Diffusion-convection equation in 2d plane

(Dated: July 31, 2017)

The diffusion-convection equation reads

$$D\nabla^2 g + \nabla \cdot (\frac{\boldsymbol{f}}{\gamma_0} g) + \nabla \cdot (\boldsymbol{u} g) = 0 \tag{1}$$

with boundary condition

$$g(r \to \infty) = 1 \tag{2}$$

For simplicity, set $\gamma_0 = D = 1$.

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$$\partial_x^2 g + \partial_x[-(\partial_x V)g] + U_x \partial_x g + \partial_y^2 g + \partial_y[-(\partial_y V)g] + U_y \partial_y g = 0 \tag{3}$$

with BC

$$g(r \to \infty) = 1 \tag{4}$$

The potential is

$$V(x, y) = \frac{K}{2}(x^2 + y^2) = v(x)v(y), \text{ with } v(x) = \frac{K}{2}x^2. \tag{5}$$

A. Separation of variables(NOT appliable!)

Eq.(3) may be solved by separating the varibles as $g(x, y) = F(x)W(y)$. So Eq.(3) is transformed to

$$WF'' + Wv_y(-v_x' F)' + WU_x F' + FW'' + FU_y W' + Fv_y(-v_y' W)' = 0 \tag{6}$$

However, the above eq. can not be seprated. Noting that

$$F''/F + v_y(-v_x' F)'/F + U_x F'/F + W''/W + U_y W'/W + v_y(-v_y' W)'/W = 0 \tag{7}$$

Due to interaction, the term $v_y(-v_x' F)'$ still depends on y. Physically, the equation is not separable because the interaction couples the equation in different dimensions.