Lecture21_QHardware_class16

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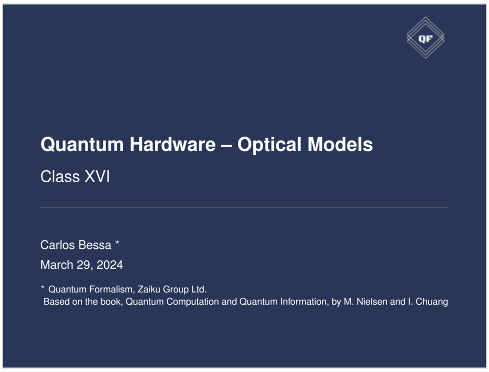
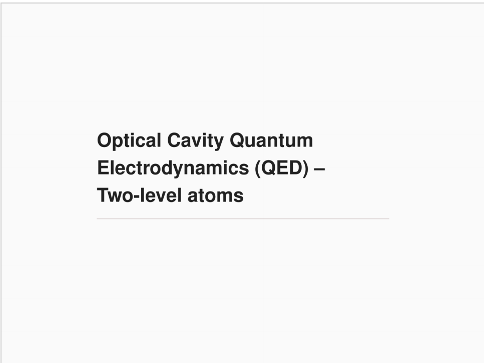
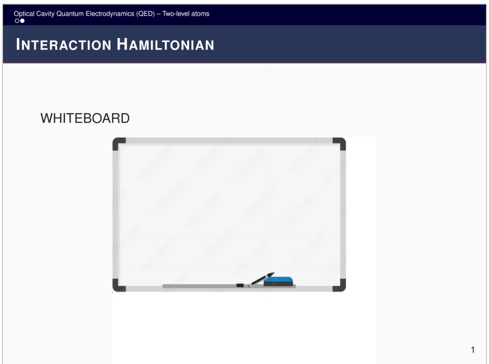
Lecture21...







REMEMBER: $H_{FIELD} = \hbar \omega a^{\dagger} a$

$H_{ATOM} = \frac{\hbar \omega_0}{2} Z$

THE INTERACTION HAMILTONIAN

– THE INTERACTION BETWEEN THE ATOM AND A CAVITY CONFINED ELECTRIC FIELD CAN BE WRITTEN BY THE TWO-LEVEL APPROXIMATION OF THE ATOM USING THE QUANTIZATION OF THE FIELD IN THE CAVITY

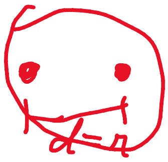
OF THE ATOM USING THE QUANTIZATION OF THE FIELD IN THE CAVITY

$$\vec{p}_{\text{Atom}} = q\vec{d}$$

$$\vec{d}\cdot\vec{E} \to \hat{r}\hat{E}, \quad (7.56) \Rightarrow \hat{E}(r) = i\epsilon E_0 \left[\hat{a}\, e^{ikr} - \hat{a}^{+} e^{-ikr}\right]$$

$$H_I \to \hat{d}\hat{E} \propto \hat{r}\, i\epsilon E_0 \left[\hat{a}\, e^{ikr} - \hat{a}^{+} e^{-ikr}\right], \quad r \to \text{POSITION}$$

$-$ INITIALY $r=0$



ATOM

$$H_I \to -i\hat{r}E_0[a-a^{+}], \quad E_0 = \epsilon\sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}$$

MATRIX REPRESENTATION

$$H_I = H_{ij} = -i\langle\psi_i|\hat{r}|\psi_j\rangle E_0(a-a^{+}) = -i\, r_{ij} E_0 (a-a^{+})$$

$$r_{ij} = r_0 X \to r_{ij} \approx r_0 Y \quad (7.65)$$

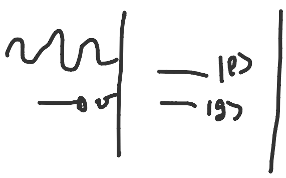
$$r_0 = 2\sqrt{\frac{\hbar}{8m\omega}}\, \operatorname{sen}\theta\, (\cos\varphi + \operatorname{sen}\varphi)$$

$$\varphi = \theta = \pi/2$$

$$r_0 = \sqrt{\frac{\hbar}{2m\omega}}$$

$$H_I = -i r_0 E_0 Y(a-a^{+}) = -igY(a-a^{+}), \quad g = r_0 E_0$$

$$g = \frac{\hbar\epsilon}{2\sqrt{m\epsilon_0 V}}$$



$-$ NOTE THAT THE H_I IS HERMITIAN

$$H_I = -igY(a-a^{+}) \qquad H_I^{+} = igY^{+}(a^{+}-a) = -igY(a-a^{+}) = H_I$$

$$H_I = -ig\gamma(a-a^\dagger) \quad , \quad H_I^\dagger = ig\gamma^\dagger(a^\dagger - a) \simeq -ig\gamma(a-a^\dagger) = H_I$$

