Introduction to Quantum Error Correction

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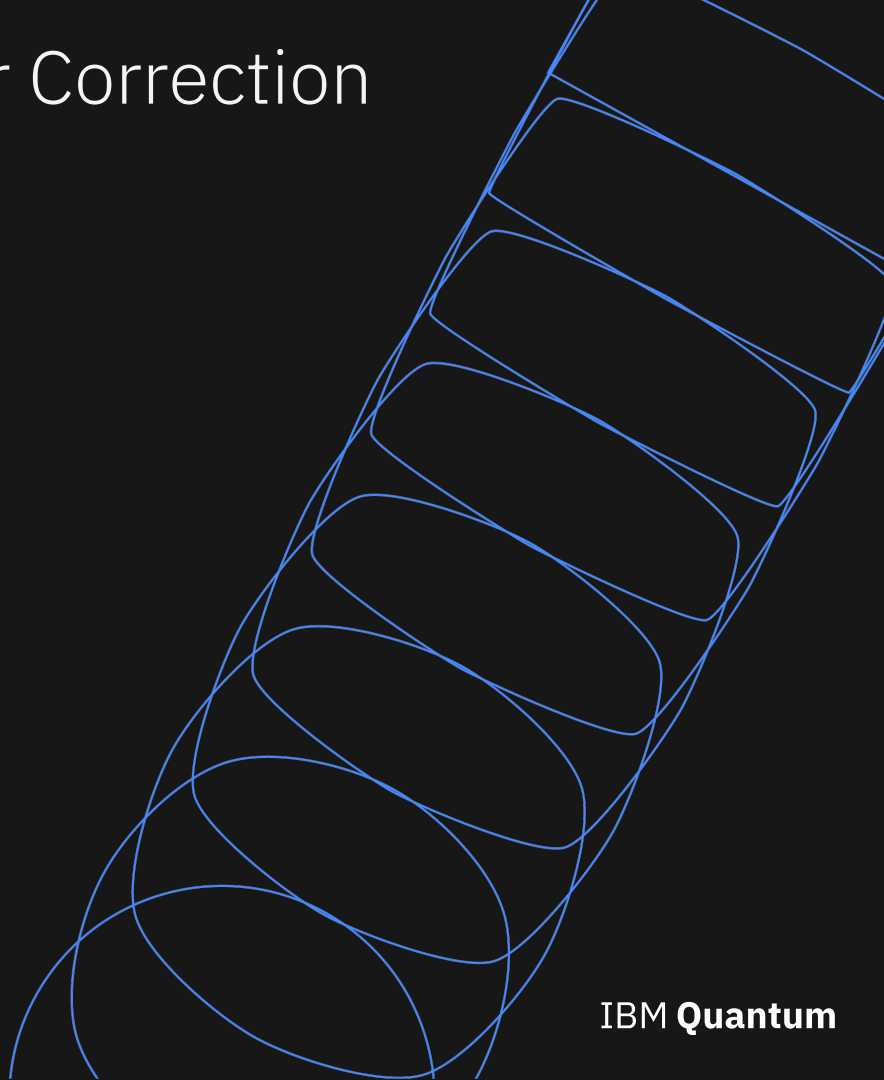
Qubit Wrangler



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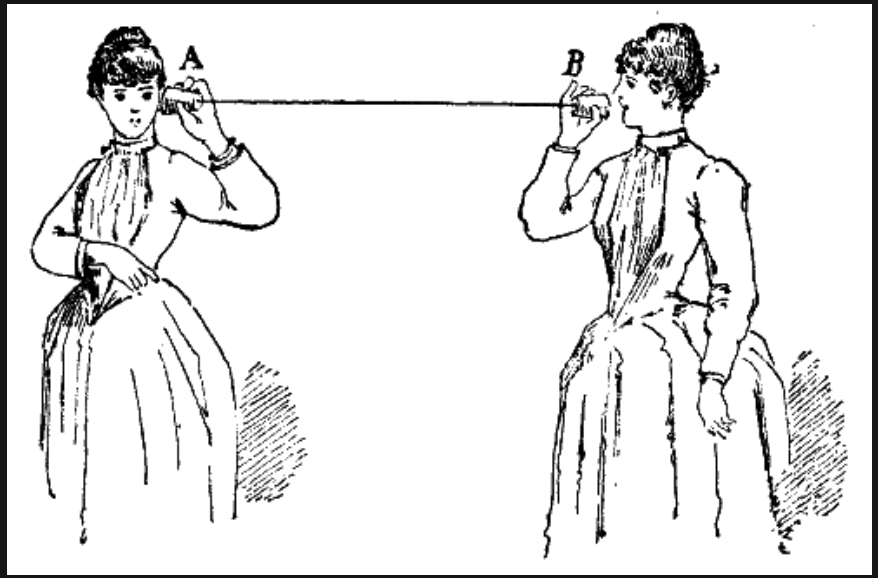
What is Error Correction?

- Before we explain *quantum* error correction, we should think about the general idea
- A simple example: you are talking on the phone, and need to answer a question with ‘yes’ or ‘no’.
- Two important things to consider:
  - How likely is it that you will be misheard?

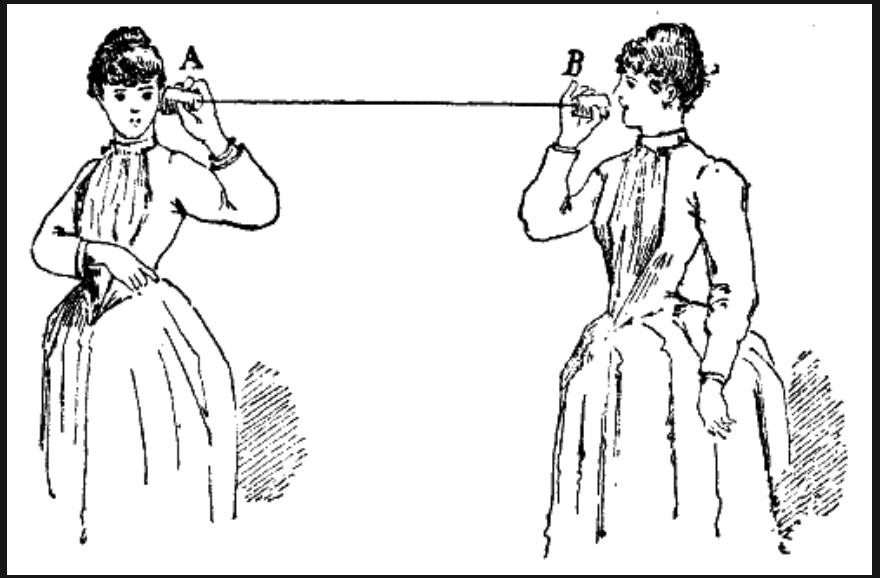
    p = probability that ‘no’ sounds like ‘yes’, etc
  - How much do you care about being misunderstood?

    P_a = maximum acceptable error probability



What is Error Correction?

- Usually $p \ll P_a$, so we don't need to worry.
- But what if we are being asked life-or-death questions over a noisy line?
- How can we make sure we are understood?



The Repetition Code

- We could repeat ourselves.
- With a lot of ‘no’s, it’s obvious we mean ‘no’.
- Same for mostly ‘no’s with a few random ‘yes’s thrown in.
- With this *encoding* of our message, it has become tolerant to small faults

The Repetition Code

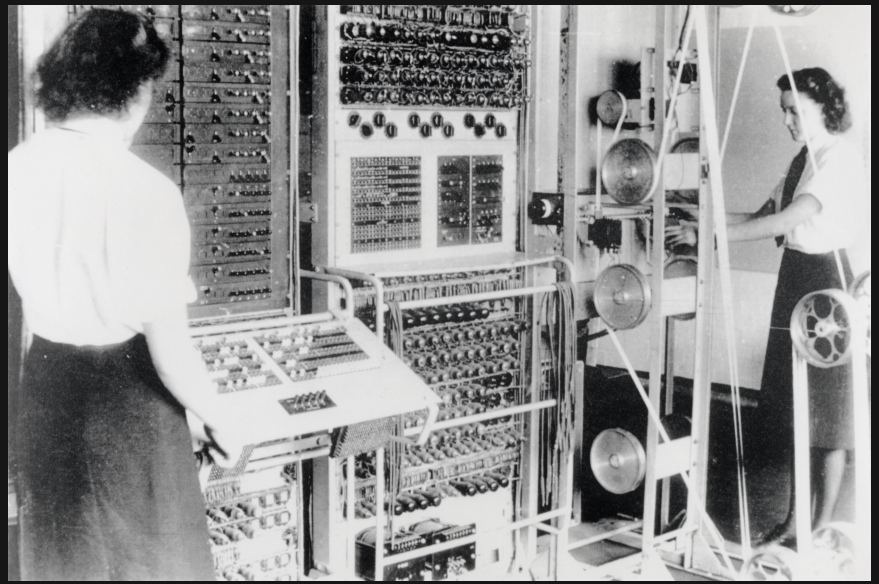
- The receiver will need to *decode* the message.
- A sensible option is majority voting.
- A misunderstanding only happens when the majority of copies are flipped
- For d repetitions

$$P = \sum_{n=0}^{\lceil d/2 \rceil} \binom{d}{n} p^n (1-p)^{d-n} \sim \left(\frac{p}{(1-p)}\right)^{\lceil d/2 \rceil}$$

- *P* decays exponentially with *d*
- With enough repetitions, we can make *P* as small as we like

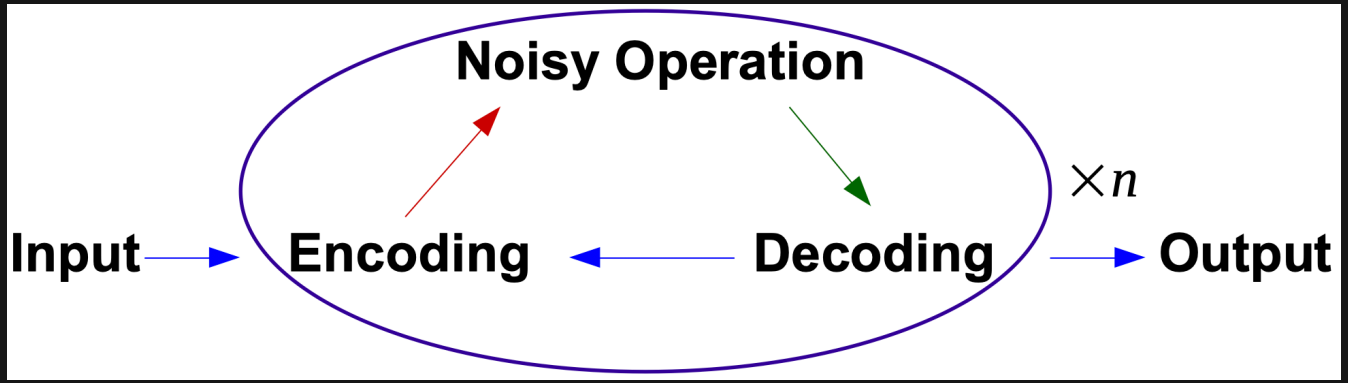
Encoding and Decoding

- This example contained all the basic features of any protocol for quantum error correction.
  - **Input:** Some information to protect.
  - **Encoding:** Transform the information to make it easier to protect.
  - **Errors:** Random perturbations of the encoded message.
  - **Decoding:** Trying to deduce the input from the perturbed message



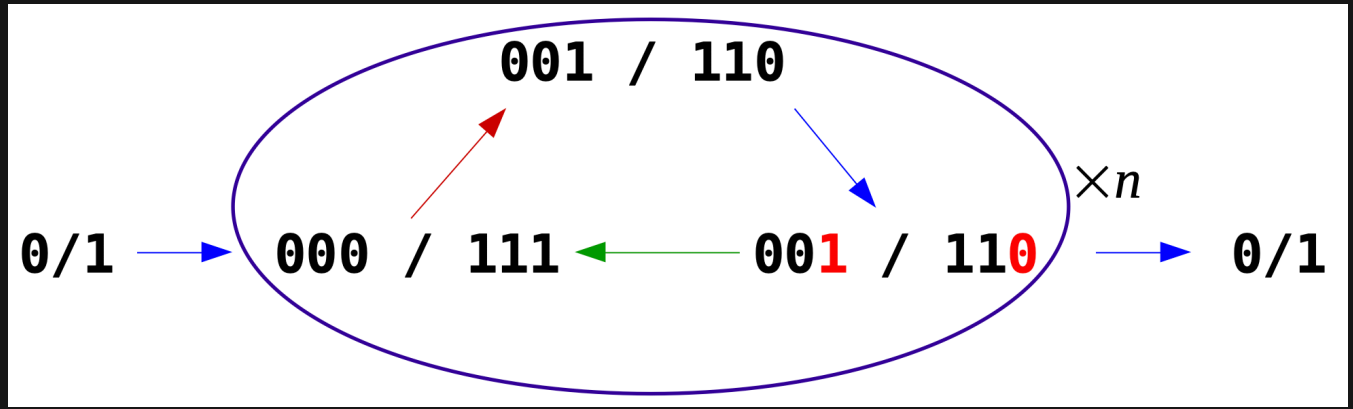
Computation

- The example we just considered was one of communication, with errors occurring during transmission.
- For computation, errors are introduced whenever we perform an operation.
- We need to keep corrected errors as they are introduced.
- Can be done by constantly decoding and re-encoding



Computation

- For example, here's a single bit using the Repetition code
- The 'noisy operation' here is just doing nothing (with some errors)
- Here we don't completely decode and re-encode, but just do it enough to find and fix the errors.

