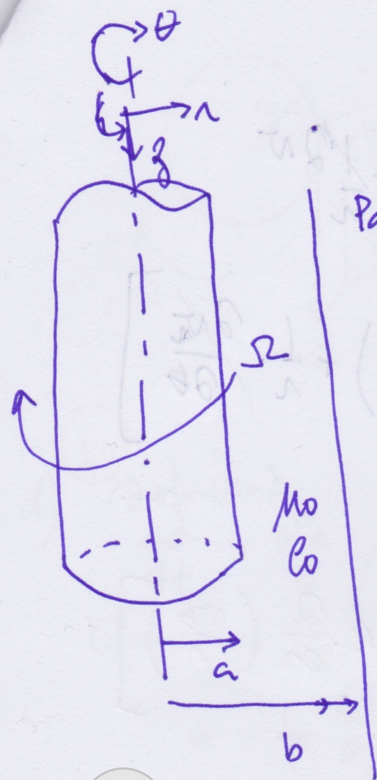
1º TESTE MFI (MAer)
31/10/2014



II (8 Val.)

Ω laminar, estacionário, incompressível

Newtoniano

Axissimétrico

completamente desenvolvido em z.

a) $v_r = ?$ $\qquad \nabla \cdot \vec{v} = \frac{1}{r} \cdot \frac{\partial}{\partial r}(r v_r) + \underbrace{\cancel{\frac{1}{r} \cdot \frac{\partial v_\theta}{\partial \theta}}}_{\text{Axissimétrico}} + \underbrace{\cancel{\frac{\partial v_z}{\partial z}}}_{\text{completamente desenvolvido}}$

Continuidade:

$$\frac{\partial}{\partial r}(r v_r) = 0 \iff r v_r = \text{const.}$$

na parede $r = a$: $v_r = 0 \Rightarrow v_r = 0 \ \forall_r$

b) $\frac{D v_\theta}{Dt} = ?$ $\qquad \underbrace{\cancel{\frac{\partial v_\theta}{\partial t}}}_{\text{Estacionário}} + \underbrace{\cancel{v_r \cdot \frac{\partial v_\theta}{\partial r}}}_{v_r = 0} + \underbrace{\cancel{\frac{v_\theta}{r} \cdot \frac{\partial v_\theta}{\partial \theta}}}_{\text{Axissimétrico}} + \underbrace{\cancel{\frac{v_r \cdot v_\theta}{r}}}_{v_r = 0} + \underbrace{\cancel{v_z \cdot \frac{\partial v_\theta}{\partial z}}}_{\text{completamente desenvolvido}} =$

$$= \underbrace{\cancel{-\frac{1}{\rho} \frac{1}{r} \cdot \frac{\partial P}{\partial \theta}}}_{\text{Axissimétrico}} + \frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r} \cdot \frac{\partial (r v_\theta)}{\partial r}\right) + \underbrace{\cancel{\frac{1}{r^2} \cdot \frac{\partial^2 v_\theta}{\partial \theta^2}}}_{\text{Axissimétrico}} + \underbrace{\cancel{\frac{2}{r^2} \cdot \frac{\partial v_r}{\partial \theta}}}_{\text{Axissimétrico}} + \underbrace{\cancel{\frac{\partial^2 v_\theta}{\partial z^2}}}_{\text{completamente desenvolvido}}\right] + \underbrace{\cancel{g_\theta}}_{g_\theta = 0}$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r} \cdot \frac{\partial (r v_\theta)}{\partial r}\right) = 0$$

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}(r v_\theta) = C_1$$

$$\frac{\partial}{\partial r}(r v_\theta) = C_1 \cdot r$$

$$r v_\theta = C_1 \frac{r^2}{2} + C_2$$

$$\boxed{v_\theta(r) = C_1 \frac{r}{2} + \frac{C_2}{r}}$$

$$\tau_{r\theta} = \mu\left[\cancel{\frac{\partial v_\theta}{\partial z}} + \cancel{\frac{1}{r}\partial v}\right]$$

$$\tau_{r\theta} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\cancel{\frac{\partial v_r}{\partial \theta}}\right]$$

Axissimetria.

$$\tau_{r\theta} = \mu\left[r\cdot\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]$$

Para os 2 fluidos i.e

fluido o: $\overset{o}{v}_\theta(r) = C_{1o}\frac{r}{2} + \frac{C_{2o}}{r}$

fluido a: $\overset{a}{v}_\theta(r) = C_{1a}\frac{r}{2} + \frac{C_{2a}}{r}$

$$\frac{v_\theta(r)}{r} = \frac{C_1}{2} + \frac{C_2}{r^2}$$

$$\frac{\partial}{\partial r}\left[\frac{v_\theta(r)}{r}\right] = \phi - \frac{2rC_2}{r^4} = -\frac{2C_2}{r^3}$$

