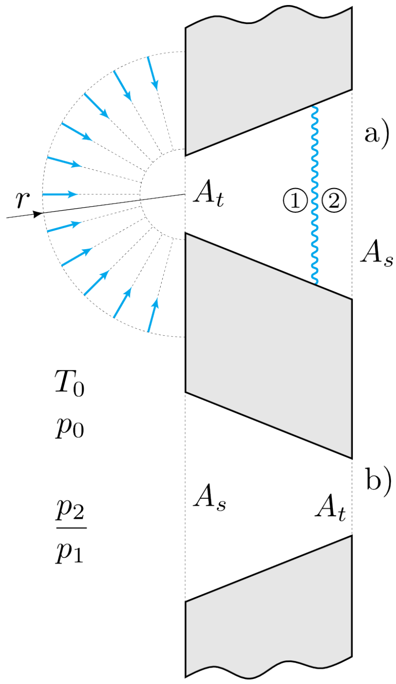


Resolução do grupo II do 2º Teste de MF1 / MEAer

a)

Como existe onda de choque o escoamento é sónico na garganta, M = 1, logo

$$\frac{T^*}{T_0} = \frac{2}{\gamma+1} = 0.8333$$

$$T^* = 0.8333 \times 333 = 277.5\,\text{K}$$

$$u^* = c^* = \sqrt{\gamma R T^*} = 333.9\,\text{m/s}$$

b)

Como as linhas de corrente são radiais a 2.5 cm da entrada (ver figura), podemos assumir que o escoamento é uniforme na semi-esfera, m. Escoamento é sónico na garganta o que implica que $A_t = A_1^*$. Logo a razão de áreas entre a semi-esfera e a garganta, A_m/A_1^*, está relacionada com o número de Mach na semi-esfera, M_m,

$$\frac{A_m}{A_1^*} = \frac{1}{\mathrm{M}_m}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}\mathrm{M}_m^2\right)\right]^{\frac{1}{2}\frac{\gamma+1}{\gamma-1}}$$

Usando as tabelas de escoamento isentrópico ou resolvendo numéricamente, obtemos

$$\frac{A_m}{A_1^*} = \frac{2\pi\, 2.5^2}{3} \Rightarrow \mathrm{M}_m = 0.04423$$

Sabendo o número de Mach na semi-esfera, podemos conhecer a velocidade se soubermos a temperatura

$$T_m = T_0\left(1+\frac{\gamma-1}{2}\mathrm{M}^2\right)^{-1} = 333\left(1+\frac{\gamma-1}{2}0.04423^2\right)^{-1} = 332.86\,\text{K}$$

$$u_m = \mathrm{M}_m c_m = \mathrm{M}_m\sqrt{\gamma R T_m} = 16.18\,\text{m/s}$$

c)

Usando as tabelas de escoamento isentrópico

$$\frac{p_2}{p_1} = 4.5 \Rightarrow \mathrm{M}_1 = 2,\ \mathrm{M}_2 = 0.5774,\ \frac{A_2^*}{A_1^*} = \frac{p_{01}}{p_{02}} = 1.3872,\ \frac{p_{02}}{p_{01}} = 0.7209$$

d)

$$\frac{A_s}{A_2^*} = \frac{A_s}{A_1^*}\frac{A_1^*}{A_2^*} = \frac{6}{3}\frac{1}{1.3872} = 1.4418$$

Sabendo que

$$\frac{A_s}{A_2^*} = \frac{1}{\mathrm{M}_s}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}\mathrm{M}_s^2\right)\right]^{\frac{1}{2}\frac{\gamma+1}{\gamma-1}}$$

usando as tabelas de escoamento isentrópico ou resolvendo numéricamente, obtemos

$$\frac{A_s}{A_2^*} = 1.4418 \Rightarrow \mathrm{M}_s = 0.4528$$

(solução subsónica após a onda de choque). Consultando as tabelas de escoamento isentrópico ou aplicando as relações isentrópicas

$$T_s = T_0\left(1+\frac{\gamma-1}{2}\mathrm{M}_s^2\right)^{-1} = 319.9\,\mathrm{K}$$

Sabendo que T_0 é constante (escoamento adiabático), a pressão de saída é função de p_{02}

$$p_{02} = \frac{p_{02}}{p_{01}}p_{01} = 0.7209\times 1.6\times 10^5 = 115344\,\mathrm{Pa}$$

logo

$$\frac{p_{02}}{p_s} = \left(\frac{T_0}{T_s}\right)^{\frac{\gamma}{\gamma-1}} = 1.151$$

donde

$$p_s = \frac{115344}{1.151} = 1.0021\times 10^5\,\mathrm{Pa}$$

e)

Podemos calcular o caudal na garganta.

$$\dot{m}_{\mathrm{I}} = \rho^* u^* A_1^*$$

A massa volúmica na garganta é dada por

$$\rho^* = \rho_0\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} = \frac{p_0}{RT_0}\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} = 1.06\,\mathrm{kg/m^3}$$

logo (velocidade calculada na alínea a)

$$\dot{m}_{\mathrm{I}} = \rho^* u^* A_1^* = 0.106\,\mathrm{kg/s}$$

f)

$$\frac{p_e}{p_0} = \frac{10^5}{1.6\times 10^5} = 0.625 > \frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 0.5283$$

O escoamento não está bloqueado na garganta. Usando a relação isentrópica entre a pressão de estagnação do reservatório e a pressão de saída

$$\frac{p_0}{p_e} = \left(1+\frac{\gamma-1}{2}\mathrm{M}_e^2\right)^{\frac{\gamma}{\gamma-1}} = 1.6$$

obtemos

$$\mathrm{M}_e = \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{p_0}{p_e}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} = 0.848$$

$$T_e = T_0 \left(1 + \frac{\gamma - 1}{2}\mathrm{M}_e^2\right)^{-1} = 291.1\,\mathrm{K}$$

Como sabemos a pressão e a temperatura de saída conhecemos o caudal facilmente,

$$\rho_e = \frac{p_e}{RT_e} = 1.197\,\mathrm{kg/m^3}$$

$$u_e = \mathrm{M}_e c_e = \mathrm{M}_e\sqrt{\gamma R T_e} = 290\,\mathrm{m/s}$$

logo

$$\dot{m}_{\mathrm{II}} = \rho_e u_e A_t = 0.104\,\mathrm{kg/s}$$

g)

Como o escoamento se encontra bloqueado $\dot{m}_{\mathrm{I}}$ corresponde ao caudal máximo, logo $\dot{m}_{\mathrm{I}} > \dot{m}_{\mathrm{II}}$.

Exame MFI(MAem) · 2º EXAME (2012/2013)

a) $V_e(x) = K.x$ $P(x) = ?$ $\frac{\partial P}{\partial x} = ?$

P_0 = Pressão Est. (Referência)

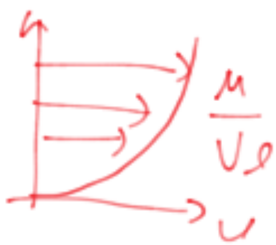
$$P_0 = P_e(x) + \frac{1}{2}\rho V_e^2(x) \quad (=)$$

$$P_e(x) = P_0 - \frac{1}{2}\rho U_e^2(x) = P_0 - \frac{1}{2}\rho\left[K.x\right]^2 =$$

$$= P_0 - \frac{1}{2}\rho K^2.x^2 \;//$$

$$\frac{\partial P_e}{\partial x} = -\frac{1}{2}\rho K^2.2.x = -\rho K^2.x$$

b) $$\frac{u(x,y)}{V_e(x)} = a.\sin\left[b\left(\frac{y}{\delta(x)}\right)\right] \qquad a,b \text{ Const.}$$



$\frac{u}{V_e} = a\sin(b\eta)$

$y = \delta \;:\; u = V_e$

$y = 0 \;:\; u = \phi$

$y = \delta \;:\; \frac{\partial u}{\partial y} = \phi$

$$\frac{u(x,y)}{v_e(x)} = a.\sin\left[b\left(\frac{y}{\delta(x)}\right)\right] \qquad a,b \text{ const.}$$

$$\frac{u}{V_e} = a\sin(b\eta)$$

$$y=\delta : \; u = v_e \qquad \text{i)}$$

$$y=0 : \; u = \phi \qquad \text{ii)}$$

$$y=\delta : \; \frac{\partial u}{\partial y} = \phi \qquad \text{iii)}$$

$$\left.\begin{array}{ll} \text{i)} & \eta=1 : \; a.\sin(b)=1 \\ \text{ii)} & \eta=0 : \; a.\sin(b.\phi)=0 \\ \text{iii)} & \eta=1 : \; ab\cos(b)=\phi \end{array}\right\} \quad a=1,0 \qquad b=\frac{\pi}{2}$$

$$\boxed{\frac{u(x,y)}{v_e(x)} = \sin\left[\frac{\pi}{2}.\eta\right]}$$

$$f'(0) = \left.\frac{\partial f}{\partial \eta}\right|_{\eta=0} = \frac{\pi}{2}.\cos\left[\frac{\pi}{2}.\phi\right] = \frac{\pi}{2} \;//$$

$$H = \frac{\delta^*}{\theta}$$

$$\frac{\delta^*}{\delta} = \int_0^1 [1 - f(\eta)]\, d\eta = \int_0^1 \left[1 - \sin\left(\frac{\pi}{2}\right)\right] d\eta =$$

$$= \left[\eta + \frac{2}{\pi} \cdot \cos\left(\frac{\pi}{2}\eta\right)\right]_0^1 = 1 + \frac{2}{\pi}\cos\left(\frac{\pi}{2}\right) - \frac{2}{\pi} \cdot \cos(\phi) =$$

$$= 1 - \frac{2}{\pi} = \frac{\pi - 2}{\pi} \quad //$$

$$\frac{\theta}{\delta} = \int_0^1 f(\eta)\,[1 - f(\eta)]\, d\eta = \int_0^1 \sin\left(\frac{\pi}{2}\eta\right)\left[1 - \sin\left(\frac{\pi}{2}\eta\right)\right] d\eta$$

$$= \int_0^1 \left[\sin\left(\frac{\pi}{2}\eta\right) - \sin^2\left(\frac{\pi}{2}\eta\right)\right] d\eta =$$

$$= \left[-\frac{2}{\pi}\cos\left(\frac{\pi}{2}\eta\right) - \frac{\eta}{2} + \frac{\sin(\pi\eta)}{4 \cdot \pi/2}\right]_0^1 = \frac{4 - \pi}{2\pi}$$

$$\int \sin^2(a.x)\,dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax)$$

$$\int \sin(ax)\,dx = -\frac{\cos(ax)}{a}$$

$$H = \frac{\delta^*}{\theta} = \frac{\frac{\pi-2}{\pi}}{\frac{4-\pi}{2\pi}} = \frac{2(\pi-2)}{\cancel{\pi}} \cdot \frac{\cancel{\pi}}{4-\pi} = \frac{2(\pi-2)}{4-\pi}$$

$$= 2{,}659 \simeq 2{,}66 /\!/$$

$$\frac{\theta}{\delta} = \frac{4-\pi}{2\pi} = 0{,}136 /\!/$$

e) $$\frac{d\theta}{dx} + \frac{\theta(H+2)}{v_e(x)}\frac{dv_e}{dx} = \frac{C_f}{2}$$

$$\frac{C_f}{2} = \frac{\tau_\omega}{\rho v_e^2} = \frac{\mu \left.\frac{\partial u}{\partial y}\right|_{y=0}}{\rho v_e^2(x)}$$

