Trying out quantum error correction on IBM Quantum hardware

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NCCR SPIN

IBM **Quantum**

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arXiv.org > quant-ph > arXiv:2109.13308

Quantum Physics

[Submitted on 27 Sep 2021]

Hexagonal matching codes with 2-body measurements

James R. Wootton

Matching codes are stabilizer codes based on Kitaev's honeycomb lattice model. The hexagonal form of these codes are particularly well-suited to the heavy-hexagon device layouts currently pursued in the hardware of IBM Quantum. Here we show how the stabilizers of the code can be measured solely through the 2-body measurements that are native to the architecture. The process is then run on 27 and 65 qubit devices, to compare results with simulations for a standard error model. It is found that the results correspond well to simulations where the noise strength is similar to that found in the benchmarking of the devices. The best devices show results consistent with a noise model with an error probability of around $1.5\% - 2\%$.

Journal of Physics A: Mathematical and Theoretical

PAPER

A family of stabilizer codes for $D(\mathbb{Z}_2)$ anyons and Majorana modes

James R Wootton[1]

Published 6 May 2015 • © 2015 IOP Publishing Ltd

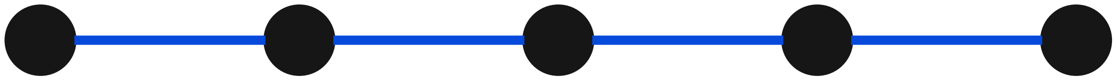
Journal of Physics A: Mathematical and Theoretical, Volume 48, Number 21

Citation James R Wootton 2015 *J. Phys. A: Math. Theor.* **48** 215302

Stabilizer Codes

Given *n* noisy qubits, how can we make *k* noiseless ones?

- Set n-k constraints (commuting, frustration free and independent)
- Constantly measure whether the constraints are satisfied
- Use the results to detect and correct errors



For an (overly) simple example: the repetition code

- 1 bit value encoded
- n-1 independent checks on neighbouring qubits: $\{|00\rangle, |11\rangle\}$ or $\{|01\rangle, |10\rangle\}$

Pauli measurements

The states $|0\rangle$ and $|1\rangle$ are eigenstates of Z with different eigenvalues

$$Z|0\rangle = |0\rangle = (-1)^0|0\rangle, \qquad Z|1\rangle = -|1\rangle = (-1)^1|1\rangle$$

The standard measurement, which distinguishes between these is called a Z measurement

Similarly, X has eigenstates

$$X|+\rangle = |+\rangle, \qquad X|-\rangle = -|-\rangle, \qquad |\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

We can also implement a so-called X measurement to distinguish these

Similarly there are Y measurements, etc

Pauli measurements

The same principle can be used to define multiqubit measurements, such as ZZ

$$Z_j Z_k|00\rangle = |00\rangle, \quad Z_j Z_k|11\rangle = (-1)^2|11\rangle = |11\rangle,$$
$$Z_j Z_k|01\rangle = -|01\rangle, \quad Z_j Z_k|10\rangle = -|10\rangle,$$

This extracts a singe bit of information about two qubits

- Are they in the +1 eigenspace? $\{|00\rangle, |11\rangle\}$ → give outcome 0
- Or the -1 eigenspace? $\{|01\rangle, |10\rangle\}$ → give outcome 1

Similarly we can have *XX* and *YY* measurements, *ZZZZ* measurements and so on

Commutation and Anticommutation

The X and Z gates anticommute: $XZ = -ZX$

This translates into their measurements being incompatible

- Measuring one makes the other random: $|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$

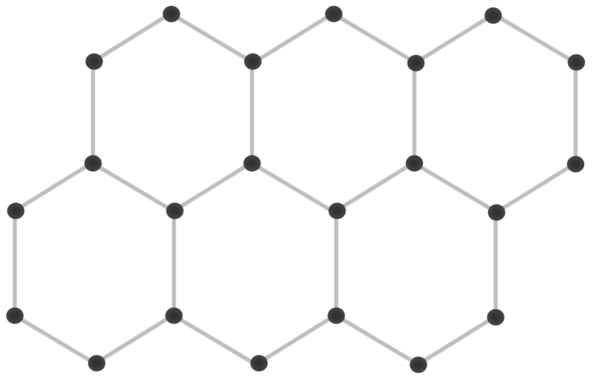
However, note that $Z_j Z_k$ and $\mathrm{X_j X_k}$ commute $X_j X_k\ Z_j Z_k = (-1)^2 Z_j Z_k\ X_j X_k$

This means that there are states with well-defined outcomes for both, e.g.

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}} = \frac{|++\rangle + |--\rangle}{\sqrt{2}}$$

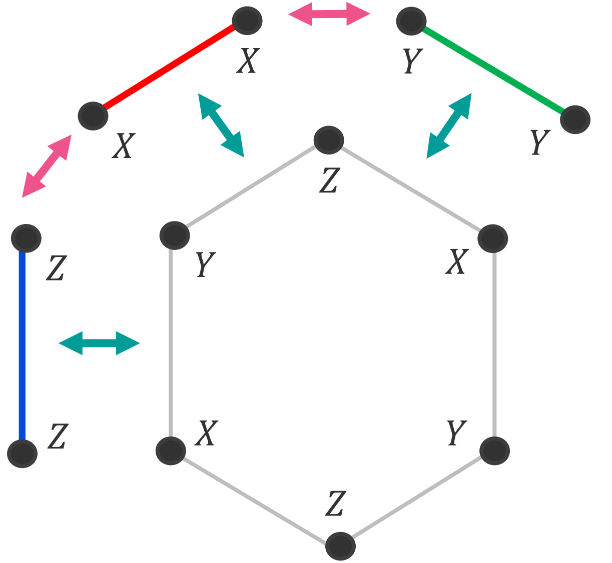
Matching codes

We are going to look at some codes on a hexagonal lattice



Three kinds of measurement will be important:

- 6 qubit ‘plaquette’ measurements
- 2 qubit ZZ measurements on vertical links
- XX and YY measurements on the other links



Matching codes

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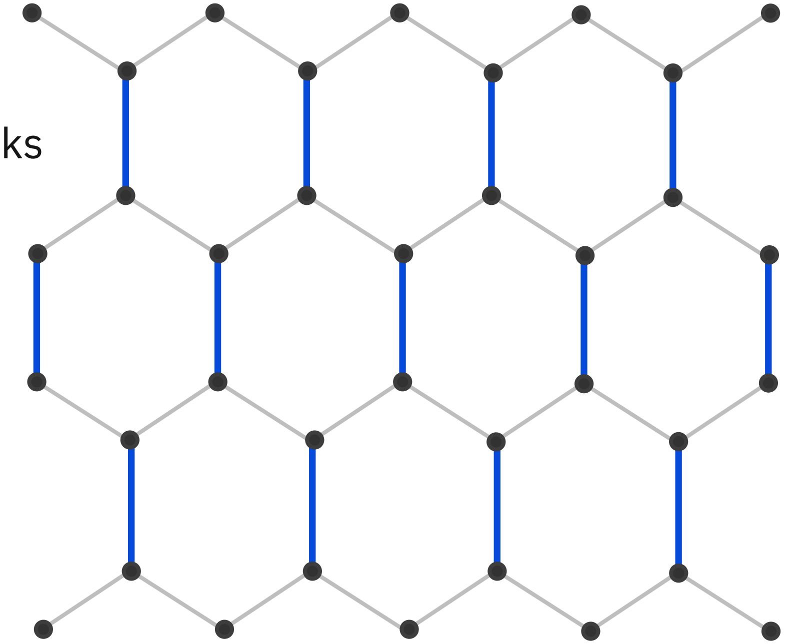
We'll define a code with

