1D Diffusion-convection Equation

I. MOTIVATION

To understand how does convection flow affect the interaction force in the fast rotation limit.

II. EQUATION

The number density current is

$$j(x,t) = -D\partial_x g + (-\frac{V'}{\gamma} - u)g \tag{1}$$

The conservation of particle number requires that

$$\partial_t g + \partial_x j = 0 \tag{2}$$

In steady state $\partial_t g = 0$, and eq.(2) is reduced to

$$\partial_x^2 g + \partial_x \left[(\frac{V'}{\gamma} + u)g\right] = 0, \tag{3}$$

where we have set $D = \gamma = 1$ for simplicity.

In addition, We assume the potential force half-harmonic, i.e.

$$V(x) = \begin{cases} \frac{K}{2}x^2, & x < 0, \\ 0 & x \geq 0 \end{cases} \tag{4}$$

III. SOLUTION

A. Direct solution?

For $x > 0$, $V' = 0$, eq.(3) is

$$\left[g' + ug\right]' = 0 \tag{5}$$

So $g' + ug = C$. Using boundary condition (BC) $g(x \to \infty) = 1$, $(g'(x \to \infty) = 0)$, we get $C = u$ and

$$g' + ug = u \tag{6}$$

The general solution is

$$g = Ae^{-ux} + 1 \tag{7}$$

one can easily convince that eq.(7) is the solution of eq.(5). A should be dertermined in the case $x < 0$.