$$\mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu \left.\frac{\partial(v_e f)}{\partial(\delta \cdot \eta)}\right|_{\eta=0} = \mu \cdot \frac{v_e(x)}{\delta(x)} \left.\frac{\partial f}{\partial \eta}\right|_{\eta=0}$$

$$\tau_\omega = \mu \frac{v_e(x)}{\delta(x)} \cdot \frac{\pi}{2} //$$

$$\frac{C_f}{2} = \mu \frac{v_e(x)}{\delta(x)} \cdot \frac{\pi}{2} \times \frac{1}{\rho v_e^2(x)} = \nu \cdot \frac{\pi}{2} \cdot \frac{1}{\delta(x) v_e(x)}$$

$$= \nu \frac{\pi}{2} \frac{1}{\delta(x) \cdot K \cdot x}$$

$$\frac{d\theta}{dx} = \frac{d}{dx}\left[\frac{\theta}{\delta} \cdot \delta\right] = \left[\frac{\theta}{\delta}\right]\frac{d\delta}{dx}$$

$$\frac{d\theta}{dx} + \frac{\theta(H+2)}{v_e(x)}\frac{dv_e}{dx} = \frac{C_f}{2}$$

$$\frac{C_f}{2} = \nu\frac{\pi}{2}\frac{1}{\delta(x)K.x} \qquad \frac{d\theta}{dx} = \left[\frac{\theta}{\delta}\right]\frac{d\delta}{dx}$$

$$\frac{\theta(H+2)}{v_e(x)}\frac{dv_e}{dx} = \left[\frac{\theta}{\delta}\right]\frac{\delta(x)[H+2]}{K.x}.K$$

$$\frac{dv_e}{dx} = \frac{d}{dx}[K.x] = K$$

Juntando tudo:

$$\left[\frac{\theta}{\delta}\right]\frac{d\delta}{dx} + \left[\frac{\theta}{\delta}\right]\frac{\delta(x)(H+2)}{K.x}.K = \nu\frac{\pi}{2}\frac{1}{\delta(x)Kx}$$

$$\left[\frac{\theta}{\delta}\right]\frac{d\delta}{dx}+\left[\frac{\theta}{\delta}\right]\frac{\delta(x)(H+z)}{K.x}.K = J\frac{\pi}{2}\frac{1}{\delta(x)Kx}$$

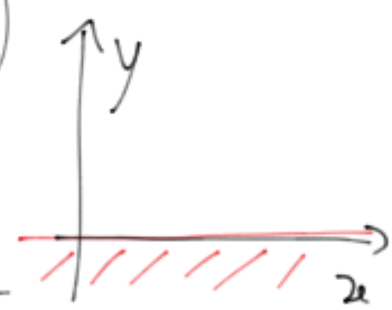
d) c/ $\underline{\delta(x)=C}$, temos, $\frac{df}{dx}=\phi$.

$$\left[\frac{\theta}{\delta}\right]\cancel{\frac{df}{dx}}+\left[\frac{\theta}{\delta}\right]\frac{c(H+z)}{\cancel{K}.x}.\cancel{K} = J\,\frac{\pi}{2}\cdot\frac{1}{c.K.x}$$

$$\left[\frac{\theta}{\delta}\right]\frac{c(H+z)}{\cancel{x}} = J\frac{\pi}{2}\,\frac{1}{cK.\cancel{x}}$$

$$C=\sqrt{\frac{J.\pi}{2K(H+z)\left[\frac{\theta}{\delta}\right]}}$$

e) $F_H, F_V = ?$ $\quad (x=0 \longrightarrow x=L)$



$$F_V = -\int P\vec{n}\cdot dA = -b\int_0^L P(x)\cdot dx =$$

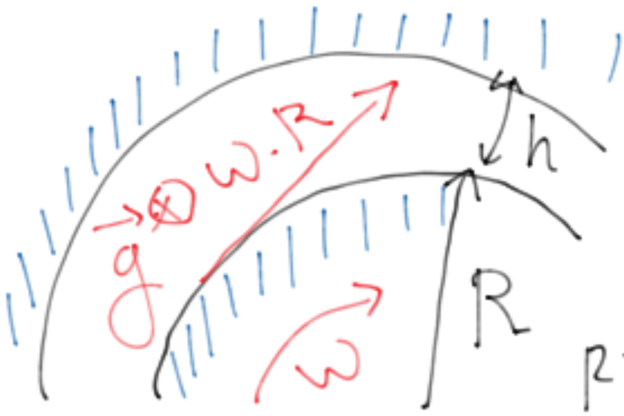
$$= -b\int_0^L \left(P_0 - \frac{1}{2}\rho K^2 x^2\right)dx = -b\left[P_0 . x - \frac{1}{2}\rho K^2 \frac{x^3}{3}\right]_0^L$$

$$= -b\left[P_0 . L - \frac{1}{6}\rho K^2 L^3\right] //$$

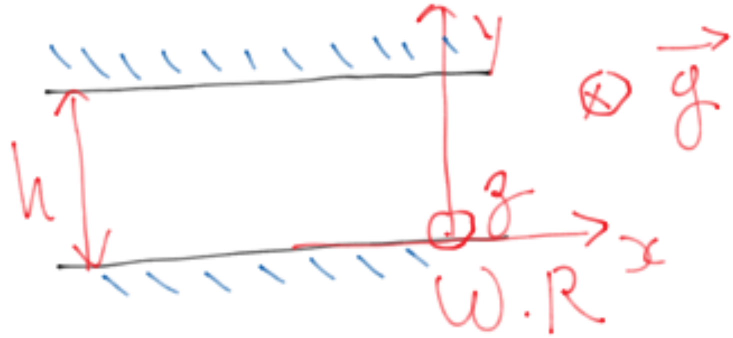
$$F_H = b\int_0^L \tau_w(x)\,dx = b\mu\frac{\pi}{2}\int_0^L \frac{v_e(x)}{\delta(x)}\,dx$$

$$= \mu b\frac{\pi}{2}\int_0^L \frac{K.x}{c}.dx = \frac{\mu b \pi K}{2c}\left[\frac{x^2}{2}\right]_0^L =$$

$$= \frac{\mu b K L^2}{2c}\left(\frac{\pi}{2}\right)$$

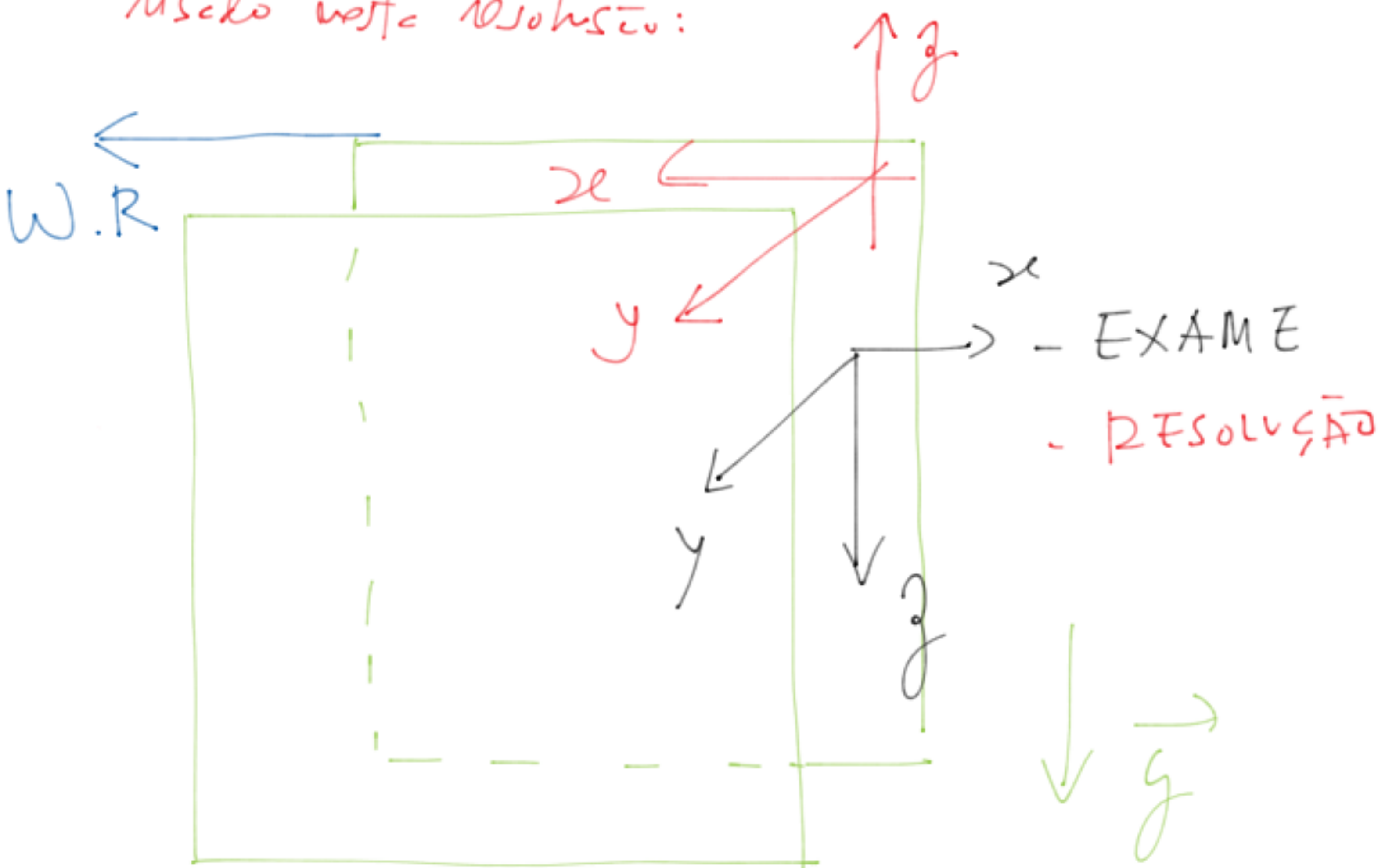


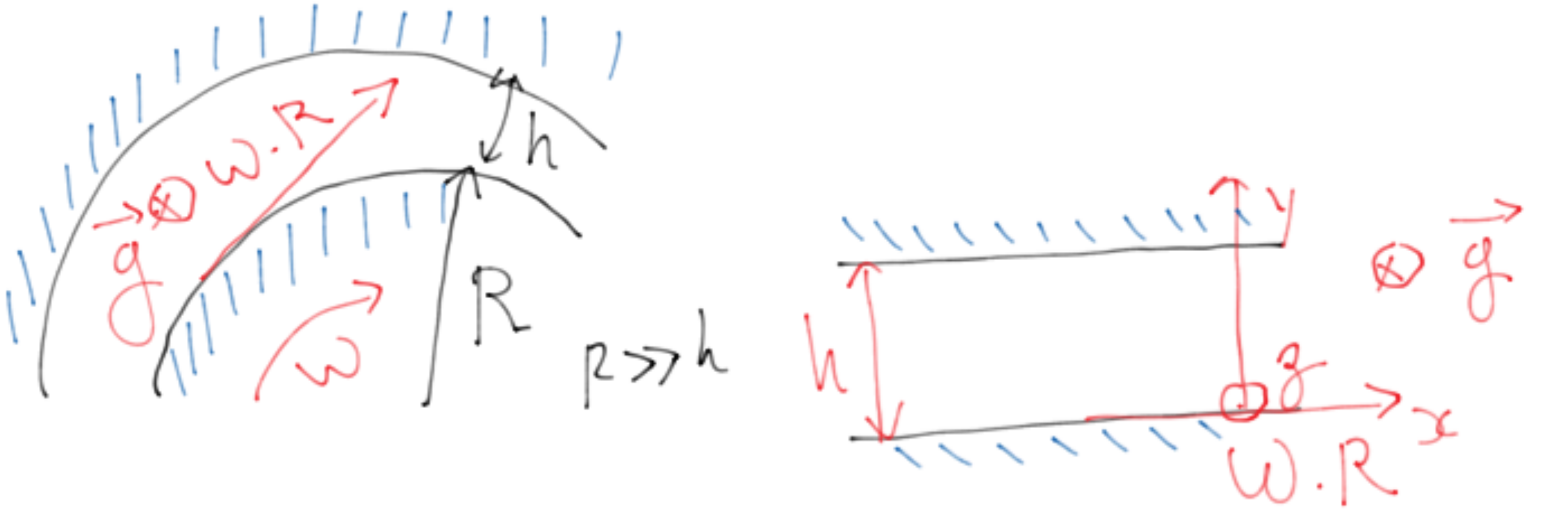
$R \gg h$



PROBLEMA I
2º EXAME MFI
MAIO 2012/2013

ATENÇÃO:
o material colocado no exame é diferente do usado nesta resolução:





$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Esc. completamente desenvolvido:

$$\frac{\partial u}{\partial x} = 0 \; ; \; \frac{\partial w}{\partial z} = 0 \Rightarrow \frac{\partial v}{\partial y} = 0 \qquad y = h : v(h) = 0 \Rightarrow v(y) = 0 \;\; \forall y.$$

$$\cancel{\frac{\partial u}{\partial t}} + u\cancel{\frac{\partial u}{\partial x}} + v\cancel{\frac{\partial u}{\partial y}} + w\cancel{\frac{\partial u}{\partial z}} = -\frac{1}{\rho}\cancel{\frac{\partial p}{\partial x}} + \nu\left[\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right] + \cancel{g_x}$$

Est. $v = 0$ desenvolvido $\nabla p = 0$ comp desenvolvido $= 0$

$$\frac{\partial^2 u}{\partial y^2} = 0 \Rightarrow \frac{\partial u}{\partial y} = \text{Const.} \Rightarrow u = C_1 \cdot y + C_2$$

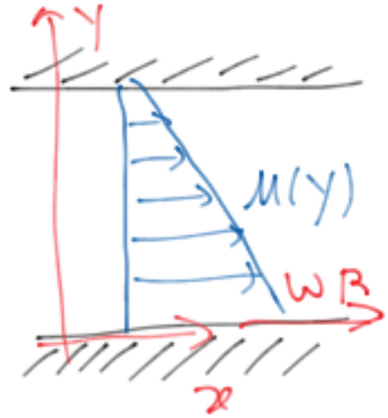
Nos fluidos:

$y = 0 \rightarrow u = \omega . R \Leftrightarrow \omega . R = C_2$

$y = h \rightarrow u = 0 \Leftrightarrow 0 = C_1 . h + \omega . R \Rightarrow C_1 = -\dfrac{\omega . R}{h}$

$$u(y) = -\frac{\omega . R}{h} . y + \omega . R =$$

$$\boxed{u(y) = \omega . R \left[-\frac{y}{h} + 1 \right]}$$



$v = 0$ Desenv.

$$\cancel{\frac{\partial w}{\partial t}} + u \cancel{\frac{\partial w}{\partial x}} + \cancel{v \frac{\partial w}{\partial y}} + w \cancel{\frac{\partial w}{\partial z}} = -\frac{1}{\rho} \cancel{\frac{\partial p}{\partial z}} +$$

Est. Desenv. $\nabla P = 0$

$$\nu \left[\cancel{\frac{\partial^2 w}{\partial x^2}} + \frac{\partial^2 w}{\partial y^2} + \cancel{\frac{\partial^2 w}{\partial z^2}} \right] + g_z$$

Desenvolvidos

$$\nu \frac{\partial^2 w}{\partial y^2} + g = 0$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{-g}{\nu}$$

$$\frac{\partial w}{\partial y} = -\frac{g}{\nu} . y + C_3$$

$$w(y) = \frac{-g}{\nu} . \frac{y^2}{2} + C_3 . y + C_4$$

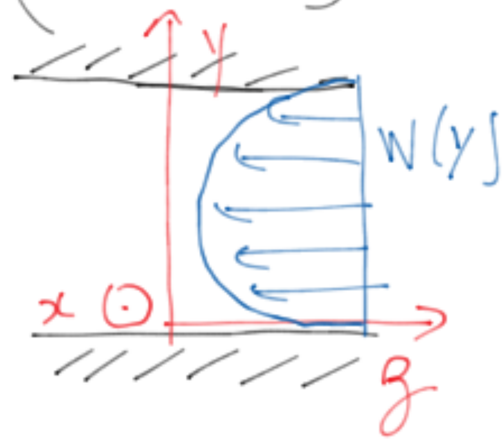
Condisões fronteira:

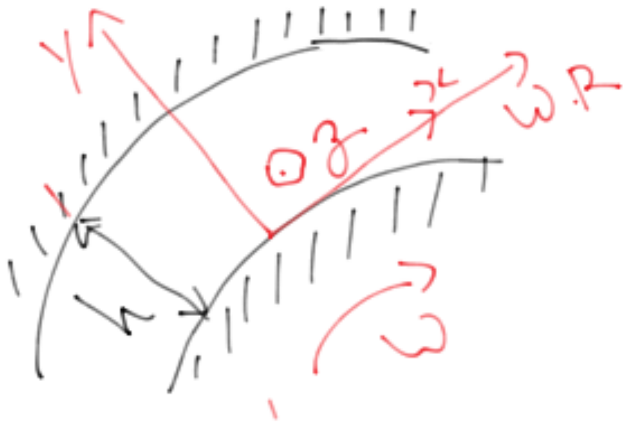
$y = 0 \Rightarrow w = 0 \Leftrightarrow 0 = \frac{-g}{\nu} . 0 + 0 + C_4 \Rightarrow C_4 = 0$

$y = h \Rightarrow w = 0 \Leftrightarrow 0 = \frac{-g}{\nu} . \frac{h^2}{2} + C_3 . h \Rightarrow C_3 = gh/_{2\nu}$

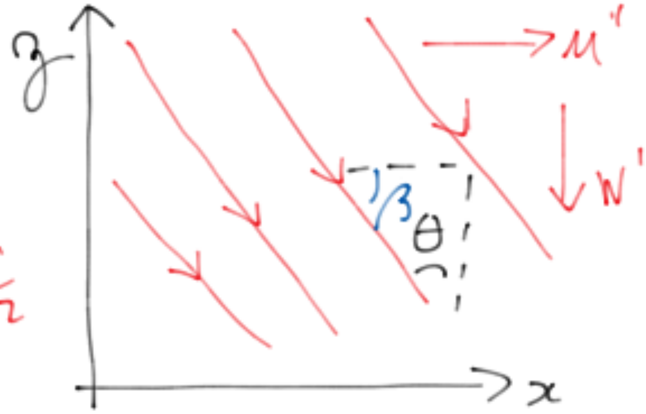
$$W(y) = -\frac{g}{\nu}\cdot\frac{y^2}{2} + \frac{gh}{2\nu}\cdot y = -\frac{g}{2\nu}\left(y^2 - h\cdot y\right)$$

$$\boxed{W(y) = -\frac{gh}{2\nu}\left(\frac{y^2}{h} - y\right)}$$





linhas de corrente no plano $(x, y = \frac{h}{2}, z)$



$y = \frac{h}{2}$:

$$u' = \frac{\omega \cdot R}{2}$$

$$\tan\theta = \frac{u'}{w'} \qquad \tan\beta = \frac{w'}{u'}$$

$$w' = -\frac{gh}{2\nu}\left(\frac{h^2}{4h} - \frac{h}{2}\right) = -\frac{gh}{2\nu}\left(\frac{h}{4} - \frac{h}{2}\right) =$$

$$= -\frac{gh}{2\nu}\left(\frac{h-2h}{4}\right) = \frac{2gh^2}{8\nu} = \frac{gh^2}{4\nu}$$

Momento T_z p/ Unidade de Comprimento Vertical :

$$T_z = \int_0^{2\pi}\int_0^{z} (\vec{r} \times \tau_w)\, dA = 2\pi \cdot \int_0^{z} |\vec{r}| \cdot \tau_w \, dA$$

Módulo tensão cortante parietal $|\vec{\tau_w}|$ p/ $y = \phi$

$$|\vec{\tau_w}| = \sqrt{(\tau_{xy}^{w})^2 + (\tau_{zy}^{w})^2}$$

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \cancel{\frac{\partial v}{\partial x}}\right)$$

$$\tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial w}{\partial y} + \cancel{\frac{\partial v}{\partial z}}\right)$$

Esc. completamente desenvolvido.

$$\frac{\partial w}{\partial y} = \frac{g}{\nu} \cdot y - \frac{g \cdot h}{\nu}$$

$$\frac{\partial u}{\partial y} = -\frac{\omega \cdot R}{h}$$

$$\frac{\partial w}{\partial y} = -\frac{g}{\nu}\cdot y + \frac{g\cdot h}{2\nu}$$

$$\frac{\partial u}{\partial y} = -\frac{\omega\cdot R}{h}$$

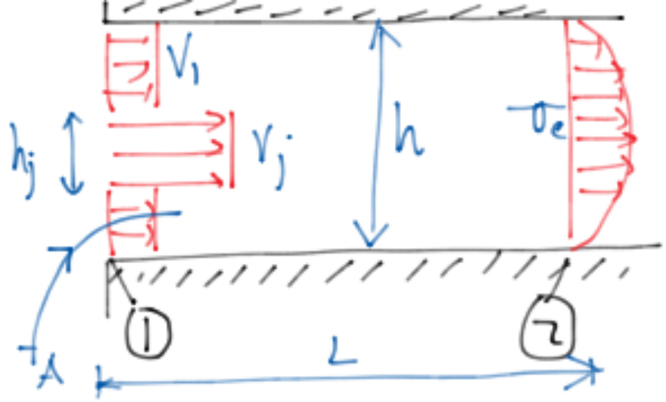
Na placa fixa $y=h$:

$$\tau_{yz} = \mu\cdot\left[-\frac{g}{\nu}\cdot h + \frac{g\cdot h}{2\nu}\right] = \frac{\mu g h}{\nu}\left(-1+\frac{1}{2}\right) =$$

$$= -\frac{\mu g h}{2\nu} = -\frac{\rho g h}{2}$$

$$\tau_{yx} = \mu\left[-\frac{\omega\cdot R}{h}\right] = -\frac{\mu\omega R}{h}$$

PROBLEMA I
31/10/2013 - MAno



- laminar
- Estacionário
- incompressível, ρ
- Newtoniano, μ 2D

h_j h V_j V_1

1) A: $p = p_0$ $y' \simeq 0$ $p_1 = ?$ [1,0 Val.]
$|\vec{V}| \simeq 0$

linhas de corrente suaves.
efeitos de viscosidade desprezíveis
. Bernoulli:

$$\frac{p_A}{\rho g} + \cancel{\frac{V_A^2}{2g}} + \cancel{z_A} = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1}$$

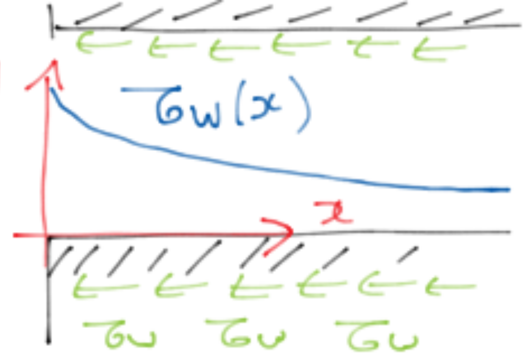
$z_1 \simeq z_A$
$V_A \simeq 0$
$p_A = p_0$

$$\frac{p_0}{\rho g} = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} \Leftrightarrow$$

$$\frac{p_1}{\rho} = \frac{p_0}{\rho} - \frac{V_1^2}{2} \Leftrightarrow \boxed{p_1 = p_0 - \frac{1}{2}\rho V_1^2}$$

2) $\tau_w(x) = \tau_0 . e^{(-x/L)}$

$D_\mu = ?$ [1,5 Val.]