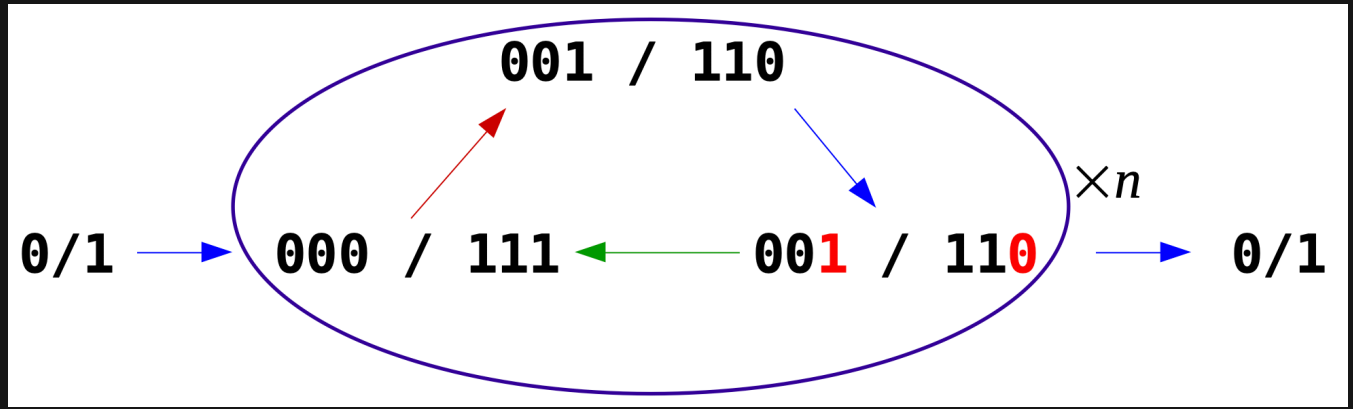


Quantum Computation

- This method works great for bits, but not for qubits
- Suppose we wanted to encode some superposition state

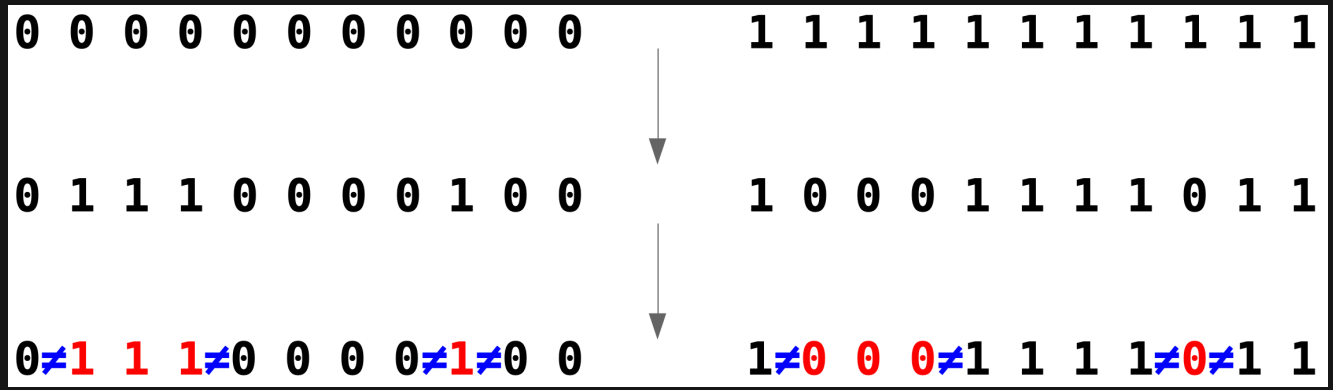
$$a|0\rangle + b|1\rangle \rightarrow a|000\rangle + b|111\rangle$$

- Decoding requires measurement, and that destroys the superposition.
- To protect against one bad thing, we caused another!



Quantum Computation

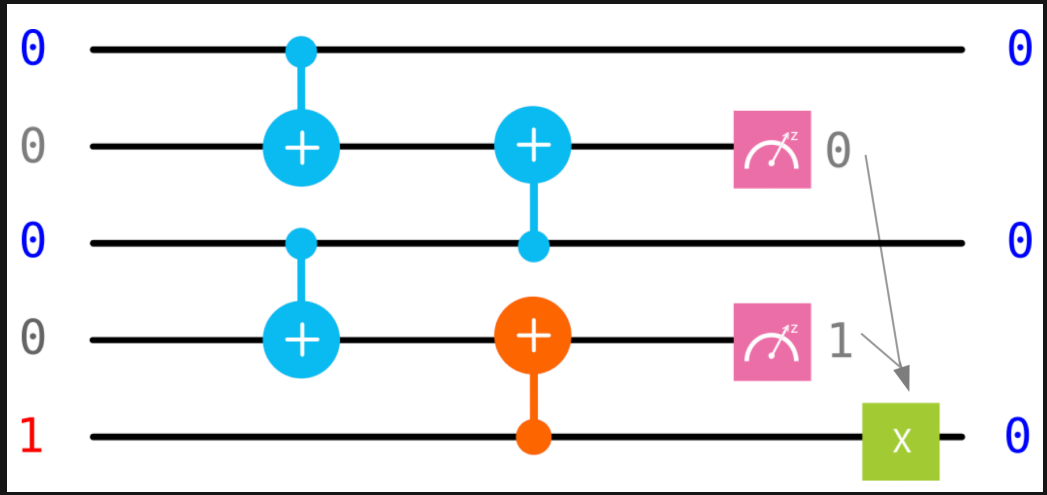
- To solve this, we need to be more careful with our measurements.
- We do need to measure, to get information about errors. But we must avoid learning about the encoded information
- For this, note that we don't actually need the bit values. We only need to know which ones have a different value to the rest.



Quantum Repetition Code

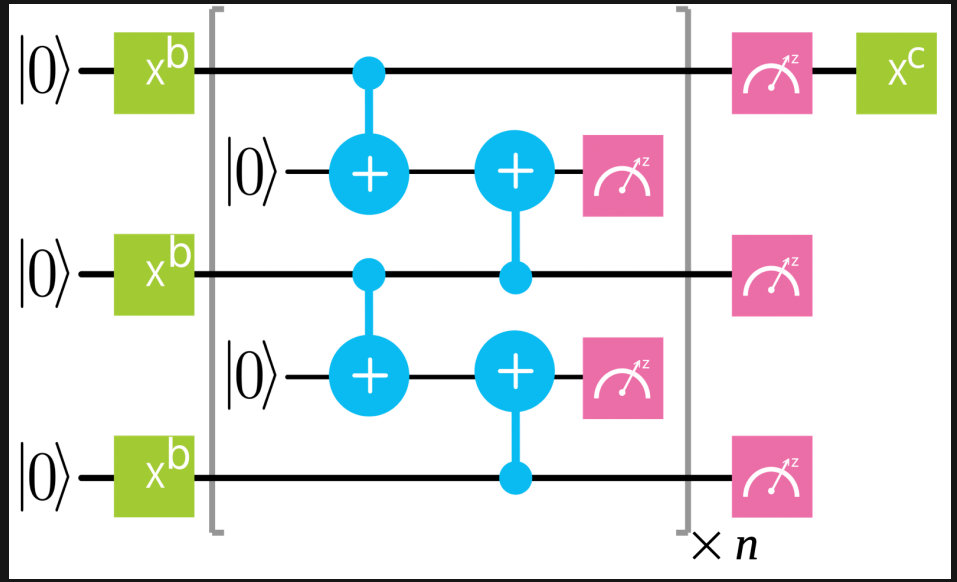
- Can be done with some extra qubits: one for each pair of code qubits.
- They are are initialized in state $|0\rangle$, and are used as the target for two CX gates
- The net effect is to measure the observable $Z_j Z_{j+1}$: the Z basis parity of the two qubits
- In short: whether they are the same or different

$$\text{cx}\,|00\rangle = |00\rangle$$
$$\text{cx}\,|01\rangle = |00\rangle$$
$$\text{cx}\,|10\rangle = |11\rangle$$
$$\text{cx}\,|11\rangle = |10\rangle$$



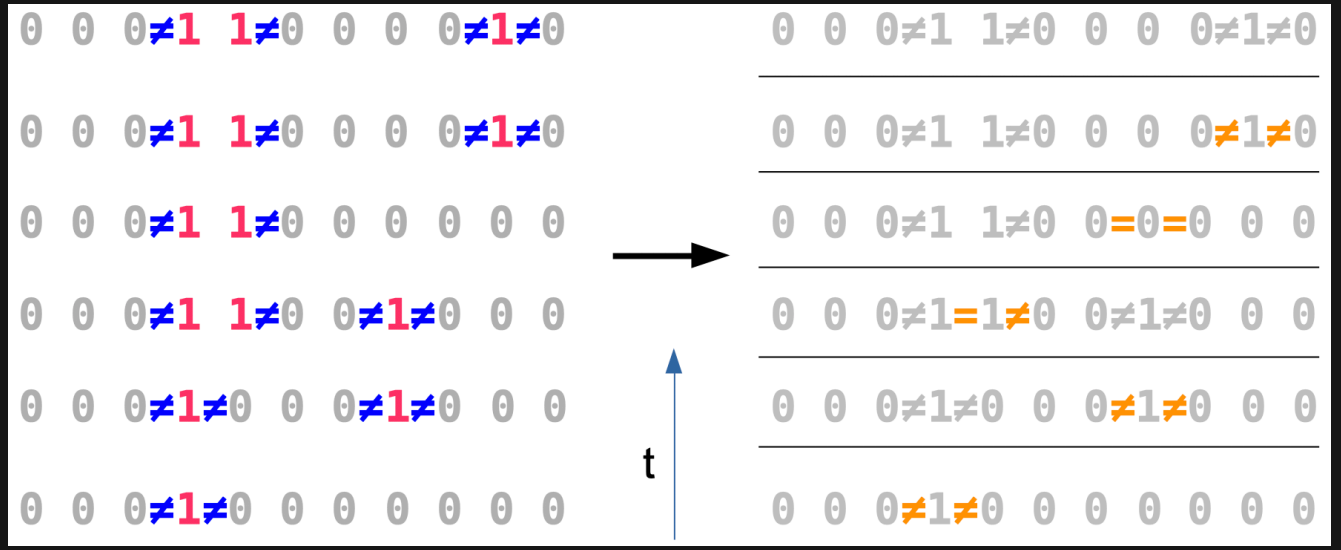
Quantum Repetition Code

- The parity measurements are repeated over the course of the computation
- The results are used to identify where errors likely occurred and how to remove their effects
- This is done by means of a classical algorithm: a decoding algorithm
- With this we can protect against bit flip errors for an arbitrarily long time



