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Lecture11...

Quantum Hardware – Optical Models

Class VI

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Based on the book, Quantum Computation and Quantum Information, by M. Nielsen and I. Chuang

Optical Photon Quantum Computer

We saw in the last lecture that one scheme for generating single photons in the laboratory is attenuating the output laser. In this lecture, we will apply some of this previous knowledge and start to discuss some devices important to quantum hardware computation as phase shifters, beamsplitters, and non-linear optical slabs.

Ψ) A Laser outputs a state known as coherent state $|\alpha\rangle$

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \qquad (7.16)$$

where $|n\rangle \rightarrow$ n-photon energy eigenstate.

Ψ) We also proved that the mean energy is proportional to $\Rightarrow \langle\alpha|n|\alpha\rangle = |\alpha|^2$

Ψ) When attenuated, a coherent state just becomes a weaker state, and a weaker coherent state can be made to have just phew photons

Coherent states – some properties

Ψ) Poisson distribution

$$P(n) = \langle n|\alpha\rangle\langle\alpha|n\rangle = \frac{|\alpha|^{2n}e^{-\langle n\rangle}}{n!} = \frac{|n|^{n}e^{-\langle n\rangle}}{n!}$$

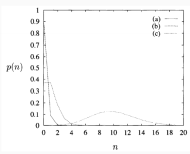


Figure 1: The photon distribution P(n) for a coherent state with (a) $|\alpha|^2 = 0.1$, (b)$|\alpha|^2 = 1$, (c) $|\alpha|^2 = 10$. It is seen that, for $|\alpha|^2 \leq 1$, $P(n)$ is maximum at $n = 0$, whereas, for $|\alpha|^2 > 1$, $P(n)$ has a peak at $n = |\alpha|^2$. Figure from the book Quantum Optics, by M.O. Scully and M.S. Zubairy.

Coherent states – some properties

- Ψ) Minimum uncertainty state

$$\Delta p \Delta r = \frac{\hbar}{2}$$

- Ψ) Completeness relation

$$\frac{1}{\pi}\int |\alpha\rangle\langle\alpha| d^2\alpha = 1$$

- Ψ) The wave function that corresponds most closely to a classical field with a minimum wave packet displaced by an amount q in the direction of r positive is

$$\psi(r,0) = \left(\frac{w}{\pi\hbar}\right)^{1/4} e^{-\frac{w}{2\hbar}(r-q)^2}$$

  we note that it oscillates back and forth in a SHO potential without changing its shape, *i.e.,* it sticks together or coheres.

One Example: Using Eq. (7.16) for $|\alpha|^2 = 0.1$, the source has a rate of 0.1 photons per unit time, thus

$$|\alpha\rangle \equiv \sqrt{0.9}|0\rangle + \sqrt{0.09}|1\rangle + \sqrt{0.002}|2\rangle + \ldots$$

Note that 90% of the time, no photons come through at all

$$|\langle 0|\alpha\rangle|^2 = 0.9 = 90\%$$

- Ψ) We can use a technique called parametric down-conversion to readout the photons
- Ψ) This involves sending photons of frequency w_0 into a non-linear optical medium to generate photon pairs at frequencies $w_0 = w_1 + w_2$ and wave numbers $\vec{k}_0 = \vec{k}_1 + \vec{k}_2$ (energy and momentum conserved)
- Ψ) So when a single w_2 photon is detected, then a single photon w_1 is known to exist.
- Ψ) By coupling this to a gate which is opened only when a single photon (as opposed to two or more) is detected

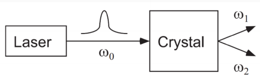


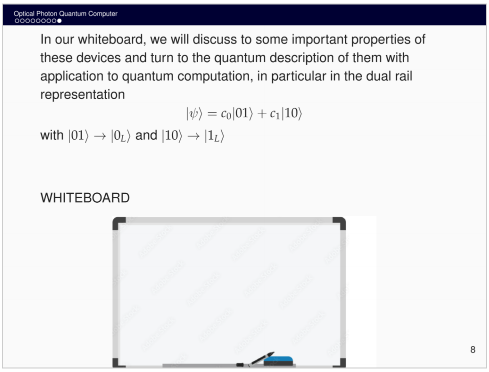
Figure 7.2. Parametric down-conversion scheme for generation of single photons.

