Lecture18_QHardware_class13

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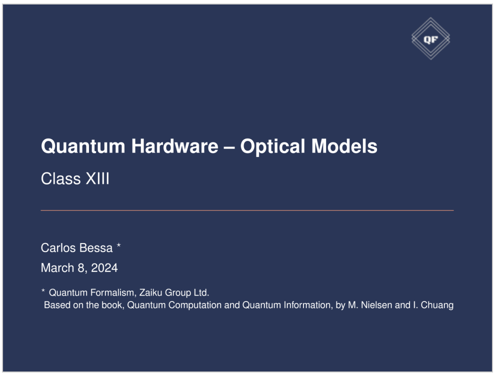
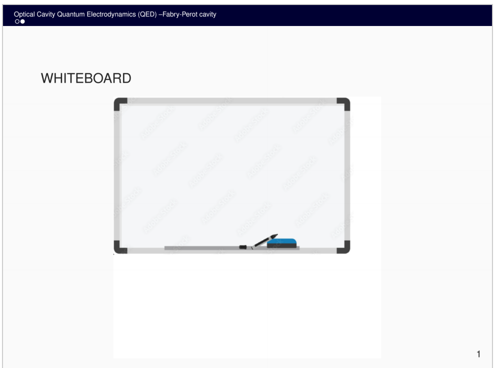
Lecture18...





BOX 7.5 THE FABRY-PEROT CAVITY

- WE COMMENTED IN THE LAST LECTURE THAT FABRY-PEROT CAVITY IS AN IMPORTANT TOOL FOR REALIZING A LARGE ELECTRIC FIELD IN NARROW BAND OF FREQUENCIES AND IN A SMALL VOLUME

- IN THE LAST LECTURE WE SAW THAT WE CAN REPRESENT A MONOCHROMATIC ELECTRIC FIELD IN THE FORM

REPRESENT A MONOCHROMATIC ELECTRIC FIELD IN THE FORM

$$\vec{E}(r) = i\vec{e}E_0\left[a\,e^{ikr} + a^{\dagger}\,e^{-ikr}\right] \qquad (7.56)$$

– A BASIC COMPONENT OF A FABRY-PEROT CAVITY IS A PARTIALLY SILVERED MIRROR, OFF WHICH INCIDENT LIGHT E_a, E_b, IS PARTIALLY REFLECTED AND PARTIALLY TRASMITED

– THIS PRODUCES OUTPUT FIELDS $E_{a'}$, $E_{b'}$

– THEY ARE RELATED BY THE UNITY TRANSFORM

$$\begin{pmatrix} E_{a'} \\ E_{b'} \end{pmatrix} = \begin{pmatrix} \sqrt{R} & \sqrt{T} \\ \sqrt{T} & -\sqrt{R} \end{pmatrix} \begin{pmatrix} E_a \\ E_b \end{pmatrix}$$

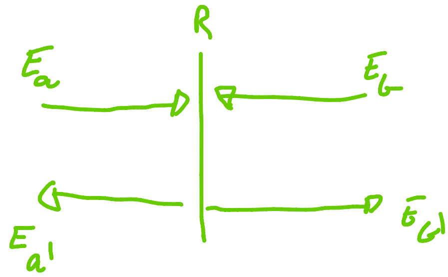
$R \to$ REFLECTIVITY OF THE MIRROR,

$T \to$ TRANSMITIVITY " " "

