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Lecture9_...

Quantum Hardware – Optical Models

Class IV

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Based on the book, Quantum Computation and Quantum Information, by M. Nielsen and I. Chuang

Optical Photon Quantum Computer - Part I

In this lecture, we will use some of the techniques we learned when quantizing the SHO, and in particular, we will discuss coherent states, which are quantum states that describe ideal laser light.

- Ψ) An attractive physical system for representing a qubit is the optical photon
- Ψ) Photons are chargeless particles and do not interact very strongly with each other, or even with the matter
- Ψ) Their importance to quantum computation is related to the fact that they can be guided along distances with low loss using optical tools like phase shifters and beamsplitters
- Ψ) Photons exhibit signature quantum phenomena such as the interference produced in two-slit experiments
- Ψ) Furthermore, photons can be made to interact with each other, using non-linear optical media which mediate interactions
- Ψ) We will learn many things in the following lectures studying the optical photon quantum computer's components, architecture, and drawbacks!

Let us begin by considering what single photons are, how they represent quantum states, and the experimental components for manipulating them.

- Ψ) We saw in the discussion of the SHO that the energy in an electromagnetic cavity is quantized in units of $\hbar\omega$
- Ψ) Each such quantum is called a photon, and a cavity can contain a superposition of zero or one photon which could be expressed as a qubit:

$$c_0|0\rangle + c_1|1\rangle$$

- Ψ) But instead, we will do something different

- We will consider two cavities, whose total energy is $\hbar w$
- The two cavity states of a qubit are represented by $|01\rangle$ if the photon is present in a cavity, or $|10\rangle$ if the photon is present in the other
- The physical state of a superposition would thus be written as

$$c_0|01\rangle + c_1|10\rangle$$

- This is called the dual-rail representation, Where the logical qubit states could be represented by: $|0_L\rangle = |01\rangle$ and $|1_L\rangle = |10\rangle$.

Coherent States

- To generate photons in the laboratory we make use of lasers and a single photon is generated by attenuating the output of a laser
- A laser outputs a state known as a coherent state $|\alpha\rangle$
- We define a coherent state as radiation emitted by a classical current distribution in such a state
- A coherent wave packet always has minimum uncertainty, and resembles the classical field as nearly as QM permits
- The corresponding state vector $|\alpha\rangle$ is the eigenstate of the positive frequency part of the electric field operator, or, equivalently, the eigenstate of the destruction operator of the field

- The displaced SHO ground state wave function satisfies this property and the wave packet oscillates in the SHO potential without changing shape

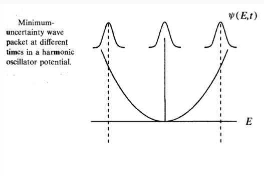

Figure 1: See Quantum Optics by M.O. Scully and M.S. Zubairy, page 47

- Classically an EM field consists of waves with well-defined amplitude and phase
- Such is not the case when we treat the EM field quantum mechanically
- There are fluctuations associated with both the amplitude and phase of the field

Let's study some of these properties on the whiteboard...

WHITEBOARD

— FOR A FIELD IN A CAVITY OF SIZE L

- THE CLASSICAL ELECTRIC FIELD CAN BE EXPANDED IN TERMS OF PLANE WAVES

$$\vec{E}(\vec{r},t) = \sum_{\vec{k}} \hat{\epsilon}_k \mathcal{E}_{\vec{k}} \alpha_{\vec{k}} e^{-i\omega_k t + i\vec{k}\cdot\vec{r}} + c.c.$$

c.c. → COMPLEX CONJUGATE , $\mathcal{E}_{\vec{k}} = \left(\frac{\omega_k}{2\epsilon_0 V}\right)^{1/2}$, $\omega_k = \frac{k\pi c_0}{L}$

$\hat{\epsilon}_k$ → UNIT POLARIZATION VECTOR ,

$\alpha_{\vec{k}}$ → DIMENSIONLESS AMPLITUDE , V → VOLUME , ϵ_0 → FREE SPACE PERMITIVITY

c_0 → SPEED OF LIGHT IN VACUUM

$k_i = \frac{2\pi n_i}{L}$, $i \in \{x, y, z\}$

→ MAXWELL EQUATION REQUIRES $\vec{k}\cdot\hat{\epsilon}_k = 0$

→ THE FIELDS ARE TRANSVERSE , BUT THERE ARE TWO INDEPENDENT POLARIZATION DIRECTIONS OF ϵ_k FOR EACH $\vec{k}$.

