Lecture23_QHardware_class18

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Lecture23...

Quantum Hardware – Optical Models

Class XVIII

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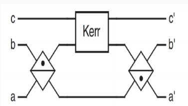
Based on the book, Quantum Computation and Quantum Information, by M. Nielsen and I. Chuang

Optical Cavity Quantum Electrodynamics (QED) – Three-level atoms

- Ψ) A natural application of the atom-photon interaction is to study what happens when two different photon modes (each mode containing at most one photon), interact with the same atom
- Ψ) This can give rise to non-linear interaction between the two modes
- Ψ) Recall from section 7.4.2 (see notes from lectures 15 & 16) that nonlinear Kerr media can be described phenomenologically as media that induce a cross-phase modulation with Hamiltonian of the form of Eq. (7.40): $H = \chi a^\dagger a b^\dagger b$, where a, b are two modes propagating through the medium

EXAMPLE: FREDKIN GATE



- Ψ) In this case, we did not see how that non-linear effect arises from fundamental interactions
- Ψ) Using the present formalism, the origin of the Kerr effect can be illustrated by using a simple model, in which two polarizations of light interacts with a three-level atom

THREE-LEVEL ATOM

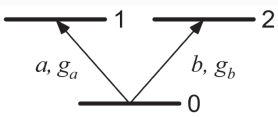
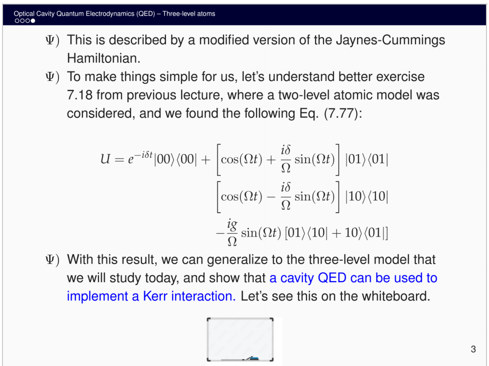


Figure 7.4. Three level atom (with levels 0, 1, and 2) interacting with two orthogonal polarizations of light described by the operators a and b. The atom–photon couplings are respectively g_a and g_b. The energy differences between 0 and 1, and between 0 and 2 are assumed to be nearly equal.



EXERCISE 7.18

$$U = e^{-iHt} = e^{-it\begin{pmatrix} \delta & 0 & 0 \\ 0 & \delta & g \\ 0 & g & -\delta \end{pmatrix}}$$

$H \to$ FOR TWO-LEVEL ATOM

LET'S WRITE H IN TERMS OF PAULI MATRIX SINCE WE MIGHT USE EQ. (7.78) $\to e^{i\vec{n}\cdot\vec{\sigma}} = \cos|n| + i\hat{n}\cdot\vec{\sigma}\sin|n|$

$\hookrightarrow$ SEE CAP. 2

$$-iHt = -it\begin{pmatrix} \delta & 0 & 0 \\ 0 & \delta & g \\ 0 & g & -\delta \end{pmatrix} = -it\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - it\begin{pmatrix} 0 & 0 & 0 \\ 0 & \delta & 0 \\ 0 & 0 & -\delta \end{pmatrix}$$

$$-it\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & g \\ 0 & g & 0 \end{pmatrix}$$

USING $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, AND $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$-iHt = -it\begin{array}{cc} & |00\rangle \\ \langle 00| & \begin{pmatrix} \delta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array} - it\begin{pmatrix} 0 & 0 & 0 \\ 0 & \delta Z & \\ 0 & & \end{pmatrix} - it\begin{pmatrix} 0 & 0 & 0 \\ 0 & gX & \\ 0 & & \end{pmatrix}$$

SO, $U = e^{-iHt} = e^{-it\delta}|00\rangle\langle 00| + \ldots e^{-it\tilde{H}} \ldots$

