# Lecture24_QHardware_class19

sexta-feira, 19 de abril de 2024 13:54

Lecture24...

## Quantum Hardware – Optical Models

Class XIX


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April 19, 2024

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Based on the book, Quantum Computation and Quantum Information, by M. Nielsen and I. Chuang

## Optical Cavity Quantum Electrodynamics (QED) – Application: Quantum Computation



$\Psi$) Broadly speaking, cavity QED tecniques can be used to perform quantum computation in a number of different ways. Two of which are the following

- $i$) Quantum information can be represented by photon states,using cavities with atoms to provide non-linear interactions between photons
- $ii$) Quantum information be represented using atoms, using photons to communicate between the atoms

$\Psi$) Our goal is (in the next lecture) to describe an experiment which demonstrates the first of these methods to realize a quantum logic gate

$\Psi$) In Section 7.4.2, we saw that a quantum computer might be constructed using single photon states, phase shifters, beamsplitters, and non-linear Kerr media

$\Psi$) However, in general the cross phase modulation, discussed in this section, is nearly infeasible with standard bulk non-linear optics techniques





$\Psi$) Cavity QED can be used to implement a Kerr (non-linear) interaction, as shown in last lecture (section 7.5.3)

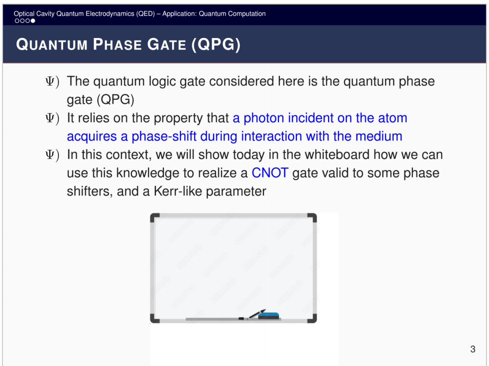
$\Psi$) Unlike for bulk media, this can have a very strong effect even at the single photon level, because of the strong field provided by a Fabry-Perot type cavity

$\Psi$) In next lecture we will study a cavity QED experiment which was performed to demonstrate the potential for realizing a logic gate with unitary transform

$\Psi$) Today, we will understand the theory behind this experiment where a non-lineaar model (as discussed in section 7.5.3) can be used to demonstrate the principle of a single quantum logic transformation

$\Psi$) Remember that in quantum logic, the goal is to control the quantum state of one particle with the quantum state of another particle

$\Psi$) When the particles are simple two-state systems, each state can be assigned a truth table value




# QUANTUM COMPUTATION

$|\text{FIELD } a, \text{ FIELD } b, \text{ ATOM}\rangle$

$|\text{FIELD } a, \text{ FIELD } b\rangle$


## CNOT-GATE

- IN THE CNOT GATE, THE TARGET QUBIT IS FLIPPED IF AND ONLY IF THE CONTROL QUBIT IS "1"

- THE TRUTH TABLE OF CNOT GATE IS DESCRIBED BY

CONTROL (a) — TARGET (b)

$$\begin{cases} |0\rangle_a |0\rangle_b \xrightarrow{CNOT} |0\rangle_a |0\rangle_b \\ |0\rangle_a |1\rangle_b \xrightarrow{CNOT} |0\rangle_a |1\rangle_b \\ |1\rangle_a |0\rangle_b \xrightarrow{CNOT} |1\rangle_a |1\rangle_b \\ |1\rangle_a |1\rangle_b \xrightarrow{CNOT} |1\rangle_a |0\rangle_b \end{cases}$$

$|00\rangle_{ab} = |0\rangle_a |0\rangle_b = |0\rangle_a \otimes |0\rangle_b$

## QUANTUM PHASE GATE (QPG)

- WRITE IN THE BASIS $\{|0\rangle_{a,b}, |1\rangle_{a,b}\}$ THE FOLLOWING QPG TO FIND A CNOT GATE