→ THE RADIATION FIELD IS QUANTIZED BY THE IDENTIFICATION

$$\left.\begin{aligned}\alpha_k &\to a_k \\ \alpha_k^* &\to a_k^\dagger\end{aligned}\right\}$$ THE SHO OPERATORS THAT THEY SATISFIES THE COMMUTATION RELATION $[a_k, a_k^\dagger] = 1$

$$E(\vec{r},t) = \sum_k \epsilon_k \mathcal{E}_k a_k e^{-i\omega t + i\vec{k}\cdot\vec{r}} + H.C.$$

- WE CAN INTERPRET THIS FIELD AS AN INFINITE SET OF OSCILLATORS ATTACHED TO EACH POINT r

- IN THIS CASE, <u>COHERENT STATES</u> ARE NATURALLY RADIATED FROM OSCILLATORS , SUCH AS LASERS, WHEN PUMPED ABOVE ITS LASING THRESHOLD

- FORMALLY, LET'S DEFINE A COHERENT STATE AS THE RADIATION EMITTED BY A CLASSICAL CURRENT DISTRIBUTION IN SUCH STATE

- BY CLASSICAL , WE MEAN THAT THE CURRENT CAN BE DESCRIBED BY A VECTOR $\vec{J}(\vec{r},t)$ (NOT AN OPERATOR)

$\vec{J} \to$ DIPOLE CURRENT

— THE IDEA IS CONSIDER THE COUPLE OF THIS CURRENT TO THE VECTOR POTENTIAL OPERATOR (A)

$$A(\vec{r},t) = -i \sum_{\vec{k}} \frac{1}{w_k} \epsilon_{\vec{k}} \mathcal{E}_{\vec{k}} a_{\vec{k}} e^{-iw_{\vec{k}}t + i\vec{k}\cdot\vec{r}} +$$

$$+ i \sum_{\vec{k}} \frac{1}{w_k} \epsilon_{\vec{k}} \mathcal{E}_{\vec{k}} a^{\dagger}_{k} e^{iw_k t - i\vec{k}\cdot\vec{r}} \qquad (*)$$

— DEFINING THE HAMILTONIAN THAT DESCRIBES THE INTERACTION BETWEEN THE FIELD AND THE CURRENT

$$H(t) = \int \vec{J}(\vec{r},t) \cdot \hat{\vec{A}}(\vec{r},t)\, d^3\vec{r} \qquad (**)$$

THE STATE VECTOR $|\psi(t)\rangle$ FOR THIS SYSTEM OBEYS THE SCHRÖDINGER EQUATION

$$\frac{d}{dt}|\psi(t)\rangle = \frac{-i}{\hbar} H |\psi(t)\rangle$$

— THE VECTOR $\vec{J}(\vec{r},t)$ COMMUTES WITH ITSSELF AT DIFFERENT TIMES, BUT THE OPERATOR $A(\vec{r},t)$ DOES NOT (HOMEWORK!)

$$|\psi(t)\rangle = e^{-\frac{i}{\hbar}\int_0^{t'} dt\, H(t)} |\psi(0)\rangle$$

— LET'S USE EQUATION $(*)$ AND $(**)$ TO SIMPLIFY THE INTEGRAL

$$-\frac{i}{\hbar}\int_0^{t'} dt\, H(t) = -\frac{i}{\hbar}\int\int dt\, d^3r\; \vec{J}(\vec{r},t)\cdot\vec{A}(\vec{r},t)$$

$$= \int\int dt\, d^3r\; \vec{J}(\vec{r},t) \cdot \Big(- \sum \frac{1}{\;} \epsilon_{k} \mathcal{E}_{\vec{k}}\, \hat{a}_{\vec{k}}\, e^{-iw_k t + i\vec{k}\cdot\vec{r}}$$

$$= \int\int dt\, d^3 r\ \vec{J}(\vec{r},t) \cdot \Big[- \sum_k \frac{1}{\omega_k \hbar} \vec{\epsilon}_k \mathcal{E}_{\vec{k}}\, \hat{a}_{\vec{k}}\, e^{\cdots}$$

$$+ \sum_k \frac{1}{\omega_k \hbar} \vec{\epsilon}_k \mathcal{E}_{\vec{k}}\, \hat{a}_k^{\dagger}\, e^{i\omega_k t - i\vec{k}\cdot\vec{r}} \Big]$$

DEFINING A NEW COMPLEX TIME-DEPENDENT AMPLITUDE $\alpha_{\vec{k}}$

$$\alpha_{\vec{k}} = \frac{1}{\omega_{\vec{k}} \hbar} \mathcal{E}_{\vec{k}} \int dt \int d^3\vec{r}\ \vec{\epsilon}_{\vec{k}} \cdot \vec{J}(\vec{r},t)\, e^{i\omega_{\vec{k}} t - i\vec{k}\cdot\vec{r}}$$