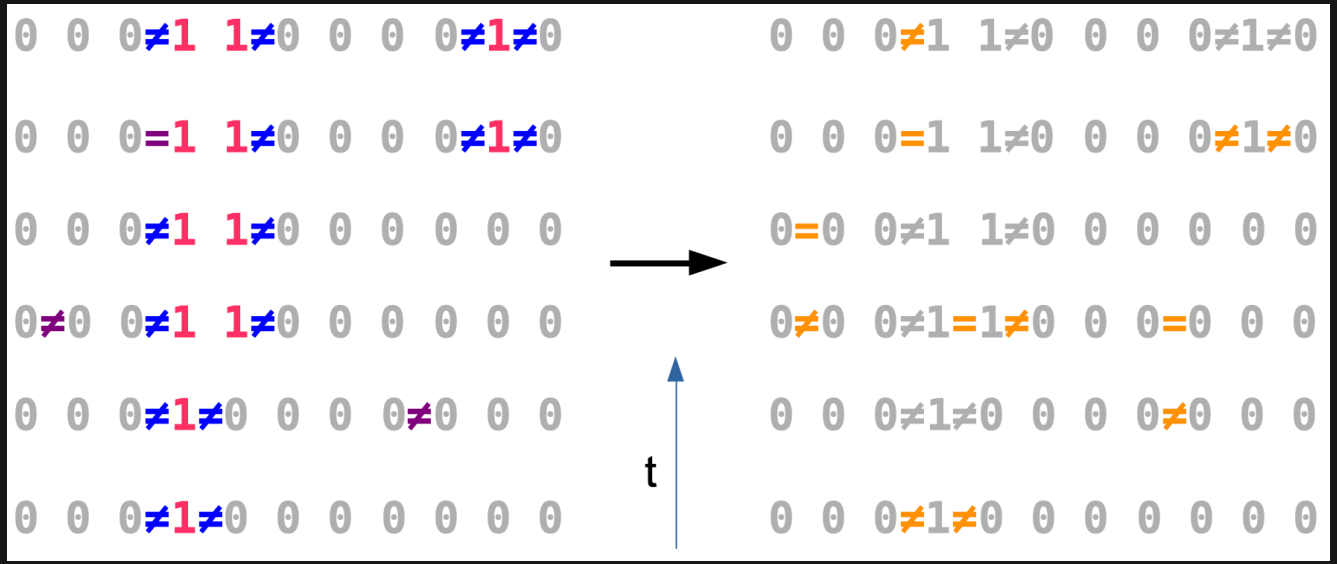
Decoding

- Start with an unrealistically simple case: errors between parity measurements only (not during)
- Focus on identifying errors for now (not correcting)
- Look for changes between rounds
- Errors create pairs of ‘defects’. Majority voting can be used to find a minimal pairing.



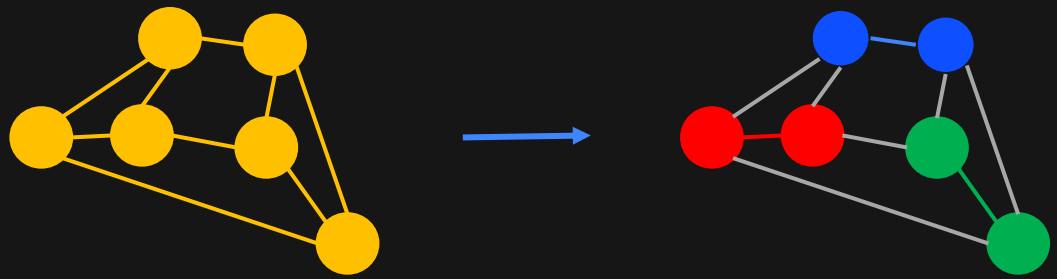
Decoding

- Next, a simple model of noise in the measurements: they randomly lie
- Again, look for changes between rounds
- Bit flips create defects with space-like separation, measurement errors with time-like
- Pairing is now a 2D problem, majority voting won't work



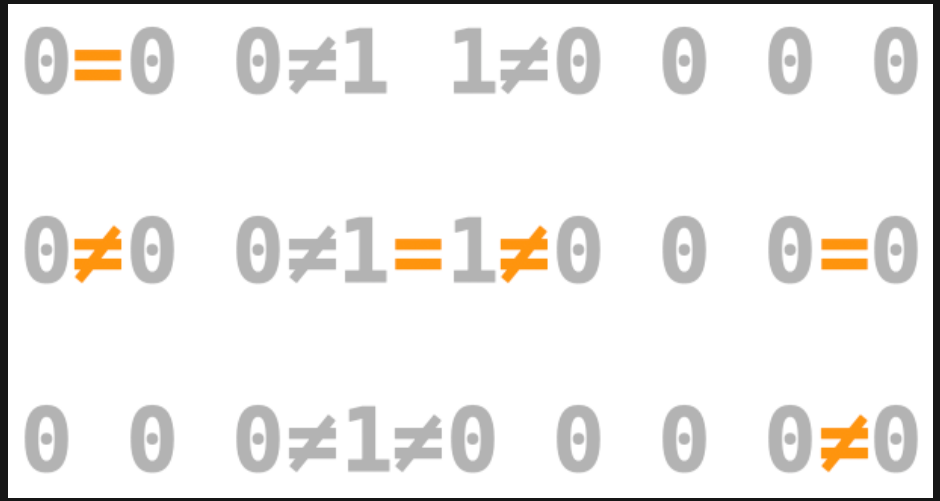
Decoding

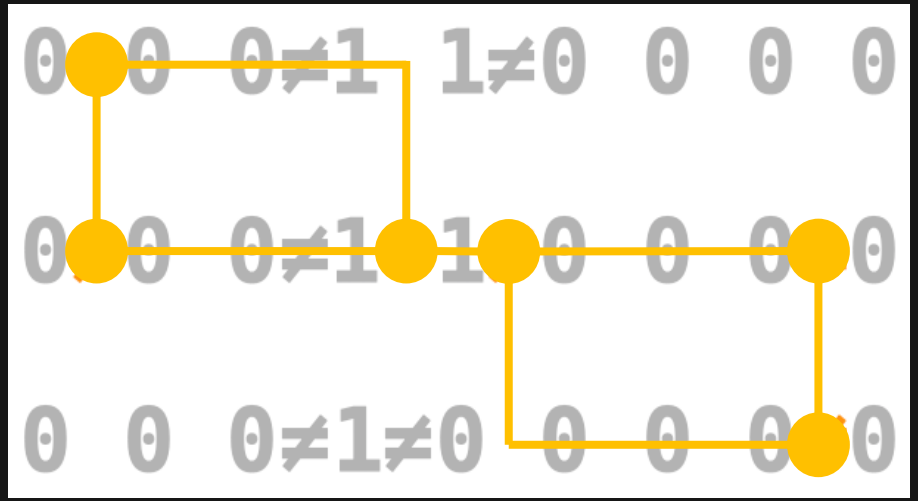
- There are many ways to do this, all with pros and cons
- One of the best is to think of the problem as a graph
  - Defects are nodes
  - The number of errors required to link them are weighted edges
- A likely set of errors corresponds to the ‘minimum weight perfect matching’ of the graph
- Efficiently computable using the ‘Blossom’ algorithm



Decoding

- Here it is in action for a portion of the example from earlier





Quantum Repetition Code

- The repetition code allows us to detect and correct bit fips, and only bit flips
- Though we made sure that it doesn't cause superpositions to collapse, it doesn't protect them either
- Also, becomes hard to perform most gates on the encoded information
- An x gate is easy
  - Just do an x on all code qubits
  - The error correction even corrects imperfections
- An Rx gate is not so easy
  - The code needs to essentially be taken apart and put back together
- Though the Repetition Code is a good first example of QEC, it cannot give us fault-tolerant quantum computation

$$R_x(2\theta)|0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$$

$$\rightarrow \cos\theta\,|000\rangle + \sin\theta|111\rangle$$

Towards a good quantum code

- The problem with the repetition code is that it treats z basis states very different to x and y basis states
- Example 1: z basis states are product states, the others are entangled

$$|0\rangle \rightarrow |000\rangle$$
$$|+\rangle \rightarrow \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

- Example 2: Distinguishing encoded z basis requires a single qubit measurement, distinguishing x basis states requires *d*
- Example 2: Flipping between z basis states requires *d* gates, but the x basis only takes one

$$|0\rangle \rightarrow |1\rangle \ : \quad X_0 X_1 X_2 |000\rangle = |111\rangle$$
$$|+\rangle \rightarrow |-\rangle \ : \quad Z_j \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)$$

- It is not good when things are too easy, because they are easy for errors too!

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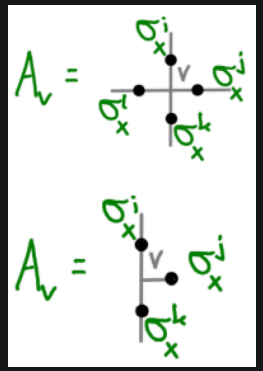
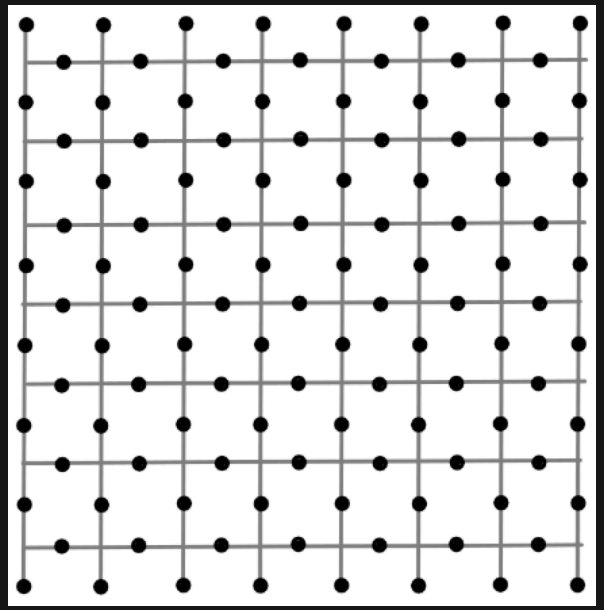
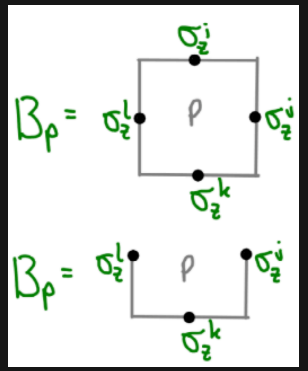
The Surface Code

- Quantum error correcting codes are defined by the measurements we make
- Let's move beyond the simple $Z_j Z_{j+1}$ of the repetition code
- In the surface code we use a 2D lattice of code qubits, and define observables for plaquettes and vertices