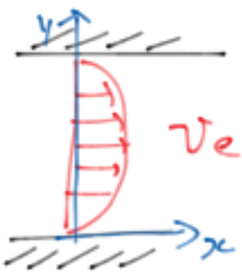


$$D_\mu = 2\int_0^L \tau_w(x)\,dx = 2\tau_0 \int_0^L e^{-x/L}\,dx =$$

$$= 2\tau_0 \left[-L . e^{-x/L} \right]_0^L = -2L\tau_0 \left(e^{-1} - e^{0} \right) =$$

$$\boxed{= 2L\tau_0 \left(1 - \frac{1}{e} \right)}$$

3) $v_e = ?$ $\quad u(y) = a + by + cy^2$ [2,0 Val.]



$$\left.\begin{array}{lll} y=0 : & u=0 & \Rightarrow \quad a=0 \\ y=h : & u=0 & \Rightarrow \quad 0 = b.h + c.h^2 \\ y=\frac{h}{2} : & u=v_e & \Rightarrow \quad v_e = b\left(\frac{h}{2}\right) + c\left(\frac{h}{2}\right)^2 \end{array}\right\}$$

$$\left\{\begin{array}{l} 0 = b + c.h \quad \Leftrightarrow \quad b = -c.h \\ \frac{v_e}{h} = b\left(\frac{1}{2}\right) + ch\left(\frac{1}{4}\right) \quad \Leftrightarrow \quad \frac{v_e}{h} = -c.h\left(\frac{1}{2}\right) + ch\left(\frac{1}{4}\right) \quad \Leftrightarrow \end{array}\right.$$

$$\Leftrightarrow \frac{v_e}{h^2} = -c\left(\frac{1}{2}\right) + c\left(\frac{1}{4}\right) \Leftrightarrow \frac{4v_e}{h^2} = -2c + c = -c$$

$$c = -\frac{4 v_c}{h^2}$$

$$b = -c \cdot h = \frac{4 v_c}{h}$$

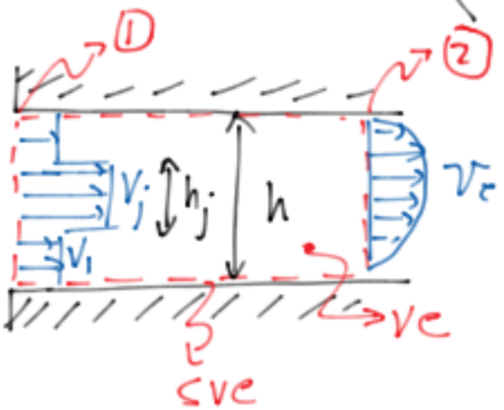
$$a = 0$$

$$u(y) = a + by + cy^2 =$$

$$= \frac{4 v_c}{h} \cdot y - \frac{4 v_c}{h^2} \cdot y^2$$

$$\frac{u(y)}{v_c} = 4\left(\frac{y}{h}\right) - 4\left(\frac{y}{h}\right)^2$$

$$\left(\begin{array}{l} y = 0;\ y = h \ :\ u = 0 \\ y = \frac{h}{2} \ :\ u = v_c \end{array} \right)$$



Balanço de Massa:

$$\cancel{\frac{d}{dt}\iiint_{vc} \rho \, dVol} + \iint_{svc} \cancel{\rho} (\vec{v} \cdot \vec{n}) \, ds = 0$$

Estacionário

incompressível

$$\iint (\vec{v} \cdot \vec{n}) \, ds = 0 \iff$$

$$-V_j h_j - V_1 (h - h_j) + \iint_{S_2} (\vec{v} \cdot \vec{n}) \, ds = 0$$

$$\iint_{S_2} (\vec{v} \cdot \vec{n}) \, ds = \int_0^h u(y) \cdot dy$$

$$u(y) = 4 v_c \left[\frac{y}{h} - \left(\frac{y}{h}\right)^2 \right]$$

$$\int_0^h u(y)\,dy = 4v_c\int_0^h\left[\frac{y}{h}-\left(\frac{y}{h}\right)^2\right]dy = 4v_c\left[\frac{y^2}{2h}-\frac{y^3}{3h^2}\right]_0^h =$$

$$= 4v_c\left(\frac{h^2}{2h}-\frac{h^3}{3h^2}\right) = 4v_c h\left(\frac{1}{2}-\frac{1}{3}\right) =$$

$$= 4v_c h/6 = \frac{2}{3}v_c h$$

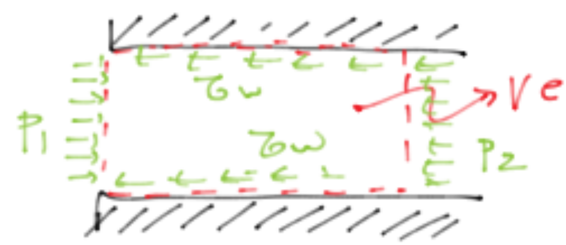
$$-V_j\cdot h_j - V_1(h-h_j) + \frac{2}{3}v_c . h = 0$$

$$V_j h_j + V_1 . h - V_1 . h_j = \frac{2}{3}v_c . h$$

$$v_c = \frac{3}{2}\cdot\frac{1}{h}\left[V_j h_j + V_1 . h - V_1 . h_j\right]$$

$$= \frac{3}{2}\left[V_j \frac{h_j}{h} + V_1 - V_1 . \frac{h_j}{h}\right]$$

$$\boxed{v_c = \frac{3}{2}\left[V_1 + \frac{h_j}{h}(V_j - V_1)\right]}$$



4) $p_2 = ?$ [1,5 Val.]

$$\vec{F} = \cancel{\frac{d}{dt}\iiint_{vc}\vec{v}\rho\, dVol} + \iint_{svc}\vec{v}\rho(\vec{v}\cdot\vec{n})\,dS$$

Estacionário

$$F_x = \int_{①} u\rho(\vec{v}\cdot\vec{n})\,ds + \int_{②} u\rho(\vec{v}\cdot\vec{n})\,ds \Leftrightarrow$$

$$\frac{p_1 . h - p_2 . h - D_x}{\rho} = -\int_{①} u^2(y)\,dy + \int_{②} u^2(y)\,dy$$

$$\int_0^h u_{①}^2(y)\,dy = \rho V_1^2 . (h - h_j) + \rho\, V_j^2 . h_j$$

$$\int_0^h u_{②}^2(y)\,dy = \int_0^h 4^2 v_c^2 \left[\frac{y}{h} - \left(\frac{y}{h}\right)^2\right]^2 dy = 16\, v_c^2 \int_0^h \left[\left(\frac{y}{h}\right)^2 + \left(\frac{y}{h}\right)^4 - 2\left(\frac{y}{h}\right)^3\right] \times dy$$

$$= 16\, v_c^2 \left[\frac{y^3}{3h^2} + \frac{y^5}{5h^4} - 2 . \frac{y^4}{4h^3}\right]_0^h = 16\, v_c^2 h \left[\frac{1}{3} + \frac{1}{5} - \frac{1}{2}\right] =$$

$$= 16\, v_c^2 . h \left[\frac{10 + 6 - 15}{30}\right] = v_c^2 . h \left(\frac{16}{30}\right) = \frac{8}{15} v_c^2 . h$$

$$\frac{p_1 . h - p_2 . h - D_x}{\rho} = \frac{8}{15} v_c^2 . h - V_1^2 (h - h_j) + V_j^2 h_j$$

$$\frac{p_1}{\rho} - \frac{p_2}{\rho} - \frac{D_x}{\rho} = \frac{8}{15} v_c^2 - V_1^2\left(1 - \frac{h_j}{h}\right) + V_j^2\left(\frac{h_j}{h}\right)$$

$$\boxed{\frac{p_2}{\rho} = \frac{p_1}{\rho} - \frac{D_\mu}{\rho} - \frac{8}{15} V_c^2 - V_1^2\left(1 - \frac{h_j}{h}\right) + V_j^2\left(\frac{h_j}{h}\right)}$$

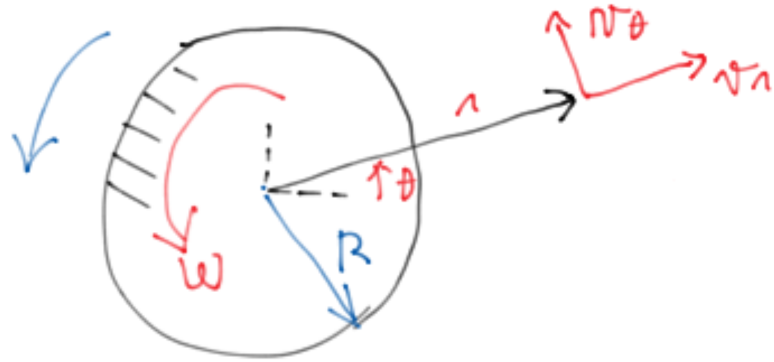
5) $h_f = ?$ [1,0 val.]

dissipação total devido à viscosidade:

$$\boxed{\frac{D_\mu}{\rho g} = \frac{2L\tau_0}{\rho g}\left(1 - \frac{1}{e}\right)}$$

- - -

PROBLEMA II
31/10/2013 - MAero



R, ω, ρ, μ

- Estacionário $\left| \dfrac{\partial}{\partial t} = 0 \right.$
- 2D $\left| \begin{array}{l} v_z = \dfrac{\partial}{\partial z} = \emptyset \\ g_\theta = g_r = \emptyset \end{array} \right.$

1) $v_r = ?$ Axissimétrico $\Rightarrow \dfrac{\partial}{\partial \theta} = 0.$ [1,0 Val.]