- All plaquette measurements
- 'Link measurements' for a commuting subset of links

For each pair of qubits, we can define

- 1 unique plaquette
- 1 unique link



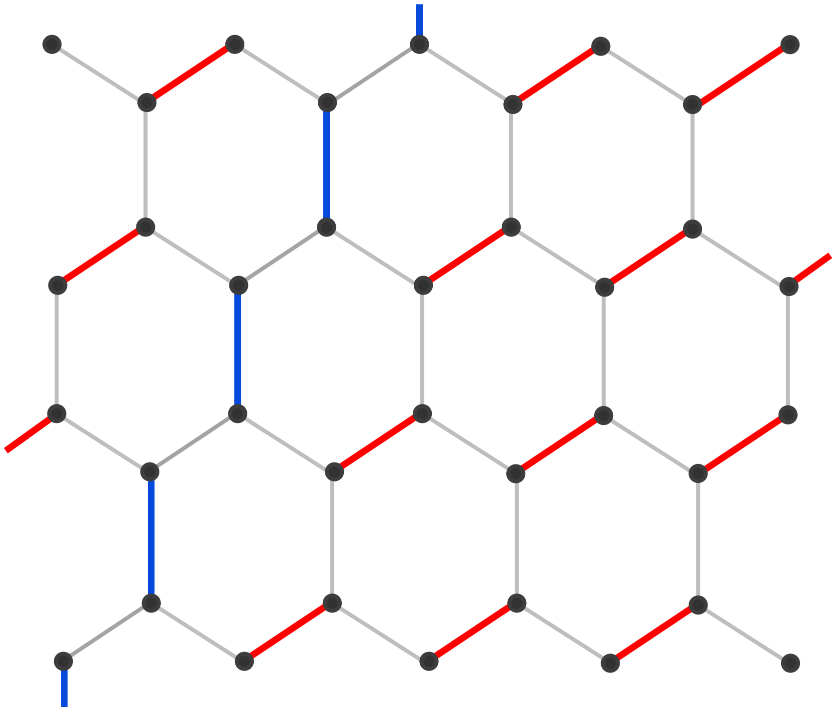
Matching codes

There are as many constraints as there are qubits

This means there is no space to store information

We need to make some space!

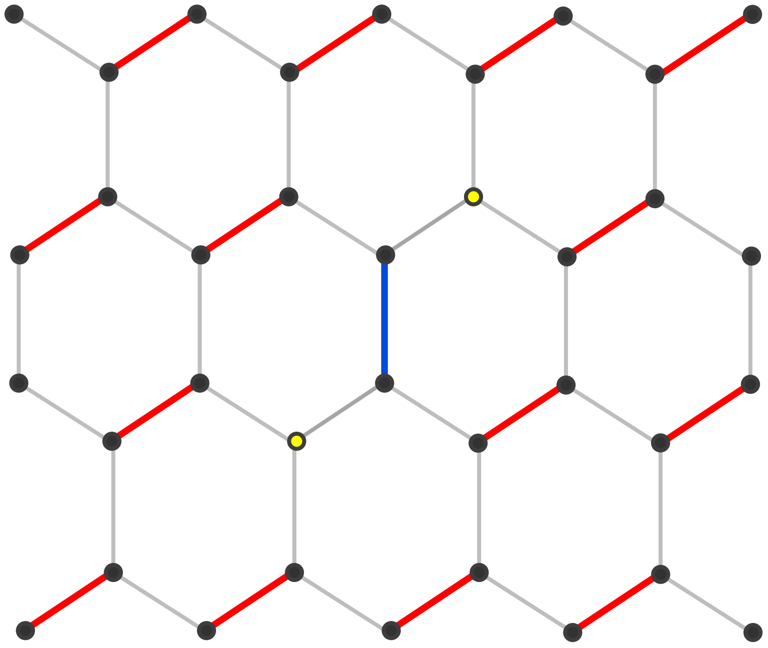
To see how, note that we could equally have used a different set of link operators



Matching code defects

We can also choose a set of links that leave two ‘defects’