- Ψ) Single photons can be detected with a high quantum efficiency for a wide range of wavelengths using a variety of technologies
- Ψ) The most important characteristic of a detector is its capability of determining with high probability whether zero or one photon exists in a particular spatial mode

- Ψ) In practice, imperfections reduce the probability of being able to detect a single photon. The quantum efficiency $0 \leq \eta \leq 1$ is the probability that a single photon incident on the detector generates a pair.
- Ψ) Four of the most experimentally accessible devices for manipulating photon states are
- Mirrors;
- Phase shifters;
- Beamsplitters;
- Non-linear optical devices;
- etc.

Experimental devices – Mirrors

- Ψ) High reflectivity mirrors reflect photons and change their propagation direction in space
- Ψ) Mirrors with 0.01% are not unusual
- Ψ) We will take these for granted in our scenario



QUANTUM COMPUTATION

– ARBITRARY UNITARY TRANSFORMS CAN BE APPLIED TO QUANTUM INFORMATION, ENCODED WITH SINGLE PHOTONS IN THE DUAL RAIL REPRESENTATION

$$|\psi\rangle = c_0|01\rangle + c_1|10\rangle$$

USING PHASE-SHIFTERS, BEAMSPLITTERS AND NON-LINEAR OPTICAL KERR MEDIA

– THIS WORKS BY GIVING A QUANTUM MECHANICAL HAMILTONIAN DESCRIPTION OF EACH OF THESE DEVICES

– WE ALSO KNOW THAT THE TIME EVOLUTION OF A CAVITY MODE OF ELECTROMAGNETIC RADIATION IS MODELED QUANTUM MECHANICALLY WITH THE HELP OF A HARMONIC OSCILLATOR

$|0\rangle \to$ VACUUM STATE, $|1\rangle = a^\dagger|0\rangle \to$ SINGLE PHOTON STATE

AND IN GENERAL: $|n\rangle = \dfrac{(a^\dagger)^n}{\sqrt{n!}}|0\rangle$

– THIS IS THE n-PHOTON STATE AND $a^\dagger$ IS THE CREATION OPERATOR FOR THE MODE

– IN FREE SPACE, THE EVOLUTION IS DESCRIBED BY THE HAMILTONIAN :

$$H = \hbar \omega \underbrace{a^{+} a}_{\hat{N}}$$

APPLYING Eq. (7.13) $\Rightarrow$ $|\psi(t)\rangle = e^{-iHt/\hbar} |\psi(0)\rangle$

AND USING $|\psi(0)\rangle = c_0|0\rangle + c_1|1\rangle$, THE TIME EVOLUTION IS

$$|\psi(t)\rangle = e^{-iHt/\hbar}\left[c_0|0\rangle + c_1|1\rangle\right]$$

$$= e^{-i\omega t a^{+}a}\left[c_0|0\rangle + c_1|1\rangle\right]$$

$a^{+}a a^{+}a|1\rangle$

$a^{+}a a^{+}a|0\rangle = 0$

$$= \left(1 - i\omega t\, a^{+}a + \dots\right)\left[c_0|0\rangle + c_1|1\rangle\right]$$

$$= c_0|0\rangle + c_1|1\rangle - i\omega t\left[c_0 \overbrace{a^{+}a|0\rangle}^{=0} + c_1 a^{+}a|1\rangle\right] + \dots$$

$$= c_0|0\rangle + c_1|1\rangle - i\omega t\, c_1|1\rangle + \dots =$$

$$|\psi(t)\rangle = c_0|0\rangle + \left(1 - i\omega t + \dots\right)c_1|1\rangle = c_0|0\rangle + e^{-i\omega t} c_1|1\rangle$$

PHASE SHIFTERS

– THEY ARE SLABS OF TRANSPARENT MEDIUM WITH INDEX OF REFRACTION $\eta \neq \eta_0$, $\eta_0 \rightarrow$ INDEX OF REFRACTION OF FREE SPACE

Ex.: BOROSILICATE GLASS :

$$\eta \simeq 1.5\, \eta_0$$

– IN GENERAL, A PROPAGATION IN A MEDIUM THROUGH A DISTANCE L CHANGES A PHOTON PHASE BY e^{ikL}

$n\omega$. — SPEED OF LIGHT IN THE VACUUM

$k = \frac{nw}{c_0}$, c_0 → SPEED OF LIGHT IN THE VACUUM

— SO, A PHOTON PROPAGATING THROUGH A PHASE SHIFTER WILL EXPERIENCE A SHIFT OF

$$e^{i\Delta n L w/c_0} = e^{i(n-n_0)Lw/c_0}$$

COMPARED TO A PHOTON GOING THE SAME DISTANCE THROUGH FREE SPACE

QUANTUM DESCRIPTION

— A PHASE SHIFTER OPERATOR P ACTS JUST LIKE NORMAL TIME EVOLUTION, BUT AT A DIFFERENT RATE, AND LOCALIZED TO ONLY MODE GOING THROUGH IT $(P = e^{-i\Delta N})$

