Lenguajes de Programación 2020-1
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Sean

- `btw := ` λx `:Nat.`λp `:Nat×(Nat×Nat).((fst p < x) and (x < (snd (snd p))))`
- `e := btw 1 (pred 1, (0, suc 1))`

Responder

1. Expresar `e` en sintaxis abstracta.

```
e = app(app(btw, 1), pair(pred(1), pair(0, suc(1))))
  = app(app(lam(Nat, x.lam(prod(Nat,prod(Nat,Nat)),
  p.and(<(fst(p), x), <(x, snd(snd(p)))))), 1),
  pair(pred(1), pair(0, suc(1))))
```

2. Mostrar paso a paso la evaluación de `e` utilizando la semántica dinámica $\to^\star$.

   Tomemos `pr = (pred 1, (0, suc 1))` $\to^\star$ `(0, (0, 2)) = pr’`.

   En el proceso, se saltaron las sustituciones al ser muy estorbozas y sencillas.

$$\mathtt{e} \to^\star \mathtt{(fst\ pr\ <\ 1)\ and\ (1\ <\ snd\ (snd\ pr))} \to^\star \mathtt{(fst\ pr'\ <\ 1)\ and\ (1\ <\ snd\ (snd\ pr'))}$$
$$\to^\star \mathtt{(0\ <\ 1)\ and\ (1\ <\ 2)} \to^\star \mathtt{true\ and\ true} \to^\star \mathtt{true}$$

3. Mostrar paso a paso que $\vdash e : Bool$.

   Primero, encontremos el tipo de `btw`. Sea $\Gamma = \{x : Nat, p : Nat \times (Nat \times Nat)\}$.

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \mathtt{p} : Nat \times (Nat \times Nat)}}{\Gamma \vdash \mathtt{fst(p)} : Nat} \quad \dfrac{}{\Gamma \vdash \mathtt{x} : Nat}}{\Gamma \vdash \mathtt{fst(p)} < x : Bool} \quad \dfrac{\dfrac{}{\Gamma \vdash \mathtt{x} : Nat} \quad \dfrac{\dfrac{\dfrac{}{\Gamma \vdash \mathtt{p} : Nat \times (Nat \times Nat)}}{\Gamma \vdash \mathtt{snd(p)} : Nat \times Nat}}{\Gamma \vdash \mathtt{snd(snd(p))} : Nat}}{\Gamma \vdash \mathtt{x\ <\ snd(snd(p))} : Bool}}{\Gamma \vdash \mathtt{(fst(p)\ <\ x)\ and\ (x\ <\ snd(snd(p)))} : Bool}}{\dfrac{\{x : Nat\} \vdash \lambda\mathtt{p}{:}Nat \times (Nat \times Nat).\mathtt{(fst(p)\ <\ x)\ and\ (x\ <\ snd(snd(p)))} : (Nat \times (Nat \times Nat)) \mapsto Bool}{\varnothing \vdash \lambda\mathtt{x}{:}Nat.\lambda\mathtt{p}{:}Nat \times (Nat \times Nat).\mathtt{(fst(p){<}x)\ and\ (x{<}snd(snd(p)))} : Nat \mapsto ((Nat \times (Nat \times Nat)) \mapsto Bool)}}$$

   Ahora, revisemos el tipo de `e`.

$$\frac{\dfrac{\dfrac{\textit{Anterior}}{\varnothing \vdash \mathtt{btw} : Nat \mapsto ((Nat \times (Nat \times Nat)) \mapsto Bool)} \quad \dfrac{}{\varnothing \vdash 1 : Nat}}{\varnothing \vdash \mathtt{btw\ 1} : (Nat \times (Nat \times Nat)) \mapsto Bool} \quad \dfrac{\dfrac{\dfrac{}{\varnothing \vdash 1 : Nat}}{\varnothing \vdash \mathtt{pred\ 1} : Nat} \quad \dfrac{\dfrac{}{\varnothing \vdash 0 : Nat} \quad \dfrac{\dfrac{}{\varnothing \vdash 1 : Nat}}{\varnothing \vdash \mathtt{suc\ 1} : Nat}}{\varnothing \vdash \mathtt{(0,\ suc\ 1)} : Nat \times Nat}}{\varnothing \vdash \mathtt{(pred\ 1,\ (0,\ suc\ 1))} : Nat \times (Nat \times Nat)}}{\varnothing \vdash \mathtt{btw\ 1\ (pred\ 1,\ (0,\ suc\ 1))} : Bool}$$