- Part of plaquette measurements
- Not part of any link measurements



Then we have one less constraint than qubits

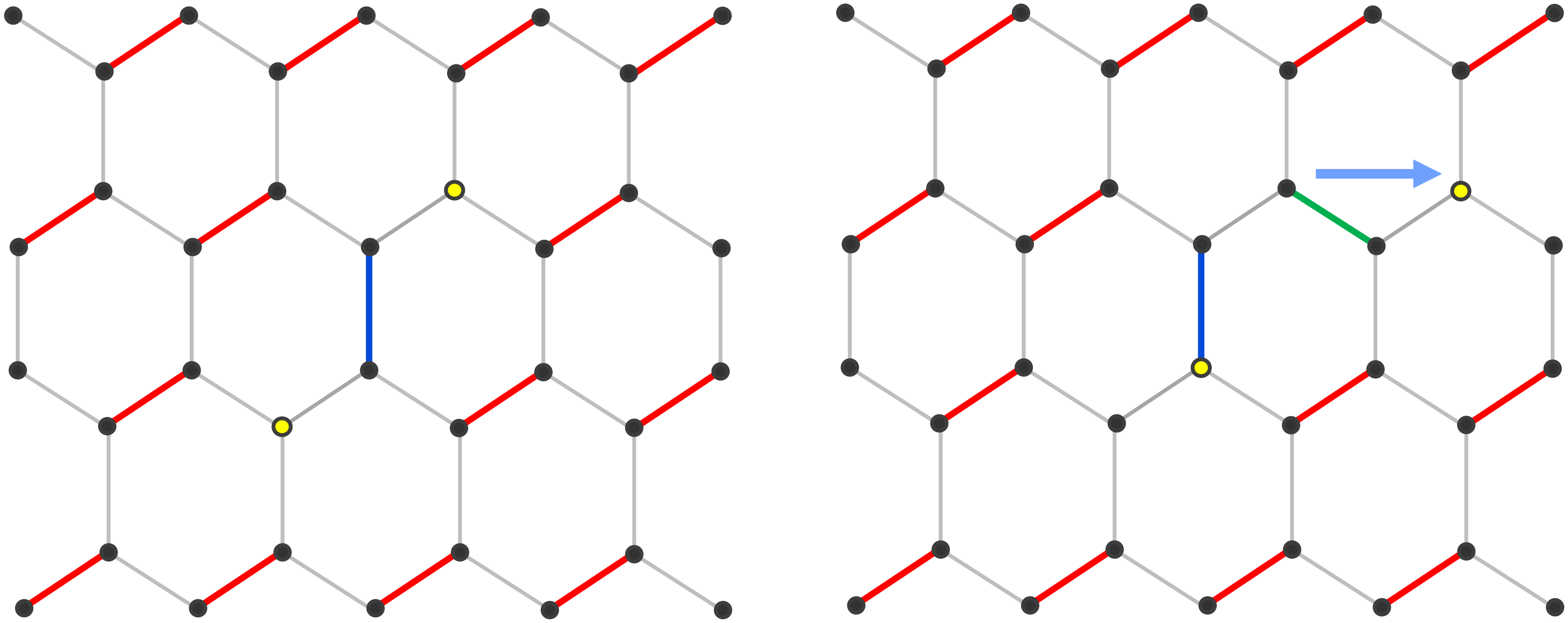
Space for one logical qubit

Code distance is the distance between the defects

IBM Quantum / © 2021 IBM Corporation

Matching code defects

By changing the measurements made, the defects can be moved



Matching code defects

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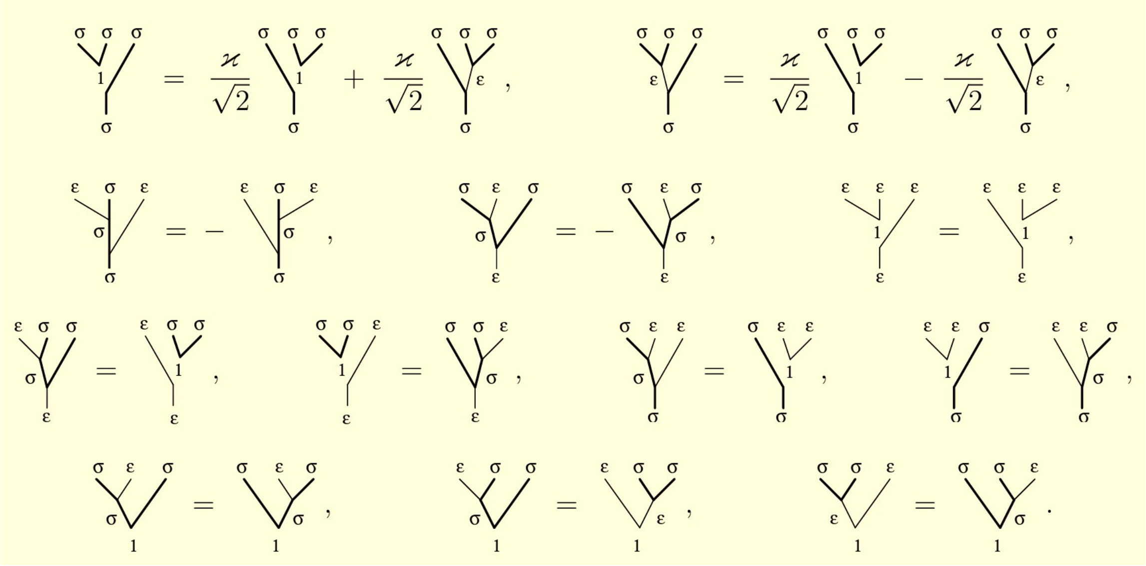
The defects behave as Majorana modes (aka Ising anyons)

Allows gates to be performed via topological quantum computation



A. Kitaev, Annals of Physics 321 (2006) 2–111

Matching codes

All this can be found in my paper on matching codes

Journal of Physics A: Mathematical and Theoretical

PAPER

A family of stabilizer codes for $D(\mathbb{Z}_2)$ anyons and Majorana modes

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Braiding matching code defects

This also contains a proposal for a minimal implementation of Majorana braiding

The experiment was done 1 year later, thanks to IBM Quantum!

Minimal Demonstration of Braiding

The simplest example of an exchange of two computational Majoranas is shown in Fig. 5. The system is composed of six qubits, corresponding to three adjacent z-links in the honeycomb lattice and the x and y links that connect them. The only stabilizer is $S = K_C$. The link operators $\pi_A = K_A$ and $\pi_B = K_B$ are taken to be parity operators for their respective pairs. The fusion space of the computational anyons is four-dimensional. A basis $\{|k_A, k_B\rangle\}$ can be defined using the eigenstates of the parity operators, labelled by the eigenvalues $k = \pm 1$

A clockwise exchange of the computational Majorana at 1 with that at 2 can be achieved by first measuring K_D, then K_E, and then finally the stabilizer $S = K_C$. In a proof of principle experiment, we may simply post-select the results for each of these gives the outcome $+1$.

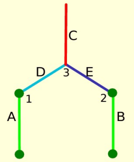


FIG. 5. Two pairs of computational Majoranas, with parity operators given by links A and B. Link C denotes a stabilizer.

Single and double exchanges have the following effect on the basis states for the computational Majoranas

$$R : |k_A, k_B\rangle \rightarrow \frac{1}{\sqrt{2}}(|k_A, k_B\rangle + i\,|-k_A, -k_B\rangle)$$

$$R^2 : |k_A, k_B\rangle \rightarrow |-k_A, -k_B\rangle \quad (20)$$

By preparing and then measuring these basis states, the effects of the braiding may then be shown.

PAPER

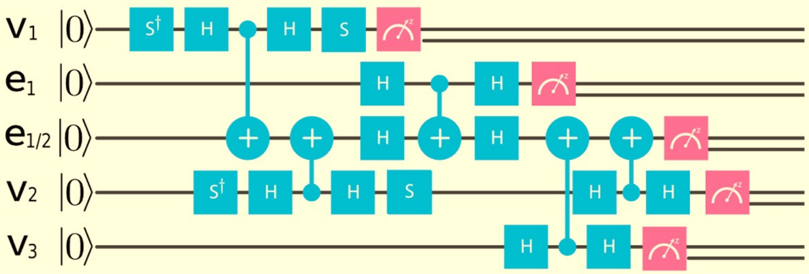
Demonstrating non-Abelian braiding of surface code defects in a five qubit experiment

James R Wootton[1]

Published 7 March 2017 • © 2017 IOP Publishing Ltd

Quantum Science and Technology, Volume 2, Number 1

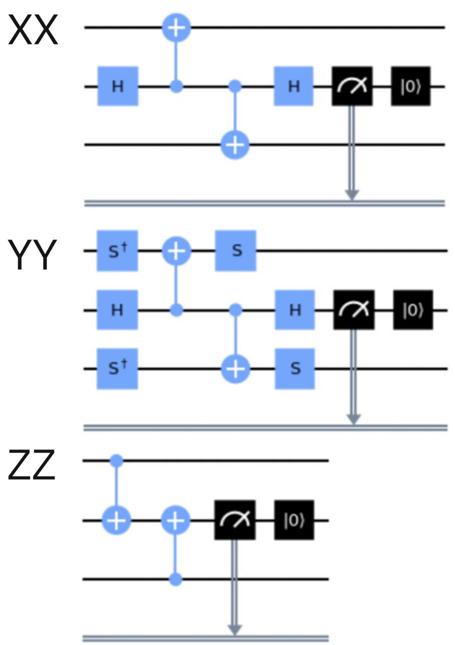
Citation James R Wootton 2017 *Quantum Sci. Technol.* **2** 015006