{ THE LOCATION OF THE NEGATIVE SIGN IS JUST A CONVENTION

$$R + T = 1 \Rightarrow T = 1 - R$$

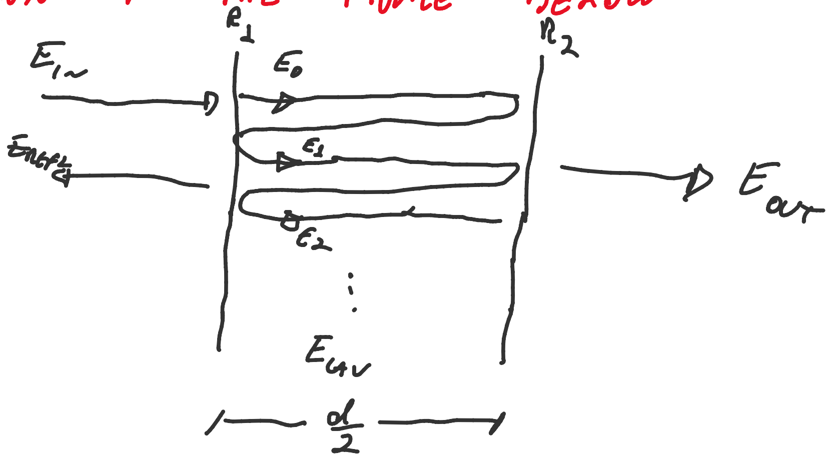
$$\begin{pmatrix} E_{a'} \\ E_{b'} \end{pmatrix} = \begin{pmatrix} \sqrt{R} & \sqrt{1-R} \\ \sqrt{1-R} & -\sqrt{R} \end{pmatrix} \begin{pmatrix} E_a \\ E_b \end{pmatrix} \qquad (7.53)$$



– IN THE CASE OF A FABRY-PEROT CAVITY, IT IS MADE FROM TWO PLANE PARALLEL MIRRORS OF REFLECTIVITIES R_1 AND R_2

– UPON WHICH LIGHT E_{in} IS INCIDENT FROM OUTSIDE

- UPON WHICH LIGHT E_{IN} IS INCIDENT FROM OUTSIDE, AS SHOWN IN THE FIGURE BELOW



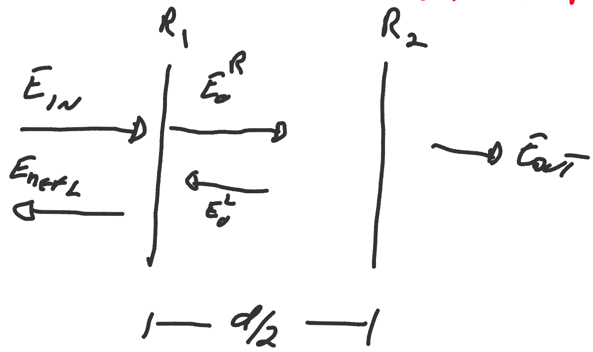
- INSIDE THE CAVITY, THE LIGHT BOUNCES BACK AND FORTH BETWEEN THE TWO MIRRORS SUCH THAT THE FIELD ACQUIRES A PHASE SHIFT $e^{i\varphi}$ ON EACH ROUND-TRIP

- THIS IS BECAUSE THE LIGHT WAVES ARE HERE TAKEN AS TRAVELLING WITH ELECTRIC FIELDS OF THE FORM

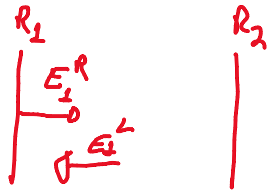
$$E_{in} \sim A e^{ikr}$$

- LET'S CONSIDER FIRST THE TRANSMISSION PROPERTIES OF AN EMPTY CAVITY FOR A LINEARLY POLARIZED LIGHT BEAM WAVE VECTOR k THAT PROPAGATE PARALLEL TO AXIS

- THE LEFTWARDS AND RIGHTWARDS FIELDS INSIDE THE CAVITY AT THE LEFT-HAND MIRROR ARE RELATED BY THE PROPAGATION PHASE FACTOR OVER TWICE THE CAVITY LENGTH

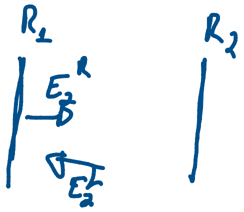


$$\begin{cases} E_0^R = \sqrt{T_1}\, E_{IN} = \sqrt{1-R_1}\, E_{IN} \\ E_0^L = -\sqrt{R_2}\, E_0^R\, e^{ikl} = -\sqrt{R_2}\, E_0^R\, e^{i\omega d/c} \end{cases} \quad , \quad k = \frac{\omega}{c}$$



$$E_1^R = \sqrt{R_1}\, E_0^L = -\sqrt{R_1 R_2}\, E_0^R\, e^{i\omega d/c}$$

$$E_1^L = -\sqrt{R_2}\, E_1^R\, e^{i\omega d/c} = R_2 \sqrt{R_1}\, E_0^R\, e^{2i\omega d/c}$$



