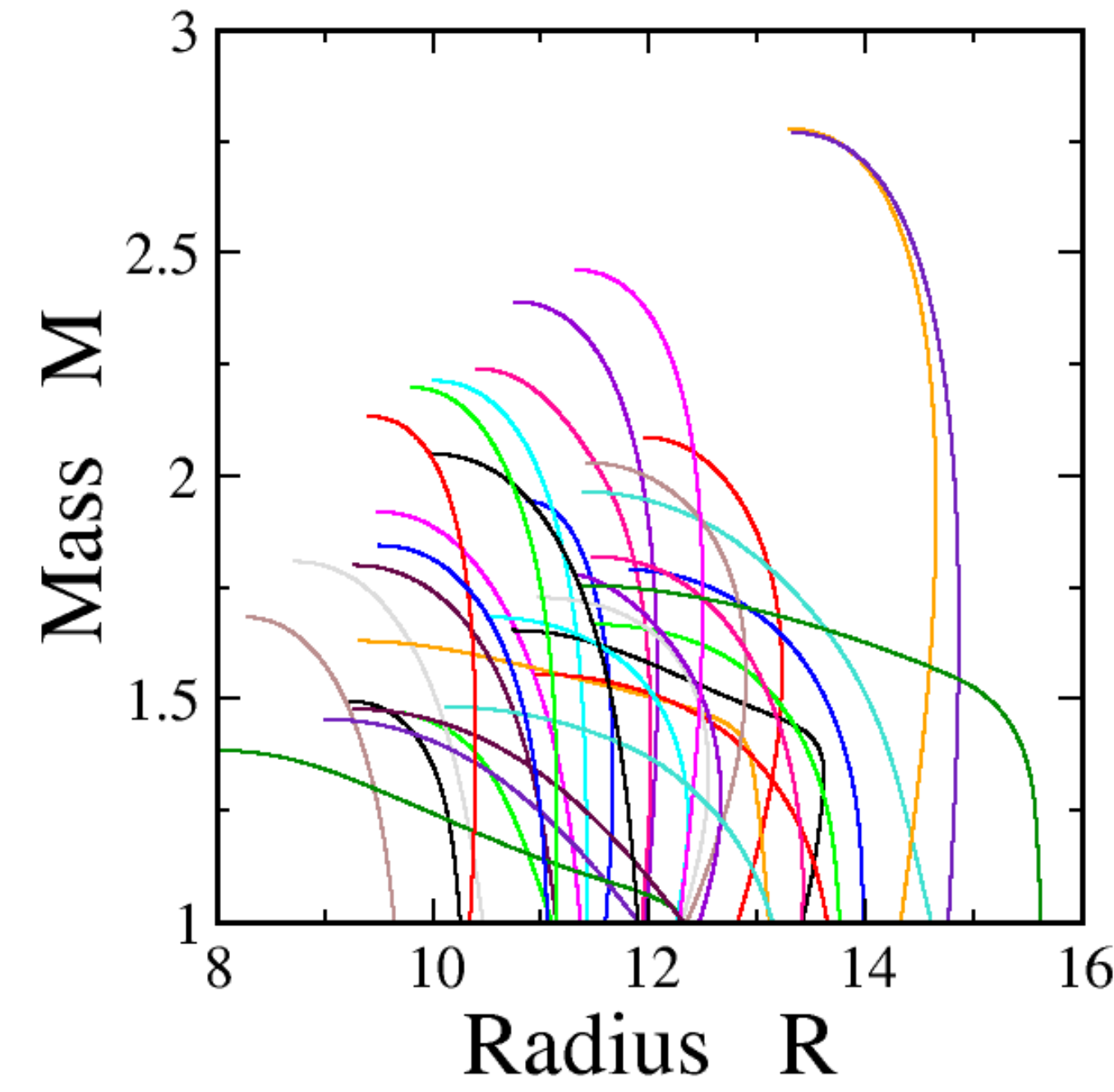


Figure 2: These curves correspond to the equations of state in Figure 1, showing the relationship between mass and radius for different equations of state.