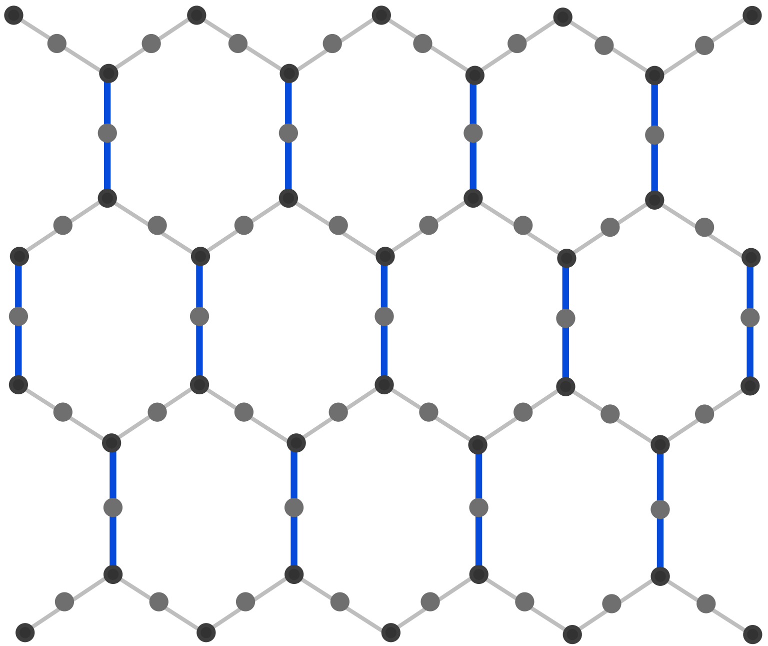


Measuring the links

Measuring the link operators is fairly easy

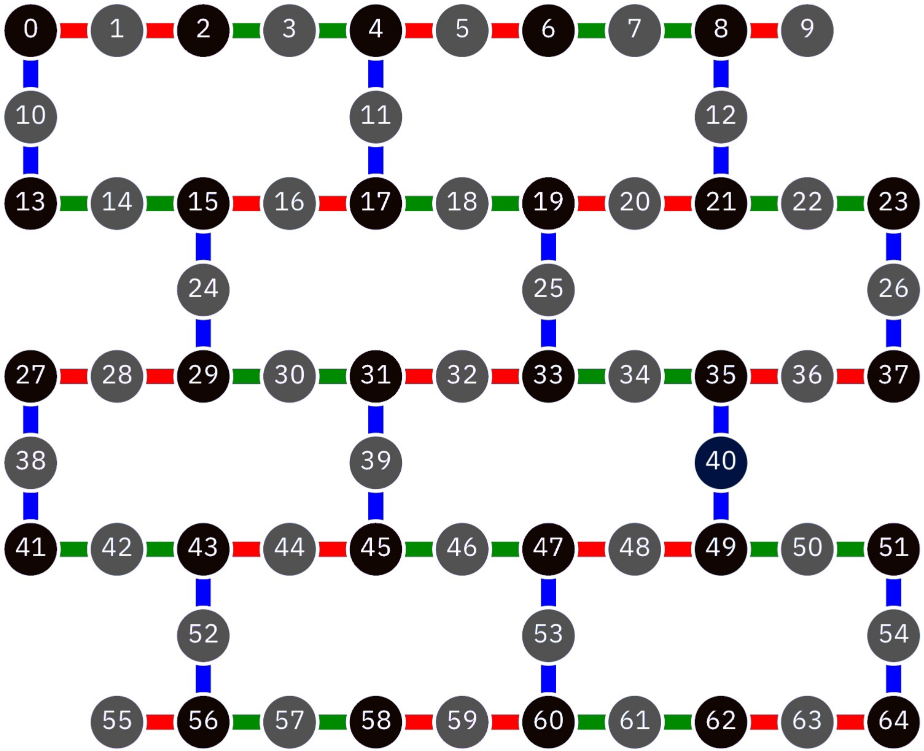
Just need an extra qubit on each link





Measuring the links

This is exactly what the connectivity of IBM Quantum devices gives us

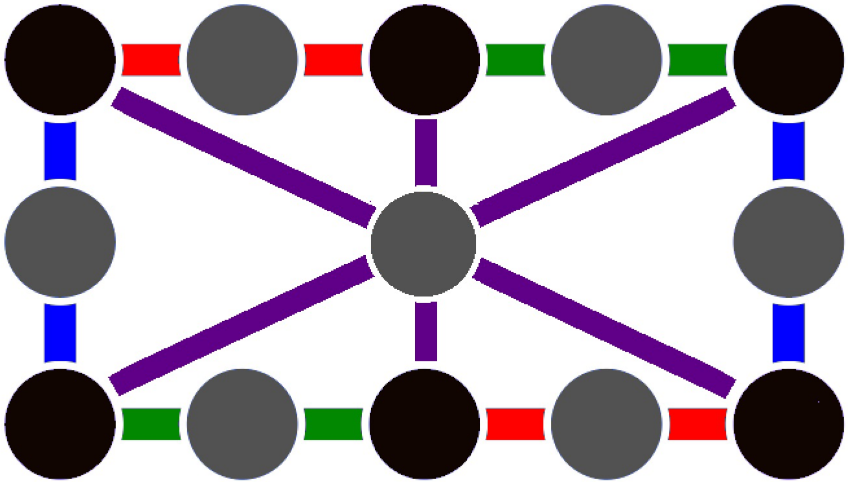


Measuring the plaquettes

The 6-qubit plaquette measurement could be done similarly

Would require an extra qubit in each plaquette, connected to 6 of its neighbours

I’ve written it on my Christmas list, but I don’t expect any miracles



IBM Quantum / © 2021 IBM Corporation

Measuring the plaquettes

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We need to find another way

Note that each plaquette op is a product of link ops

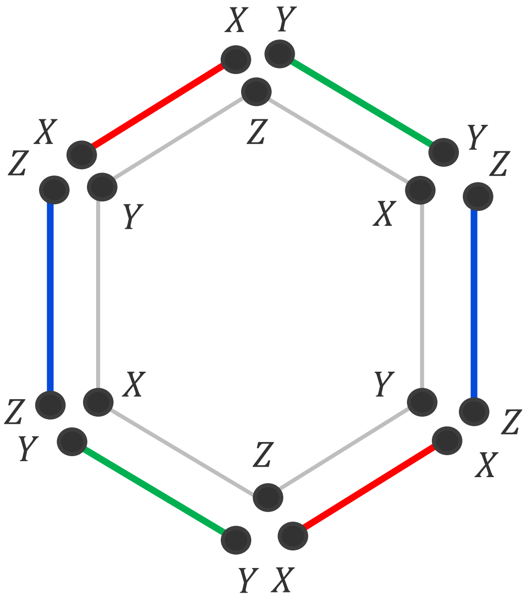
Naïve question

- Can we just measure all the link ops and combine results?

