

A Givens rotations on modes j and k performs the transformation

$$\begin{pmatrix} c_j^\dagger \\ c_k^\dagger \end{pmatrix} \mapsto \begin{pmatrix} \cos \theta & -e^{i\varphi} \sin \theta \\ \sin \theta & e^{i\varphi} \cos \theta \end{pmatrix} \begin{pmatrix} c_j^\dagger \\ c_k^\dagger \end{pmatrix} = \begin{pmatrix} \cos \theta c_j^\dagger - e^{i\varphi} \sin \theta c_k^\dagger \\ \sin \theta c_j^\dagger + e^{i\varphi} \cos \theta c_k^\dagger \end{pmatrix}. \quad (1)$$

In the two-qubit subspace corresponding to modes j and k , with $j < k$, under the JWT, we have

$$|00\rangle = |\text{vac}\rangle \quad (2)$$

$$|01\rangle = c_k^\dagger |\text{vac}\rangle \quad (3)$$

$$|10\rangle = c_j^\dagger |\text{vac}\rangle \quad (4)$$

$$|11\rangle = c_j^\dagger c_k^\dagger |\text{vac}\rangle. \quad (5)$$

Plugging in Eq. (1), we get that the Givens rotation performs the transformation

$$|00\rangle \mapsto |00\rangle \quad (6)$$

$$|01\rangle \mapsto e^{i\varphi} \cos \theta |01\rangle + \sin \theta |10\rangle \quad (7)$$

$$|10\rangle \mapsto -e^{i\varphi} \sin \theta |01\rangle + \cos \theta |10\rangle \quad (8)$$

$$|11\rangle \mapsto e^{i\varphi} |11\rangle. \quad (9)$$

So the corresponding two-qubit gate is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\varphi} \cos \theta & -e^{i\varphi} \sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & e^{i\varphi} \end{pmatrix}. \quad (10)$$