Casual Spectral Representation

- The sound speed of the baryonic fluid is defined by $v^2 = dp/d\epsilon$;
 - To ensure the causality of the fluid, we define a new sound speed function
- $$\Upsilon = \frac{c^2 - v^2}{v^2}; \quad (4)$$
- We could either represent using a pressure-based or enthalpy-based approach;
 - For pressure-based approach, reduce the sound speed equation to

$$\epsilon(P) = \epsilon_0 + (P - P_0) + \int_{P_0}^P \Upsilon(P') dP'; \quad (5)$$

- Set Υ to be expressed by spectral method,
- $$\Upsilon(P, v_k) = \Upsilon_0 \exp \left\{ \sum_{a=0}^{N_{\text{parms}}-1} v_a (1+z) T_a(z) \right\}, \quad (6)$$
- where v_k is a set of spectral parameters;

- Define

$$z = -1 + 2 \log(P/P_0) / \log(P_{\text{max}}/P_0). \quad (7)$$

- to ensure $-1 \leq z \leq 1$ for $P_0 \leq P \leq P_{\text{max}}$;
- Enthalpy-based representation is similar to pressure-based representation by re-expressing $P = P(h)$ and $\epsilon = \epsilon(P(h)) = \epsilon(h)$ where h is the enthalpy.

Testing The Representations

- Define the error function $\chi^2 = \frac{1}{N} \sum_{i=0}^N [\log(\frac{\epsilon(v_a)}{\epsilon_i})]^2$;
- Using datasets from the first-order and second-order phase transitions to optimize the error function with the best-fitted sets of spectral coefficients;

