$$\boxed{\begin{array}{c}1^{\text{º}}\ \text{TESTE}\\ 31/10/2014\end{array}}$$

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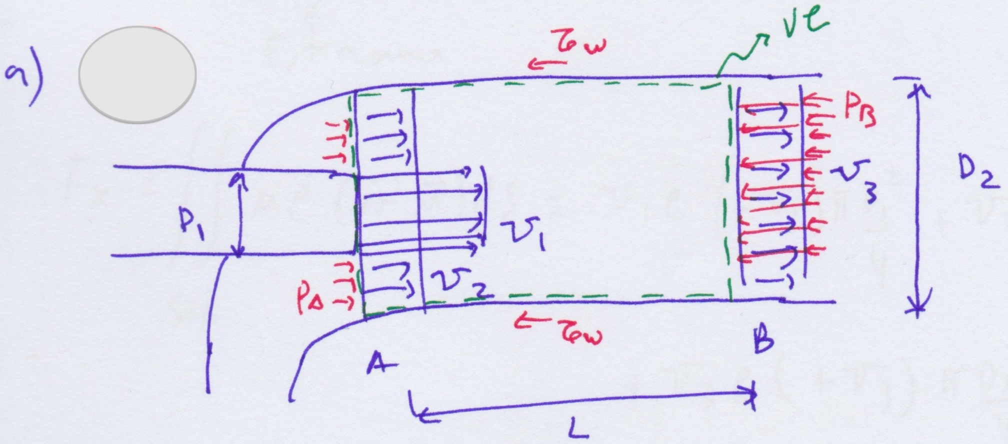
ρ, ν $\quad D_1, D_2$ $\quad v_1, v_2$ $\quad$ incompressível, estacionário, adiabático. $\quad L$ (Misturado) $\quad v_3$ $\quad \vec{g} \simeq \vec{0}$

a)



$v_3 = ?$

Balanço de massa:

$$\frac{d}{dt}\underbrace{\cancel{\iiint_{vc} \rho \, dVol}}_{\text{Estacionário}} + \iint_{svc} \rho(\vec{v}\cdot\vec{n})\,dS = 0 \quad (\Leftrightarrow)$$

(incompressível)

$$\iint \vec{v}\cdot\vec{n}\,dS = 0 \quad (\Leftrightarrow)$$

$$-v_1 \frac{\pi D_1^2}{4} - v_2\left(\frac{\pi D_2^2}{4} - \frac{\pi D_1^2}{4}\right) + v_3 \frac{\pi D_2^2}{4} = 0 \quad (\Leftrightarrow)$$

$$-v_1 D_1^2 - v_2\left(D_2^2 - D_1^2\right) + v_3 D_2^2 = 0 \quad (\Leftrightarrow)$$

$$v_3 = v_1\left(\frac{D_1}{D_2}\right)^2 + v_2\left[1 - \left(\frac{D_1}{D_2}\right)^2\right] \quad //$$

$$= \left(\frac{D_1}{D_2}\right)^2 (v_1 - v_2) + v_2$$

$\Delta P = P_B - P_A = ?$

Balanço de Momento linear:

$$\vec{F} = \cancel{\frac{d}{dt}\iiint_{ve} \vec{v}\, \rho \, dvol} + \iint_{sve} \vec{v}\, \rho\, (\vec{v}\cdot\vec{n})\, ds$$

Estacionário

$$F_x = \iint_{sve} u\, \rho\, (\vec{v}\cdot\vec{n})\, ds = v_1 \rho\, (-v_1)\, \frac{\pi D_1^2}{4} + v_2\, \rho\, (-v_2)\left(\frac{\pi D_2^2}{4} - \frac{\pi D_1^2}{4}\right)$$

$$+ v_3\, \rho\, (+v_3)\, \frac{\pi D_2^2}{4}$$

$$F_x = \rho\frac{\pi}{4}\left[-v_1^2 D_1^2 - v_2^2 \cdot (D_2^2 - D_1^2) + v_3^2 \cdot D_2^2\right]$$

$$F_x = P_A \cdot \frac{\pi D_2^2}{4} - P_B\, \frac{\pi D_2^2}{4} - \left[\int_0^L \tau_w(x)\, dx\right] \times 2\pi \frac{D_2}{2}$$

$\mu_{c)}$ $\quad \bar{\tau}_w = \frac{1}{L}\int_0^L \tau_w(x)$

$$= (P_A - P_B)\left(\frac{\pi D_2^2}{4}\right) - 2\pi \frac{D_2}{2}\underbrace{\left[\int_0^L \tau_w(x)\, dx\right]}_{\bar{\tau}_w \cdot L}$$

$$\frac{\rho\pi}{4}\left[-v_1^2 D_1^2 - v_2^2 (D_2^2 - D_1^2) + v_3^2 D_2^2\right] = (P_A - P_B)\left(\frac{\pi D_2^2}{4}\right) - 2\pi \frac{D_2}{2} \bar{\tau}_w \cdot L$$

$$\rho\left[v_1^2 D_1^2 + v_2^2 (D_2^2 - D_1^2) - v_3^2 \cdot D_2^2\right] = (P_B - P_A)\, D_2^2 + 8\frac{D_2}{2}\bar{\tau}_w \cdot L$$

$$P_B - P_A = \frac{\rho\left[v_1^2 D_1^2 + v_2^2(D_2^2 - D_1^2) - v_3^2 . D_2^2\right] - 4.L\, D_2 . \overline{\tau_w}}{D_2^2}$$

$$P_B - P_A = \rho\left[v_1^2\left(\frac{D_1}{D_2}\right)^2 + v_2^2\left(1 - \left(\frac{D_1}{D_2}\right)^2\right) - v_3^2\right] - \frac{4L\overline{\tau_w}}{D_2} \quad //$$

c) u_A

$u_B = ?$ $\qquad h_f = (u_B - u_A)/g$

$h_f = ?$

Balanço de energia: $\displaystyle \cancel{\frac{d}{dt}\iiint_{vc}\left(u + \frac{v^2}{2} + gz\right)\rho\, dVol} + \iint_{svc}\rho\left(u + \frac{v^2}{2} + gz\right)(\vec{v}.\vec{n})\, dA =$