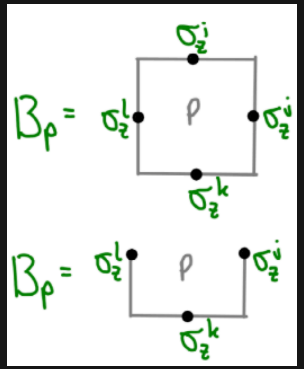


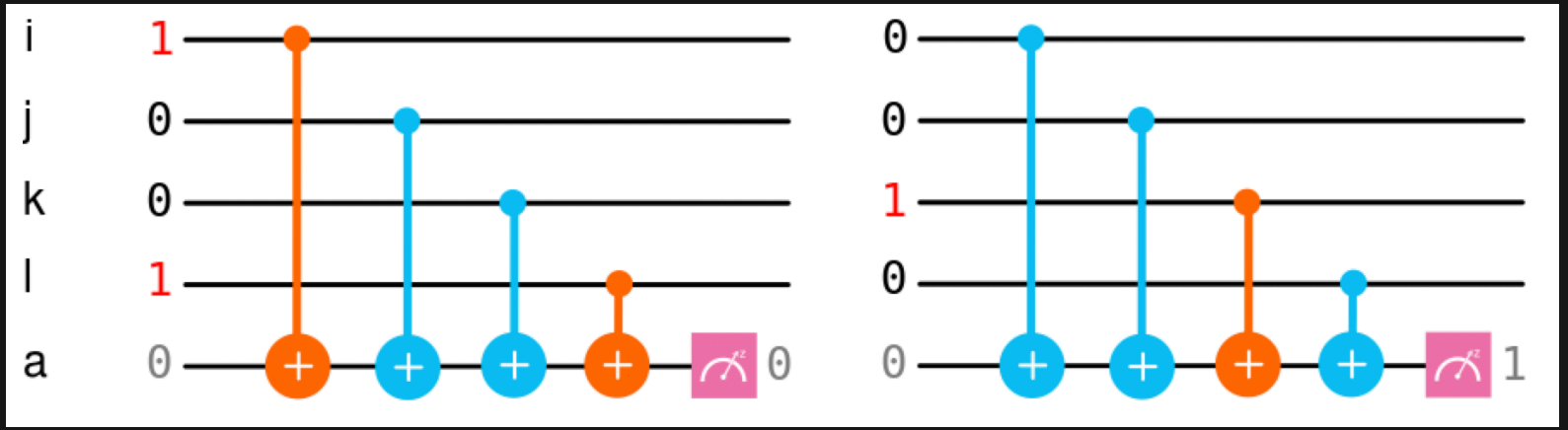




Plaquette Stabilizers

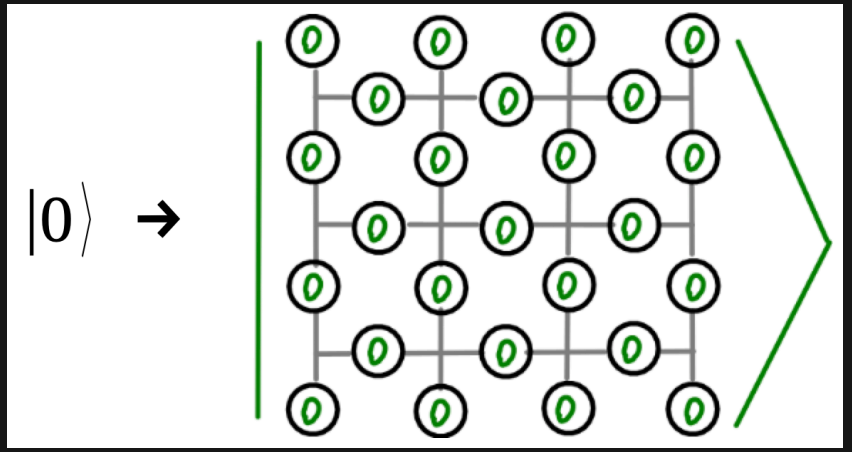
- First let's focus on the plaquette stabilizers
- These are similar to the two qubit measurements in the repetition code
- Instead we measure the parity around plaquettes in the lattice
- Can again be done with CX gates and an extra qubit





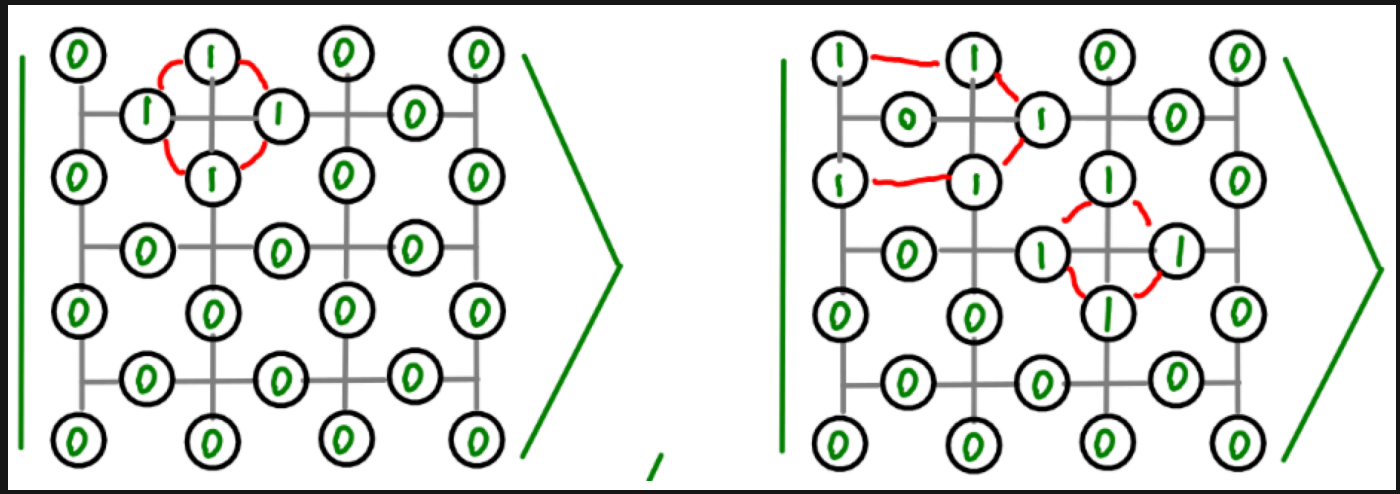
Plaquette Stabilizers

- We can define a classical code (storing only a bit) based on the plaquette stabilizers alone
- Valid states are those with trivial outcome for all plaquette stabilizer measurements:
  Even parity on all plaquettes
- How to store a 0 in this?
- How about the state where every code qubit is $|0\rangle$?



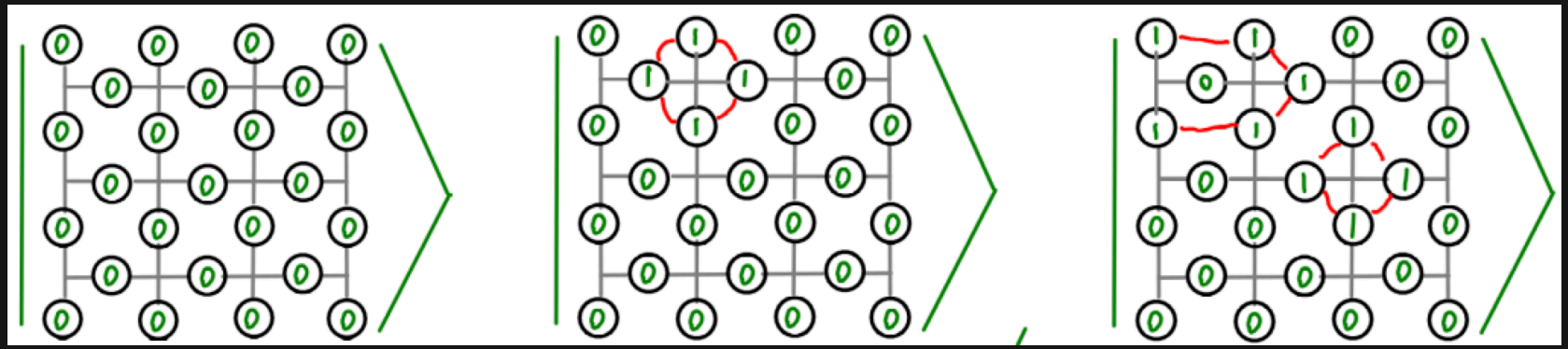
Plaquette Stabilizers

- There are ‘nearby’ states that also have even parity on all plaquettes
- These can’t be a different encoded state: they are only a few bit flips away from our encoded 0 state
- We’ll treat them as alternative ways to store a 0



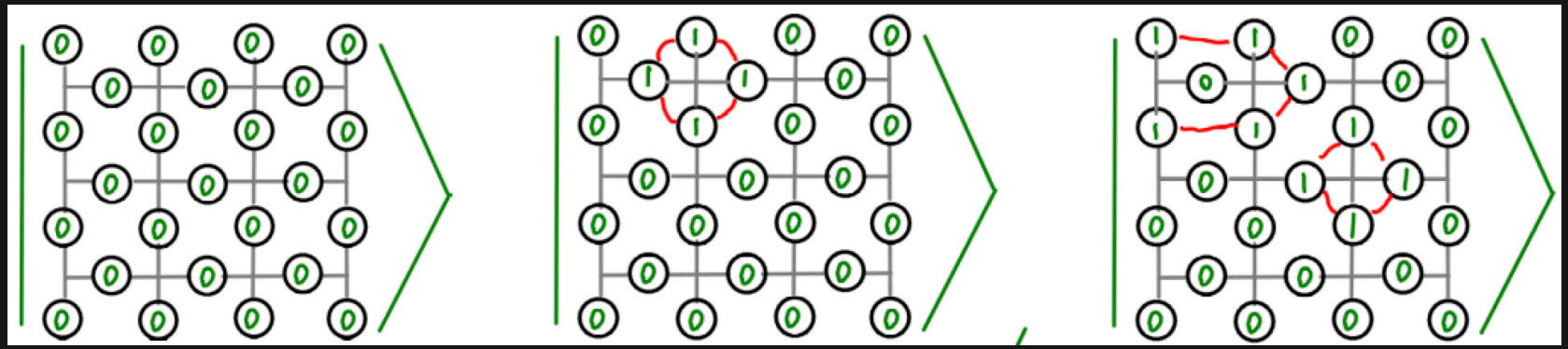
Plaquette Stabilizers

- Given any state for an encoded 0
  - Pick a vertex
  - Apply bit flips around that vertex
- Now you have another valid state for 0
- This generates an exponentially large family



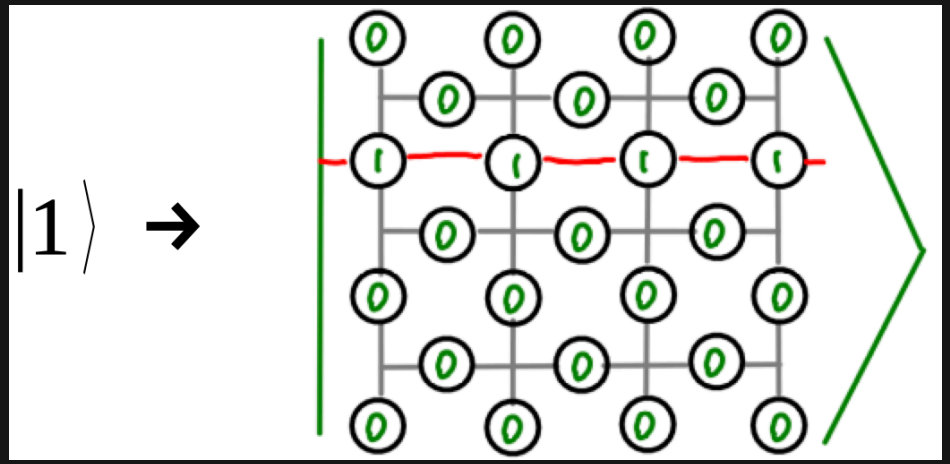
Plaquette Stabilizers

- The states in this family can be very different
- But they all share a common feature
  - Any line from top to bottom (passing along edges) has even parity
- This is how we can identify an encoded 0
- And it gives us a clue about how to encode a 1



Plaquette Stabilizers

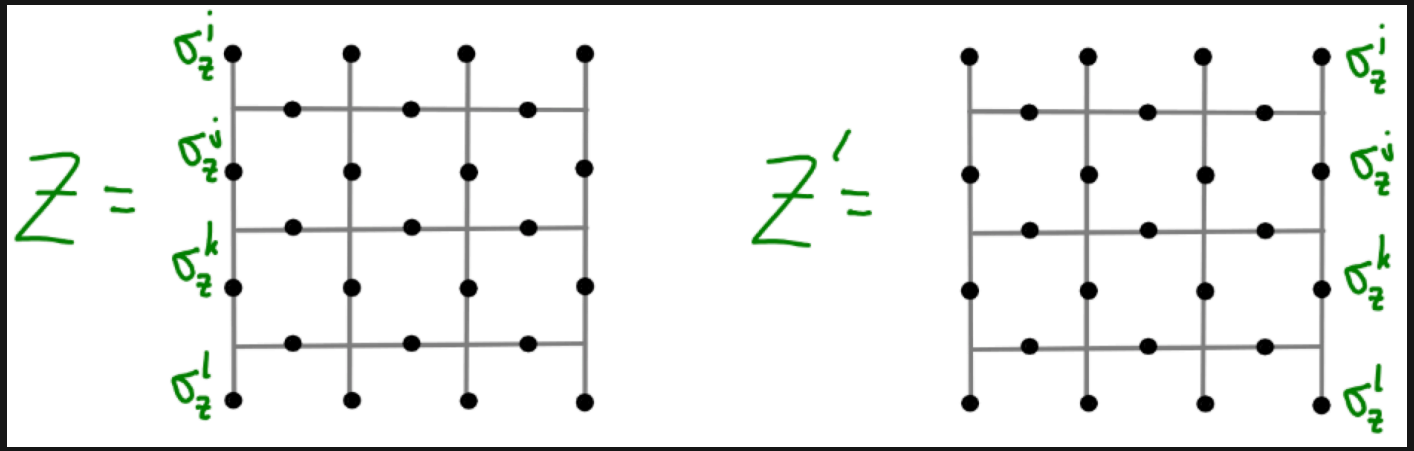
- For our basic encoded 1, we use a bunch of 0s with a line from left to right (passing through plaquettes)



- This also spawns an exponentially large family
- All have *odd* parity for a line from top to bottom
- Unlike the repetition code, distinguishing encoded 0 and 1 requires some effort (which is good!)