— H_I CAN BE SIMPLIFIED FURTHER BY RECOGNIZING THAT IT CONTAINS TERMS WHICH ARE GENERALLY SMALL

— TO SEE THIS IT IS USEFUL TO DEFINE THE PAULI RAISING AND LOWERING OPERATORS

• $\sigma \to$ LOWERING ATOM OPERATOR

$$\sigma = |g\rangle\langle e| \Rightarrow \sigma|e\rangle = |g\rangle\langle e|e\rangle = |g\rangle$$

$$\sigma = |0\rangle\langle 1| \Rightarrow \sigma|1\rangle = |0\rangle$$

$$\sigma|0\rangle = |0\rangle\langle 1|0\rangle = 0$$

MATRIX REPRESENTATION : $\sigma = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

WE CAN WRITE σ IN TERMS OF X AND Y- PAULI GATES

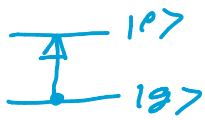
$$\sigma = \frac{X+iY}{2} = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \frac{i}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\sigma|0\rangle = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0$$

$$\sigma|1\rangle = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

• $\sigma^\dagger \to$ RAISING OPERATOR

$$\sigma^\dagger = |e\rangle\langle g| = |1\rangle\langle 0|$$



$$\sigma^\dagger|g\rangle = |e\rangle$$

$$\sigma^\dagger = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\sigma = \frac{x+iy}{2} \Rightarrow \sigma^+ = \frac{x-iy}{2} = \frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix} - \frac{i}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix} = \begin{pmatrix}0&0\\1&0\end{pmatrix}$$

$$\sigma^+|0\rangle = \begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} = |1\rangle$$

$$\sigma^+|1\rangle = \begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} = 0$$

$$\boxed{\cancel{\text{BOOK} \quad \sigma_\pm = \frac{x\pm iy}{2}}}$$

– WE CAN RE-EXPRESS EQ. (7.67) IN TERMS OF σ AND σ^+

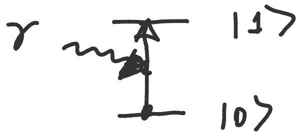
$$\sigma^+ - \sigma = \frac{x-iy}{2} - \frac{(x+iy)}{2} = -iy$$

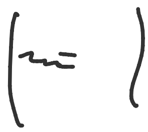
FROM (7.67) $\Rightarrow$ $H_I = -iyg(a-a^+) = g(\sigma^+ - \sigma)(a-a^+)$ (7.69)

$$H_I = g(\sigma^+ a - \sigma^+ a^+ - \sigma a + \sigma a^+)$$

LET'S ANALYSE EACH TERM OF H_I

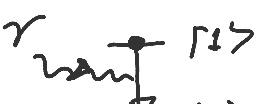
i) $\sigma^+ a$: (ABSORB) ANNIHILATE A PHOTON AND RAISE THE ATOM

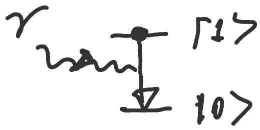




ii) $\sigma^+ a^+$: (EMISSION) CREATE A PHOTON AND RAISE THE ATOM

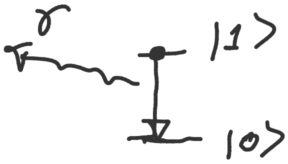
iii) σa : (ABSORB) ANNIHILATE A PHOTON AND LOWER THE ATOM

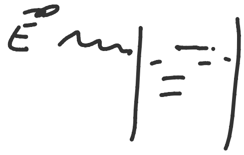




(EMISSION)

iv) $\sigma a^{\dagger}$: CREATE A PHOTO AND LOWER THE ATOM





– IN THE TERMS (ii) AND (iii) THE ENERGY IS NOT CONSERVED, THUS WE MUST DROP THEM OUT

$$H_I = g\left(\sigma^{+} a + \sigma a^{\dagger}\right) \qquad (*)$$

$$g \to \underline{g_k} \;,\; a \to a_k$$

– THE TOTAL HAMILTONIAN

$$H = H_{ATOM} + H_{FIELD} + H_I$$

$$H = \underbrace{\hbar \omega a^{\dagger} a}_{FIELD} + \underbrace{\frac{\hbar \omega_0}{2} z}_{ATOM} + \underbrace{g\left(\sigma^{+} a + \sigma a^{\dagger}\right)}_{INTERACTION} \qquad (7.10)$$