$\rho =$ constante $\Rightarrow \nabla \cdot \vec{v} = 0$ (incompressível)

$$\nabla \cdot \vec{v} = \frac{1}{r} \cdot \frac{\partial}{\partial r}(r v_r) + \frac{1}{r} \cdot \underbrace{\cancel{\frac{\partial v_\theta}{\partial \theta}}}_{\frac{\partial}{\partial \theta} = 0} + \underbrace{\cancel{\frac{\partial v_z}{\partial z}}}_{v_z = 0} = 0 \iff$$

$$\frac{\partial}{\partial r}(r v_r) = 0$$

$$r v_r = \text{constante.}$$

Cond. de impermeabilidade na parede do cilindro:

$$r = R \Rightarrow v_r = 0 \Rightarrow r v_r(R) = 0 \Rightarrow \boxed{v_r = 0 \quad \forall r.}$$

2) Simplifique: $\frac{Dv_r}{Dt}$ [1,0 Val.]

$$\cancel{\frac{\partial v_r}{\partial t}} + \cancel{v_r}\cdot\frac{\partial v_r}{\partial r} + \cancel{\frac{v_\theta}{r}\cdot\frac{\partial v_r}{\partial \theta}} - \frac{v_\theta^2}{r} + \cancel{v_z\cdot\frac{\partial v_r}{\partial z}} =$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial r} + \cancel{\frac{\mu}{\rho}}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\cdot\frac{\partial(r v_r)}{\partial r}\right) + \cancel{\frac{1}{r^2}\cdot\frac{\partial^2 v_r}{\partial \theta^2}} - \cancel{\frac{2}{r^2}\cdot\frac{\partial v_\theta}{\partial \theta}} + \cancel{\frac{\partial^2 v_r}{\partial z^2}}\right] \cancel{+ g_r}$$

$\frac{\partial}{\partial t} = 0$ $\qquad$ $\frac{\partial}{\partial \theta} = 0$ $\qquad$ $v_r = 0$ $\qquad$ $g_r = 0$

$v_z = \frac{\partial}{\partial z} = 0$

$$\boxed{\frac{\partial p}{\partial r} = \rho\cdot\frac{v_\theta^2}{r}}$$

3) Simplifique $\frac{Dv_\theta}{Dt}$ [1,5 Val.]

$$\cancel{\frac{\partial v_\theta}{\partial t}} + \cancel{v_r}\cdot\frac{\partial v_\theta}{\partial r} + \cancel{\frac{v_\theta}{r}\cdot\frac{\partial v_\theta}{\partial \theta}} + \frac{\cancel{v_r}\cdot v_\theta}{r} + \cancel{v_z\cdot\frac{\partial v_\theta}{\partial z}} =$$

$$\cancel{-\frac{1}{\rho}\cdot\frac{1}{r}\cdot\frac{\partial p}{\partial \theta}} + \frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\cdot\frac{\partial(r v_\theta)}{\partial r}\right) + \cancel{\frac{1}{r^2}\cdot\frac{\partial^2 v_\theta}{\partial \theta^2}} + \cancel{\frac{2}{r^2}\cdot\frac{\partial v_r}{\partial \theta}} + \cancel{\frac{\partial^2 v_\theta}{\partial z^2}}\right] \cancel{\pm g_\theta}$$

$$\boxed{\cancel{\frac{\mu}{\rho}}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\cdot\frac{\partial(r v_\theta)}{\partial r}\right)\right] = 0}$$

4) Cond. fronteira (2) [1,0 Val.]

1. ñ escorregamento : $\boxed{v_\theta (r=R) = W \cdot R}$

2. Repouso no infinito : $\boxed{r \to \infty \quad v_\theta(r) \to \phi.}$

5) $v_\theta(r) = ?$ [1,5 Val.]

$$\frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\cdot\frac{\partial(r v_\theta)}{\partial r}\right)\right] = 0 \iff$$

$$\frac{1}{r}\,\frac{\partial(r v_\theta)}{\partial r} = C_1 \iff$$

$$\frac{\partial(r v_\theta)}{\partial r} = C_1 \cdot r \iff$$

$$r v_\theta = C_1 \cdot \frac{r^2}{2} + C_2 \iff$$

$$\boxed{v_\theta(r) = C_1 \cdot \frac{r}{2} + \frac{C_2}{r}}$$

C.F. 1) $v_\theta(R) = W \cdot R \iff$

$$W \cdot R = C_1 \cdot \frac{R}{2} + \frac{C_2}{R} \,//$$

C.F. 2) $r \to \infty : v_\theta = 0 \iff$

$$\lim_{r \to \infty}\left(C_1 \cdot \frac{r}{2} + \frac{C_2}{r}\right) = 0$$

$\Rightarrow \quad C_1 = 0$ donde,

$$W \cdot R = \frac{C_2}{R} \iff$$

$C_2 = W \cdot R^2$, o que dá

$$\boxed{v_\theta(r) = \frac{W \cdot R^2}{r}}$$

b) $M_0 = ?$ p/ $\omega = $ constante. [1,0 Val.]

$$\tau_{r\theta} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\cdot\cancel{\frac{\partial v_r}{\partial \theta}}\right]$$

$$\frac{\partial}{\partial \theta} = 0$$

$$v_\theta(r) = \frac{\omega . R^2}{r}$$

$$\tau_{r\theta} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{\omega . R^2}{r^2}\right)\right] = \mu\,\omega . R^2 . r\left(-\frac{2}{r^3}\right)$$

$$\tau_{r\theta}(r) = -2\frac{\mu\,\omega R^2}{r^2}$$

$r = R$: $\tau_{r\theta}(R) = -2\mu\omega.$

$$M_0 = M_z = \tau_{r\theta}(R).(2\pi . R). R$$

$$= -4\pi\mu\omega R^2$$

(aplicado no cilindro pelo fluido)

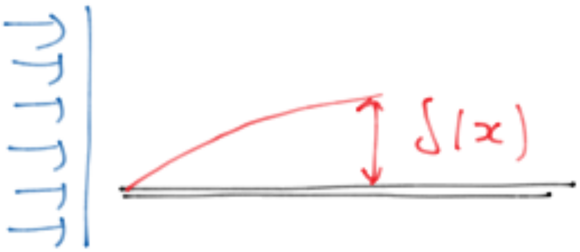
O Momento aplicado no cilindro é:

$$\boxed{M_z = +4\pi\mu\omega R^2}$$

2º TESTE MFI (MAero) – 20/12/2013

$\rho = 1{,}26\ Kg\,m^{-3}$ $\qquad P_\infty = 101{,}3\ KPa.$

$\nu = 1{,}5\times10^{-5}\ m^2.s^{-1}$



$v = 20\ m/s$

$L = 1\ m$

$b = 0{,}2\ m$

a) [0,5]

$Re_T = 5{,}5\times10^5$

$$Re_x = \frac{v.x}{\nu} \Rightarrow Re_T = \frac{v.x_T}{\nu} \Leftrightarrow x_T = \frac{Re_T.\nu}{v} =$$

$$x_T = \frac{5{,}5\times10^5 \, . \, 1{,}5\times10^{-5}}{20} = 0{,}4125$$

$$x_T = 0{,}413\ m$$

- - - - - - -

b) [1,0]

$\frac{dP}{dx} = 0$; laminar até x_T

(BLASIUS)

$$\frac{\delta(x)}{x} = \frac{5{,}0}{\sqrt{Re_x}} \Leftrightarrow$$

$$\delta(x_T) = \frac{5{,}0 . x_T}{\sqrt{Re_T}} = \frac{5{,}0 . 0{,}413}{\sqrt{5{,}5\times10^5}} = 0{,}0027\ m$$

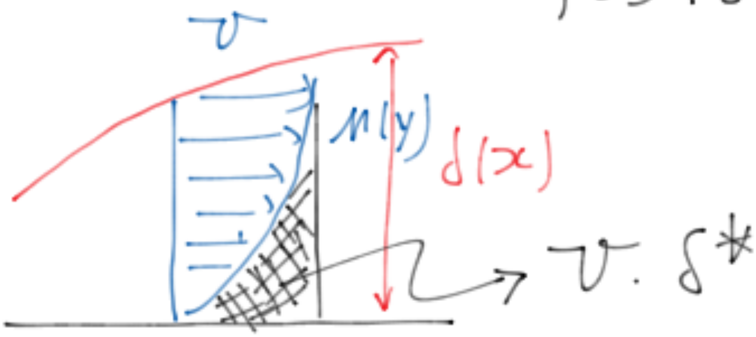
$$\frac{\delta^*(x)}{x} = \frac{1{,}72}{\sqrt{Re_x}} \quad : \quad \delta^*(L) = \frac{1{,}72 \cdot 0{,}413}{\sqrt{5{,}5\times10^5}} = 9{,}58\times10^{-4} \ (m)$$

Caudal recorrido dentro da capa límite.

$$Q = \int_0^{\delta(x_T)} u(y)\,dy = v\left(\delta(x_T) - \delta^*(x_T)\right) =$$

$$= 20 \cdot \left(2{,}7\times10^{-3} - 9{,}58\times10^{-4}\right) =$$

$$= 0{,}0348 \ m^3/s/m$$



c) [1,0]

$$y = \frac{\delta(x_T)}{2} = \frac{0{,}0027}{2} = 0{,}00135 \ m$$

$$\eta = y\sqrt{\frac{v}{\nu \cdot x_T}} = 0{,}00135 \cdot \sqrt{\frac{20}{1{,}5\times10^{-3} \cdot 0{,}413}} = 2{,}243$$

SOL. BLASIUS:

$\eta = y\sqrt{\frac{v}{\nu x}}$	$f'(\eta) = \frac{u(y)}{v}$	$f''(\eta)$
⋮	⋮	⋮
2	0,6298	0,2668
3	0,8461	0,1614
⋮	⋮	⋮

$$\eta = 2{,}243 \rightarrow f'(\eta) = 0{,}6824$$

$$f''(\eta) = 0{,}2412$$

$$u = v \cdot f'(\eta) = 20 \cdot 0{,}6824 = 13{,}68 \text{ m/s}$$

$$\tau(y) = \mu \frac{\partial u}{\partial y} = \nu \rho v \sqrt{\frac{v}{\nu \cdot x}} \cdot f''(\eta) =$$

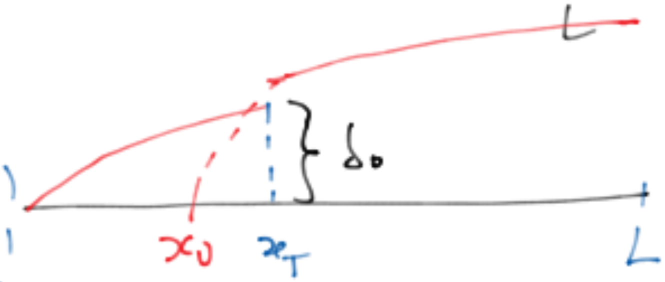
$$= 1{,}5 \times 10^{-5} \cdot 1{,}26 \cdot 20 \cdot \sqrt{\frac{20}{1{,}5 \times 10^{-5} \cdot 0{,}413}} \times 0{,}2412$$

$$= 0{,}164 \text{ N} \cdot \text{m}^{-2}$$

d) [1,5] O escoamento é turbulento p/ $x > x_T$.