### ANSATZ

$|0\rangle_a |0\rangle_b \xrightarrow{QPG} |0\rangle_a |0\rangle_b$

$|0\rangle_a |1\rangle_b \xrightarrow{QPG} e^{i\varphi_a} |0\rangle |1\rangle$


$|0\rangle \otimes |0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$|0\rangle \otimes |1\rangle = ( 0$

$$|0\rangle_a|1\rangle_b \xrightarrow{QPG} e^{i\varphi_a}|0\rangle_a|1\rangle_b$$

$$|1\rangle_a|0\rangle_b \xrightarrow{QPG} e^{i\varphi_b}|1\rangle_a|0\rangle_b$$

$$|1\rangle_a|1\rangle_b \xrightarrow{QPG} e^{i(\varphi_a+\varphi_b+\Delta)}|1\rangle_a|1\rangle_b$$

$$|0\rangle\otimes|1\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$|1\rangle\otimes|0\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

$$|1\rangle\otimes|1\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

– THIS LEADS TO A UNITARY TRANSFORM

$$QPG = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & e^{i\varphi_a} & 0 & 0\\ 0 & 0 & e^{i\varphi_b} & 0\\ 0 & 0 & 0 & e^{i(\varphi_a+\varphi_b+\Delta)}\end{pmatrix} \qquad (7.87)$$

HERE, $\varphi_a$ AND $\varphi_b$ ARE THE 1-QUBIT (PHOTON) PHASE, AND $\Delta$ PARAMETRIZES THE NON-LINEAR 2-QUBIT CONDITIONAL PHASE

– IF $\Delta \neq 0$ THE QPG IS CAPABLE OF UNIVERSAL QUANTUM COMPUTATION SINCE IT SATISFIES WHAT IS TALKING TO BE A NECESSARY CRITERION FOR QUANTUM LOGIC, WHICH IS THE CREATION OF ENTANGLEMENT BETWEEN QUBITS

$$(|0\rangle_a + |1\rangle_a)\otimes(|0\rangle_b+|1\rangle_b) = (|0\rangle_a|0\rangle_b + |0\rangle_a|1\rangle_b + |1\rangle_a|0\rangle_b + |1\rangle_a|1\rangle_b)$$

$$\xrightarrow{QPG} |0\rangle_a|0\rangle_b + e^{i\varphi_a}|0\rangle_a|1\rangle_b + e^{i\varphi_b}|1\rangle_a|0\rangle_b + e^{i(\varphi_a+\varphi_b+\Delta)}|1\rangle_a|1\rangle_b$$

$\Delta \neq 0$, THIS EQUATION IS ENTANGLED OUTPUT STATE

$$\neq (|0\rangle_a + e^{i\varphi_a}|1\rangle_a)(|0\rangle_b + e^{i\varphi_b}|1\rangle_b)$$

– TO IMPLEMENT A CNOT GATE WE WILL CONSIDER THE LIMITING CASES ($\Delta = \pi$, $\varphi_a = 0$, $\varphi_b = \pi$)

$$(|0\rangle_a + |1\rangle_a) \otimes (|0\rangle_b + |1\rangle_b) = (|0\rangle_a |0\rangle_b + |0\rangle_a |1\rangle_b + |1\rangle_a |0\rangle_b + |1\rangle_a |1\rangle_b)$$

$$\xrightarrow{QPG(\delta=\pi)} |0\rangle_a |0\rangle_b + e^{i\varphi_b} |0\rangle_a |1\rangle_b + e^{i\varphi_a} |1\rangle_a |0\rangle_b + e^{i(\varphi_a+\varphi_b+\pi)} |1\rangle_a |1\rangle_b$$

$$= |0\rangle_a |0\rangle_b + e^{i\varphi_b} |0\rangle_a |1\rangle_b + e^{i\varphi_a} |1\rangle_a |0\rangle_b - e^{i(\varphi_a+\varphi_a)} |1\rangle_a |1\rangle_b$$

– TO IMPLEMENT A CNOT GATE, LET'S CONSIDER THE FOLLOWING STEPS:

1) CONSIDER A $\frac{\pi}{4}$ ROTATION IN PHOTON (QUBIT) $b$

TO DO THIS LET'S REMEMBER THE FOLLOWING GATE ($R$)

$$R(\theta, \phi, \lambda) = \begin{pmatrix} \cos(\theta/2) & -e^{i\lambda} \sin(\frac{\theta}{2}) \\ e^{i\phi} \sin(\theta/2) & e^{i(\phi+\lambda)} \cos(\frac{\theta}{2}) \end{pmatrix}$$

$$\left\{ \begin{array}{l} |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{array} \right.$$

for $\phi = \lambda = 0$ AND $\theta = \pi/4$

$$R(\pi/4) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \Rightarrow \left\{ \begin{array}{l} \bullet\ R(\pi/4)\, |0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} +1 \\ 1 \end{pmatrix} = +|+\rangle \\ \bullet\ R(\pi/4)\, |1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = -|-\rangle \end{array} \right.$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

