Logistic Regression

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February 2023

1 Derivation

We start with

$$p(x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}$$

Let us simplify the notation by letting

$$y = \beta_0 + \beta_1 x$$

We can now write

$$p(x) = \frac{1}{1 + e^{-y}}$$

Rearranging gives

$$\frac{1}{p(x)} = 1 + e^{-y}$$

$$\frac{1 - p(x)}{p(x)} = e^{-y}$$

$$\frac{p(x)}{1 - p(x)} = e^{y}$$

$$\log\left(\frac{p(x)}{1 - p(x)}\right) = y$$

By the definition of the odds, this is

odds $= \frac{p(x)}{1-p(x)}$

$$\log(\text{odds}) = y$$

This gets us finally to

$$\boxed{\log(\text{odds}) = \beta_0 + \beta_1 x}$$

2 Interpreting the coefficients

We start with

$$\log(\text{odds}) = \beta_0 + \beta_1 x$$

In other words,

$$\text{odds} = \exp[\beta_0 + \beta_1 x]$$

Rewritten, this is:

$$\text{odds} = e^{\beta_0} e^{\beta_1 x} \tag{1}$$

$\boxed{\text{We see that } e^{\beta_0} \text{ is the odds when } x = 0}$

Now suppose we go from x to $x + 1$:

$$e^{\beta_0 + \beta_1 (x+1)}$$

This is the same as:

$$e^{\beta_0} e^{\beta_1 x} e^{\beta_1}$$

Substituting the result from Equation 1, we get

$$\text{odds} \times e^{\beta_1}$$

Final result: e^{β_1} tells you the multiplicative increase in the odds (with each unit of increase in x)