Physics 20 Lab 3

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1 Part 1

1.1 Explicit Euler Method

For all graphs, we will use the initial conditions of $x_{initial} = 0$ and $v_{initial} = 10$. Plotting x and v against t with $h = 0.001$, we have the following plots:

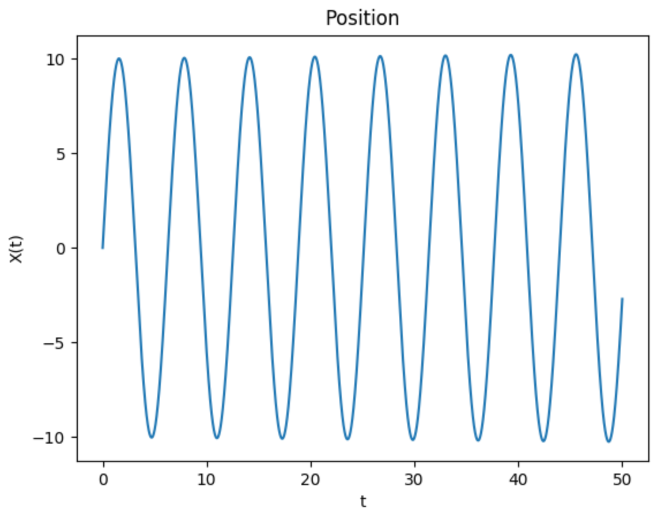

Figure 1: x vs. t, $h = 0.001$, $x_{init} = 0$, $v_{init} = 10$.

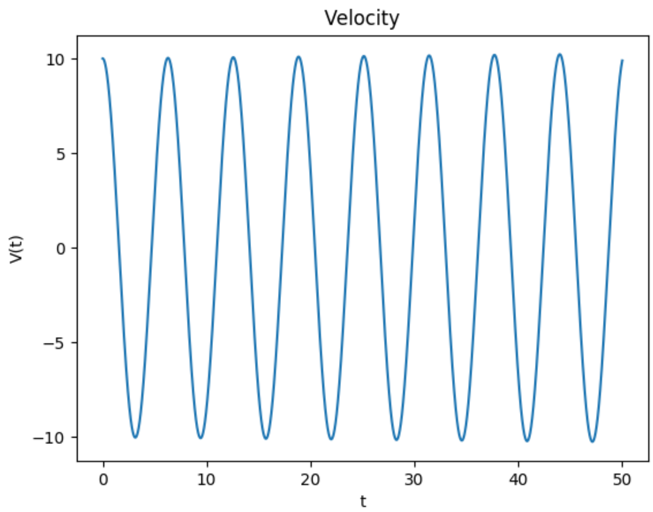

Figure 2: v vs. t, $h = 0.001$, $x_{init} = 0$, $v_{init} = 10$.

Given the equation $F = -kx = m\frac{d^2x}{dt^2}$ from applying Newton's second law to a spring, we can can solve the differential equation (setting $k/m = 1$) to find that $x(t) = x_{init}\cos(t) + v_{init}\sin(t)$. Accordingly $x'(t) = v(t) = -x_{init}\sin(t) + v_{init}\cos(t)$. Subtracting the explicit Euler solutions from the analytic solution, we can plot the error for $h = 0.001$:

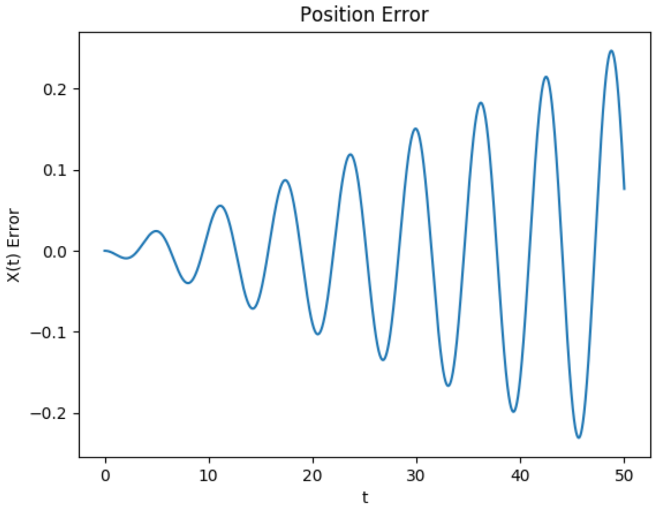

Figure 3: $x(t)$ error vs. t, $h = 0.001$, $x_{init} = 0$, $v_{init} = 10$.

