Problema 2

Datos $R = 287 \ J/(kg\,K)$

$\gamma = 1,4$

$\beta = \frac{1}{2}\frac{\gamma+1}{\gamma-1} = 3$

$A_1 = 0,1 \ m^2$

$A_2 = 0,14 \ m^2$

$A_3 = 0,2 \ m^2$

$T_0 = 353 \ K$

$T_1 = 323 \ K$

$\dot{m} = 200 \ kg/s$

a)

$$\frac{T_0}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2 \Rightarrow M_1 = 0,681$$

$$\frac{\dot{m}}{A_1}\frac{\sqrt{T_0}}{P_0} = \sqrt{\frac{\gamma}{R}}\, M_1 \left[\left(1 + \frac{\gamma-1}{2} M_1^2\right)\right]^{-\beta}$$

$$\Rightarrow P_0 = 1030543,8 \ Pa$$

b) Caudal máximo $\Rightarrow A_1 = A_1^*$

$$\frac{A_3}{A_1^*} = \frac{1}{M_3}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M_3^2\right)\right]^{\beta} \Rightarrow M_3 = 2,197$$

$$P_3 = \frac{P_0}{\left[1 + \frac{\gamma-1}{2} M_3^2\right]^{\gamma/\gamma-1}} = 96801,7 \ Pa = P_4$$

c) $\frac{A_2}{A_{2A}^*} = \frac{1}{M_{2A}}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_{2A}^2\right)\right]^{\beta}$ com $A_{2A}^* = A_1^*$

$M_{2A} = 1{,}763$ (imediatamente antes da O.C.)

$$M_{2B} = \sqrt{\frac{(\gamma-1)M_{2A}^2+2}{2\gamma M_{2A}^2-(\gamma-1)}} = 0{,}625$$

$$\frac{A_{2B}^*}{A_{2A}^*} = \frac{M_{2B}}{M_{2A}}\left[\frac{2+(\gamma-1)M_{2A}^2}{2+(\gamma-1)M_{2B}^2}\right]^{\beta} = 1{,}206$$

$$P_{oB} = \frac{P_{oA}}{\left(\frac{A_{2B}^*}{A_{2A}^*}\right)} = 85\,41\,66{,}6 \text{ Pa com } P_{oA} = P_o$$

$$A_{2B}^* = \frac{A_{2B}^*}{A_{2A}^*} \cdot A_{2A}^* = 0{,}121 \text{ m}^2$$

$$\frac{A_3}{A_{2B}^*} = \frac{1}{M_3}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_3^2\right)\right]^{\beta} \rightarrow M_3 = 0{,}380$$

$$P_3 = \frac{P_{oB}}{\left[1+\frac{\gamma-1}{2}M_3^2\right]^{\gamma/\gamma-1}} = 77\,30\,44{,}4 \text{ Pa} > P_3 \text{ medido}$$

$\Rightarrow$ a O.C. ocorre a jusante da secção 2

d)