$$\tau_{r\theta} = \mu\left[-2C_2\frac{r}{r^3}\right] = -\frac{2\mu C_2}{r^2}$$

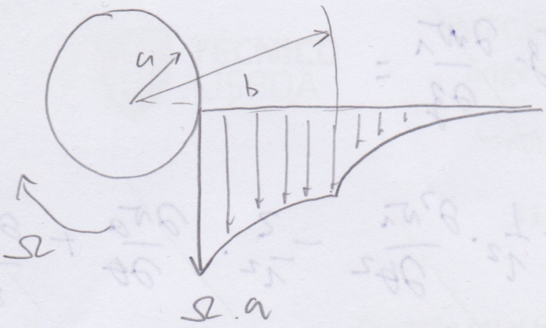
e) i) $\overset{o}{v}_\theta(a) = \Omega a \Leftrightarrow \boxed{C_{1o}\cdot\frac{a}{2} + \frac{C_{2o}}{a} = \Omega a}$

ii) $\overset{o}{v}_\theta(b) = \overset{a}{v}_\theta(b) \Leftrightarrow \boxed{C_{1o}\frac{b}{2} + \frac{C_{2o}}{b} = C_{1a}\frac{b}{2} + C_{2a}\cdot\frac{1}{b}}$

iii) $\overset{o}{\tau}_{r\theta}(a) = -\overset{a}{\tau}_{r\theta}(a) \Leftrightarrow \boxed{-2\mu_o\frac{C_{2o}}{a^2} = +2\mu_a\frac{C_{2a}}{a^2}}$

iv) $\lim_{r\to\infty} \overset{a}{v}_\theta(r) = 0 \Leftrightarrow \boxed{\lim_{r\to\infty} C_{1a}\cdot\frac{r}{2} + \frac{C_{2a}}{r} = 0}$

) (cont.)



$\mu_a < \mu_b$

d) $$\frac{\partial v_z}{\partial t} + v_r \cdot \cancel{\frac{\partial v_z}{\partial r}} + \frac{v_\theta}{r} \cdot \cancel{\frac{\partial v_z}{\partial \theta}} + v_z \cdot \cancel{\frac{\partial v_z}{\partial z}} =$$

Estacionário $\quad v_r = 0 \quad$ Axissimétrico $\quad$ escoamento desenvolvido

$$= -\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{\mu}{\rho}\left[\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\cdot\cancel{\frac{\partial^2 v_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right] + g_z.$$

Axissimétrico $\quad$ escoamento desenvolvido

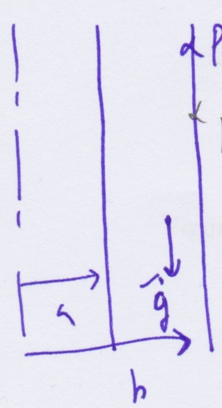
$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{\mu}{\rho}\left[\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right] + g$$

fluido o: $$0 = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{\mu_o}{\rho_o}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right] + g$$

fluido a: $$0 = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{\mu_a}{\rho_a}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right] + g$$

$\frac{\partial P}{\partial r} = ?$ $\quad$ Eq $\frac{D v_r}{Dt}$

e) Axissimétrico: $\frac{\partial P}{\partial \theta} = 0$



$P(r=b) = P_a$:

$\frac{\partial P}{\partial z} = 0$ pois

$P(z, r=b) = P_a$

$$\frac{\partial v_r}{\partial t} + v_r \cdot \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \cdot \frac{\partial v_r}{\partial z} =$$

$$= -\frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_r)}{\partial r}\right) + \frac{1}{r^2}\cdot\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + g_r$$

Eq v_r:

$$\cancel{\frac{\partial v_r}{\partial t}} + v_r \cancel{\frac{\partial v_r}{\partial r}} + \frac{v_\theta}{r}\cdot\cancel{\frac{\partial v_r}{\partial \theta}} - \frac{v_\theta^2}{r} + v_z \cancel{\frac{\partial v_r}{\partial z}} =$$

Estacionario $\quad v_r=0 \quad$ Axisimetrico $\quad v_r=0$

$$= -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{\rho}\left[\cancel{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(r v_r)}{\partial r}\right)} + \frac{1}{r^2}\cdot\cancel{\frac{\partial^2 v_r}{\partial\theta^2}} - \frac{2}{r^2}\cdot\cancel{\frac{\partial v_\theta}{\partial\theta}} + \cancel{\frac{\partial^2 v_r}{\partial z^2}}\right] + g_r$$

$v_r=0 \quad$ Axisimetrico $\quad$ Axisimetrico $\quad$ axial desarrollado $\quad g_r=0$

$$\Rightarrow \quad \frac{\partial P}{\partial r} = \rho\frac{v_\theta^2}{r}$$

$P = P(r,\theta,z) = P(r)$ apenas $\quad \dfrac{\partial P}{\partial r} = \rho\cdot\dfrac{v_\theta^2}{r}$

f) $M_z = ?$

Para manter a roda $\quad M_z = \tau^{\circ}_{r\theta}(a)\cdot 2\pi a\cdot \underset{\text{braço}}{a}$

$$\tau^{\circ}_{r\theta}(a) = -\frac{2\mu C_2^{\circ}}{a^2}$$

$$M_z = \left|-\frac{2\mu C_2^{\circ}}{a^2}\right|(2\pi a)\cdot a = \frac{4\pi}{a}\cdot\left(\mu C_2^{\circ}\right)\cdot a = 4\pi\mu C_2^{\circ}$$