Naïve answer

- No! They don't commute.

But if we're careful, there is a way…



Measuring the plaquettes

One way to measure a plaquette op

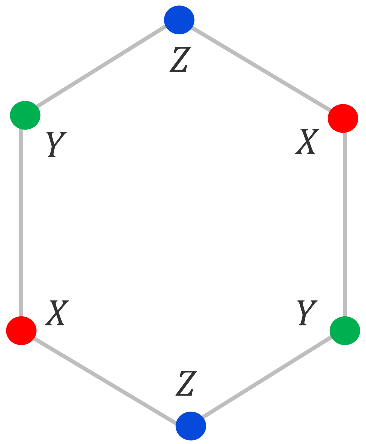
- Do 6 single qubit measurements (2X, 2Y and 2Z)
- Add up all the results (mod 2)

It works because

- We are measuring things that combine to make the plaquette op
- They all commute

But it's bad because

- It disentangles the qubits
- It does not commute with other neighbouring plaquette ops
- It does not commute with neighbouring link ops

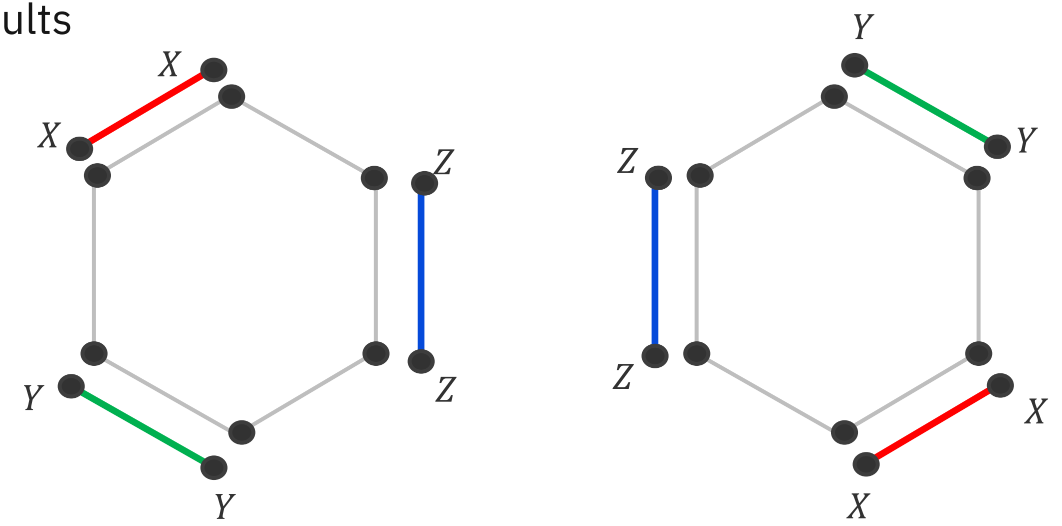


Measuring the plaquettes

Instead we could split the plaquette operator into two halves

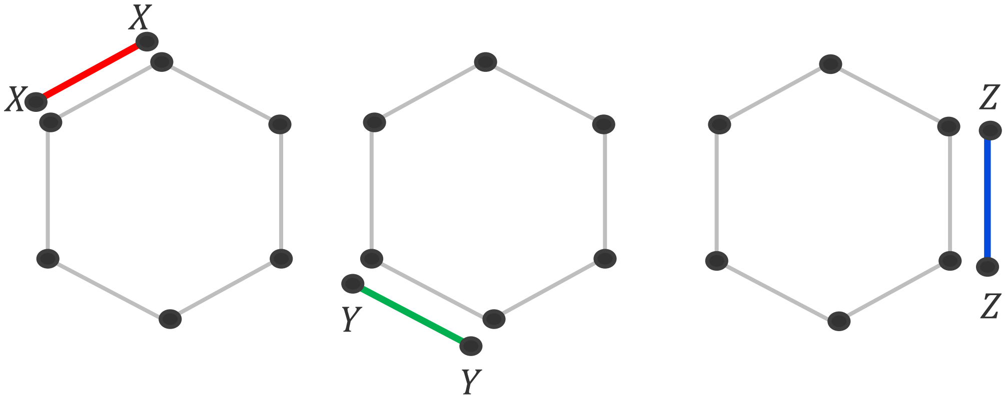
- Each made up of half the link operators
- They commute
- Can measure both and combine results

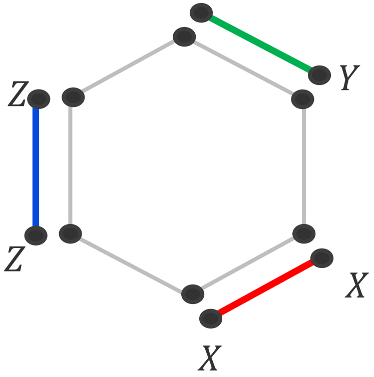
But how do we measure these?



Measuring the plaquettes

To measure the first half, we measure its three link ops



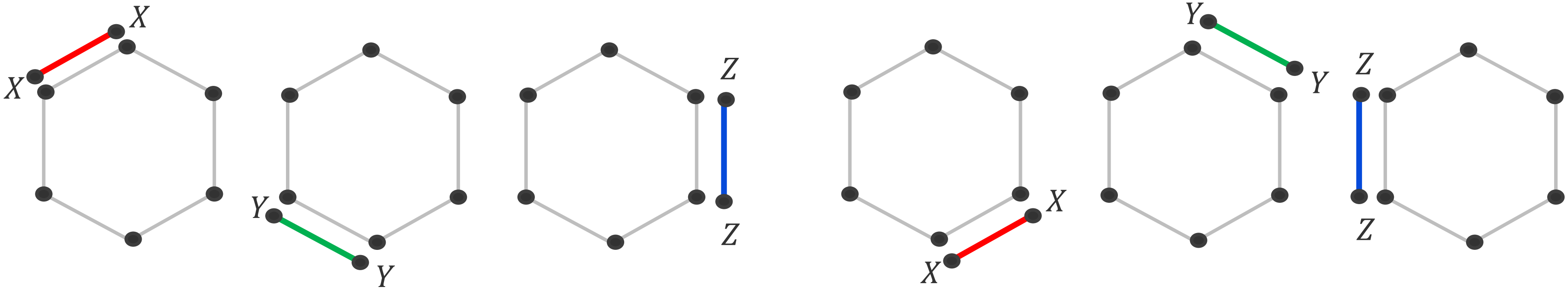


This works because these commute with each other

They also commute with the second half, and so don’t disturb its subsequent measurement