X and Z for encoded qubits

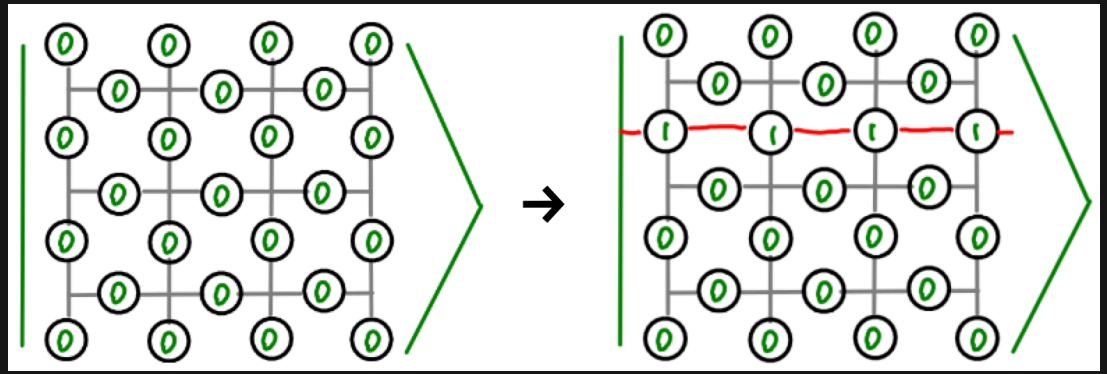
- Distinguishing 0 and 1 corresponds to measuring Z on the encoded qubit
- The following observables detect what we need



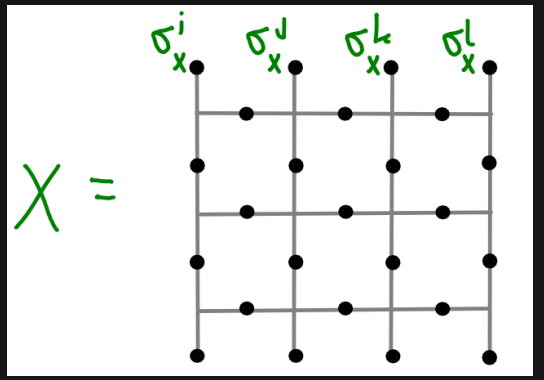
- Or the same on any line from top to bottom
- Uses the edges has a nice advantage: we can think of them as large (unenforced) plaquettes

X and Z for Encoded Qubits

- To flip between 0 and 1, we can flip a line of qubits

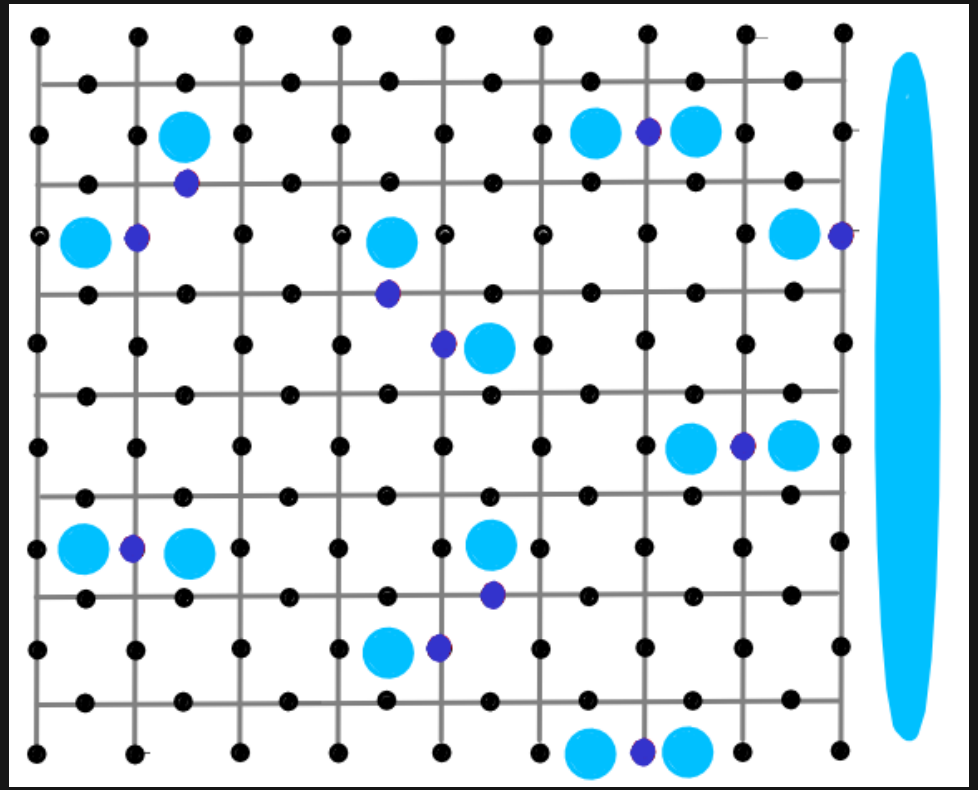


- Such lines of flips act as an X on the encoded qubit



Effects of Errors

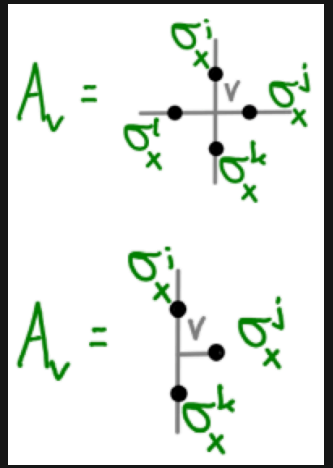
- Applying an X to any code qubit changes the parity of its two plaquettes
- An isolated X creates a pair of defects
- Further Xs can be move a defect, or annihilate pairs of them
- A logical X requires many errors to stretch across the lattice

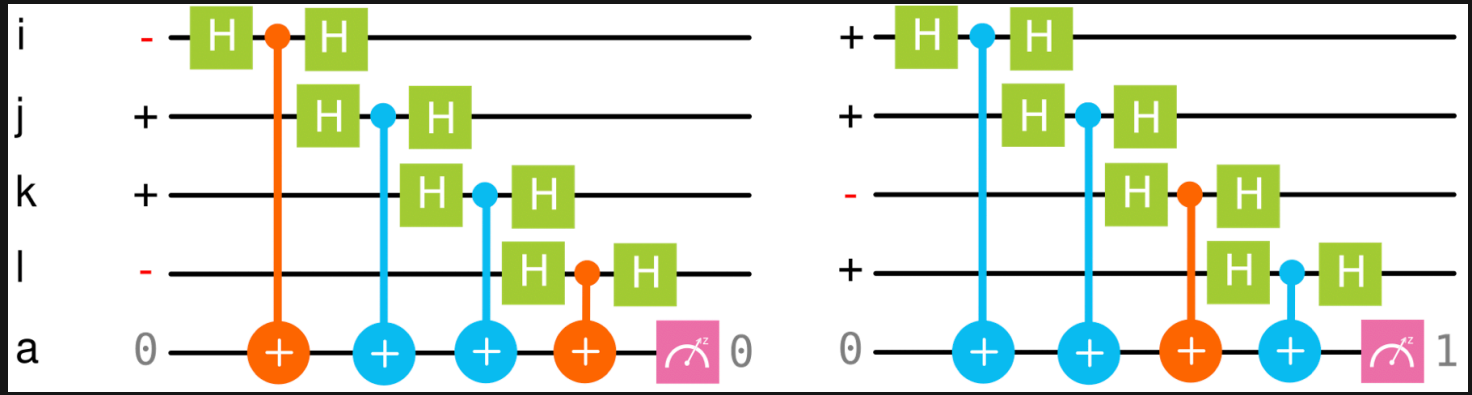


- With the plaquette operators, we can encode and protect a *bit*

Vertex Stabilizers

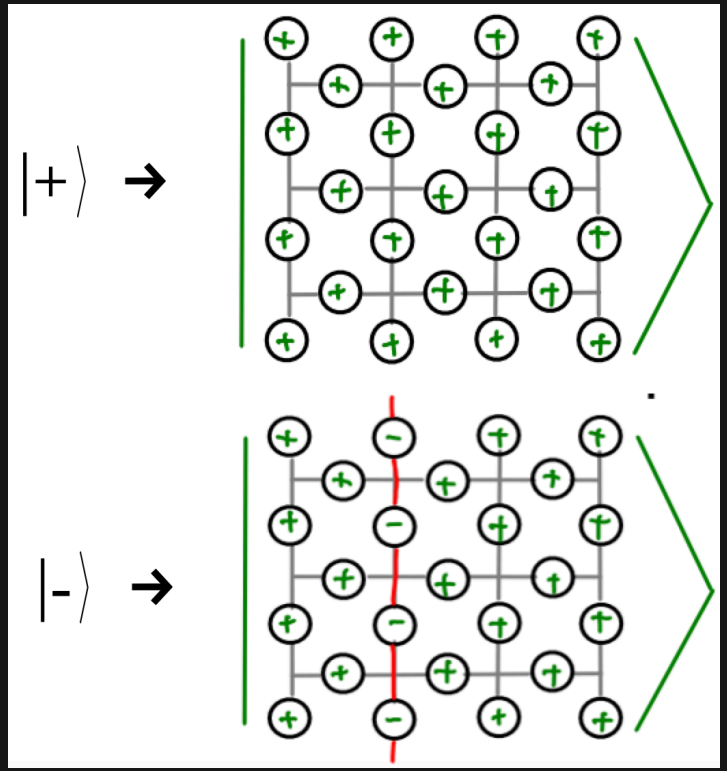
- Now forget the plaquettes and focus on vertices
- These observables can also be measured using CX gates an an ancilla
- In this case they look at the $|+\rangle$ and $|-\rangle$ states, and count the parity of the number of $|-\rangle$s





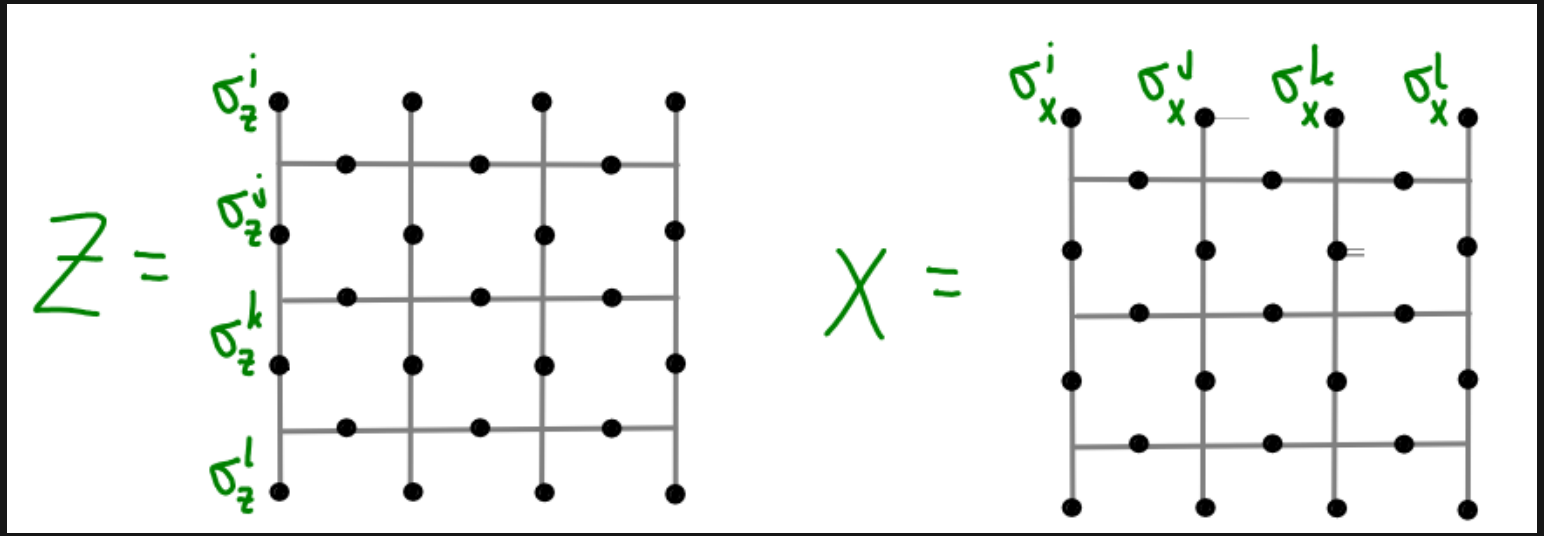
Plaquette Stabilizers

- These operators also allow is to encode and protect a bit value
- In this case, let's use + and - to label the two states
- They are encoded using suitable patterns of $|+\rangle$ and $|-\rangle$ states for the code qubits
- As with the plaquettes, these also correspond to exponentially large families of states



