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Lecture17...

Quantum Hardware – Optical Models

Class XII

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Based on the book, Quantum Computation and Quantum Information, by M. Nielsen and I. Chuang

Optical Cavity Quantum Electrodynamics (QED)

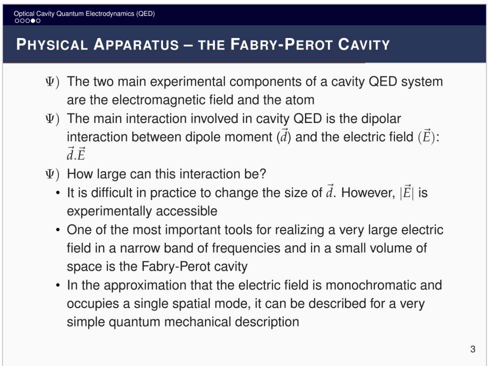
Today we start the study of a new model based on optical devices together with Quantum Electrodynamics (QED). We will see some advantages related to the previous models and the importance of the previous knowledge we learned, which will be applied here.

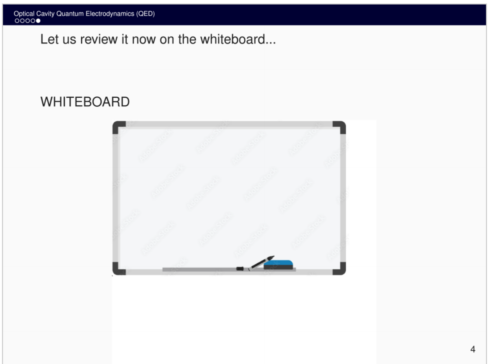
- Ψ) Cavity QED is a field of study that accesses an important regime involving the coupling of single atoms to only a few optical modes
- Ψ) Experimentally, this is made possible by placing single atoms in optical cavities
- Ψ) Because only one or two electromagnetic modes exist within the cavity and each of these has a very high electric field strength, the dipole coupling between the atom and the field is very high
- Ψ) Because of that, photons within the cavity have an opportunity to interact many times with the atoms before escaping
- Ψ) Theoretically, this technique presents an opportunity to control single quantum systems – in particular quantum computers which are the system we are interested in here

- Ψ) The single-atom cavity QED method offers a potential solution to the main problem related to the optical quantum computer described in the previous lectures

  Single photons can be good carriers of quantum information, but they require some other medium (Kerr medium) to interact with each other

- Ψ) Because non-linear optical Kerr media are compound by a large number of atoms or molecules, they are not good to satisfy the need for the photons to be good carriers of quantum information – DECOHERENCE
- Ψ In this context, well-isolated single atoms might not necessarily suffer from the same decoherence effects
- Ψ) Moreover, they could also provide cross-phase modulation between the photons
- Ψ) Thus, the idea is to transfer the state of single photons to single atoms with a controlled interaction





QUANTIZATION OF THE ELECTROMAGNETIC FIELD

– THE ELECTROMAGNETIC FIELD IS QUANTIZED BY THE ASSOCIATION OF THE QUANTUM MECHANICAL HARMONIC OSCILLATOR WITH EACH MODE $\vec{k}_\lambda$ OF THE RADIATION FIELD IN THE CAVITY

– THE MODES TO WHICH THE QUANTUM MECHANICAL OPERATORS REFER ARE INDICATED BY THE SUBSCRIPTS $\vec{k}_\lambda$

REMEMBER:

$$a|n\rangle = \sqrt{n}\,|n-1\rangle \,, \quad a^{\dagger}|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

(HARMONIC OSCILLATOR)

ELECTROMAGNETIC FIELD

THE ANNIHILATION AND CREATION OPERATORS FOR A CAVITY MODE $\vec{k}_\lambda$ TAKES SIMILAR FORM

$$a_{\vec{k}_\lambda}|n_{\vec{k}_\lambda}\rangle = \sqrt{n_{\vec{k}_\lambda}}\,|n_{\vec{k}_\lambda}-1\rangle \,, \quad a^{\dagger}_{\vec{k}_\lambda}|n_{\vec{k}_\lambda}\rangle = \sqrt{n_{\vec{k}_\lambda}+1}\,|n_{\vec{k}_\lambda}+1\rangle$$