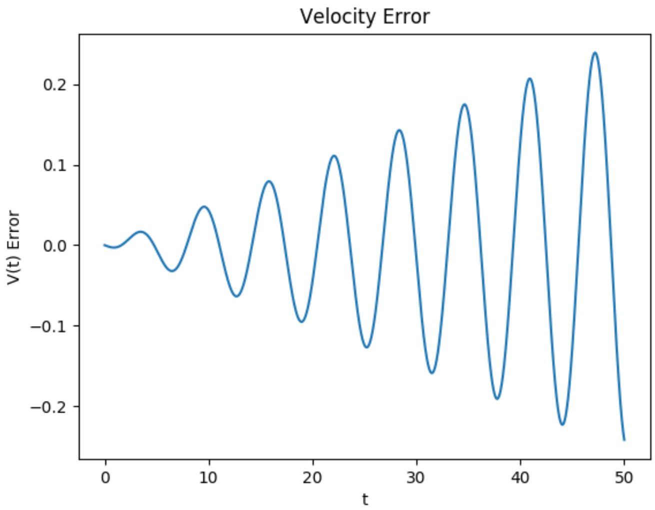

Figure 4: $v(t)$ error vs. t, $h = 0.001$, $x_{init} = 0$, $v_{init} = 10$.

To evaluate the truncation error, we plotted the maximum error of the Eulerian solution to $x(t)$ compared to the analytic solution from $t = 0$ to $t = 50$ for values of $h = h_0, h_0/2, h_0/4, h_0/8, h_0/16$ starting from $h_0 = 0.001$. The plot is shown below:

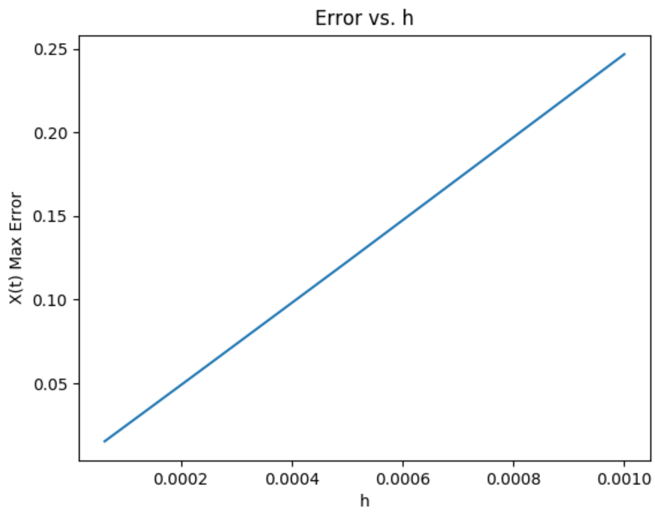

Figure 5: $x(t)$ max error vs. h, t ranges from 0 to 50, $x_{init} = 0$, $v_{init} = 10$.

Computing the normalized total energy (shown below), we found that E increases over time, starting close to the proper value (100) and gradually getting larger. This increased energy error corresponds to the error in position and velocity increasing over time.

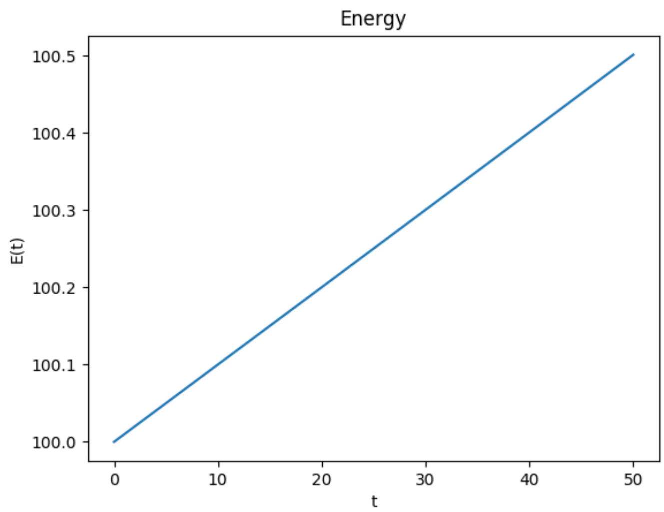

Figure 6: E vs. t, $h = 0.001$, $x_{init} = 0$, $v_{init} = 10$.