– THUS, WHEN ROTATE QUBIT $b$ BY $\pi/4$, WE WILL OBTAIN

i) $|0\rangle_a |0\rangle_b \xrightarrow{R_b(\pi/4)} |0\rangle_a |+\rangle_b$, iii) $|1\rangle_a |0\rangle_b \xrightarrow{R_b(\pi/4)} |1\rangle_a |+\rangle_b$

i) $|0\rangle_a|0\rangle_b \longrightarrow |0\rangle_a|+\rangle_b$ , iii) $|1\rangle_a|0\rangle_b \longrightarrow |1\rangle_a|+\rangle_b$

ii) $|0\rangle_a|1\rangle_b \xrightarrow{R_b(\pi/4)} -|0\rangle_a|-\rangle_b$ , iv) $|1\rangle_a|1\rangle_b \xrightarrow{R_b(\pi/4)} -|1\rangle_a|-\rangle_b$

2) IN THE SECOND STEP, APPLY THE QPG (MATRIX 7.87)

i) $|0\rangle_a|+\rangle_b = \frac{1}{\sqrt{2}}|0\rangle_a\left(|0\rangle_b+|1\rangle_b\right) \xrightarrow{QPG} \frac{1}{\sqrt{2}}\left(|0\rangle_a|0\rangle_b + e^{i\varphi_a}|0\rangle_a|1\rangle_b\right)$

ii) $-|0\rangle_a|-\rangle_b = -\frac{1}{\sqrt{2}}|0\rangle_a\left(|0\rangle_b-|1\rangle_b\right) \xrightarrow{QPG} -\frac{1}{\sqrt{2}}\left(|0\rangle_a|0\rangle_b - e^{i\varphi_a}|0\rangle_a|1\rangle_b\right)$

iii) $|1\rangle_a|+\rangle_b = \frac{1}{\sqrt{2}}|1\rangle_a\left(|0\rangle_b+|1\rangle_b\right) \xrightarrow{QPG(\Delta=\pi)} \frac{1}{\sqrt{2}}\left(e^{i\varphi_b}|1\rangle_a|0\rangle_b - e^{i(\varphi_a+\varphi_b)}|1\rangle_a|1\rangle_b\right)$

iv) $-|1\rangle_a|-\rangle_b = -\frac{1}{\sqrt{2}}|1\rangle_a\left(|0\rangle_b-|1\rangle_b\right) \xrightarrow{QPG(\Delta=\pi)}$

$\longrightarrow -\frac{1}{\sqrt{2}}\left(e^{i\varphi_b}|1\rangle_a|0\rangle_b + e^{i(\varphi_a+\varphi_b)}|1\rangle_a|1\rangle_b\right)$

3) AS THIRD STEP, APPLY A ROTATION BY $-\pi/4$ IN QUBIT b

$$R(\theta=-\pi/4) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

- $R(-\pi/4)|0\rangle_b = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
- $R(-\pi/4)|1\rangle_b = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$$\text{i)}\ \frac{1}{\sqrt{2}}\left(|0\rangle_a|0\rangle_b + e^{i\varphi_a}|0\rangle_a|1\rangle_b\right) \xrightarrow{R_b(-\pi/4)} \frac{1}{\sqrt{2}}\left(|0\rangle_a|-\rangle_b + e^{i\varphi_a}|0\rangle_a|+\rangle_b\right)$$

$$= \frac{|0\rangle_a}{\sqrt{2}}\left(\frac{|0\rangle_b - |1\rangle_b}{\sqrt{2}}\right) + \frac{e^{i\varphi_a}|0\rangle_a}{\sqrt{2}}\left(\frac{|0\rangle_a + |1\rangle_b}{\sqrt{2}}\right)$$

$$= \frac{|0\rangle_a|0\rangle_b}{2}\left(1+e^{i\varphi_a}\right) - \frac{|0\rangle_a|1\rangle_b}{2}\left(1-e^{i\varphi_a}\right)$$

$$\varphi_a = 0$$

$$= |0\rangle_a|0\rangle_b$$

$$\text{i)}\ |0\rangle_a|0\rangle_b \xmapsto{R_b(\pi/4)} \xmapsto{QPG(\Delta=\pi)} \xmapsto{R_b(-\pi/4)} |0\rangle_a|0\rangle_b$$