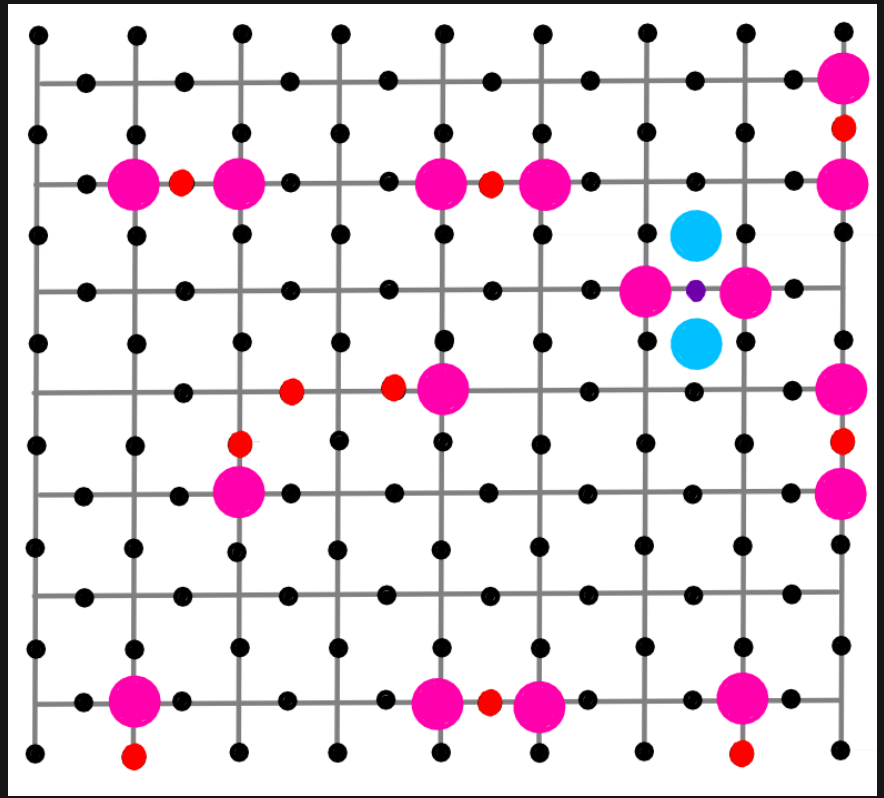
X and Z for Encoded Qubits

- What is the X operator (distinguish between $|+\rangle$ and $|-\rangle$)?
- What is the Z operator (flip between $|+\rangle$ and $|-\rangle$)?
- Turns out they are exactly the same as before!



Effects of Errors

- Applying a Z to any code qubit changes the X parity of its two vertices
- An isolated Z creates a pair of defects
- Further Zs can be move a defect, or annihilate pairs of them
- A logical Z requires many errors to stretch across the lattice



- With the vertex operators, we can encode and protect a *bit*

Putting it all Together

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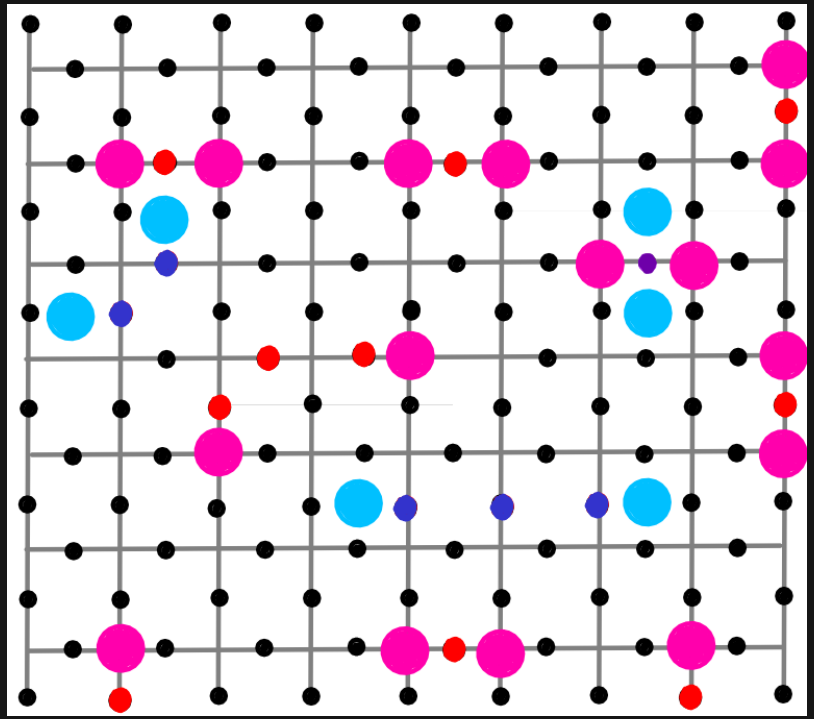
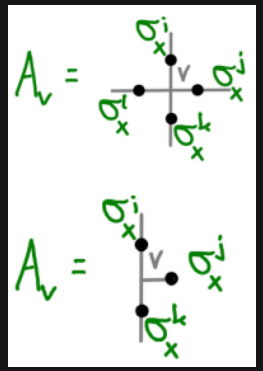
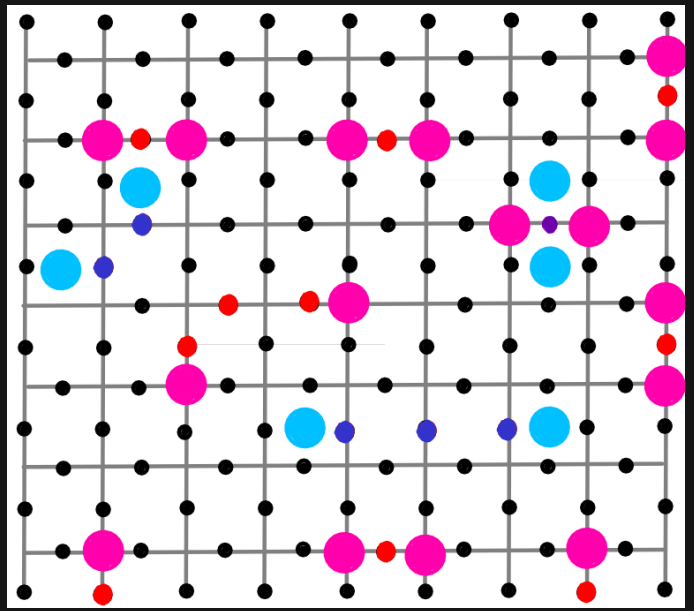


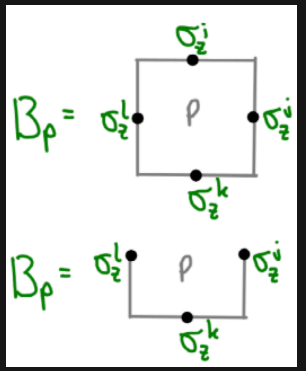
- With the vertex operators, we can encode and protect a *bit*

Putting it all Together

- The plaquette and vertex operators commute
- This allows us to detect both X and Z errors
- Since Y~XZ, we can detect Y errors too







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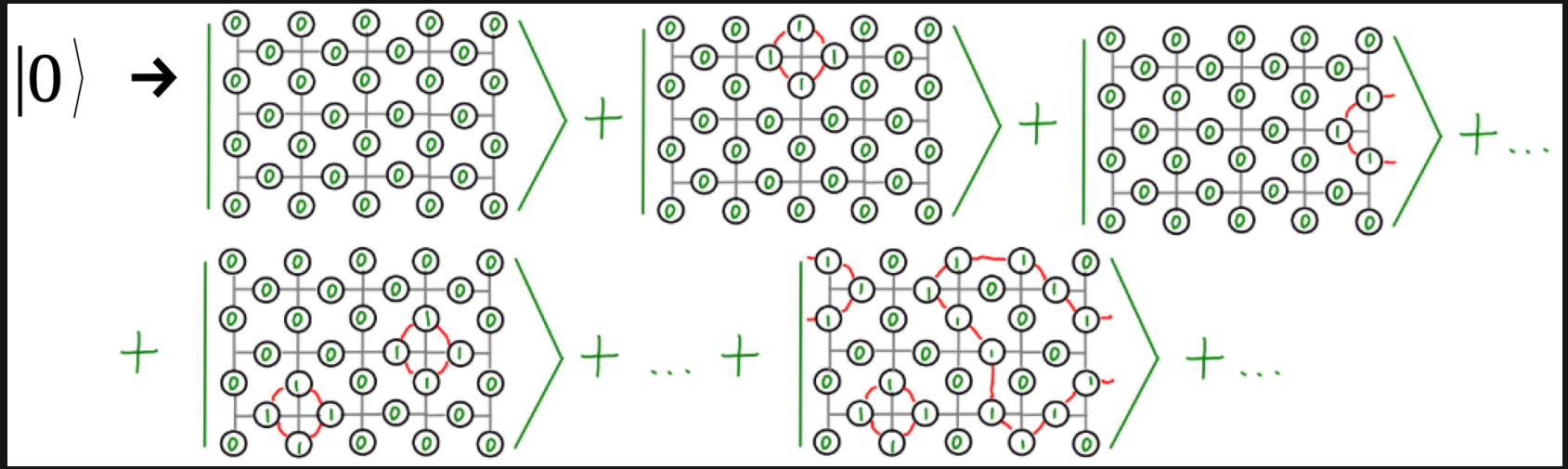
Putting it all Together

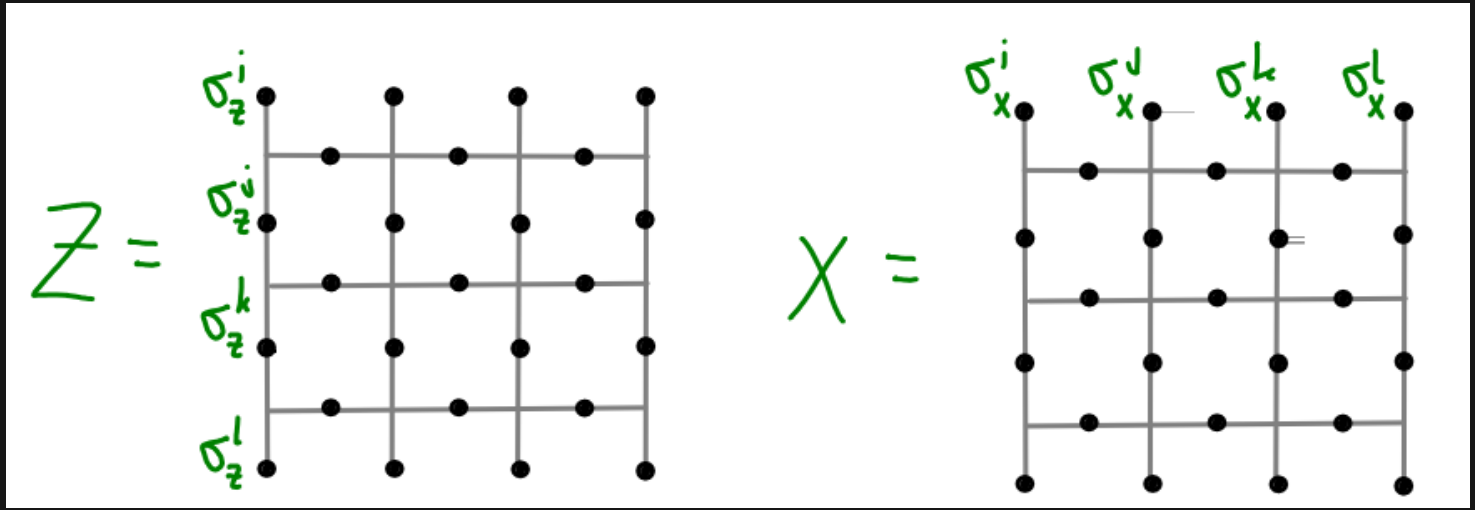
- Encoded states now unique: superposition of all previous solutions
- For example, the encoded 0