— THIS IS BECAUSE LIGHT SLOWS DOWN IN A MEDIUM WITH LARGER MEDIUM OF REFRACTION $(n > n_0)$

— SPECIFICALLY, IT TAKES IN THE VACUUM A TIME TO PROPAGATE A DISTANCE L

— IN A MEDIUM LIGHT WOULD EXPERIENCE A DIFFERENCE $\Delta = \frac{(n-n_0)Lw}{c_0}$ TO PROPAGATE A DISTANCE L IN THE MEDIUM WITH INDEX OF REFRACTION n

EXAMPLE!

— THE ACTION OF P ON THE VACUUM IS TO DO NOTHING

$$P = e^{-i\Delta N} = e^{-i\Delta a^{\dagger} a}$$

• $P|0\rangle = e^{-i\Delta a^\dagger a}|0\rangle = (1 - \Delta a^\dagger a + \dots)|0\rangle = |0\rangle$

• $P|1\rangle = (1 - i\Delta a^\dagger a + \dots)|1\rangle = e^{-i\Delta}|1\rangle$

— P PERFORMS A USEFUL LOGICAL OPERATION ON A DUAL RAIL REPRESENTATION

$$P[c_0|01\rangle + c_1|10\rangle] \to c_0 e^{i(n_0-n)L\omega/c_0}|01\rangle + c_1 e^{i(n-n_0)L\omega/c_0}|10\rangle$$

• $P|10\rangle = P|1\rangle \otimes |0\rangle = e^{i(n-n_0)L\omega/c_0}|10\rangle$

• $P|01\rangle = P|0\rangle \otimes |1\rangle = e^{i(n_0-n)L\omega/c_0}|01\rangle$

— PLACING A PHASE SHIFTER IN ONE MODE RETARDS ITS PHASE EVOLUTION WITH RESPECT TO ANOTHER MODE

— THUS, WE COULD ARRANGE THIS IN THE FOLLOWING

$$P[c_0|01\rangle + c_1|10\rangle] = e^{-i\Delta}c_0|01\rangle + e^{i\Delta}c_1|10\rangle$$

— TO DO QUANTUM COMPUTATION, IT IS CONVENIENT TO REDEFINE $\Delta \to \Delta/2$

— BECAUSE WE COULD RELATE THE PHASE SHIFTER OPERATOR P WITH THE ROTATION OPERATOR

En EQUATION (4.6) $\Rightarrow R_z(\theta) = e^{-i\theta Z/2}$

$$P[c_0|01\rangle + c_1|10\rangle] \to e^{-i\Delta/2}c_0|01\rangle + e^{i\Delta/2}c_1|10\rangle$$

$$P = R_z(\Delta) = e^{-i\Delta Z/2}$$

— LET'S SEE IF THIS DEFINITION WORKS!

$|01\rangle \to |0_L\rangle$, $|10\rangle \to |1_L\rangle$

$$R_z(\Delta) = e^{-iZ\Delta/2} = \begin{pmatrix} e^{-i\Delta/2} & 0 \\ 0 & e^{i\Delta/2} \end{pmatrix}$$

- $R_z(\Delta)|0_L\rangle = \begin{pmatrix} e^{-i\Delta/2} & 0 \\ 0 & e^{i\Delta/2} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = e^{-i\Delta/2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = e^{-i\Delta/2}|0_L\rangle = e^{-i\Delta/2}|01\rangle$
- $R_z(\Delta)|1_L\rangle = \begin{pmatrix} e^{-i\Delta/2} & 0 \\ 0 & e^{i\Delta/2} \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = e^{i\Delta/2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = e^{i\Delta/2}|1_L\rangle = e^{i\Delta/2}|10\rangle$

— WE CAN THINK OF P AS A RESULTING FROM TIME EVOLUTION UNDER THE HAMILTONIAN

$$H = (\eta_0 - \eta)Z \quad , \quad Z \to \text{PAULI-}Z \text{ GATE} \quad (7.22)$$

$$P = e^{-iHt/t_0}$$

HOMEWORKS: EXERCISES: 7.6, 7.7 AND 7.8