SUMMARY :

σ^{+}, σ	RAISING AND LOWERING OPERATORS ACTING ON THE TWO-LEVEL ATOM
ω_0	FREQUENCY OF THE ATOM 
$a^{\dagger}, a$	CREATE AND ANNIHILATING OPERATORS ACTING ON THE SINGLE MODE FIELD
ω	FREQUENCY OF THE FIELD
g	COUPLING CONSTANT FOR THE INTERACTION BETWEEN

ω | ... OF THE FIELD

g | COUPLING CONSTANT FOR THE INTERACTION BETWEEN THE FIELD AND THE ATOM

– THESE ARE THE FUNDAMENTAL THEORETICAL TOOLS IN THE STUDY OF CAVITY QED

– JAYNES-CUMMINGS HAMILTONIAN (H) → DESCRIBES INTERACTIONS BETWEEN TWO-LEVEL ATOMS AND THE ELECTROMAGNETIC FIELD

– WE CAN RE-WRITE H IN TERMS OF COMMUTING PARTS

$$H = \hbar\omega a^\dagger a + \frac{\hbar\omega z}{2} - \frac{\hbar\omega}{2} z + \frac{\hbar\omega_0}{2} z + g(\sigma^+ a + \sigma a^\dagger)$$

$$H = \hbar\omega\left(a^\dagger a + \frac{z}{2}\right) + \frac{\hbar}{2}(\omega_0 - \omega) z + g(\sigma^+ a + \sigma a^\dagger)$$

$\delta = \dfrac{\omega_0 - \omega}{2}$ → DETUNING FREQUENCY

$N = a^\dagger a + \dfrac{z}{2}$

missing in NIELSEN-CHUANG

$$H = \hbar\omega N + \hbar\delta z + g(\sigma^+ a + \sigma a^\dagger) \qquad (7.74)$$

δ → FREQUENCY DIFFERENCE BETWEEN THE FIELD AND THE ATOMIC RESONANCE

EXERCISE 7.17 (EIGENSTATES OF THE JAYNES-CUMMINGS HAMILTONIAN)

$$|\chi_n\rangle = \frac{1}{\sqrt{2}}\left[\,|n,1\rangle + |n+1,0\rangle\,\right] \qquad (7.72)$$

$$|\bar{\chi}_n\rangle = \frac{1}{\sqrt{2}}\left[\,|n,1\rangle - |n+1,0\rangle\,\right] \qquad (7.73)$$

$$|\text{FIELD, ATOM}\rangle = |\text{FIELD}\rangle \otimes |\text{ATOM}\rangle$$

$\omega = \delta = 0$ in Eq. (7.71)

$$H = g\,(a^{+}\sigma + a\sigma^{+})$$

$$H|\chi_n\rangle = \frac{g}{\sqrt{2}}\,(a^{+}\sigma + a\sigma^{+})\,(|n,1\rangle + |n+1,0\rangle)$$

$$= \frac{g}{\sqrt{2}}\Big(a^{+}\sigma\,|n,1\rangle + a^{+}\sigma\,|n+1,0\rangle + a\sigma^{+}|n,1\rangle + a\sigma^{+}|n+1,0\rangle\Big)$$

$\sigma|1\rangle = |0\rangle$; $\sigma|0\rangle = 0$

$$a^{+}\sigma\,|n\rangle \otimes |1\rangle = a^{+}|n\rangle \otimes \sigma|1\rangle$$

$$\begin{cases} \sigma|1\rangle = |0\rangle \\ a^{+}|n\rangle = \sqrt{n+1}\,|n+1\rangle \end{cases} \quad , \quad \sigma^{+}|0\rangle = |1\rangle \quad , \quad \sigma^{+}|1\rangle = 0 \quad , \quad a|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$H|\chi_n\rangle = \frac{g}{\sqrt{2}}\left(\sqrt{n+1}\,|n+1,0\rangle + 0 + 0 + \sqrt{n+1}\,|n,1\rangle\right)$$

$$H|\chi_n\rangle = g\sqrt{n+1}\,\underbrace{\frac{1}{\sqrt{2}}\left(|n+1,0\rangle + |n,1\rangle\right)}_{|\chi_n\rangle}$$

$$H|\chi_n\rangle = \sqrt{n+1}\, g\, |\chi_n\rangle \qquad (7.74)$$

$|\chi_n\rangle \rightarrow$ EIGENSTATE OF H WITH EIGENVALUES $\sqrt{n+1}\, g$

HOMEWORK

$$H|\bar{\chi}_n\rangle = -g\sqrt{n+1}\, |\bar{\chi}_n\rangle \qquad (7.75)$$