THE PHYSICAL INTERPRETATION NOW IS THAT THE OPERATORS DESTROY

THE PHYSICAL INTERPRETATION NOW IS THAT THE OPERATORS DESTROY OR CREATE ONE PHOTON OF ENERGY $\hbar w$ IN THE $\vec{k}_\lambda$

$\vec{k}_\lambda \rightarrow$ WAVE VECTOR , $\lambda = (1, 2) \rightarrow$ IS THE POLARIZATION INDEX

$n_{\vec{k}\lambda} \rightarrow$ PHOTON EXCITED IN THE CAVITY

$$N_{\vec{k}\lambda} = a^+_{\vec{k}\lambda} \, a_{\vec{k}\lambda}$$

$$N_{\vec{k}\lambda} \, | n_{\vec{k}\lambda} \rangle = a^+_{\vec{k}\lambda} \, a_{\vec{k}\lambda} \, | n_{\vec{k}\lambda} \rangle = n_{\vec{k}\lambda} \, | n_{\vec{k}\lambda} \rangle$$

$| n_{\vec{k}\lambda} \rangle \rightarrow$ ORTHONORMAL EIGENSTATES AND ARE KNOWN AS PHOTON NUMBER STATES OR FOCK STATES

- THE CAVITY MODES ARE INDEPENDENT AND THEIR ASSOCIATED OPERATORS COMMUTE

$$\left[a_{\vec{k}\lambda} \, , \, a^+_{\vec{k}'\lambda'} \right] = \delta_{\vec{k}\lambda \, \vec{k}'\lambda'}$$

HARMONIC OSCILLATOR: $H = \hbar w \left(a^+ a + 1/2 \right)$

- LETS WRITE THIS HAMILTONIAN IN A MORE APPROPRIATE FORM!

$$[a, a^+] = 1 \Rightarrow a a^+ - a^+ a = 1 \Rightarrow \frac{1}{2} = \frac{1}{2} a a^+ - \frac{1}{2} a^+ a$$

$$H = \hbar w \left(a^+ a + \frac{1}{2} a a^+ - \frac{1}{2} a^+ a \right) = \frac{1}{2} \hbar w \left(a^+ a + a a^+ \right)$$

- LET'S DEFINE A SIMILAR HAMILTONIAN FOR EACH MODE IN THE CAVITY

$$H_{\vec{k}\lambda} = \frac{1}{2} \hbar w_k \left(a^+_{\vec{k}\lambda} \, a_{\vec{k}\lambda} + a_{\vec{k}\lambda} \, a^+_{\vec{k}\lambda} \right)$$

LET'S ASSOCIATE THIS HAMILTONIAN WITH THE RADIATIVE ENERGY FOR

LET'S ASSOCIATE THIS HAMILTONIAN WITH THE RADIATIVE ENERGY FOR THE ELECTROMAGNETIC FIELD

$$\mathcal{E}_{\vec{k}\lambda} = \epsilon_0 V w_k^2 \left(A_{\vec{k}\lambda} A^*_{\vec{k}\lambda} + A^*_{\vec{k}\lambda} A_{\vec{k}\lambda} \right)$$

$A_{\vec{k}\lambda} \rightarrow$ MODES COEFFICIENTS , $V \Rightarrow$ VOLUME OF THE CAVITY

$\omega_k = ck \rightarrow$ ANGULAR FREQUENCY , $c = (\epsilon_0 \mu_0)^{-1/2}$

$\epsilon_0 \rightarrow$ ELECTRIC PERMITIVITY IN THE FREE SPACE

$\mu_0 \rightarrow$ MAGNETIC " " " " "