$\delta(L) = ?$ O escoamento é laminar em $\frac{x_T}{L} = 41\%$ da placa:

$D(L) = ?$



$$Re_L = \frac{V \cdot L}{\nu} = \frac{20 \cdot 1}{1{,}5\times10^{-5}} = 1{,}33\times10^{6}$$

c.l. turbulenta desde x_T

$$\delta = \left[\delta_0^{5/4} + 0{,}292(x - x_T)\left(\frac{\nu}{V}\right)^{1/4}\right]^{4/5} =$$

$$\delta(L) = \left[0{,}0027^{5/4} + 0{,}292(1{,}0 - 0{,}413)\left(\frac{1{,}5\times10^{-5}}{20}\right)^{1/4}\right]^{4/5}$$

$$= 0{,}0159 \text{ m} \;//$$

$$C_D = \left[\left(1{,}328 \cdot Re_{x_1}^{-0{,}5} \cdot \frac{x_1}{L}\right)^{1{,}25} + 0{,}0377\, Re_L^{-1/4}\left(1 - \frac{x_1}{L}\right)\right]^{4/3} =$$

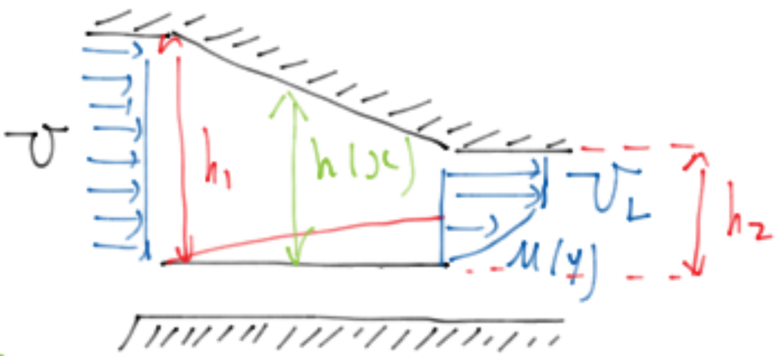
$$C_D = \left[\left(1{,}328 \cdot (3{,}5\times10^{5})^{-0{,}5} \cdot 0{,}413\right)^{1{,}25} + 0{,}0377 \cdot (1{,}33\times10^{6})^{-1/4} \cdot (1 - 0{,}413)\right]^{4/5}$$

$$= \left[1{,}214\times10^{-4} + 6{,}516\times10^{-4}\right]^{4/3} = 3{,}240\times10^{-3}$$

$$C_D = \frac{D}{\frac{1}{2}\rho v^2 b.L}$$

$$D = \frac{1}{2}\cdot \rho v^2 . b.L.\ C_D = \frac{1}{2}.1,26.20^2.0,2.1.3,240\times10^{-3} =$$

$= 0,163\ N$



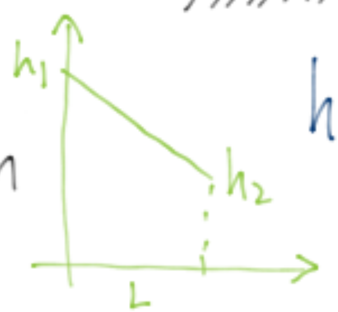
e) $[1,0]$

$v = 1\ m/s$

$h_1 = 0,5\ m$

$h_2 = 0,1\ m$

$Re_T = 5,5\times10^5$



$$h(x) = -\frac{(h_1 - h_2)}{L}.x + h_1$$

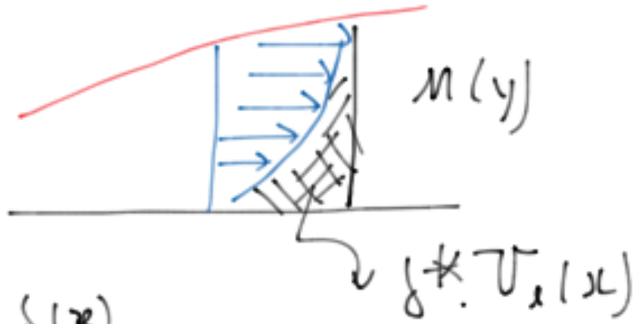
$$Re_T = \frac{v.x_T}{\nu} \Rightarrow x_T = \frac{Re_T.\nu}{v} = \frac{5,5\times10^5.1,5\times10^{-3}}{1,0}$$

$= 8,25\ m < L$: Esc. laminar

No fim do placa: $x = L$ $\quad Re_L = \frac{v.L}{\nu} = \frac{1,0.1,0}{1,5\times10^{-5}} = 0,66\times10^5$

Se $\frac{dP_e}{dx} = 0$ (BLASIUS)

$$\int_0^{h_1} v\cdot dy = \int_0^{h(x)} u(y)\,dy \Leftrightarrow$$



$$v\cdot h_1 = V_e(x)\left[h(x) - \delta(x)\right] + \underbrace{\int_0^{\delta(x)} u(y)\,dy}_{V_e(x)\cdot\left(\delta(x) - \delta^*(x)\right)} =$$

$$v\cdot h_1 = V_e\left(h(x) - \delta\right) + V_e\left(\delta - \delta^*\right) \Leftrightarrow$$

$$v\cdot h_1 = V_e\cdot h(x) - \cancel{V_e\cdot\delta(x)} + \cancel{V_e\cdot\delta(x)} - V_e\cdot\delta^*(x)$$

$$v\cdot h_1 = V_e\cdot h - V_e\cdot\delta^* = V_e\cdot\left[h - \delta^*\right]$$

$$V_e(x) = \frac{v\cdot h_1}{h - \delta^*} = \frac{0{,}5}{h(x) - \delta^*(x)} = \frac{1}{\frac{h(x)}{0{,}5} - 2\cdot\delta^*(x)}$$

i.e.

$$V_e(x) = \frac{v\cdot h_1}{-\frac{(h_1 - h_2)}{L}\cdot x + h_1 - \delta^*(x)}$$

$$\frac{dP_e}{dx} = -\rho\, v_e(x)\, \frac{dV_e}{dx}(x) =$$

$$= -\rho \frac{v.h_1}{\left[-\frac{(h_1-h_2)}{L}.x+h_1-\delta^*(x)\right]} \cdot \frac{v.h_1\left[\frac{h_1-h_2}{L} - \frac{d\delta^*}{dx}\right]}{\left[-\frac{(h_1-h_2)}{L}.x+h_1-\delta^*(x)\right]^2}$$

$$= \frac{\rho v^2 h_1^2\left(\frac{h_1-h_2}{L} - \frac{d\delta^*}{dx}\right)}{\left[-\frac{(h_1-h_2)}{L}.x+h_1-\delta^*(x)\right]^3}$$

f) [2,0]

$$\frac{d\theta}{dx} + \frac{\theta(H+2)}{v_e(x)} \cdot \frac{dv_e}{dx} = \frac{c_f}{2}$$

$$\boxed{f(\eta) = a + b\eta + c\eta^2}$$

$f'(1) = 0 \Rightarrow 0 = b + 2c$

$f(0) = 0 \Rightarrow a = 0$

$f(1) = 1 \Rightarrow 1 = b + c$

$$\begin{cases} b+c=1 \\ b+2c=0 \end{cases} \quad \begin{cases} c=1-b \\ b+2-2b=0 \end{cases} \quad \begin{cases} c=1-2=-1 \\ b=\dfrac{-2}{-1}=2 \end{cases}$$

$$f(\eta)=2\eta-\eta^2 \qquad f'(\eta)=2-2\eta$$

$$f'(0)=2$$

$$\frac{\delta^*}{\delta}=\int_0^1 [1-f(\eta)]\,d\eta=\int_0^1 (1-2\eta+\eta^2)\,d\eta=$$

$$=\left[\eta-\frac{2\eta^2}{2}+\frac{\eta^3}{3}\right]_0^1=1-1^2+\frac{1}{3}=\frac{1}{3}$$

$$\frac{\theta}{\delta}=\int_0^1 f(\eta)[1-f(\eta)]\,d\eta=\int_0^1 (2\eta-\eta^2)(1-2\eta+\eta^2)\,d\eta=$$

$$=\int_0^1 (2\eta-\eta^2-4\eta^2+2\eta^3+2\eta^3-\eta^4)\,d\eta=\int_0^1 (2\eta-5\eta^2+4\eta^3-\eta^4)\,d\eta$$

$$=\left[\frac{2\eta^2}{2}-\frac{5\eta^3}{3}+\frac{4\eta^4}{4}-\frac{\eta^5}{5}\right]_0^1=1-\underset{(5)}{\frac{5}{3}}+1-\underset{(3)}{\frac{1}{5}}=\frac{30-25-3}{15}=\frac{2}{15}$$

$$H = \frac{\delta^*}{\theta} = \frac{\delta^*/\delta}{\theta/\delta} = \frac{1/3}{2/15} = \frac{15}{6} = \frac{5}{2}$$

$$\frac{C_f}{2} = \frac{\tau_w}{\frac{1}{2}\rho V_e^2(x)/2} = \frac{\mu \frac{V_e(x)}{\delta(x)} \cdot f'(0)}{\rho V_e^2(x)} =$$

$$\frac{C_f}{2} = \frac{\nu \cdot f'(0)}{V_e(x) \cdot \delta(x)} = \frac{\nu \cdot f'(0) \cdot [\delta^*/\delta]}{V_e(x) \cdot \delta^*(x)}$$

$$\frac{d\theta}{dx} = \frac{d}{dx}\left[\frac{\theta}{\delta} \cdot \delta\right] = \frac{d}{dx}\left[\frac{\theta}{\delta} \cdot \frac{\delta}{\delta^*} \cdot \delta^*(x)\right] =$$

$$= \frac{d}{dx}\left[\frac{\theta/\delta}{\delta^*/\delta} \cdot \delta^*(x)\right] = \frac{1}{H} \cdot \frac{d\delta^*}{dx}$$

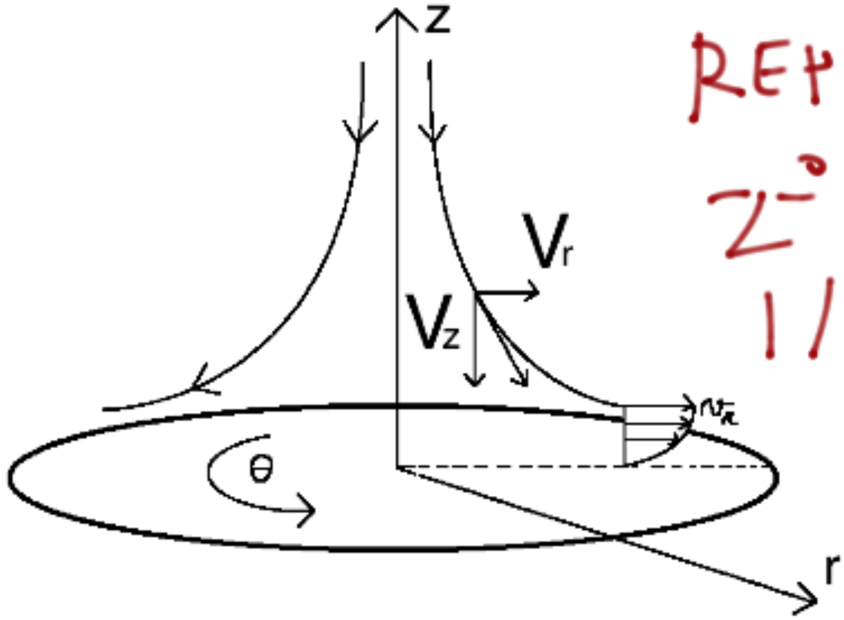
$$\frac{\theta(H+2)}{V_e(x)} \cdot \frac{dV_e(x)}{dx} = \frac{\theta}{\delta^*} \cdot \delta^*(x) \cdot (H+2) \cdot \frac{1}{V_e(x)} \cdot \frac{dV_e}{dx}$$

Ec. Von-Kármán Fic.:

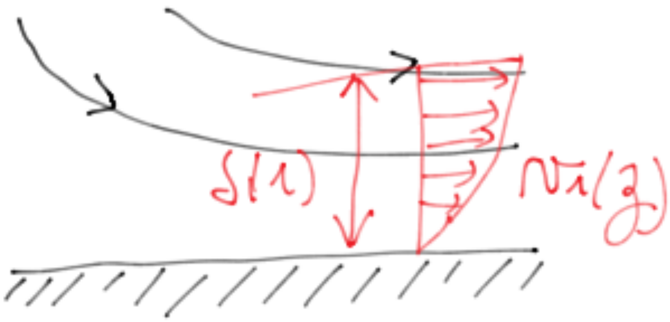
$$\frac{d\theta}{dx} + \frac{\theta(H+2)}{v_e(x)} \cdot \frac{dv_e}{dx} = \frac{c_f}{2}$$

$$\frac{1}{H} \cdot \frac{d\delta^*}{dx} + \frac{\theta}{\delta^*} \delta^*(x)(H+2)\frac{1}{v_e(x)}\frac{dv_e}{dx} =$$

$$= \frac{\nu f'(0)\left[\delta^*/\delta\right]}{v_e(x)\,\delta^*(x)}$$



REPESCAGEM
2º TESTE
1/02/2014



$$\frac{\delta}{R} \ll 1$$

$V_z = -2az$ $\quad p$ | $v_r \quad v_\theta = 0 \quad v_z$

$V_r = a.r$ | p

$V_\theta = v_\theta = 0$ |

fora da C.L. | dentro da C.L.