1.2 Implicit Euler Method

Given the analytic solutions $x_{i+1} = \frac{x_i + hv_i}{1+h^2}$ and $v_{i+1} = v_i - hx_{i+1}$, we plotted exactly the same graphs for the implicit Euler method as above. While energy increased over time for the explicit method, energy decreased over time for the implicit method. However, global error still increased, because the energy started at approximately the analytically correct value (100) before decreasing and moving further away from the correct solution.

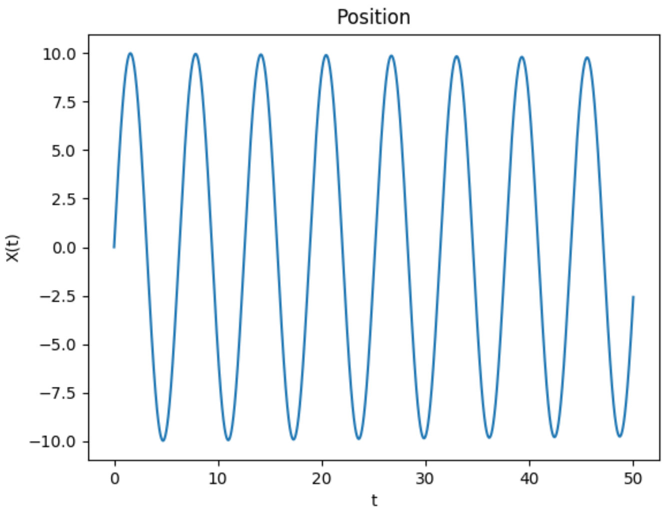

Figure 7: x vs. t, $h = 0.001$, $x_{init} = 0$, $v_{init} = 10$.

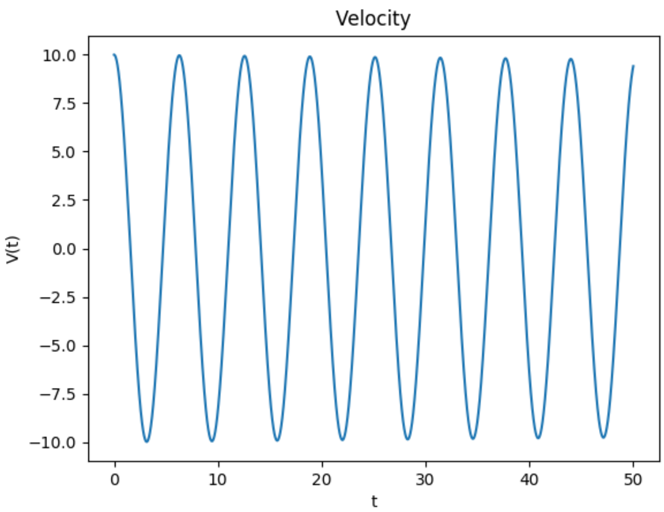

Figure 8: v vs. t, $h = 0.001$, $x_{init} = 0$, $v_{init} = 10$.

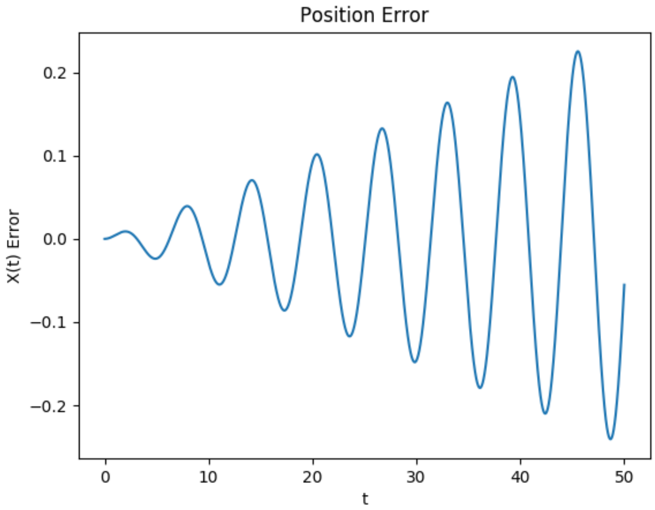

Figure 9: $x(t)$ error vs. t, $h = 0.001$, $x_{init} = 0$, $v_{init} = 10$.