– SO, WE SEE THAT THE EXPONENTIAL

$$e^{-\frac{i}{\hbar}\int_0^t dt\, H(t)} = e^{\sum_k (-\alpha^*_{\vec{k}} a_{\vec{k}} + \alpha_{\vec{k}} a^{\dagger}_{\vec{k}})} = \prod_k e^{\alpha_k a_k^{\dagger} - \alpha_k^* a_k}$$

$$e^{\sum A_i} = e^{A_1 + A_2 + A_3 + \cdots} = e^{A_1} e^{A_2} e^{A_3} \cdots = \prod_i e^{A_i}$$

– LET'S CHOOSE THE INITIAL STATE $|\psi(0)\rangle$ TO BE THE VACUUM $|0\rangle$, SO THE $|\psi(t)\rangle$

$$|\psi(t)\rangle = e^{-\frac{i}{\hbar}\int dt\, H(t)} |\psi(0)\rangle = \prod_K \left(e^{\alpha_k a_k^{\dagger} - \alpha_k^* a_k} \right) |0\rangle_{\vec{k}}$$

– THIS IS THE STATE OF THE RADIATION FIELD, AND THIS STATE IS CALLED COHERENT STATE AND IT IS DENOTED BY

$$|\alpha(t)\rangle = \prod_K |\alpha_{\vec{k}}\rangle \quad , \text{ WHERE}$$

$$|\alpha_{\vec{k}}\rangle = \left(e^{\alpha_k a_k^{\dagger} - \alpha_k^* a_k} \right) |0\rangle_{\vec{k}}$$

– HOWEVER, WE WILL BE MOSTLY CONCERNED WITH A SINGLE-MODE COHERENT STATE

$$|\alpha\rangle = \left(e^{\alpha a^{\dagger} - \alpha^* a} \right) |0\rangle \qquad (1)$$

SINGLE-MODE COHERENT STATE

$$|\alpha\rangle = \left(e^{\alpha a^\dagger - \alpha^* a}\right)|0\rangle \qquad (1)$$

– THIS EXPRESSION WAS OBTAINED BY DEFINING THE COHERENT STATE OF THE RADIATION FIELD AS A STATE OF THE FIELD GENERATED BY A CLASSICALLY OSCILLATING CURRENT DISTRIBUTION

– THE SAME EXPRESSION FOR $|\alpha\rangle$ CAN BE OBTAINED BY DEFINING IT AS AN EIGENSTATE OF THE ANNIHILATION OPERATOR a WITH EIGENVALUE α

– TO SEE THIS, NOTE THAT

$$a|\alpha\rangle = a\left(e^{\alpha a^\dagger - \alpha^* a}\right)|0\rangle = a\, e^{\alpha a^\dagger} \underbrace{e^{-\alpha^* a}|0\rangle}_{|0\rangle}$$

- $$e^{-\alpha^* a}|0\rangle = \left(1 - \alpha^* a - \frac{(\alpha^*)^2}{2}(a)^2 + \cdots\right)|0\rangle$$

$$= |0\rangle$$

- $$e^{\alpha a^\dagger}|0\rangle = \left(1 + \alpha a^\dagger + \frac{\alpha^2}{2}(a^\dagger)^2 + \cdots\right)|0\rangle$$

$$= |0\rangle + \alpha|1\rangle + \frac{\alpha^2}{2}\sqrt{2}\,|2\rangle + \cdots = \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

- $$a\left(|0\rangle + \alpha|1\rangle + \frac{\alpha^2}{2}\sqrt{2} + \cdots\right) = 0 + \alpha|0\rangle + \alpha^2|1\rangle + \cdots$$

$$= \alpha\underbrace{\left(|0\rangle + \alpha|1\rangle + \cdots\right)}_{|\alpha\rangle} = \alpha|\alpha\rangle$$

SO, $a|\alpha\rangle = \alpha|\alpha\rangle$

– TO STUDY THE COHERENT STATES MORE CAREFULLY, WE

INTRODUCE A NEW OPERATOR

$$D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$

– SO

$$|\alpha\rangle = D(\alpha)\,|0\rangle$$

– THE OPERATOR $D(\alpha)$ DISPLACES THE AMPLITUDE a BY THE THE COMPLEX NUMBER α

$$D^\dagger(\alpha)\, a\, D(\alpha) = a + \alpha$$

– TO PROVE THIS, LET'S IMAGINE THAT α IS DECOMPOSED INTO INFINITESIMAL QUANTITIES $\delta\alpha$

– SO, IN FIRST ORDER OF $\delta\alpha$

$$D^\dagger(\delta\alpha)\, a\, D(\delta\alpha) = \left(e^{\delta\alpha^* a - \delta\alpha a}\right) a \left(e^{\delta\alpha a^\dagger - \delta\alpha^* a}\right)$$