Measuring the plaquettes

To measure the second half, we measure its three link ops



This works because these commute with each other

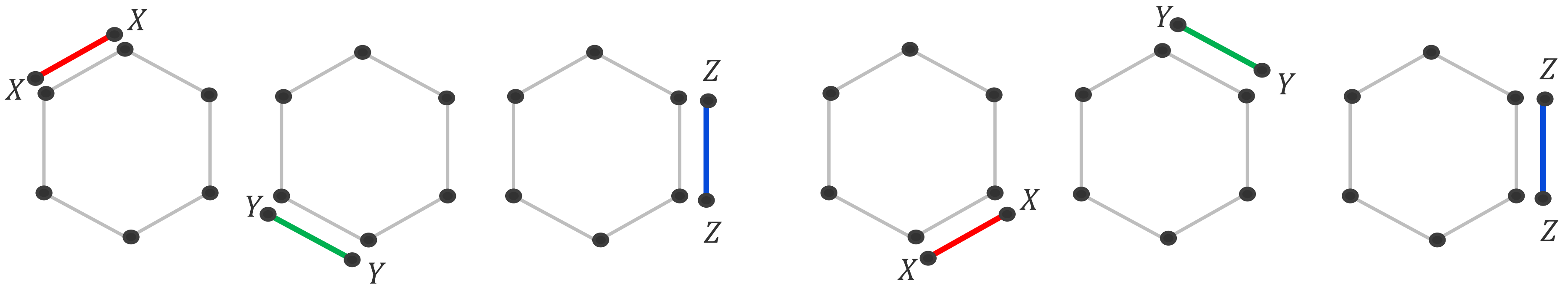
They don't commute with the first half, but that's already been measured

Measuring the plaquettes

This process:

- Commutes with the plaquette op being measured
- Commutes with all other plaquette ops

So as far as the plaquettes are concerned, this is a great way to measure!



Measuring the plaquettes

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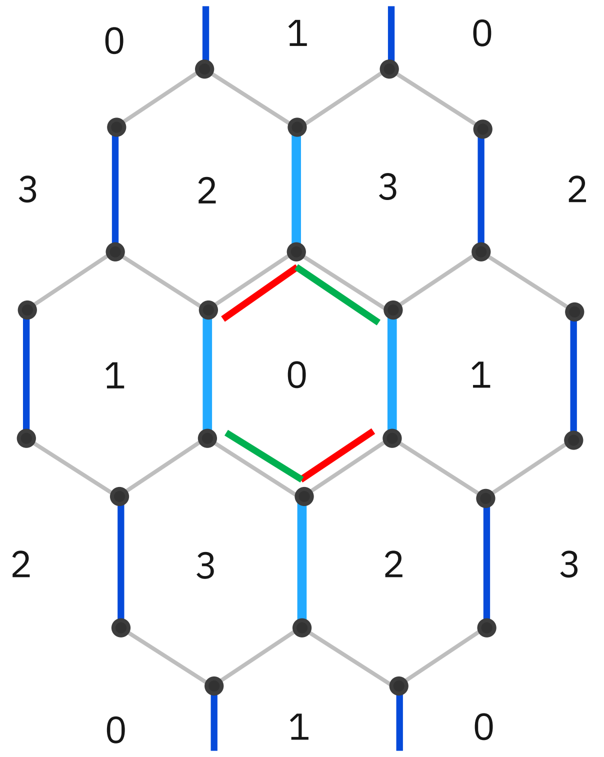
But they don't commute with the surrounding link ops

By measuring the plaquettes we make a mess

If we measuring all at once, we'd make an uncorrectable mess

So we measure them in shifts



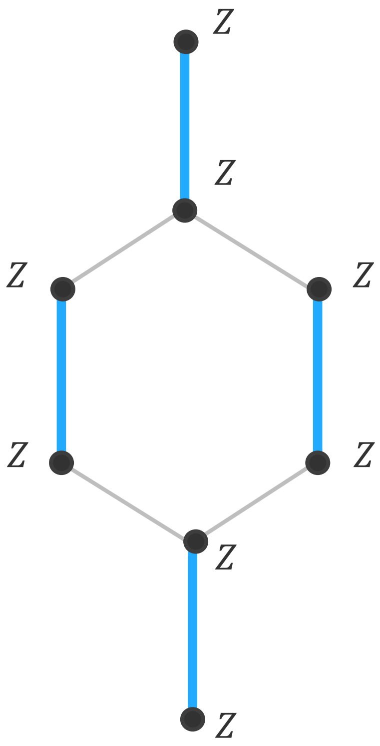
Measuring the plaquettes

Each shift causes lots of localized messes

- Effects are completely detected by link ops
- Link ops must all be measured between shifts

Product of link ops does commute

Effectively a second type of plaquette op



Running the measurements

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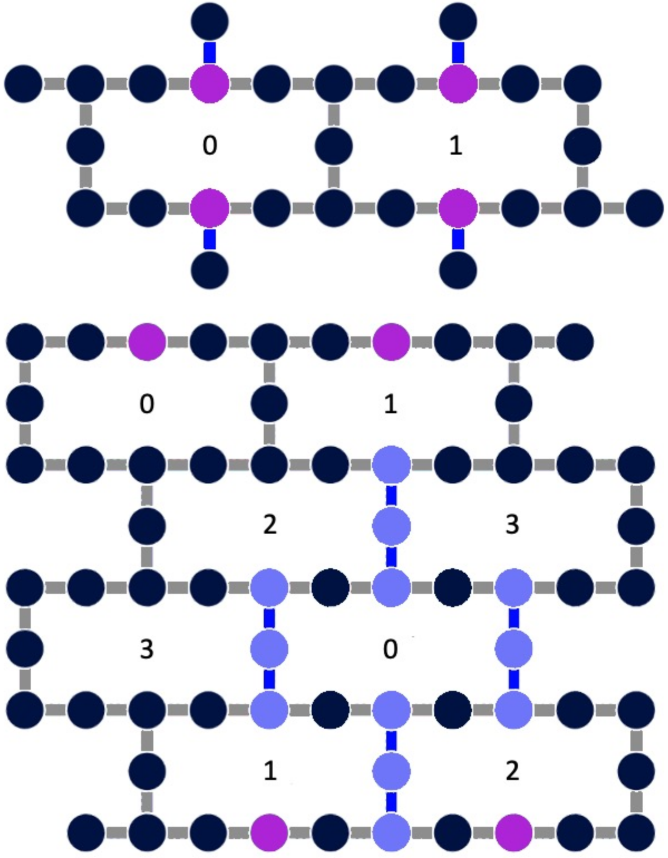
Now we can make some syndrome measurements!