So,

$$U = e^{-iHt} = e^{-it\delta}\,|00\rangle\langle 00| + \sum_{ij} e^{-it\tilde{H}}\,|ij\rangle\langle ij|$$

$$\tilde{H} = \delta Z + gX$$

$$e^{-i\tilde{H}t} = e^{i\vec{n}\cdot\vec{\sigma}} \quad , \quad \vec{n} = -t(g, 0, \delta), \quad \vec{\sigma} = (X, Y, Z)$$

$$|\vec{n}| = \sqrt{\vec{n}\cdot\vec{n}} = \sqrt{t^2g^2 + t^2\delta^2} = t\sqrt{g^2+\delta^2} = t\Omega, \qquad \Omega = \sqrt{g^2+\delta^2}$$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{-t(g, 0, \delta)}{t\Omega} \Rightarrow \hat{n} = -\frac{1}{\Omega}(g, 0, \delta)$$

$$e^{i\vec{n}\cdot\vec{\sigma}} = \cos(|n|)\,I + i\hat{n}\cdot\vec{\sigma}\,\sin(|n|)$$

$$= \cos(\Omega t)\,I - i\left(\frac{g}{\Omega}X + \frac{\delta}{\Omega}Z\right)\sin(\Omega t)$$

$$= \cos(\Omega t)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - i\sin(\Omega t)\begin{bmatrix}\delta/\Omega & g/\Omega\\ g/\Omega & -\delta/\Omega\end{bmatrix}$$

$$\tilde{U} = e^{-i\tilde{H}t} = \begin{array}{c c} & \begin{array}{cc} |01\rangle & |10\rangle \end{array} \\ \begin{array}{c}\langle 01| \\ \langle 10|\end{array} & \begin{pmatrix}\cos(\Omega t) - \frac{i\delta}{\Omega}\sin(\Omega t) & -\frac{ig}{\Omega}\sin(\Omega t)\\ -\frac{ig}{\Omega}\sin(\Omega t) & \cos(\Omega t) + \frac{i\delta}{\Omega}\sin(\Omega t)\end{pmatrix}\end{array}$$

obs.: $H = \begin{pmatrix}\delta & 0 & 0\\ 0 & \delta & g\\ 0 & g & -\delta\end{pmatrix}$, $H = \begin{pmatrix}\delta & 0\\ 0 & \tilde{H}\end{pmatrix}$, $\tilde{H} = \begin{pmatrix}\delta & g\\ g & -\delta\end{pmatrix}$

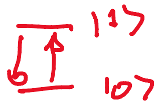
- $\cos(\Omega t) - \frac{i\delta}{\Omega}\sin(\Omega t) \longrightarrow \delta$, $-\frac{ig}{\Omega}\sin(\Omega t) \longrightarrow g$
- $\cos(\Omega t) + \frac{i\delta}{\Omega}\sin(\Omega t) \longrightarrow -\delta$

• $\underline{\cos(\Omega t) + \frac{i\delta}{\Omega}\sin(\Omega t)}$ $\longrightarrow -\delta$ $\quad$ " $\quad$ ___ $\quad$ " $\quad$ ___

$$U_{\text{TWO LEVEL}} = e^{-iHt} = e^{-i\delta t}|00\rangle\langle 00| + \left[\cos(\Omega t) - \frac{i\delta}{\Omega}\sin(\Omega t)\right]|01\rangle\langle 01|$$

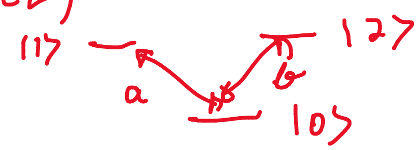
$$+ \left[\cos(\Omega t) + \frac{i\delta}{\Omega}\sin(\Omega t)\right]|10\rangle\langle 10| +$$

$$-\frac{ig}{\Omega}\sin(\Omega t)\left[|01\rangle\langle 10| + |10\rangle\langle 01|\right] \qquad (7.77)$$



$|\text{FIELD}, \text{ATOM}\rangle$

– NOW WE USE (7.77) TO FIND THE EQUIVALENT U FOR THE THREE-LEVEL ATOM (V-MODEL)