$$E_2^R = \sqrt{R_1}\, E_1^L = R_1 R_2\, E_0^R\, e^{2i\omega d/c}$$

$$\vdots$$

$$E_n^R = \left(-\sqrt{R_1 R_2}\right)^n e^{in\omega d/c}\, E_0^R \qquad (1)$$

USE EQ. (1) TO EVALUATE E_{CAV}

$$E_{CAV} = \sum_n E_n^R = \sum_n E_n \qquad \text{(NIELSEN–CHUANG NOTATION)}$$

$$E_{CAV} = \sum_{n=0}^{\infty} E_n = E_0 + E_1 + E_2 + \dots$$

$$E_{CAV} = E_0 - \sqrt{R_1 R_2}\, e^{i\varphi} E_0 + R_1 R_2\, e^{2i\varphi} E_0 + \dots$$

$$E_{CAV} = E_0 \left[1 - \sqrt{R_1 R_2}\, e^{i\varphi} + R_1 R_2\, e^{2i\varphi} + \dots \right] \qquad (2)$$

NOTE THAT EQ. (2) IS A GEOMETRIC SERIES

$\sum^{\infty} a\, r^n = \frac{a}{\dots}$ for $|r|<1$

$$E_{cav} = E_o - \sqrt{R_1 R_2}\, e \; E_o + R_1 R_2\, e \; E_o + \cdots$$

$$E_{cav} = E_o \left[1 - \sqrt{R_1 R_2}\, e^{i\varphi} + R_1 R_2\, e^{2i\varphi} + \cdots \right] \qquad (2)$$

NOTE THAT EQ. (2) IS A GEOMETRIC SERIES

$$\sum_{n=0}^{\infty} a r^n = \frac{a}{1-r}, \quad \text{FOR} \quad |r| < 1$$

DEFINE $r = -\sqrt{R_1 R_2}\, e^{i\varphi}$, $a = E_o$

$$E_{cav} = \frac{E_o}{1 + \sqrt{R_1 R_2}\, e^{i\varphi}} \qquad (7.54)$$

– EQUATION (7.54) IS A VERY IMPORTANT EQUATION TO US BECAUSE IT SHOWS THAT THE FABRY-PEROT CAVITY IS A USEFUL TOOL FOR OUR PURPOSE

– BECAUSE THE POWER IN THE CAVITY INTERNAL FIELD CAN BE WRITTEN AS A FUNCTION OF THE INPUT POWER AND FIELD FREQUENCY

$$\frac{P_{cav}}{P_{in}} = \frac{|E_{cav}|^2}{|E_{in}|^2} = \left| \frac{\sqrt{1-R_1}\, \cancel{E_{in}}}{\left[1 + \sqrt{R_1 R_2}\, e^{i\varphi}\right] \cancel{E_{in}}} \right|^2 = \frac{1 - R_1}{\left|1 + e^{i\varphi}\sqrt{R_1 R_2}\right|^2} \qquad (7.55)$$

FOR $R_1 = R_2 = 0.9$, $\varphi = \pi$ (SEE FIG. BELOW)

$$\frac{P_{cav}}{P_{in}} = 10 \Rightarrow P_{cav} = 10\, P_{in}$$

- PHYSICALLY, IT COMES ABOUT BECAUSE THE CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE BETWEEN THE CAVITY FIELD AND THE FRONT SURFACE REFLECTED LIGHT

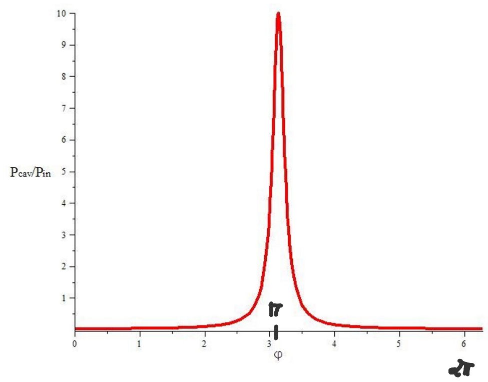
- ON RESONANCE, THE CAVITY FIELD ACHIEVES A MAXIMUM VALUE WHICH IS APPROXIMATELY

$$\frac{1}{1-R}$$

TIMES THE INCIDENT FIELD

$$R = 0.9 \Rightarrow \frac{1}{1-0.9} = \frac{1}{0.1} = 10$$

- THIS PROPERTY IS INVALUABLE FOR CAVITY QED



PLOT FROM EQ. (7.55) FOR $R_1 = R_2 = 0.9$, IN THE RANGE $0 \le \varphi \le 2\pi$