UP TO FIRST ORDER IN $\delta\alpha$ WE OBTAIN

$$= \left(1 + a\,\delta\alpha^* - a\,\delta\alpha + \cdots\right) a \left(1 + a^\dagger \delta\alpha - a\,\delta\alpha^*\right)$$

$$= a + a\left(a^\dagger \delta\alpha - a\,\delta\alpha^* + a\,\delta\alpha^* - a\,\delta\alpha + \cdots\right)$$

$$= a + \left[a, a^\dagger \delta\alpha - a\,\delta\alpha^*\right]$$

$$= a + \delta\alpha \underbrace{[a, a^\dagger]}_{=1} - \delta\alpha^* \underbrace{[a, a]}_{=0} = a + \delta\alpha$$

BECAUSE $\alpha = \Sigma\, \delta\alpha$, THE TOTAL DISPLACEMENT OPERATOR

$$D(\alpha) = \Pi\, D(\delta\alpha)$$

$$= \left(e^{a^\dagger \delta\alpha - a\,\delta\alpha^*}\right)\left(e^{a^\dagger \delta\alpha - a\,\delta\alpha^*}\right) \cdots = e^{a^\dagger \Sigma \delta\alpha - a \Sigma \delta\alpha^*}$$

SO, WE COULD APPLY THE INFINITESIMAL STEPS $\delta\alpha$ AS

OFTEN AS WE NEED TO SHOW THAT

$$D^\dagger(\alpha)\, a\, D(\alpha) = a + \sum \delta\alpha = a + \alpha \qquad (***)$$

- IF WE APPLY A NEGATIVE DISPLACEMENT TO $|\alpha\rangle$

$$a\, D(-\alpha)\, |\alpha\rangle = \underbrace{D(-\alpha)\, D^\dagger(-\alpha)}_{\text{IDENTITY (HOMEWORK)}}\, a\, D(-\alpha)\, |\alpha\rangle$$

$$D^\dagger(-\alpha)\, a\, D(-\alpha) = a - \alpha \quad , \text{ SO}$$

$$a\, D(-\alpha)\, |\alpha\rangle = D(-\alpha)\,(a-\alpha)\,|\alpha\rangle \quad , \text{ BUT } \quad a|\alpha\rangle = \alpha|\alpha\rangle$$

$$a\, D(-\alpha)\,|\alpha\rangle = D(-\alpha)\,(\alpha-\alpha)\,|\alpha\rangle = 0$$

- THIS IMPLIES THAT $D(-\alpha)|\alpha\rangle$ IS THE VACUUM STATE

- CONSEQUENTLY, A COHERENT STATE IS THE DISPLACED VACUUM

$$|\alpha\rangle = D(\alpha)\,|0\rangle$$

- NOW, LET'S STUDY SOME PARTICLE ASPECTS OF THE COHERENT STATE

- FOR THIS PURPOSE WE SEEK THE NUMBER REPRESENTATION OF $D(\alpha)|0\rangle$

(CBH)

- LET'S USE THE CAMPBELL-BAKER-HAUSDORFF FORMULA TO REPRESENT $D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$ IN A DIFFERENT FORM

CBH-FORMULA $\quad e^{A}\, e^{B} = e^{A + B + [A,B]/2 + \frac{1}{12}\{[[A,B],B] + [A,[A,B]]\} + \cdots}$

USE $\quad A = \alpha a^\dagger \;,\quad B = -\alpha^* a$

$$|\alpha|^2 \overset{=-1}{[a^\dagger, a]} \ldots \{[\overset{=-1}{[a^\dagger, a]}, a] + [a^\dagger, \overset{=-1}{[a^\dagger, a]}] \ldots$$

USE $A = \alpha a^{\dagger}$, $B = -\alpha^{*} a$

$$e^{\alpha a^{\dagger}} e^{-\alpha^{*} a} = e^{\alpha a^{\dagger} - \alpha^{*} a + \frac{|\alpha|^2}{2}\overbrace{[a^{\dagger}, a]}^{=-1} + \frac{1}{12}\left\{\left[\overbrace{[a^{\dagger}, a]}^{=-1}, a\right] + \left[a^{\dagger}, \overbrace{[a^{\dagger}, a]}^{=-1}\right]\right\} + \dots}$$

$$e^{\alpha a^{\dagger} - \alpha^{*} a} = e^{\alpha a^{\dagger}} e^{-\alpha^{*} a} e^{-\frac{1}{2}|\alpha|^2} = e^{-\frac{1}{2}|\alpha|^2} e^{\alpha a^{\dagger}} e^{-\alpha^{*} a} = D(\alpha)$$

WE ALREADY KNOW THAT • $e^{-\alpha^{*} a}|0\rangle = |0\rangle$

• $e^{\alpha a^{\dagger}}|0\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle$

$$|\alpha\rangle = D(\alpha)|0\rangle$$

$$\boxed{|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle}$$

(7.16)

NIELSE-CHUANG BOOK