a) $P(r,\theta,z) = ?$ [1,0]

$$P_0 = P(r,\theta,z) + \frac{1}{2}\rho V^2$$

$$P = P(r,z)$$

$$V^2 = V_z^2 + V_r^2 = (2az)^2 + (ar)^2 =$$

$$= 4a^2z^2 + a^2r^2 =$$

$$= a^2(4z^2 + r^2)$$

$$P_0 = P(r,z) + \frac{1}{2}\rho a^2(4z^2 + r^2)$$

$$P(r,z) = P_0 - \frac{\rho a^2}{2}(4z^2 + r^2)$$

b) $v_r \sim V_r$ $[1,5]$

$z \sim \delta \qquad v_z \sim v_r \left(\frac{\delta}{R}\right)$

$r \sim R$

$$\frac{1}{r} \cdot \frac{\partial}{\partial r}(r v_r) + \cancel{\frac{1}{r} \frac{\partial v_\theta}{\partial \theta}} + \frac{\partial v_z}{\partial z} = 0$$

Axiss.

$$\frac{1}{R} \cdot \frac{1}{\cancel{R}} \cdot V_r \cdot \cancel{R} + \frac{O[v_z]}{\delta} \sim 0$$

$$\frac{V_r}{R} \sim \frac{O[v_z]}{\delta} \qquad v_z \sim V_r \left(\frac{\delta}{R}\right)$$

c) Mostrar que $\dfrac{\partial p}{\partial \theta} = 0$ [0,5]

$$\cancel{\frac{\partial v_\theta}{\partial t}} + \cancel{v_r \frac{\partial v_\theta}{\partial r}} + \cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}} + \cancel{\frac{v_r v_\theta}{r}} + \cancel{v_z \frac{\partial v_\theta}{\partial z}} =$$

$$-\frac{1}{\rho}\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{\mu}{\rho}\left[\cancel{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right)} + \cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}} + \cancel{\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}} + \cancel{\frac{\partial^2 v_\theta}{\partial z^2}}\right] + \cancel{g_\theta}$$

/ - Estacionário.

\ - $v_\theta = 0$

/ - Assisimétrico

| - forças Mássicas desprezáveis

$$-\frac{1}{r}\cdot\frac{1}{\rho}\frac{\partial p}{\partial \theta} = 0$$

$$\frac{\partial p}{\partial \theta} = 0$$

d) Mostra que $[1,0]$

$$\frac{\delta}{R} \sim \frac{1}{\sqrt{Re}} \qquad e/ \quad Re = \frac{V_1 \cdot R}{\nu}$$

$$\cancel{\frac{\partial v_z}{\partial t}} + v_r \frac{\partial v_z}{\partial r} + \cancel{\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}} + v_z \frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}} + \frac{\partial^2 v_z}{\partial z^2}\right] + g_z$$

$$v_r \frac{\partial v_z}{\partial r} + v_z \cdot \frac{\partial v_z}{\partial z} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial z} + \nu \left[\frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r} \right) + \frac{\partial^2 v_z}{\partial z^2} \right]$$

$$V_1 V_1 \left(\frac{\delta}{R}\right)\frac{1}{R} + V_1 \left(\frac{\delta}{R}\right) V_1 \left(\frac{\delta}{R}\right)\frac{1}{\delta} \sim \frac{1}{\rho}\, O\left[\frac{\partial p}{\partial z}\right] + \nu \left[\frac{1}{R} \cdot \frac{1}{R} \cdot R V_1 \left(\frac{\delta}{R}\right) + V_1 \left(\frac{\delta}{R}\right) \frac{1}{\delta^2} \right]$$

$$V_1^2\frac{\delta}{R^2}+V_1^2\frac{\delta}{R^2}\sim O\left[\frac{1}{\rho}\frac{\partial p}{\partial z}\right]+\frac{V_1 . R}{Re}\left[\frac{V_1.}{R^2}\,\frac{\delta}{R}+\frac{V_1}{R}\cdot\frac{1}{\delta}\right]$$

$$V_1^2\frac{\delta}{R^2}+V_1^2\frac{\delta}{R^2}\sim O\left[\frac{1}{\rho}\frac{\partial p}{\partial z}\right]+\frac{1}{Re}\,V_1^2\frac{\delta}{R^2}+\frac{V_1^2}{Re}\cdot\frac{1}{\delta}\Bigg]$$

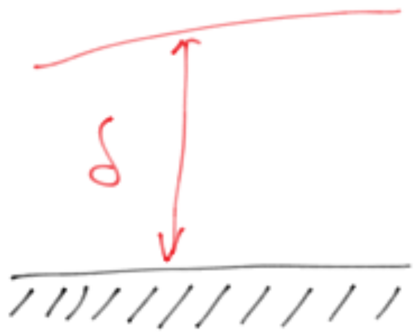
$$1+1\sim\frac{R^2}{\rho\delta V_1^2}\cdot O\left[\frac{\partial p}{\partial z}\right]+\left[\underbrace{\frac{1}{Re}}_{<<1}+\frac{1}{Re}\cdot\frac{R^2}{\delta^2}\right]$$

logo, quando Hmito:

$$\frac{1}{Re}\,\frac{R^2}{\delta^2}\sim 1 \Leftrightarrow \frac{R^2}{\delta^2}\sim Re \Leftrightarrow \frac{\delta}{R}\sim\frac{1}{Re}$$

e) $\frac{\partial p}{\partial z}\approx ?\quad [1,0]$

$$\frac{1}{\rho}\frac{R^2}{\delta V_1^2}\left[\frac{\partial p}{\partial z}\right] \sim 1$$



$$O\left[\frac{\partial p}{\partial z}\right] \sim \rho V_1^2 \cdot \frac{\delta}{R^2}$$

$$\Delta p \sim \rho V_1^2 \cdot \left(\frac{\delta}{R}\right)^2 \sim \rho \cdot V_1^2 \cdot \frac{1}{Re}$$

$$\frac{\Delta p}{\rho V_1^2} << 1 \Rightarrow \frac{\partial p}{\partial z} \approx 0$$

f) Aprox. C.L. N1 Axisimetrica [z, 0]

$$\cancel{\frac{\partial v_r}{\partial t}} + v_r \frac{\partial v_r}{\partial r} + \cancel{\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}} - \cancel{\frac{v_\theta^2}{r}} + v_z \frac{\partial v_r}{\partial z} =$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_r)}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}} - \cancel{\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}} + \frac{\partial^2 v_r}{\partial z^2}\right] + g_r$$

$$v_r \frac{\partial v_r}{\partial r} + v_z \cdot \frac{\partial v_r}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} +$$

$$\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\cdot\frac{\partial}{\partial r}(r v_r)\right) + \frac{\partial^2 v_r}{\partial z^2}\right]$$

$$V_r \cdot \frac{V_r}{R} + V_r\left(\frac{\delta}{R}\right)\frac{V_r}{\delta} \sim \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{V_r \cdot R}{Re}\left[\frac{V_r}{R^2} + \frac{V_r}{\delta^2}\right]$$

$$\frac{V_r^2}{R} + \frac{V_r^2}{R} \sim \underbrace{\frac{1}{\rho}\frac{\partial p}{\partial r}} + \frac{1}{Re}\left[\frac{V_r^2}{R} + \frac{V_r^2}{\delta}\cdot\frac{R}{\delta}\right]$$

$$V_r \cdot \frac{\partial V_r}{\partial r} \sim \frac{V_r^2}{R}$$ (Por Bernoulli)

$$1 + 1 \sim 1 + \frac{1}{Re}\left[1 + Re\right]$$

$<<1$

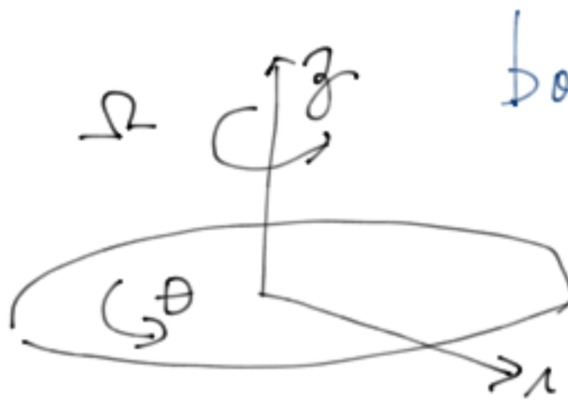
donde,

$$\begin{cases} v_r \cdot \dfrac{\partial v_r}{\partial r} + v_z \cdot \dfrac{\partial v_r}{\partial z} = -\dfrac{1}{\rho}\dfrac{dp}{dr} + \nu \cdot \underline{\underline{\dfrac{\partial^2 v_r}{\partial z^2}}} \\[2ex] \dfrac{dp}{dz} \simeq 0 \end{cases}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{\partial v_z}{\partial z} = 0$$

MFI (MAero) | resolução do 1º teste
1/02/2014

II



Bomba viscosa ou Von-Kármán

laminar

$\frac{\partial p}{\partial r}=0$ Estacionário: $\frac{\partial}{\partial t}=0$ Axissimétrico: $\frac{\partial}{\partial \theta}=0$

incompressível ρ

Newtoniano $\nu=\mu/\rho$ desprezar forças mássicas

$$z^*=\frac{z}{\sqrt{\nu/\Omega}} \qquad F(z^*)=\frac{v_r}{r\Omega}$$

$$G(z^*)=\frac{v_\theta}{r\Omega} \qquad P(z^*)=\frac{p}{\rho\nu\Omega}$$

$$H(z^*)=v_z/\sqrt{\nu\Omega}$$

a) Mostra : $2F + H' = 0$ [1,0]

$$H' = \frac{\partial H}{\partial z^*}$$

$$\frac{\partial}{\partial z} = \sqrt{\frac{\Omega}{\nu}} \cdot \frac{\partial}{\partial z^*}$$

Continuidade:

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\cancel{\frac{\partial v_\theta}{\partial \theta}} + \frac{\partial v_z}{\partial z} = 0$$

Axissimetrico

$$\frac{1}{r} \cdot \frac{\partial}{\partial r}\left[r F . r \Omega\right] + \frac{\partial\left(H\sqrt{\nu\Omega}\right)}{\partial\left(z^* \cdot \sqrt{\frac{\nu}{\Omega}}\right)} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r^2 \Omega F\right) + \frac{\sqrt{\nu\Omega}}{\sqrt{\frac{\nu}{\Omega}}} \cdot H' = 0$$

$$\frac{1}{r}\left[2r\Omega F + r^2\Omega \cancel{\frac{\partial F}{\partial r}}\right] + \Omega . H' = 0$$

$$2\Omega F + \Omega H' = 0$$

$$\boxed{2F + H' = 0}$$

b) Simplifica $\frac{Dv_r}{Dt}$ [2,0]

$$v_r \cdot \frac{\partial v_r}{\partial r} - \frac{v_\theta^2}{r} + v_z \cdot \frac{\partial v_r}{\partial z} = \nu \left[\frac{\partial}{\partial r}\left(\frac{1}{r} \underbrace{\frac{\partial (r v_r)}{\partial r}}_{2\Omega F} \right) + \frac{\partial^2 v_r}{\partial z^2} \right]$$

$$(r\Omega)F \frac{\partial}{\partial r}\left[r\Omega F \right] - \frac{1}{r} \cdot \left[(r\Omega G)^2 \right] + \sqrt{\nu\Omega}\, H \frac{\partial}{\partial z}\left[r\Omega F \right] =$$

$$= \nu \left[\frac{\partial}{\partial r}\cancel{(2\Omega F)} + \frac{\partial}{\partial z}\left(\frac{\partial}{\partial z}(r\Omega F) \right) \right] \Leftrightarrow$$