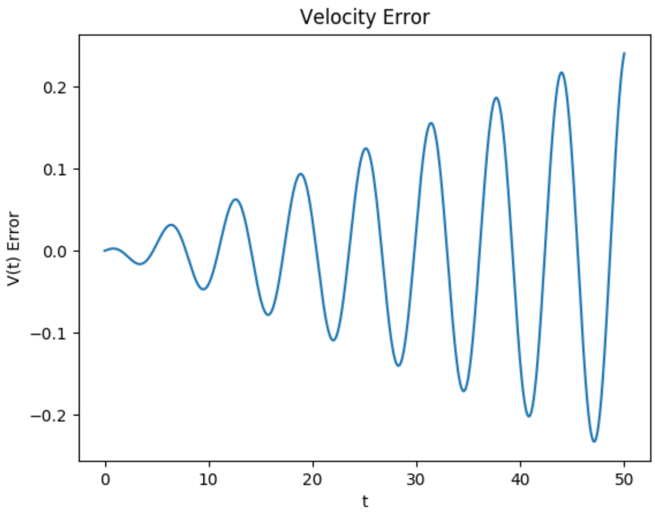

Figure 10: $v(t)$ error vs. t, $h = 0.001$, $x_{init} = 0$, $v_{init} = 10$.

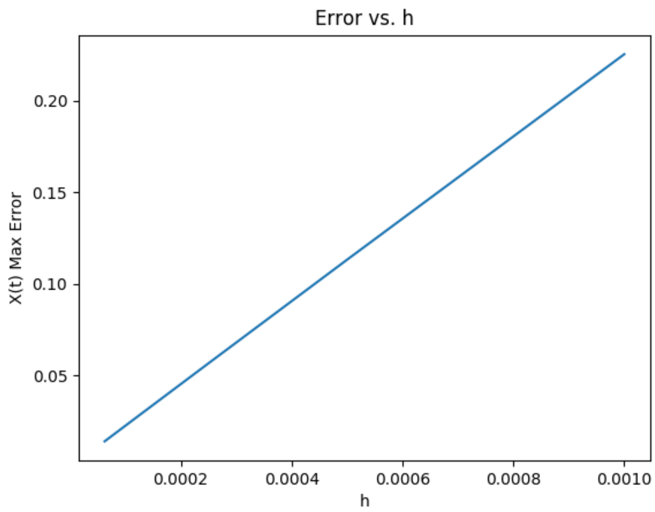

Figure 11: $x(t)$ max error vs. h, t ranges from 0 to 50, $x_{init} = 0$, $v_{init} = 10$.

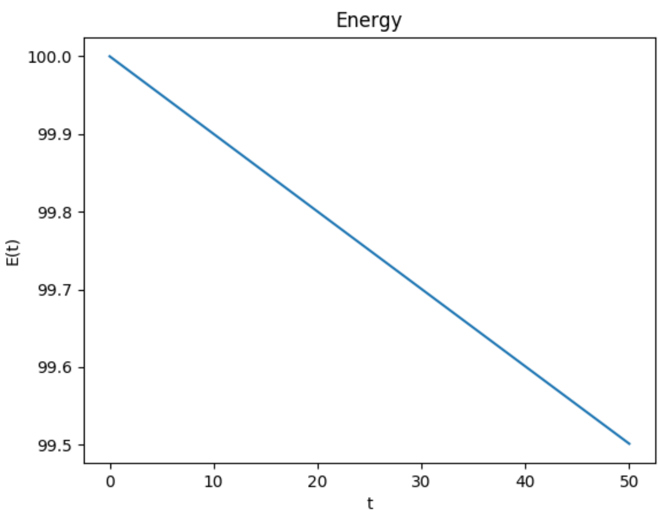

Figure 12: E vs. t, $h = 0.001$, $x_{init} = 0$, $v_{init} = 10$.

2 Part 2

2.1 Phase Space

In phase space, the explicit Eulerian method spiraled outwards, indicating increasing energy over time. In comparison, the implicit Eulerian method spiraled inwards, indicating decreasing energy. Plotting the symplectic method in phase space, however, we find that energy is conserved and a closed plot is produced.