$|a, b, \text{ATOM}\rangle$

– IN THE MATRIX FORM THE RELEVANT TERMS IN H ARE FOUND TO BE THE BLOCK-DIAGONAL MATRIX IN THE BASIS

$$H = \begin{pmatrix} H_c & 0 & 0 \\ 0 & H_1 & 0 \\ 0 & 0 & H_2 \end{pmatrix}$$

• $|a, b, \text{ATOM}\rangle = |000\rangle$ FOR H_c

• $|a, b, \text{ATOM}\rangle = |100\rangle, |001\rangle, |010\rangle, |002\rangle$ FOR H_1

• $|a, b, \text{ATOM}\rangle = |110\rangle, |011\rangle, |102\rangle$ FOR H_2

– LET'S USE EQ. (7.77) TO BUILD THE MATRIX FOR THE THREE-LEVEL (V) ATOM

– IN EQUATION (7.77) THE SIMPLEST TERM IS THE FIRST ONE

$$U_0 = e^{-i\delta t} |000\rangle\langle 000|$$

SINCE $U = e^{-iHt} \Rightarrow U_0 = e^{-iH_0 t} \Rightarrow H_0 = \delta$

– NOW LET'S CONSIDER THE CASE WHERE AT MOST ONE PHOTON IS PRESENT

– USE EQ. (7.77) DISREGARDING U_0, SO WE HAVE

$$U_1 = \left[\cos(\Omega_a t) + \frac{i\delta}{\Omega_a}\sin(\Omega_a t)\right] |001\rangle\langle 001| +$$

$$\left[\cos(\Omega_a t) - \frac{i\delta}{\Omega_a}\sin(\Omega_a t)\right] |100\rangle\langle 100| - \frac{ig_a}{\Omega_a}\sin(\Omega_a t)\Big[|001\rangle\langle 100| + |100\rangle\langle 001|\Big]$$

$$+\left[\cos(\Omega_b t) + \frac{i\delta}{\Omega_b}\sin(\Omega_b t)\right] |002\rangle\langle 002| +$$

$$\left[\cos(\Omega_b t) - \frac{i\delta}{\Omega_b}\sin(\Omega_b t)\right] |010\rangle\langle 010| - \frac{ig_b}{\Omega_b}\sin(\Omega_b t)\Big[|002\rangle\langle 010| + |010\rangle\langle 002|\Big]$$

– USING THE RELATION THAT WE OBTAINED PREVIOUSLY

$$\left\{ \bullet \cos(\Omega t) - \frac{i\delta}{\Omega}\sin(\Omega t) \xrightarrow{U \to H} \delta \quad , \quad -\frac{ig}{\Omega}\sin\Omega t \to g \right\}$$

$$\cos(\Omega t) + \frac{i\delta}{\Omega}\sin(\Omega t) \longrightarrow -\delta$$

$$H_1 = \begin{array}{c|cccc} & |100\rangle & |001\rangle & |010\rangle & |002\rangle \\ \hline \langle 100| & \delta & g_a & 0 & 0 \\ \langle 001| & g_a & -\delta & 0 & 0 \\ \langle 010| & 0 & 0 & \delta & g_b \\ \langle 002| & 0 & 0 & g_b & -\delta \end{array} \qquad (7.84)$$

– NOW WE WISH TO BUILD THE HAMILTONIAN MATRIX WHEN TWO PHOTONS (ONE OF MODE a, ONE OF MODE b) ARE INVOLVED

– TO DO THIS LET'S USE AGAIN EQ. (7.77) (DISREGARDING U_0)

$$U_2 = \left[\cos(\Omega_b t) + i\frac{\delta}{\Omega_b}\sin(\Omega_b t)\right]|011\rangle\langle 011| +$$

$$\left[\cos(\Omega_{ab} t) - i\frac{\delta}{\Omega_{ab}}\sin(\Omega_{ab} t)\right]|110\rangle\langle 110|$$

$$- i\frac{g_{ab}}{\Omega_{ab}}\sin(\Omega_{ab} t)\left[|011\rangle\langle 110| + |110\rangle\langle 011|\right]$$

$$+ \left[\cos(\Omega_a t) + i\frac{\delta}{\Omega_a}\sin(\Omega_a t)\right]|102\rangle\langle 102|$$