- Satisfies $A_v|\psi\rangle = |\psi\rangle$ and $B_v|\psi\rangle = |\psi\rangle$, so will give the 0 outcome for all stabilizer measurements

Putting it all Together

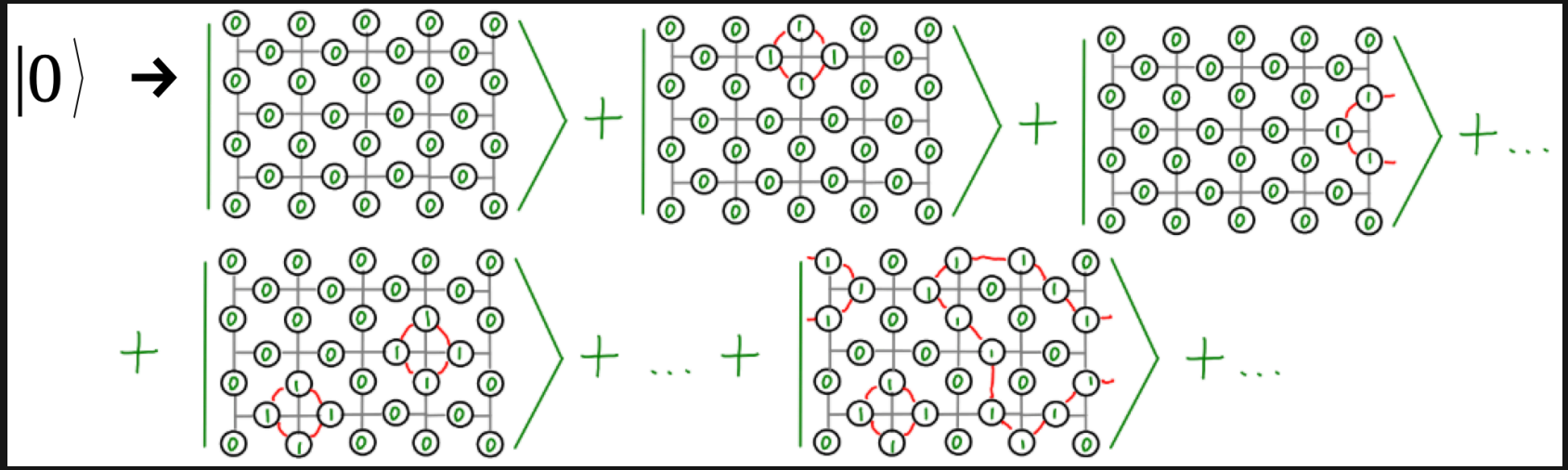
- The Z and X operators on the encoded qubit are exactly the same as before



- These, and the Hadamard, can be performed fault-tolerantly

Putting it all Together

- The states we need are highly entangled quantum states
- They are examples of topologically ordered states



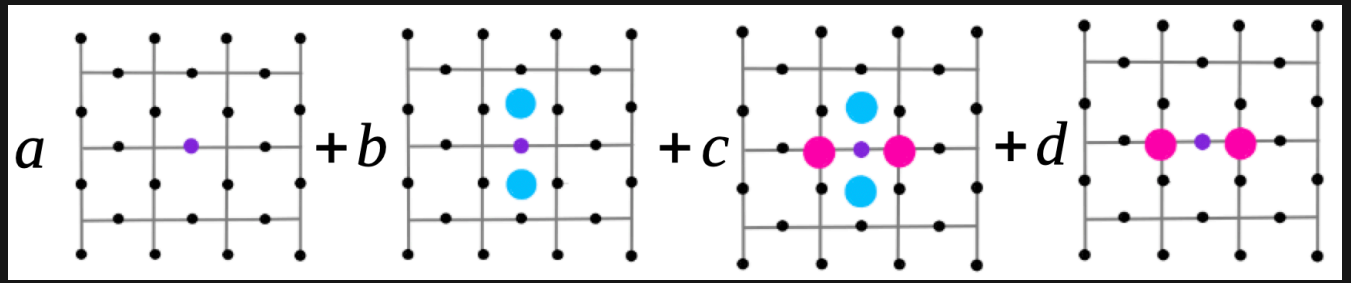
- Though such things can be hard to make, we create and protect them simply by making the stabilizer measurements

Putting it all Together

- We are not just protected against X and Z, but all local errors
- As mentioned earlier, Y~XZ
- Everything else can be expressed

$$E = a\,I + b\,X + c\,Y + d\,Z$$

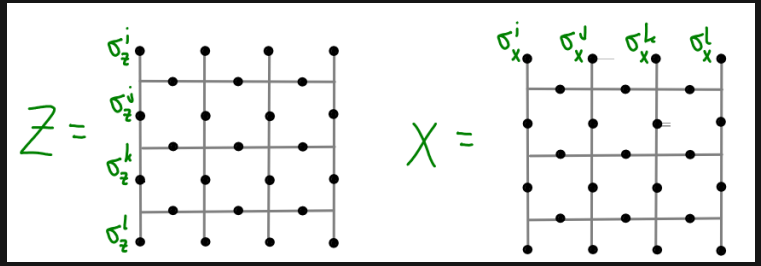
- This creates a superposition of different types of error on the surface code



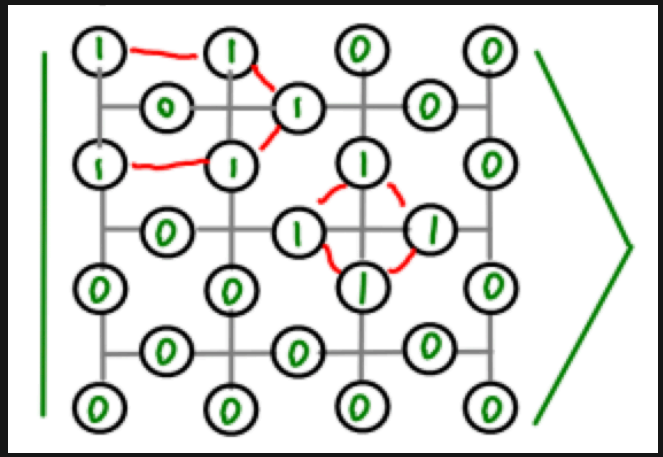
- Measuring the stabilizers collapses this to a simple X, Y or Z

Final Readout

- The logical operators are many-body observables
- So how do we read them out fault-tolerantly



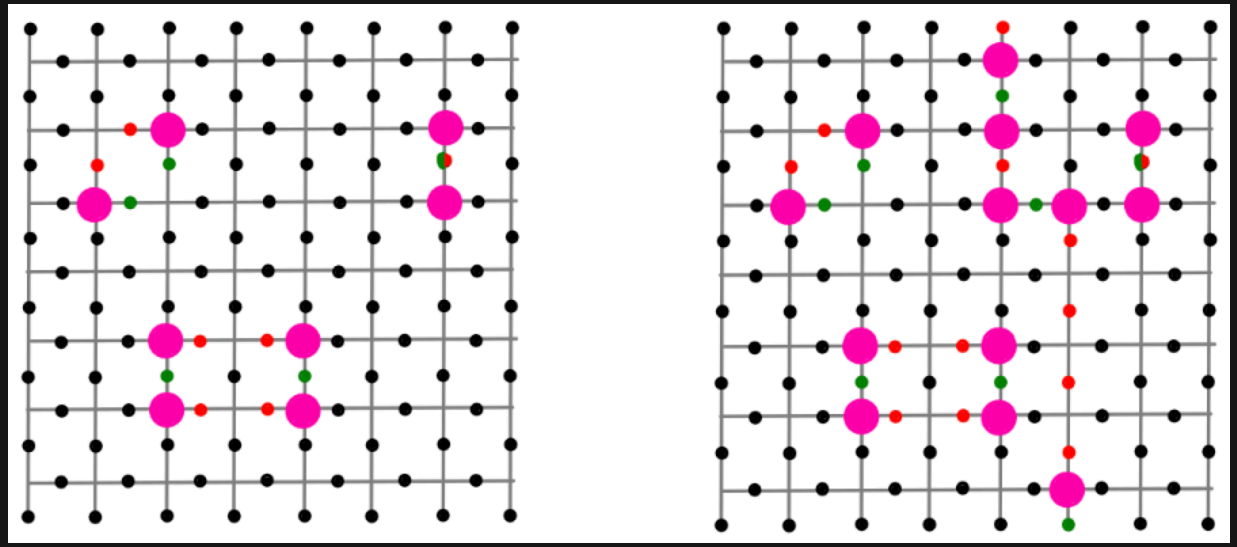
- When you decide on a basis for final measurement, you stop caring about some errors
- You can then measurement in a product basis
- Final readout and final stabilizer measurement can be constructed from the result



- Measurement errors are effectively the same as errors before measurement

Decoding

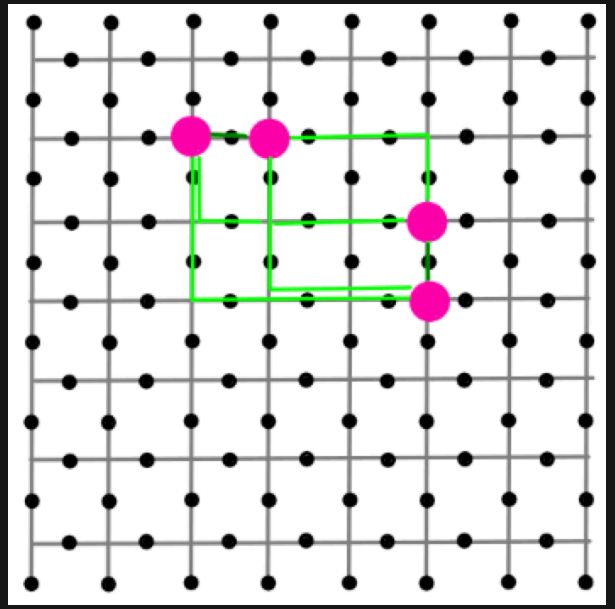
- Given the measurement results, we need to work out what errors happened
- More specifically, the ‘equivalence class’ of errors
- This is the job of the decoding algorithm