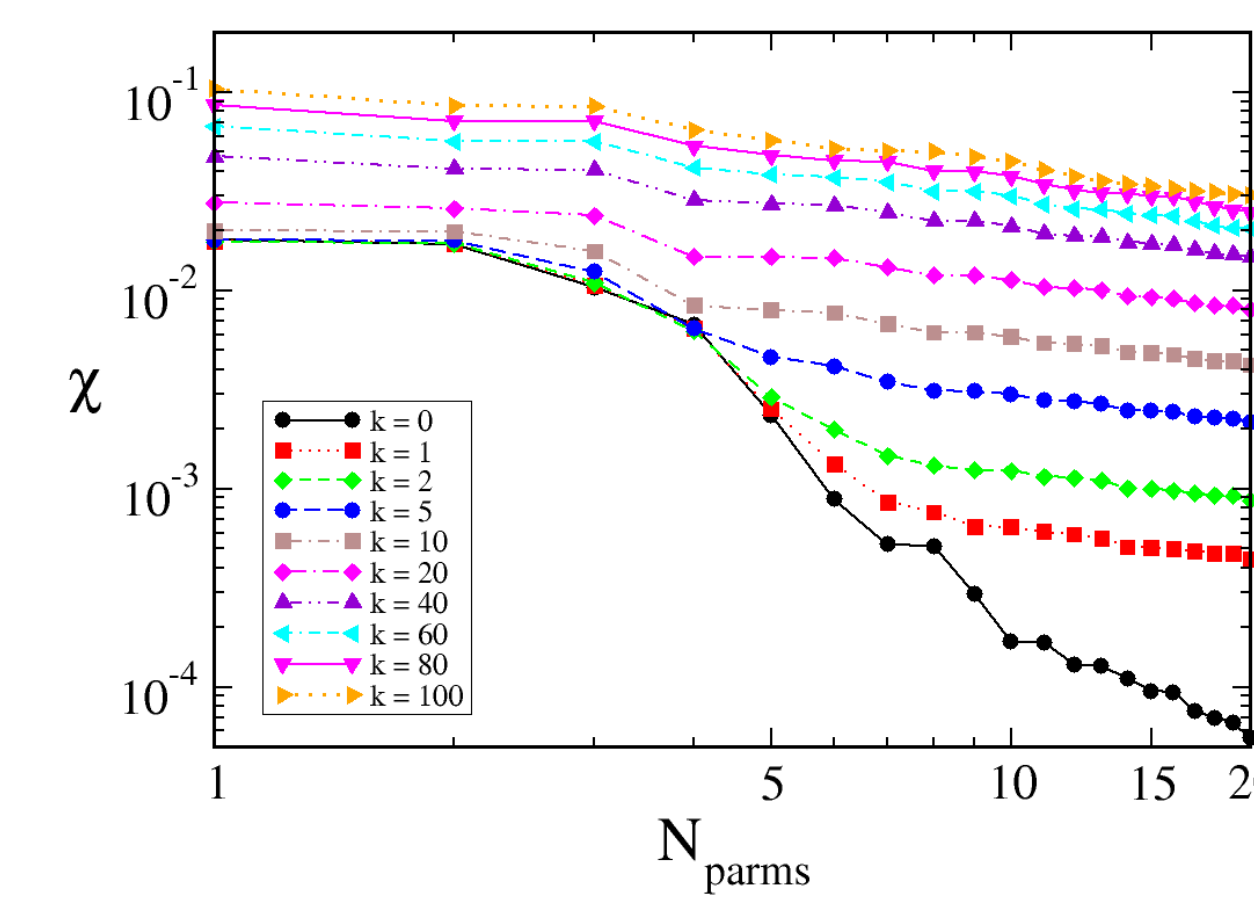


Figure 3: Modeling errors χ using pressure-based Chebyshev polynomial spectral expansions for first-order phase transitions.

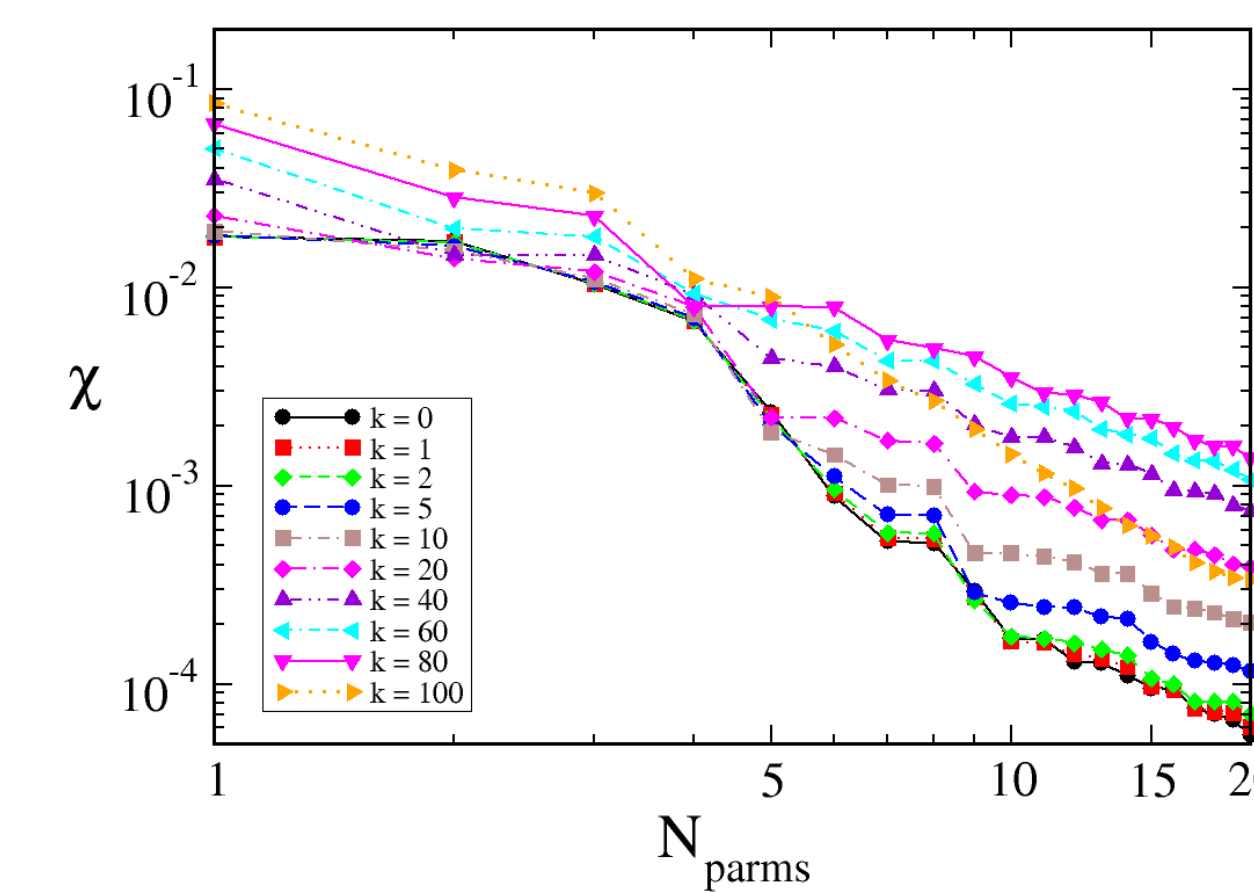


Figure 4: Modeling errors χ using pressure-based Chebyshev polynomial spectral expansions for second-order phase transitions.