$c \rightarrow$ VELOCITY OF LIGHT

— COMPARING THE EQUATIONS FOR $\hat{H}_{\vec{k}\lambda}$ AND $\mathcal{E}_{\vec{k}\lambda}$, WE MAY NOTE THAT

$$\left.\begin{array}{l} \rightarrow \epsilon_0 V \omega_k^2 \hat{A}_{\vec{k}\lambda} \hat{A}^*_{\vec{k}\lambda} = \frac{\hbar}{2} \omega_k \hat{a}_{\vec{k}\lambda} \hat{a}^\dagger_{\vec{k}\lambda} \\ \rightarrow \epsilon_0 V \omega_k^2 \hat{A}^*_{\vec{k}\lambda} \hat{A}_{\vec{k}\lambda} = \frac{\hbar}{2} \omega_k \hat{a}^\dagger_{\vec{k}\lambda} \hat{a}_{\vec{k}\lambda} \end{array}\right\rangle \left\{ \begin{array}{l} \hat{A}_{\vec{k}\lambda} \rightarrow \sqrt{\dfrac{\hbar}{2\omega_k V \epsilon_0}}\, \hat{a}_{\vec{k}\lambda} \\ \hat{A}^\dagger_{\vec{k}\lambda} \rightarrow \sqrt{\dfrac{\hbar}{2\omega_k V \epsilon_0}}\, \hat{a}^\dagger_{\vec{k}\lambda} \end{array}\right.$$

— WITH THESE RELATIONS WE CAN DEFINE THE CLASSICAL VECTOR POTENTIAL IN TERMS OF $a_{\vec{k}\lambda}$, $a^\dagger_{\vec{k}\lambda}$

$$\left\{ \begin{array}{l} A_{\vec{k}\lambda}(\vec{r},t) = A_{\vec{k}\lambda}\, e^{-i\omega_k t + i\vec{k}\cdot\vec{r}} + A^*_{\vec{k}\lambda}\, e^{i\omega_k t - i\vec{k}\cdot\vec{r}} \\ \vec{A}_{\vec{k}\lambda}(\vec{r},t) = \sum_{k\lambda(1,2)} \vec{e}_{\vec{k}\lambda} A_{\vec{k}\lambda}(\vec{r},t) \end{array}\right\} \text{CLASSICAL}$$

$$A_{\vec{k}\lambda}(\vec{r},t) \longrightarrow \hat{A}_{\vec{k}\lambda}(\vec{r},t)$$

$$A_{\vec{k}\lambda}(\vec{r},t) \longrightarrow \hat{A}_{\vec{k}\lambda}(\vec{r},t)$$

$$\hat{A}_{\vec{k}\lambda}(\vec{r},t) = \sqrt{\frac{\hbar}{2\omega_k V \epsilon_0}} \left\{ \hat{a}_{\vec{k}\lambda}\, e^{-i\omega_k t + i\vec{k}\cdot\vec{r}} + \hat{a}^{\dagger}_{\vec{k}\lambda}\, e^{i\omega_k t - i\vec{k}\cdot\vec{r}} \right\}$$

CLASSICAL : $\vec{E}_T = -\dfrac{\partial \vec{A}}{\partial t}$ $\qquad \sqrt{\dfrac{\hbar}{2\omega_k V\epsilon_0}}$

QUANTUM : $\hat{E}_{\vec{k}\lambda} = -\dfrac{\partial \hat{A}}{\partial t} = i\omega_k \left[\hat{a}_{\vec{k}\lambda}\, e^{-i\omega_k t + i\vec{k}\cdot\vec{r}} - \hat{a}^{\dagger}\, e^{i\omega_k t - i\vec{k}\cdot\vec{r}} \right]$

$$\vec{E}_T = \sum_{\vec{k}\lambda} \vec{e}_{\vec{k}\lambda}\, i\omega_k \sqrt{\frac{\hbar}{2\omega_k V \epsilon_0}} \left[a_{\vec{k}\lambda}\, e^{-i\omega t + i\vec{k}\cdot\vec{r}} - a^{\dagger} e^{i\omega_k t - i\vec{k}\cdot\vec{r}} \right]$$

for A MONOCHROMATIC MODE

$$\vec{E} = i\vec{\epsilon}\, E_0 \left[a\, e^{-i\omega t + i\vec{k}\cdot\vec{r}} - a^{\dagger} e^{i\omega_k t - i\vec{k}\cdot\vec{r}} \right]$$

$\omega t = 2\pi n$, $\quad n$

$$\vec{E} = i\vec{\epsilon}\, E_0 \left[a\, e^{i\vec{k}\cdot\vec{r}} - a^{\dagger}\, e^{-i\vec{k}\cdot\vec{r}} \right] \qquad (7.56)$$

$$E_0 = \sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}$$