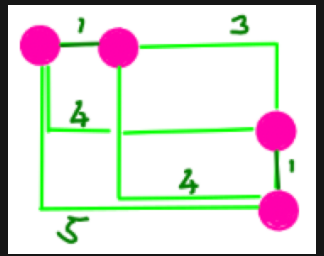


Decoding with MWPM

- A good option is MWPM
- Just as we considered for the repetition code

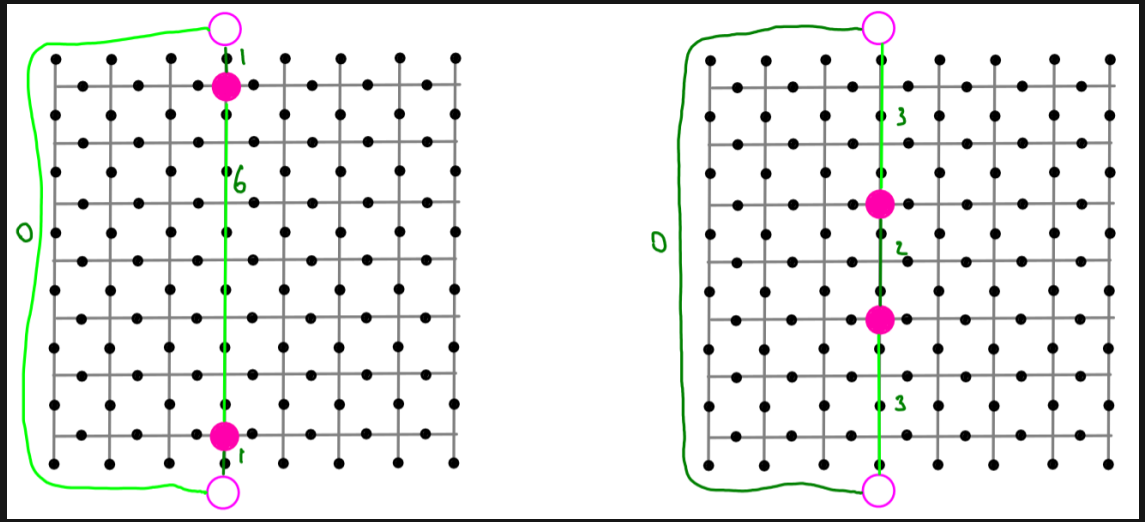
- Again we start with the simple and unrealistic case: errors only between measurement
- Each round can be decoded separately, corresponding to MWPM on a 2D graph
- Decoding for X and Z errors can also be done independently





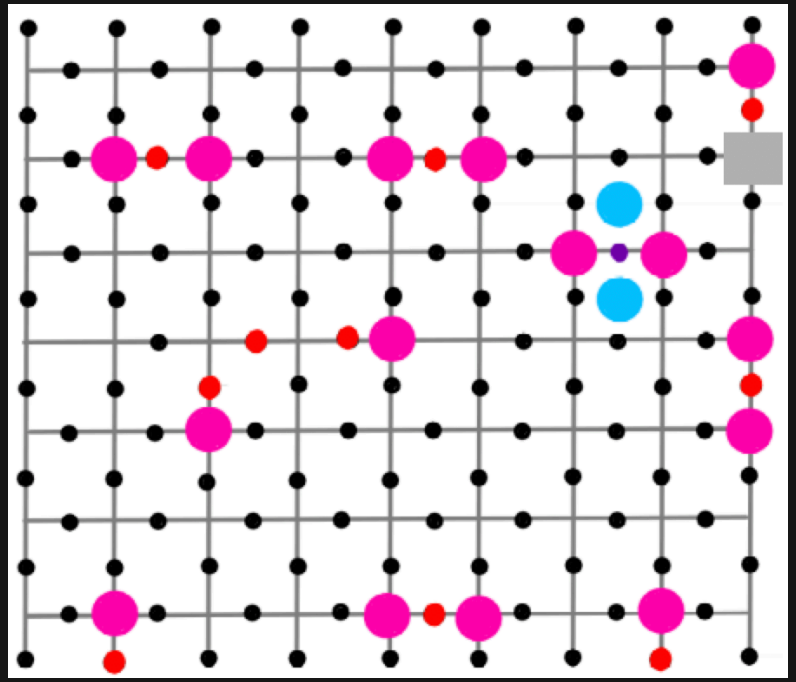
Decoding

- We need to be careful to account for the effects of the edges
- This is done by introducing extra ‘virtual nodes’
- (also required for the repetition code, but we ignored it earlier)



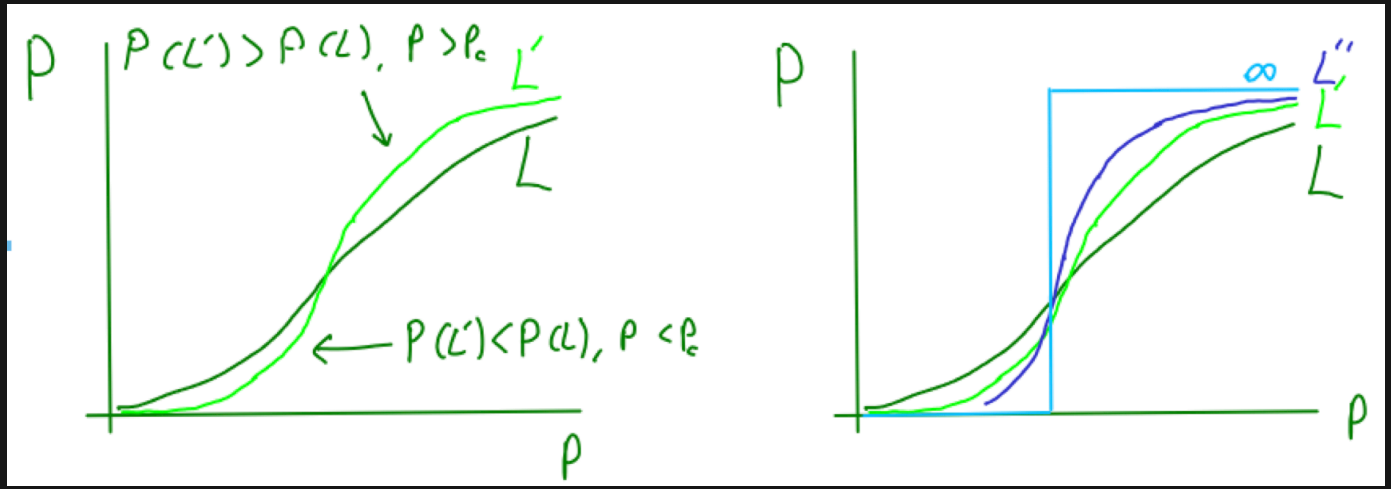
Imperfect Measurements

- Again, we have the problem of imperfect measurements
  - The measurements might lie
  - Errors on the additional qubit
  - Errors in the CX gates
- We base the decoding using syndrome changes
- This leads to a 3D MWPM problem (2D space + time)



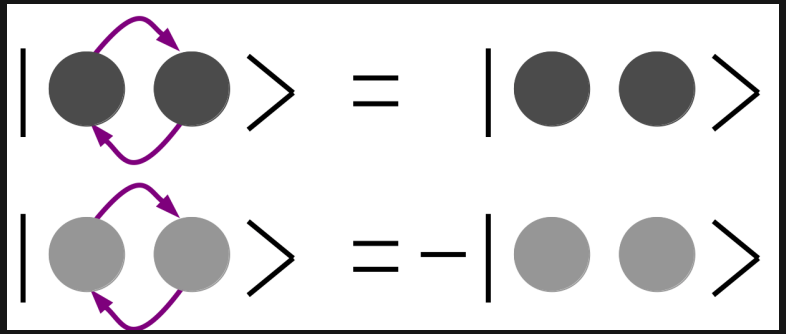
Threshold

- Correcting according to the right class removes the effects of errors
- Correcting according to the wrong class causes an operation on the encoded qubit (without our knowing)
- What is the probability of such an error, *P*, given the probability on the qubits of the code, *p*?
- We find a phase transition as *L* is increased (for an *LxL* grid)

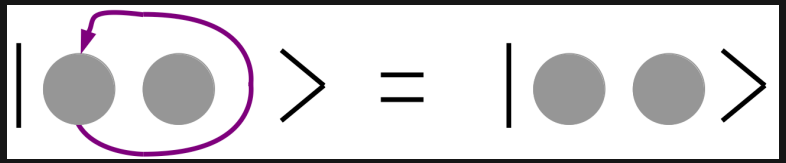


Anyons in the Surface Code

- There are many variants on how qubits can be encoded and manipulated in the surface code
- They all depend on the unique topological nature
- The ‘defects’ created by errors in the surface code behave like particles

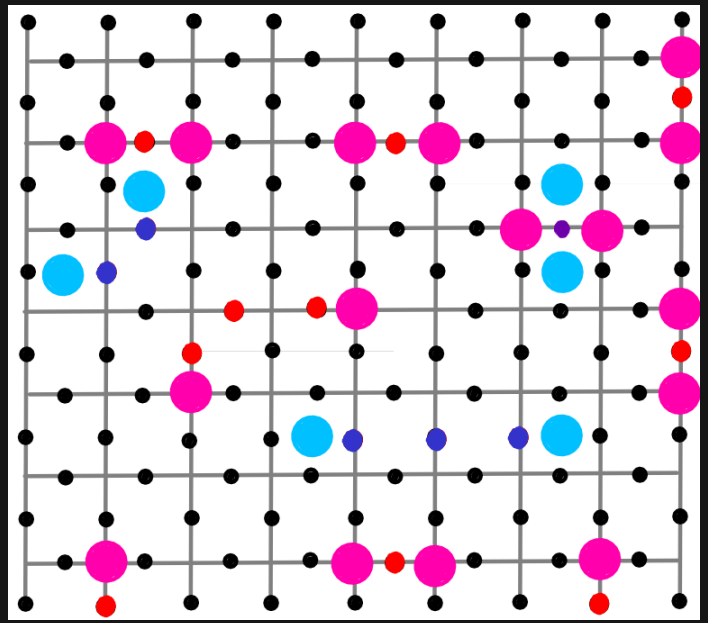


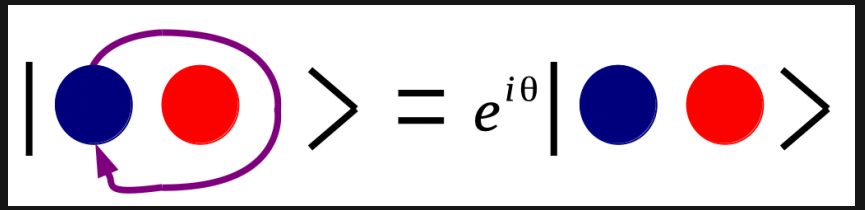
- All particles in our universe are either bosons or fermions
- This due to topological restrictions in a 3D universe



Anyons in the Surface Code

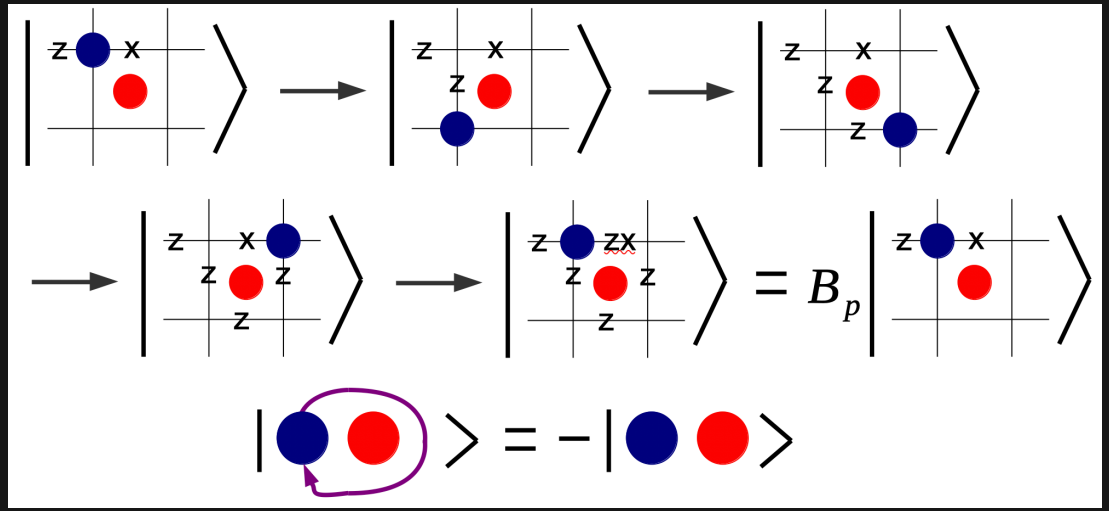
- The surface code is only a 2D ‘universe’, so doesn’t have these restrictions
- How do these particles behave?





Anyons in the Surface Code

- Braiding a particle corresponds to applying a stabilizer
- Their eigenstates defines the braiding phase
- Neither bosons nor fermions, but anyons!



Minimal Implementation

- The first surface code will probably be a small one
- The smallest that allows correction is *d=3*
- This requires ~17 qubits
- Who will be among the first to run and study this code?

- **Maybe you!**