Lenguajes de Programación 2020-1
Facultad de Ciencias UNAM
Ejercicio Semanal 8

Sandra del Mar Soto Corderi
Edgar Quiroz Castañeda

10 de octubre del 2019

1. Encontrar el tipo mas general para las siguientes expresiones de Cálculo Lambda utilizando el algoritmo de inferencia de tipos W.

   a) $\texttt{and} = \lambda x \lambda y.xy\texttt{true}$
   Primero

$$\dfrac{\dfrac{x : X \in \Gamma_1 = \{x : X\}}{\varnothing|\Gamma_1 \vdash x : X}\text{ Var} \qquad \dfrac{y : Y \in \Gamma_2 = \{y : y\}}{\varnothing|\Gamma_2 \vdash y : Y}\text{ Var} \qquad \begin{array}{c} S = \varnothing \\ Z_1 \texttt{ fresh} \\ (\varnothing \cup \{X\}) \cap (\varnothing \cup \{Y\}) = \varnothing \end{array}}{R_1 = \{X = Y \mapsto Z_1\}|\Gamma_3 = \Gamma_1 \cup \Gamma_2 \vdash xy : Z_1}\text{ App}$$

   Luego

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathit{Anterior}}{R|\Gamma_3 \vdash xy : Z_1}\text{ App} \qquad \dfrac{}{\varnothing|\varnothing \vdash \texttt{true} : \mathit{Bool}}\text{ Ax} \qquad \begin{array}{c} S = \varnothing \\ Z_2 \texttt{ fresh} \\ (\varnothing \cup \{X, Y\}) \cap (\varnothing \cup \varnothing) = \varnothing \end{array}}{R_2 = \{Z_1 = \mathit{Bool} \mapsto Z_2\} \cup R|\Gamma_3 \vdash xy\texttt{true} : Z_2}\text{ App}}{R_2|\Gamma_4 = \Gamma_3 \backslash \{y : Y\} \vdash \lambda y.xy\texttt{true} : Y \mapsto Z_2}\text{ Lam}}{R_2|\varnothing = \Gamma_4 \backslash \{x : X\} \vdash \lambda x.\lambda y.xy\texttt{true} : X \mapsto (Y \mapsto Z_2)}\text{ Lam}$$

   Con $R_2 = \{X = Y \mapsto Z_1, Z_1 = \mathit{Bool} \mapsto Z_2\}$, que unificado es $\{X = Y \mapsto (\mathit{Bool} \mapsto Z_2)\}$
   Por lo que la expresión tiene tipo

$$\lambda x.\lambda y.xy\texttt{true} : (Y \mapsto (\mathit{Bool} \mapsto Z_2)) \mapsto (Y \mapsto Z_2)$$

   b) $\texttt{or} = \lambda x \lambda y.x\texttt{true}y$
   Primero

$$\dfrac{\dfrac{x : X \in \Gamma_1 = \{x : X\}}{\varnothing|\Gamma_1 \vdash x : X}\text{ Var} \qquad \dfrac{}{\varnothing|\varnothing \vdash \texttt{true} : \mathit{Bool}}\text{ Ax} \qquad \begin{array}{c} S = \varnothing \\ Z_1 \texttt{ fresh} \\ (\varnothing \cup \{X\}) \cap (\varnothing \cup \varnothing) = \varnothing \end{array}}{R_1 = \{X = \texttt{Bool} \mapsto Z_1\}|\Gamma_1 \vdash x\texttt{true} : Z_1}\text{ App}$$

   Luego

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathit{Anterior}}{R_1|\Gamma_1 \vdash x\texttt{true} : Z_1}\text{ App} \qquad \dfrac{y : Y \in \Gamma_2 = \{y : y\}}{\varnothing|\Gamma_2 \vdash y : Y}\text{ Var} \qquad \begin{array}{c} S = \{X = \texttt{Bool} \mapsto Z_1\} \\ Z_2 \texttt{ fresh} \\ (\{X, Z_1\} \cup \{X\}) \cap (\varnothing \cup \varnothing) = \varnothing \end{array}}{R_2 = \{Z_1 = Y \mapsto Z_2\} \cup R_1|\Gamma_3 = \Gamma_1 \cup \Gamma_2 \vdash x\texttt{true}y : Z_2}\text{ App}}{R_2|\Gamma_3 \vdash x\texttt{true}y : Z_2}\text{ App}}{R_2|\Gamma_4 = \Gamma_3 \backslash \{y : Y\} \vdash \lambda y.x\texttt{true}y : Y \mapsto Z_2}\text{ Lam}}{R_2|\varnothing = \Gamma_4 \backslash \{x : X\} \vdash \lambda x.\lambda y.x\texttt{true}y : X \mapsto (Y \mapsto Z_2)}\text{ Lam}$$

Con $R_2 = \{X = \texttt{Bool} \mapsto Z_1, Z_1 = Y \mapsto Z_2\}$.
Unificando tenemos que $\{X = Bool \mapsto (Y \mapsto Z_2)\}$.
Por lo que la expresión tiene tipo

$$\lambda x.\lambda y.x\texttt{true}y : (Bool \mapsto (Y \mapsto Z_2)) \mapsto (Y \mapsto Z_2)$$

c) $\texttt{snd} = \lambda p.p\texttt{false}$

$$\dfrac{\dfrac{\dfrac{p : P \in \Gamma = \{p : P\}}{\varnothing|\Gamma \vdash p : P}\ \text{Var} \qquad \dfrac{}{\varnothing|\varnothing \vdash \texttt{false} : Bool}\ \text{Ax} \qquad \begin{array}{c} S = \varnothing \\ Z\ \texttt{fresh} \\ (\varnothing \cup \{P\}) \cap (\varnothing \cup \varnothing) = \varnothing \end{array}}{R = \{P = Bool \mapsto Z\}|\Gamma \vdash p\texttt{false} : Z}\ \text{App}}{R|\varnothing = \Gamma \backslash \{p : P\} \vdash \lambda p.p\texttt{false} : P \mapsto Z}\ \text{Lam}$$

Y como $R = \{P = Bool \mapsto Z\}$ ya está unificado, entonces el tipo de la expresión es

$$\lambda p.p\texttt{false} : (Bool \mapsto Z) \mapsto Z$$