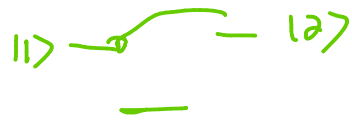
$$- i\frac{g_b}{\Omega_{ab}}\sin(\Omega_{ab} t)\left[|102\rangle\langle 110| + |110\rangle\langle 102|\right]$$

$$\Omega_{ab} = \sqrt{\delta^2 + g_a^2 + g_b^2}$$

– USING THE PREVIOUS RELATIONS WE ARE ABLE TO FIND

H_2

$$H_2 = \begin{array}{c|ccc} & |110\rangle & |011\rangle & |102\rangle \\ \hline \langle 110| & \delta & g_a & g_b \\ \langle 011| & g_a & -\delta & 0 \\ \langle 102| & g_b & 0 & -\delta \end{array} \qquad (7.95)$$



CHECK $|102\rangle\langle 011|$

$$H = \begin{pmatrix} H_0 & 0 & 0 \\ 0 & H_1 & 0 \\ 0 & 0 & H_2 \end{pmatrix}$$

– WE WISH TO SEE A NON-LINEAR RELATION BETWEEN THE THREE-LEVEL ATOM AND TO TWO PHOTONS

– WITH THE U MATRIX FOUND BEFORE, WE CAN OBTAIN THE <u>SINGLE PHOTON</u> PHASE SHIFTS

- $\underline{\varphi_a} = \arg\left(\langle 100|U|100\rangle\right) - \arg\left(\langle 000|U|000\rangle\right)$

- $\underline{\varphi_b} = \arg\left(\langle 010|U|010\rangle\right) - \arg\left(\langle 000|U|000\rangle\right)$

AND THE TWO PHOTONS PHASE SHIFTS

- $\underline{\varphi_{ab}} = \arg\left(\langle 110|U|110\rangle\right) - \arg\left(\langle 000|U|000\rangle\right)$

– NOTE THAT IN A LINEAR MEDIA, ONE WOULD EXPECT

$$\varphi_{ab} = \varphi_a + \varphi_b \quad \text{(if linear)}$$

THE TWO-PHOTON STATE WOULD HAVE TWICE THE PHASE SHIFT OF THE SINGLE PHOTON STATE

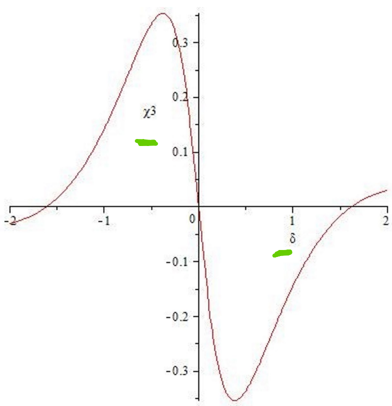
- TO TEST THIS, LET'S DEFINE

$$\chi_3 = \varphi_{ab} - \varphi_a - \varphi_b$$

, if linear $\chi_3 = 0$

- LET'S PLOT $\chi_3 \times \delta$ for $t = 0.98$ and $g_a = g_b = 1$

- $\varphi_a = \arg\left[\cos(\Omega_a t) - i\frac{\delta}{\Omega_a}\sin(\Omega_a t)\right] - \arg\left(e^{-i\delta t}\right)$
- $\varphi_b = \arg\left[\cos(\Omega_b t) - i\frac{\delta}{\Omega_b}\sin(\Omega_b t)\right] - \arg\left(e^{-i\delta t}\right)$
- $\varphi_{ab} = \arg\left[\cos(\Omega_{ab} t) - i\frac{\delta}{\Omega_{ab}}\sin(\Omega_{ab} t)\right] - \arg\left(e^{-i\delta t}\right)$



- NOTE THAT $\chi^3 \neq 0$, for most values of δ
- maximum around $\delta \approx 0.4$

→ SO OUR MODEL COULD MIMIC A KERR NON-LINEAR MATERIAL