Estacionário.

$$= \dot{Q} - \dot{W}_m - \dot{W}_p - \dot{W}_v$$

Adiabático: $\dot{Q} = 0$

desprezam-se $\dot{W}_v = 0$ $\qquad \dot{W}_p = \int_{svc} P(\vec{v}.\vec{n})\, dA$

Sem troca de trabalho c/ veio $\dot{W}_m = 0$

desprezam-se as forças mássicas $\vec{g} \simeq \vec{0}$

$$\iint_{svc}\rho\left(u + \frac{v^2}{2}\right)(\vec{v}.\vec{n})\, dA = -\iint_{svc} P(\vec{v}.\vec{n})\, dA$$

$$\iint_{svc}\rho\left[u + \frac{P}{\rho} + \frac{v^2}{2}\right](\vec{v}.\vec{n})\, dA = 0$$

$$\rho\left(u_A + \frac{p_A}{\rho} + \frac{v_1^2}{2}\right)(-v_1)\cdot\frac{\pi D_1^2}{4} +$$

$$\rho\left(u_A + \frac{p_A}{\rho} + \frac{v_2^2}{2}\right)(-v_2)\frac{\pi}{4}\left(D_2^2 - D_1^2\right) +$$

$$\rho\left(u_B + \frac{p_B}{\rho} + \frac{v_3^2}{2}\right)(+v_3)\frac{\pi}{4}\cdot D_2^2 = 0 \iff$$

$$\left(u_A + \frac{p_A}{\rho} + \frac{v_1^2}{2}\right)(-v_1)D_1^2 + \left(u_A + \frac{p_A}{\rho} + \frac{v_2^2}{2}\right)(-v_2)\left(D_2^2 - D_1^2\right) +$$

$$\left(u_B + \frac{p_B}{\rho} + \frac{v_3^2}{2}\right)(v_3)D_2^2 = 0$$

$$u_B + \frac{p_B}{\rho} + \frac{v_3^2}{2} = \left(u_A + \frac{p_A}{\rho} + \frac{v_1^2}{2}\right)\left(\frac{v_1}{v_3}\right)\left(\frac{D_1}{D_2}\right)^2 +$$

$$+\left(u_A + \frac{p_A}{\rho} + \frac{v_2^2}{2}\right)\left(\frac{v_2}{v_3}\right)\left(1 - \left(\frac{D_1}{D_2}\right)^2\right)$$

$$u_B = \frac{\left(u_A + \frac{p_A}{\rho} + \frac{v_1^2}{2}\right)\left(\frac{v_1}{v_2}\right)\left(\frac{D_1}{D_2}\right)^2 + \left(u_A + \frac{p_A}{\rho} + \frac{v_2^2}{2}\right)\left(\frac{v_2}{v_3}\right)\left(1 - \left(\frac{D_1}{D_2}\right)^2\right)}{\left(\frac{p_B}{\rho} + \frac{v_3^2}{2}\right)}$$

$$h_f = (u_B - u_A)/g$$

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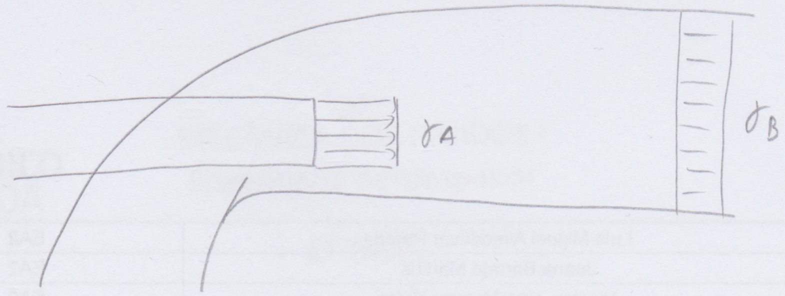
$\Gamma \quad m \quad P_1$

$\gamma = \frac{\Gamma}{m}$ $\quad \gamma_A$



$\gamma_B = ?$ $\quad \Gamma$

$$\beta = \frac{d\Gamma}{dm} = \gamma \qquad \cancel{\frac{d}{dt}\iiint_{VC} \beta \, dVol} + \iint_{SVC} \beta \rho (\vec{v} \cdot \vec{n}) \, dS = 0$$

Estacionário

(Γ é conservado)

$$\iint_{SVC} \gamma \rho (\vec{v} \cdot \vec{n}) \, dS = 0$$

$$+\gamma_A \cancel{\rho} (-v_1) \frac{\cancel{\pi} D_1^2}{\cancel{4}} + \gamma_B \cancel{\rho} (+v_3) \frac{\cancel{\pi} D_2^2}{\cancel{4}} = 0$$

$$-\gamma_A v_1 D_1^2 + \gamma_B v_3 D_2^2 = 0$$

$$\gamma_B = \gamma_A \left(\frac{D_1}{D_2}\right)^2 \left(\frac{v_1}{v_3}\right) //$$