$F \neq F(r)$

$$\cancel{r}\Omega F . \Omega F - \cancel{r^2}\Omega^2 G^2 + \sqrt{\nu\Omega}\, H \cancel{r}\Omega \frac{\partial F}{\partial z} =$$

$$= \nu \frac{\partial}{\partial z}\left[\cancel{r}\Omega \frac{\partial F}{\partial z} \right] \Leftrightarrow$$

$$\Omega^2 F^2 - \Omega^2 G^2 + \sqrt{\nu\Omega}\, H \Omega \sqrt{\frac{\Omega}{\nu}} F' = \nu\Omega \frac{\partial}{\partial z}\left[\sqrt{\frac{\Omega}{\nu}} \cdot \frac{\partial F}{\partial z^*} \right]$$

$$F^2 - G^2 + HF' = F''$$

$$\boxed{F^2 + HF' - G^2 = F''}$$

e) Cond. fronteira: [1,5]

$$z^* = 0: \quad v_r = 0 \Rightarrow F = 0$$

$$v_z = 0 \Rightarrow H = 0$$

$$v_\theta = \Omega \cdot r \Rightarrow G = \frac{v_\theta}{r\Omega} = \frac{\Omega \cdot r}{r \cdot \Omega} = 1$$

$$z^* \rightarrow +\infty: \quad v_r \rightarrow 0 \Rightarrow F \rightarrow 0$$

$$v_z \rightarrow v_\infty \Rightarrow H \rightarrow H^\infty$$

$$v_\theta \rightarrow 0 \Rightarrow G \rightarrow 0$$

d) $G, F, H = ?$ $[1,5]$

$$F = N_1/(1\Omega)$$

$$G = N_\theta/(1\Omega)$$

$$H = N_\eta/\sqrt{1\Omega}$$

e) $M_0 = ?$ $[1,0]$

$$\tau_{\theta z} = \mu \left[\frac{\partial v_\theta}{\partial z} + \cancel{\frac{1}{r} \frac{\partial v_z}{\partial \theta}} \right]$$

$$= \mu \sqrt{\frac{\Omega}{\nu}} \cdot \left[r \, \Omega \, G' \right]$$

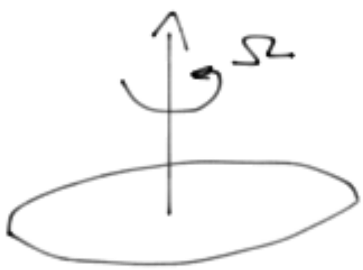
$$= \rho \cdot \nu^{-1/2} \cdot \Omega^{3/2} \cdot r \, G'$$

$$M_0 = \int_0^{2\pi} \int_0^R r \, \tau_{\theta z} \, r \, dr \, d\theta =$$

$$= 2\pi \rho \nu^{-1/2} \cdot \Omega^{3/2} \cdot G' \int_0^R r^3 dr$$

$$= 2\pi \rho \nu^{-1/2} \cdot \Omega^{3/2} \cdot G' \left[\frac{r^4}{4} \right]_0^R =$$

$$\frac{\pi}{2} \cdot \rho \cdot \nu^{-1/2} \cdot \Omega^{3/2} \cdot G' R^4 \quad //$$



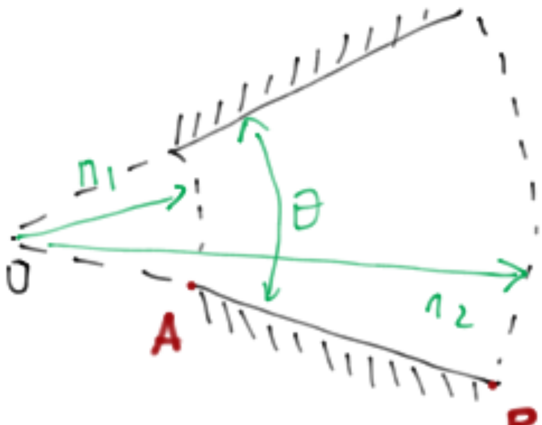
$M_0 > 0$

(Aplicado no disco p/ 1 lado R).

MFI (MAero) 11/02/2014

Repescagem do 1º teste

I



$\theta = \frac{\pi}{6}$ $r_1 = 1\,m$

$r_2 = 3\,m$

$\rho = 1{,}2\,Kg\,m^{-3}$

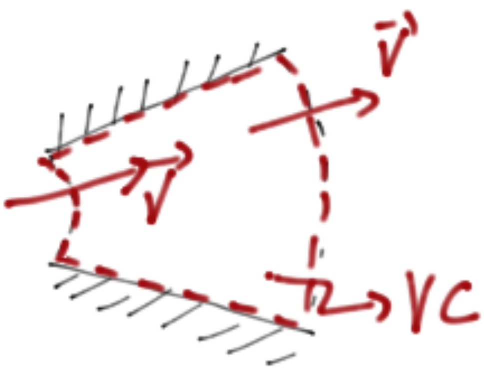
$\mu = 0$ desprezam forças mássicas.

$\vec{v} = (v_r(r),\ 0,\ 0)$

a) $v_{1r} = 10\ m/s$ [1,0]

$\dot{m} = ?$

$v_{2r} = ?$



$$\cancel{\frac{d}{dt}\iiint_{VC} \rho\, dVol} + \iint_{SVC} \rho(\vec{v}\cdot\vec{n})\,ds = 0$$

Estacionário

$$\iint \vec{v}\cdot\vec{n}\,ds = 0 \quad (\Leftrightarrow)$$

$$\dot{m} = \rho \cdot v_1(1) \cdot 1 \cdot \frac{\pi}{6}$$

$$-v_{1n} \cdot h_1 \cdot \frac{\pi}{6} + v_{2n} \cdot h_2 \cdot \frac{\pi}{6} = 0$$

$$v_1(1) = \frac{6\dot{m}}{\rho\pi} \cdot \frac{1}{1}$$

$$v_{1n} h_1 = v_{2n} \cdot h_2$$

$$v_{2n} = v_{1n}\left(\frac{h_1}{h_2}\right)$$

$$= 10 \cdot \frac{1}{3} = 3{,}33\ m\,s^{-1}$$

$$\dot{m} = \rho \cdot v_{1n} \cdot h_1 \cdot \frac{\pi}{6} =$$

$$= 1{,}2 \cdot \frac{\pi}{6} \cdot 10 \cdot 1 = 6{,}28\ kg\,s^{-1}$$

b) $P_2 = ?$ $P_1 = 10^5 \, Pa$ [1,0]

$$P_1 + \frac{1}{2}\rho v_{1r}^2 = P_2 + \frac{1}{2}\rho v_{2r}^2 = P_0$$

$$P_0 = P_1 + \frac{1}{2}\rho |v_{1r}|^2 = 10^5 + \frac{1}{2} \cdot 1{,}2 \cdot |10|^2 = 100060 \; Pa$$

$$P_2 = P_0 - \frac{1}{2} \cdot 1{,}2 \cdot |3{,}33|^2 = 99{,}9 \times 10^3 \; Pa$$

c) $v_r(r) = ?$ [2,0]

$p(r) = ?$

$$\dot{m} = \rho v_r(r) \cdot r \cdot \frac{\pi}{6} \Rightarrow v_r(r) = \frac{6\dot{m}}{\rho\pi} \cdot \frac{1}{r} = \frac{6 \times 6{,}28}{1{,}2 \cdot \pi} \cdot \frac{1}{r}$$

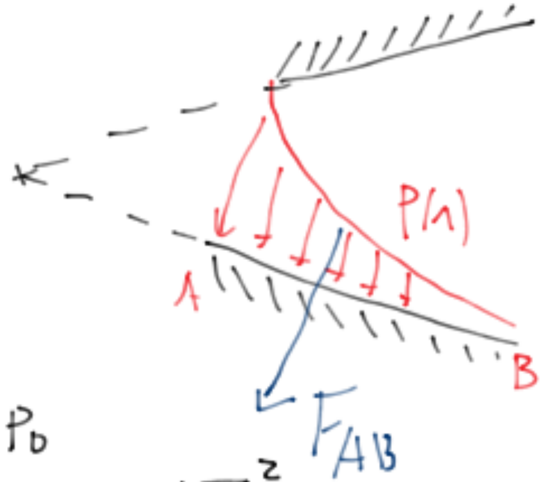
$$v_r(r) = \frac{10}{r} \; (m\,s^{-1})$$

$$P_0 = P(r) + \frac{1}{2}\rho \cdot v_r^2(r)$$

$$P(r) = P_0 - \frac{1}{2}\rho\left[\frac{10}{r}\right]^2 = 100060 - \frac{1}{2} \cdot 1{,}2 \cdot \left[\frac{10}{r}\right]^2 =$$

$$p(r) = 101060 - \frac{60}{r^2} \quad (Pa)$$

d) $F_{AB} = ?$ $[1,5]$



$$p(r) = a - \frac{b}{r^2}$$

$$a = p_0$$

$$b = \frac{1}{2}\rho\left[\frac{6\dot{m}}{\rho.\pi}\right]^2$$

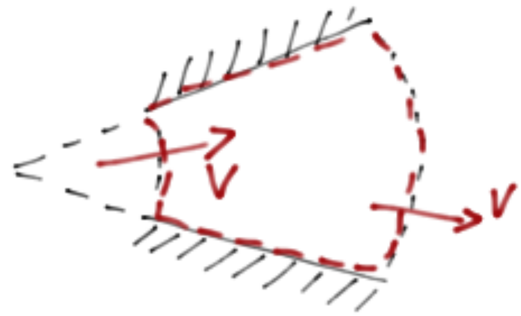
$$F_{AB} = \int_{r_1}^{r_2} p(r)\,dr = \int_{r_1}^{r_2}\left(a - \frac{b}{r^2}\right)dr = \left[a.r + \frac{b}{r}\right]_{r_1}^{r_2} =$$

$$= a(r_2 - r_1) + b\left(\frac{1}{r_2} - \frac{1}{r_1}\right) =$$

$$= 100060\,(3-1) + 60.\left(\frac{1}{3} - 1\right) = 200062{,}4 \ N$$

e) $\Delta E_{c\,flux} = ?$ [1,5]

$$\Delta\left[\int \frac{\rho v^2}{2} (\vec{v} \cdot \vec{n})\, ds\right] =$$



$$\int_1 \frac{\rho v^2}{2} (\vec{v} \cdot \vec{n})\, ds - \int_2 \frac{\rho v^2}{2} (\vec{v} \cdot \vec{n})\, ds$$

$$= \int_1 \frac{\rho v_1^3}{2} \cdot ds - \int \frac{\rho v_1^3}{2} \cdot ds = \frac{\rho}{2} \cdot \theta \left[v_{11}^3 \cdot r_1 - v_{71}^3 \cdot r_2 \right]$$

$$= \frac{1{,}2}{2} \cdot \frac{\pi}{6} \left[10^3 \cdot 1 - 3{,}33^3 \cdot 3 \right] = 279{,}4 \quad Kg\, m^{-1} s^{-3}$$