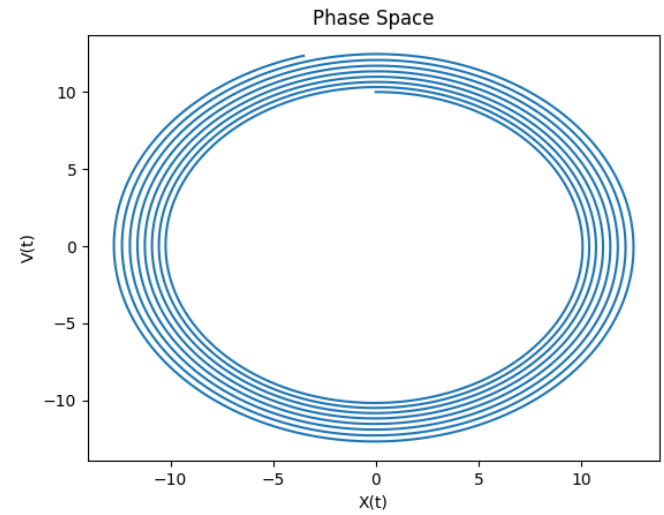

Figure 13: $v(t)$ vs. $x(t)$, $h = 0.01$, $x_{init} = 0$, $v_{init} = 10$. Explicit Euler.

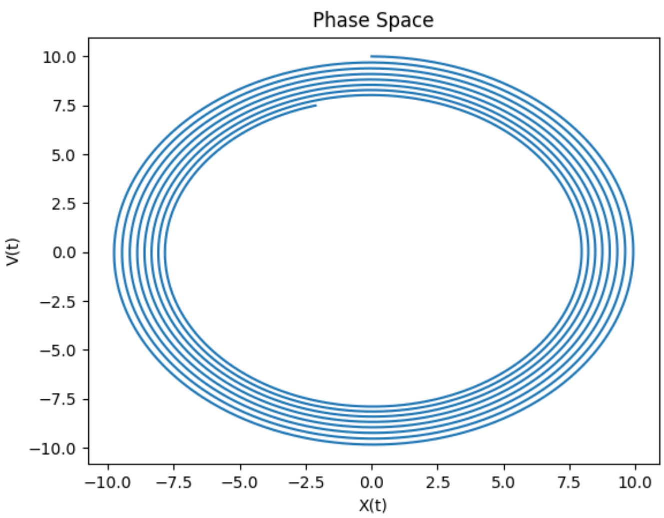

Figure 14: $v(t)$ vs. $x(t)$, $h = 0.01$, $x_{init} = 0$, $v_{init} = 10$. Implicit Euler.

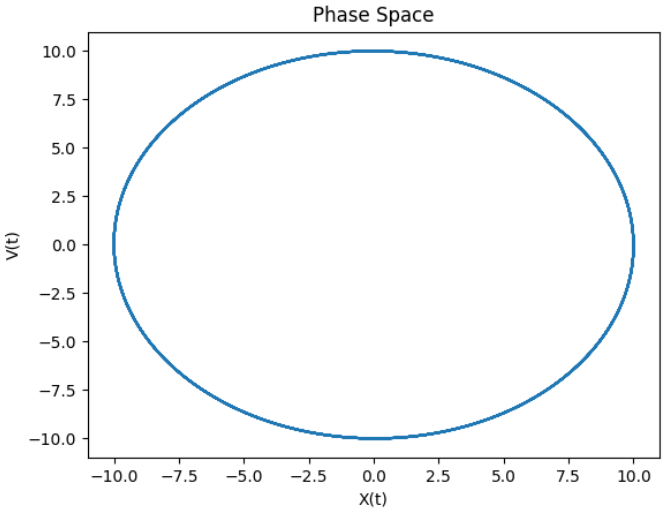

Figure 15: $v(t)$ vs. $x(t)$, $h = 0.01$, $x_{init} = 0$, $v_{init} = 10$. Symplectic Euler.

2.2 Total Energy

In the symplectic Euler method, the total energy fluctuates close to the analytic value (100), but the sum off the error is always 0 every half-period of each spring oscillation. Thus, although the symplectic Euler method deviates from conservation of energy within a given oscillation, it conserves energy in the long run instead of monotonically increasing/decreasing total energy. This corresponds to a closed circle in phase space, representing conservation of energy.

