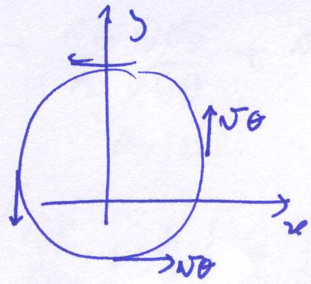
II

Axis:
laminar
permanente



$$v_\theta(r) = \frac{\Gamma_0}{2\pi r}\left[1-\exp\left(\frac{-r^2}{4\nu t}\right)\right]$$

a) $$\frac{1}{r}\frac{\partial(r v_r)}{\partial r} + \underbrace{\frac{1}{r}\cancel{\frac{\partial v_\theta}{\partial \theta}}}_{\text{Axis}} + \underbrace{\cancel{\frac{\partial v_z}{\partial z}}}_{v_z=0} = 0$$

$$\frac{1}{r}\frac{\partial(r v_r)}{\partial r} = 0 \Leftrightarrow r v_r = \text{constante}$$

p/ $r=0 \Rightarrow v_r = 0$, logo constante $=0$

e $\forall r \quad v_r = 0$.

b) Eq. v_z:

$$\underbrace{\cancel{\frac{\partial v_z}{\partial t}}}_{v_z=0} + v_r \cdot \cancel{\frac{\partial v_z}{\partial r}} + \underbrace{\frac{v_\theta}{r}\cdot\cancel{\frac{\partial v_z}{\partial \theta}}}_{\text{Axis}} + \underbrace{v_z \cancel{\frac{\partial v_z}{\partial z}}}_{v_z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\Bigg[\underbrace{\cancel{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)}}_{v_z=0} + \underbrace{\frac{1}{r^2}\cdot\cancel{\frac{\partial^2 v_z}{\partial \theta^2}}}_{\text{Axis}} + \underbrace{\cancel{\frac{\partial^2 v_z}{\partial z^2}}}_{v_z=0}\Bigg]$$

$$\Leftrightarrow \quad 0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} \Leftrightarrow \frac{\partial p}{\partial z} = 0$$

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$$\frac{\partial v_\theta}{\partial t} + \underset{v_r=0}{\cancel{v_r \frac{\partial v_\theta}{\partial r}}} + \underset{\text{Axis}}{\cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}}} + \underset{v_r=0}{\cancel{\frac{v_r v_\theta}{r}}} + \underset{v_z=0}{\cancel{v_z \frac{\partial v_\theta}{\partial z}}} =$$

$$= \underset{\text{Axis}}{\cancel{-\frac{1}{\rho}\frac{1}{r}\frac{\partial p}{\partial \theta}}} + \frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right) + \underset{\text{Axis}}{\cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}}} + \underset{\text{Axis}}{\cancel{\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}}} + \underset{v_\theta \neq v_\theta(z)}{\cancel{\frac{\partial^2 v_\theta}{\partial z^2}}}\right]$$

$$\Leftrightarrow \quad \frac{\partial v_\theta}{\partial t} = \frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right)\right] \; //$$

d) $$\underset{v_r=0}{\cancel{\frac{\partial v_r}{\partial t}}} + \underset{v_r=0}{\cancel{v_r \frac{\partial v_r}{\partial r}}} + \underset{\text{Axis}}{\cancel{\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}}} - \frac{v_\theta^2}{r} + \overset{v_z=0}{\cancel{v_z \frac{\partial v_r}{\partial z}}} =$$

$$= -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{\rho}\left[\underset{v_r=0}{\cancel{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_r)}{\partial r}\right)}} + \underset{\text{Axis}}{\cancel{\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}}} + \underset{v_r=0}{\cancel{\frac{\partial^2 v_r}{\partial z^2}}}\right]$$

$$\Leftrightarrow \quad -\frac{v_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} \quad \Leftrightarrow \quad \frac{\partial p}{\partial r} = \rho \frac{v_\theta^2}{r} \; //$$

$$\frac{\partial v_\theta}{\partial t} = \nu \left[\frac{\partial}{\partial r}\left(\frac{1}{r} \cdot \frac{\partial (r v_\theta)}{\partial r} \right) \right]$$

$$v_\theta(r) = \frac{\Gamma_0}{2\pi r}\left[1 - \exp\left(\frac{-r^2}{4\nu t} \right) \right]$$

$$\frac{\partial v_\theta}{\partial t} = -\frac{\Gamma_0}{2\pi \cancel{r}} \cdot \left(-\frac{r^{\cancel{2}}}{4\nu} \right)\left(-\frac{1}{t^2} \right) \exp\left(\frac{-r^2}{4\nu t} \right)$$

$$= -\frac{\Gamma_0 \cdot r}{\nu 8\pi t^2} \cdot \exp\left(\frac{-r^2}{4\nu t} \right) //$$

$$\frac{\partial (r v_\theta)}{\partial r} = \frac{\partial}{\partial r}\left[\frac{\Gamma_0}{2\pi} \left[1 - \exp\left(\frac{-r^2}{4\nu t} \right) \right] \right] =$$

$$= \frac{\Gamma_0}{2\pi} \cdot \frac{\partial}{\partial r}\left[1 - \exp\left(\frac{-r^2}{4\nu t} \right) \right] = \frac{\Gamma_0}{2\pi}\left(\frac{2r}{4\nu t} \right) \exp\left(\frac{-r^2}{4\nu t} \right)$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial (r v_\theta)}{\partial r} \right) = \frac{\partial}{\partial r}\left\{ \frac{\Gamma_0}{4\pi \nu t} \exp\left(\frac{-r^2}{4\nu t} \right) \right\}$$

$$= \frac{\Gamma_0}{4\pi \nu t}\left(\frac{-2r}{4\nu t} \right) \exp\left(\frac{-r^2}{4\nu t} \right) = -\frac{\Gamma_0}{8\pi \nu^2 t^2} \exp\left(\frac{-r^2}{4\nu t} \right)$$

$$\nu \left[\frac{\partial}{\partial r}\left(\frac{1}{r} \cdot \frac{\partial (r v_\theta)}{\partial r} \right) \right] = -\frac{\Gamma_0 \cdot r}{8\pi \nu t^2} \exp\left(\frac{-r^2}{4\nu t} \right) ※$$

$$// = ※$$