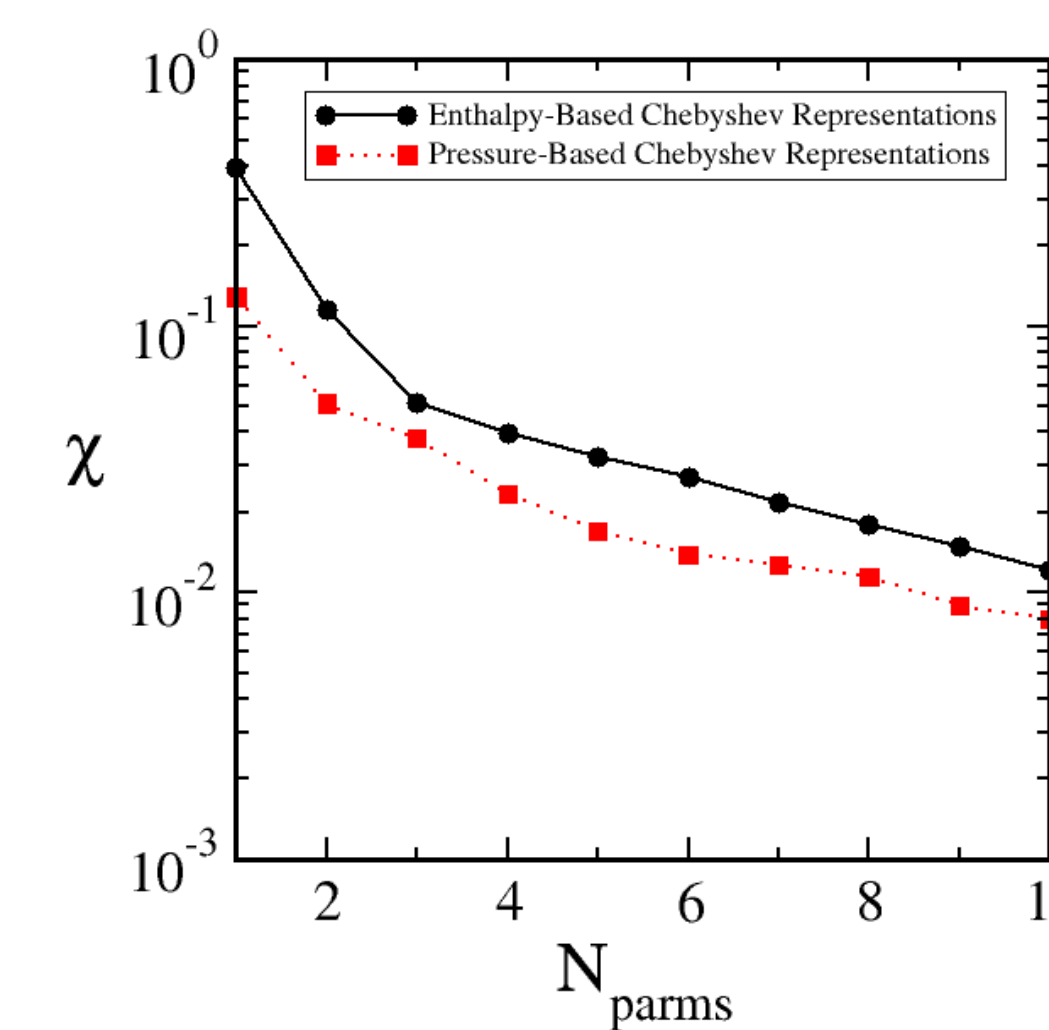


Figure 5: The average modeling error for different nuclear-theory models between Chebyshev enthalpy-based and pressure-based representations.

Conclusion

- When N_{parms} increases, the Chebyshev polynomial representation converges;
- Pressure-based representation does a better job than enthalpy-based representation;
- In the future, we will explore how well the Chebyshev polynomial pressure-based representation models the equations of state by using M and R data with noise.

References

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- [2] Lee Lindblom. Spectral representations of neutron-star equations of state. *Phys. Rev. D*, 82:103011, Nov 2010.
- [3] Lee Lindblom. Causal representations of neutron-star equations of state. *Phys. Rev. D*, 97:123019, Jun 2018.
- [4] Lee Lindblom. Improved spectral representations of neutron-star equations of state. *Phys. Rev. D*, 105:063031, Mar 2022.
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