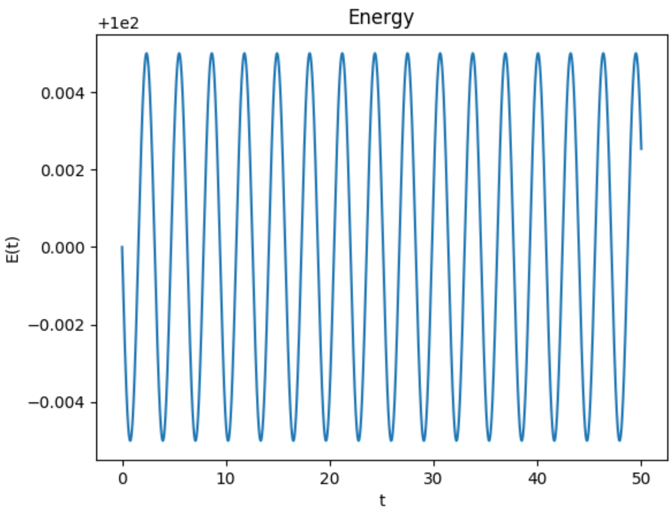

Figure 16: E vs. t, $h = 0.01$, $x_{init} = 0$, $v_{init} = 10$. Symplectic Euler.

2.3 Phase Error

Plotting from $t = 4950$ to $t = 5000$ for x vs. t and v vs. t, the phase error becomes apparent in the symplectic method. While the explicit and implicit Euler methods conserve phase but not energy, the symplectic method conserves energy but not phase. This can be seen by the difference in phase between the symplectic and analytic solutions below.

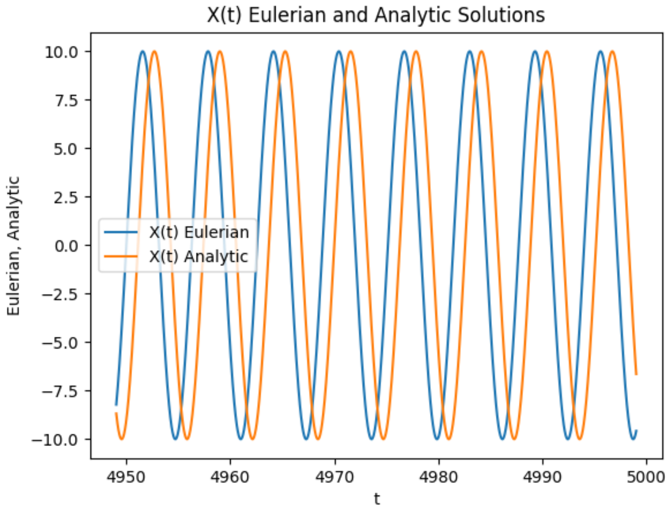

Figure 17: $x(t)$ vs. t, $h = 0.03$, $x_{init} = 0$, $v_{init} = 10$. Symplectic Euler.

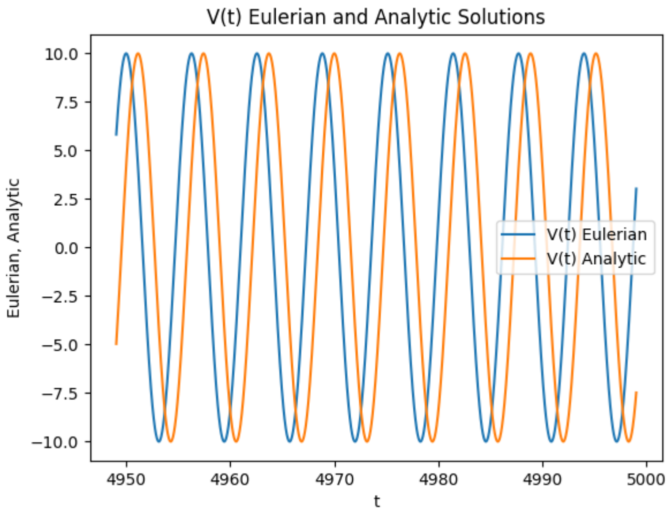

Figure 18: $x(t)$ vs. t, $h = 0.03$, $x_{init} = 0$, $v_{init} = 10$. Symplectic Euler.