- Used Falcon and Hummingbird devices
- Ran three rounds of plaquette measurements
- Calculated probs of syndrome change
  - For plaquette ops
  - For product of link ops for plaquette
- Then averaged these over all rounds and plaquettes

Gives us two numbers to characterize each device



Results

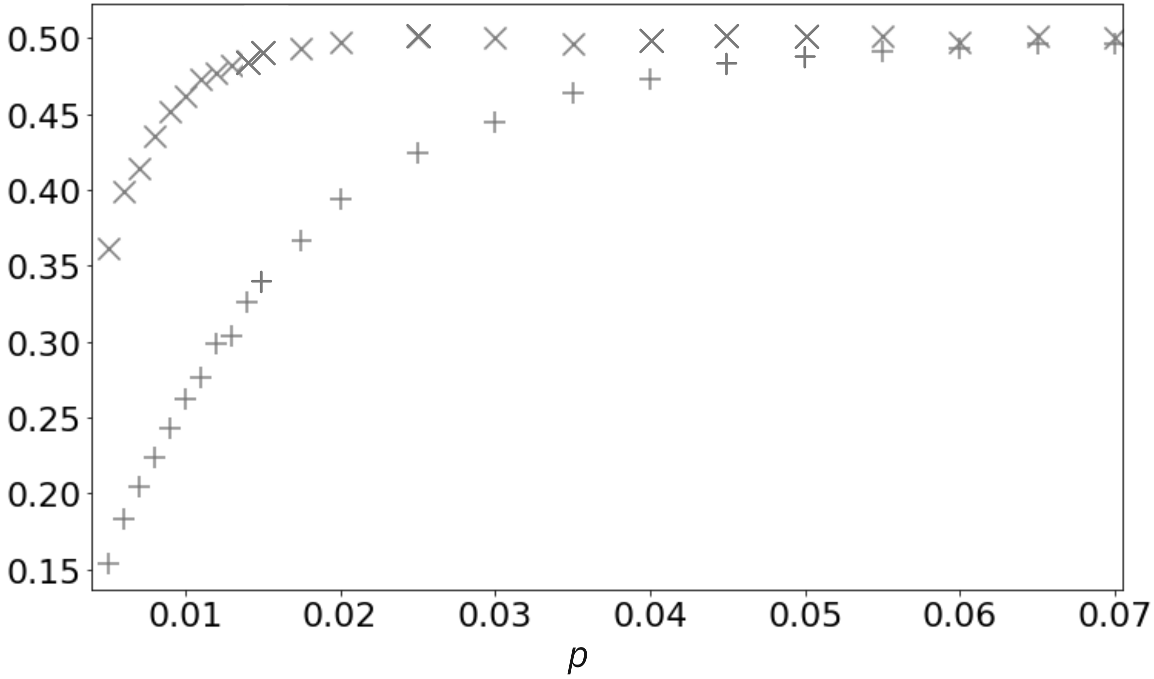
The same was done for a simulated Falcon with a standard error model

- Everything that can go wrong, goes wrong with probability *p*

Average probability of syndrome change

X Plaquette ops

+ Link ops



Results

We can extract equivalent error probabilities from the benchmarking of real devices

To distill down to a single number, we take the mean

Device	$\langle p \rangle$	σ	QV
ibm_cairo	1.47 %	1.23 %	64
ibm_hanoi	1.55 %	1.61 %	64
ibmq_brooklyn	4.73 %	5.00 %	32
ibmq_montreal	4.41 %	6.00 %	128
ibmq_toronto	5.17 %	6.45 %	32
ibmq_manhattan	18.3 %	31.9 %	32

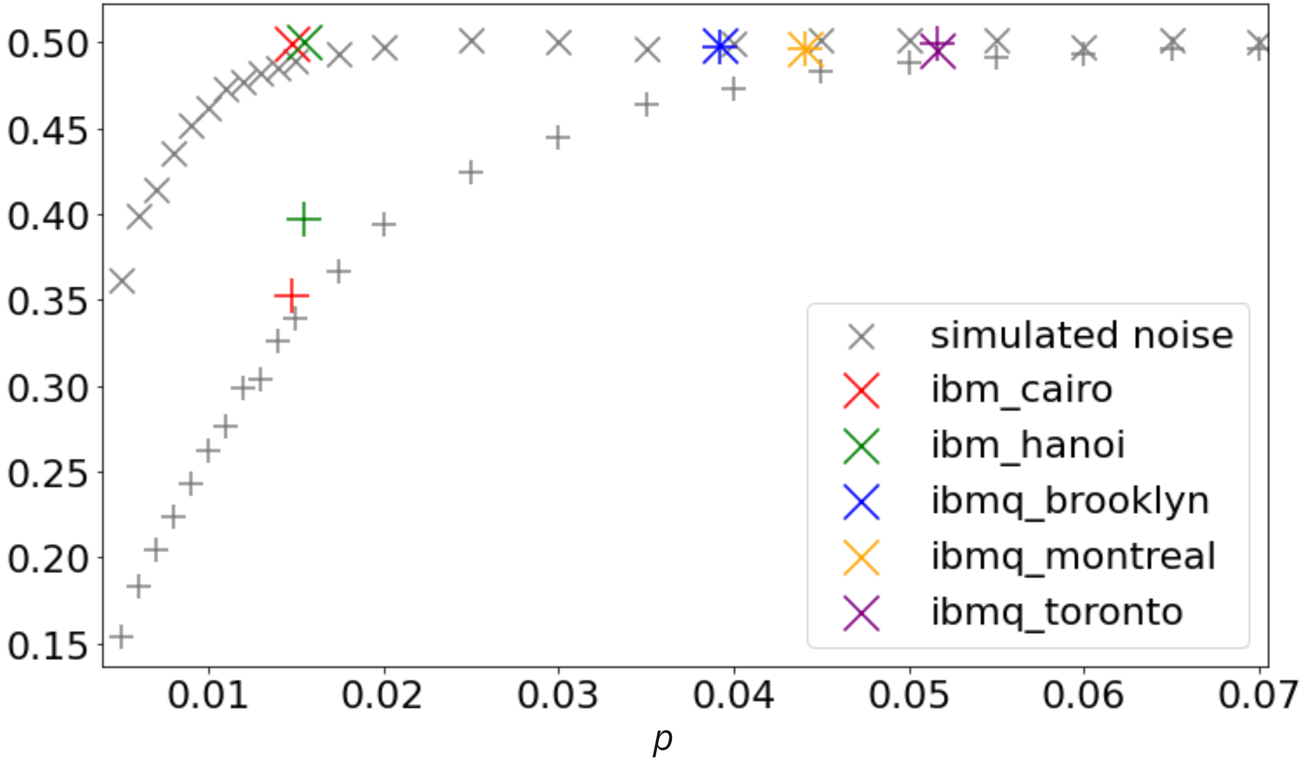
Results

Now we can add results from the real devices on the plot

Average probability of syndrome change

X Plaquette ops

+ Link ops



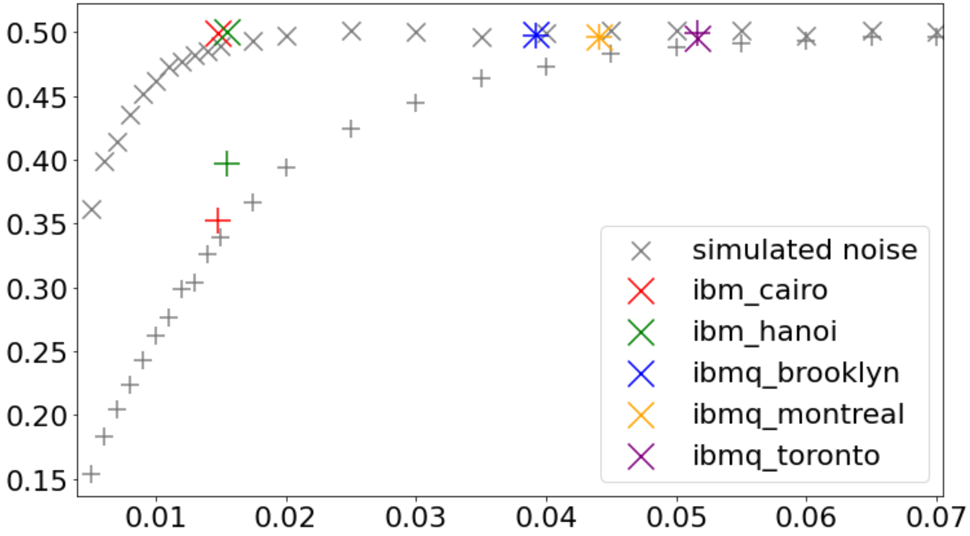
Results

Best things we can say:

- Some devices have results equivalent to 1-2% noise
- This is in the ballpark of the surface code threshold

Worst things we can say:

- Plaquette measurements always ≈50%: same as for completely random noise



To-do list (for the code)

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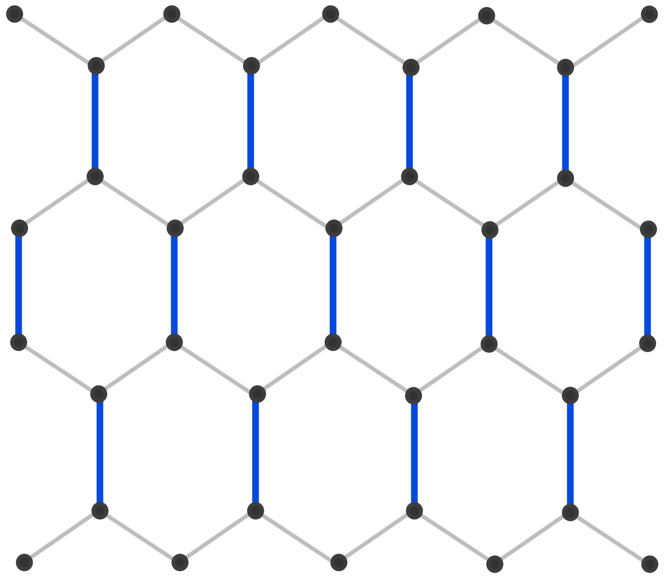
- Write a decoder
- Determine the threshold
- Compare with other codes for the heavy hexagonal lattice



arXiv.org > quant-ph > arXiv:2110.04285

Quantum Physics

[Submitted on 8 Oct 2021]

Calibrated decoders for experimental quantum error correction

Edward H. Chen, Theodore J. Yoder, Youngseok Kim, Neereja Sundaresan, Srikanth Srinivasan, Muyuan Li, Antonio D. Córcoles, Andrew W. Cross, Maika Takita

Arbitrarily long quantum computations require quantum memories that can be repeatedly measured without being corrupted. Here, we preserve the state of a quantum memory, notably with the additional use of flagged error events. All error events were extracted using fast, mid-circuit measurements and resets of the physical qubits. Among the error decoders we considered, we introduce a perfect matching decoder that was calibrated from measurements containing up to size-4 correlated events. To compare the decoders, we used a partial post-selection scheme shown to retain ten times more data than full post-selection. We observed logical errors per round of $2.2 \pm 0.1 \times 10^{-2}$ (decoded without post-selection) and $5.1 \pm 0.7 \times 10^{-4}$ (full post-selection), which was less than the physical measurement error of 7×10^{-3} and therefore surpasses a pseudo-threshold for repeated logical measurements.

To-do list (for the hardware)

IBM **Quantum**

- More qubits!
- Less noise!
  - Need to see <50% results from plaquette ops
  - Need to see <50% results from Hummingbird link ops

```
Mean error probabilities for ibm_cairo

Measurement                   0.012714814814814821
Preparation                   0.010925925925925927
cx gates                      0.010308158565038878
Idling during measurement     0.02306839108570667

Overall mean                  0.014218119258120816
```

```
Mean error probabilities for ibmq_montreal

Measurement                   0.02578888888888889
Preparation                   0.018548148148148154
cx gates                      0.015098342079556836
Idling during measurement     0.1161724110367075

Overall mean                  0.04363769427723573
```

```
Mean error probabilities for ibmq_brooklyn

Measurement                   0.030695384615384617
Preparation                   0.018538461538461542
cx gates                      0.012859957230917447
Idling during measurement     0.09753480141600156

Overall mean                  0.03919804873657737
```

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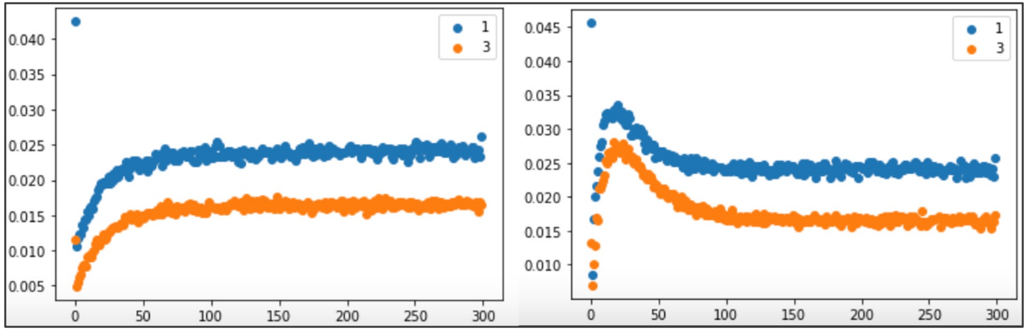
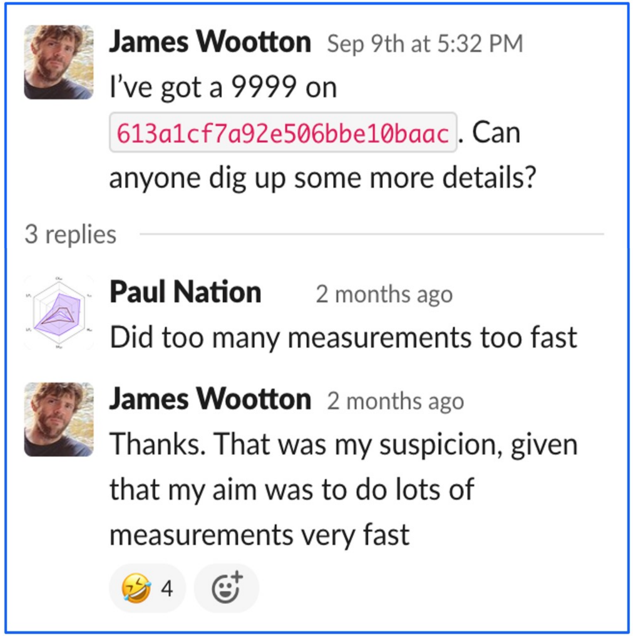
To-do list (for the control systems)

- Would be great to do many measurement rounds
  - See how error rate changes throughout circuit



J. R. Wootton, D. Miller, in preparation



- Unfortunately, anything more than 3 triggers an error

- To test (and perform) QEC, this will need to be fixed

Thanks for your attention!

qisk.it/topological_codes

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