$$\text{ii)}\ -\frac{1}{\sqrt{2}}\left(|0\rangle_a|0\rangle_b - e^{i\varphi_a}|0\rangle_a|1\rangle_b\right) \xmapsto{R_b(-\pi/4)}$$

$$-\frac{1}{\sqrt{2}}\left(|0\rangle_a|-\rangle_b - e^{i\varphi_a}|0\rangle_a|+\rangle_b\right) =$$

$$= -\frac{|0\rangle_a|0\rangle_b}{2}\left(1-e^{i\varphi_a}\right) + \frac{|0\rangle_a|1\rangle_b}{2}\left(1+e^{i\varphi_a}\right)$$

$$\varphi_a = 0$$

$$= |0\rangle_a|1\rangle_b$$

$$\text{ii)}\ |0\rangle_a|1\rangle_b \xmapsto{R_b(\pi/4)} \xmapsto{QPG(\Delta=\pi)} \xmapsto{R_b(-\pi/4)} |0\rangle_a|1\rangle_b$$

iii) $\frac{1}{\sqrt{2}}\left(e^{i\varphi_b}|1\rangle_a|0\rangle_b - e^{i(\varphi_a+\varphi_b)}|1\rangle_a|1\rangle_b\right) \xrightarrow{R_b(-\pi/4)}$

$\frac{e^{i\varphi_b}}{\sqrt{2}}|1\rangle_a|-\rangle_b - \frac{e^{i(\varphi_a+\varphi_b)}}{\sqrt{2}}|1\rangle_a|+\rangle_b =$

$= \frac{e^{i\varphi_b}}{\sqrt{2}}|1\rangle_a\left(\frac{|0\rangle_b-|1\rangle_b}{\sqrt{2}}\right) - \frac{e^{i(\varphi_a+\varphi_b)}}{\sqrt{2}}|1\rangle_a\left(\frac{|0\rangle_b+|1\rangle_b}{\sqrt{2}}\right)$

$= \frac{e^{i\varphi_b}}{2}\left(|1\rangle_a|0\rangle_b - |1\rangle_a|1\rangle_b\right) - \frac{e^{i(\varphi_a+\varphi_b)}}{2}\left(|1\rangle_a|0\rangle_b + |1\rangle_a|1\rangle_b\right)$

$= \frac{|1\rangle_a|0\rangle_b}{2}\left(e^{i\varphi_b} - e^{i(\varphi_a+\varphi_b)}\right) - \frac{|1\rangle_a|1\rangle_b}{2}\left(e^{i\varphi_b} + e^{i(\varphi_a+\varphi_b)}\right)$

$\varphi_a = 0$

$= -e^{i\varphi_b}|1\rangle_a|1\rangle_b \quad , \quad \varphi_b = \pi$

$= \underline{|1\rangle_a|1\rangle_b}$

iii) $|1\rangle_a|0\rangle_b \xmapsto{R_b(\pi/4)} \xmapsto{QPG(\delta=\pi)} \xmapsto{R_b(-\pi/4)} |1\rangle_a|1\rangle_b$

iv) $-\frac{1}{\sqrt{2}}\left(e^{i\varphi_b}|1\rangle_a|0\rangle_b + e^{i(\varphi_a+\varphi_b)}|1\rangle_a|1\rangle_b\right) \xmapsto{R_b(-\pi/4)}$

$= -\frac{|1\rangle_a|0\rangle_b}{2}\left(e^{i\varphi_b} + e^{i(\varphi_a+\varphi_b)}\right) + \frac{|1\rangle_a|1\rangle_b}{2}\left(e^{i\varphi_b} - e^{i(\varphi_a+\varphi_b)}\right)$

$\varphi_a = 0, \qquad \varphi_b = \pi$

$= \underline{|1\rangle_a} |0\rangle_b$

iv) $|1\rangle_a |1\rangle_b \xrightarrow{R_b(\pi/4)} \; \xrightarrow{QPG(\delta=\pi)} \; \xrightarrow{R_b(-\pi/4)} |1\rangle_a |0\rangle_b$

TRUTH TABLE

| | INPUT | OUTPUT |
|---|---|---|
| i) | 00 | 00 |
| ii) | 01 | 01 |
| iii) | 10 | 11 |
| iv) | 11 | 10 |

CNOT-GATE

ROTATION → BEAM SPLITS

QPG → QED-CAVITY →