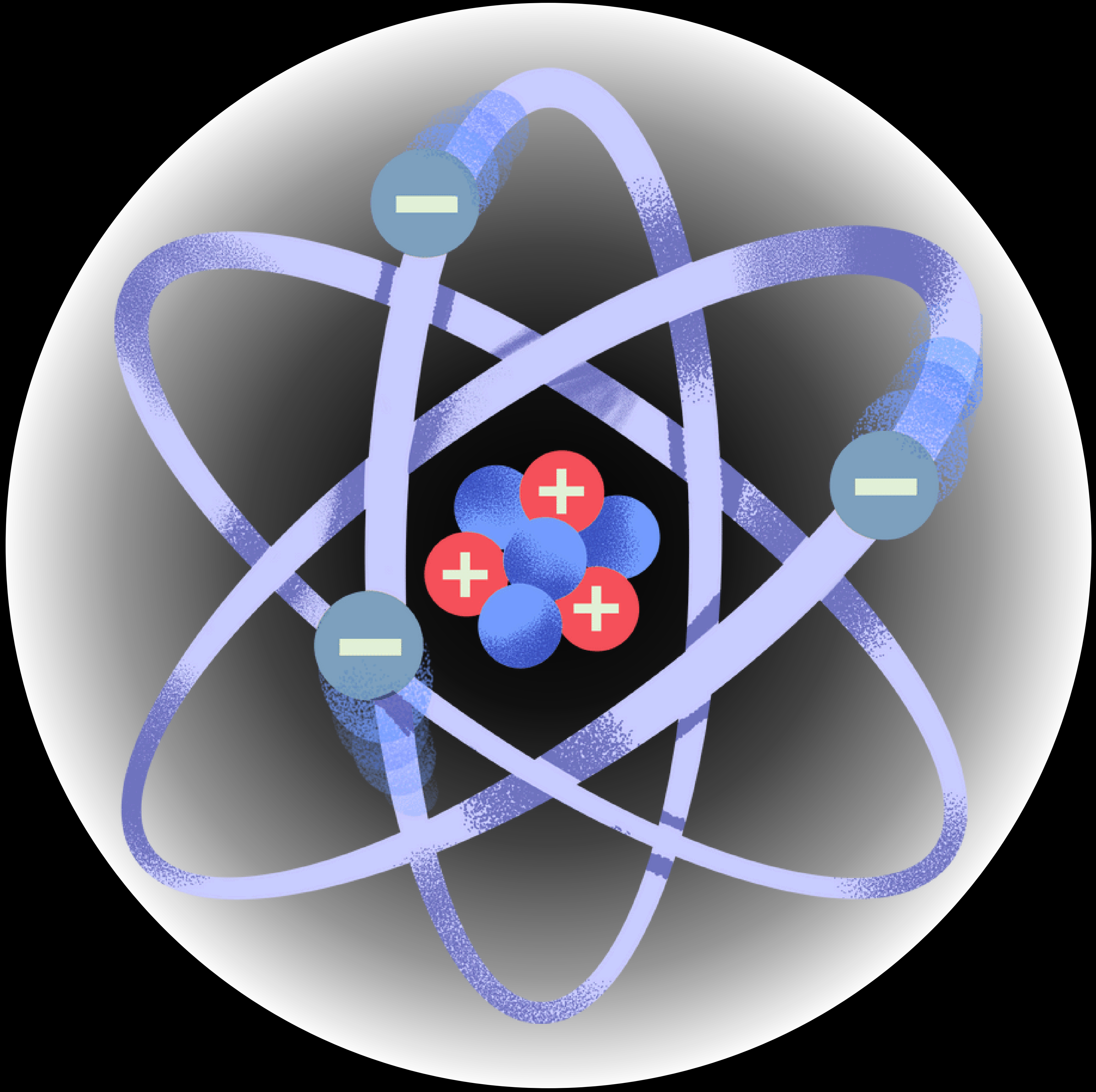
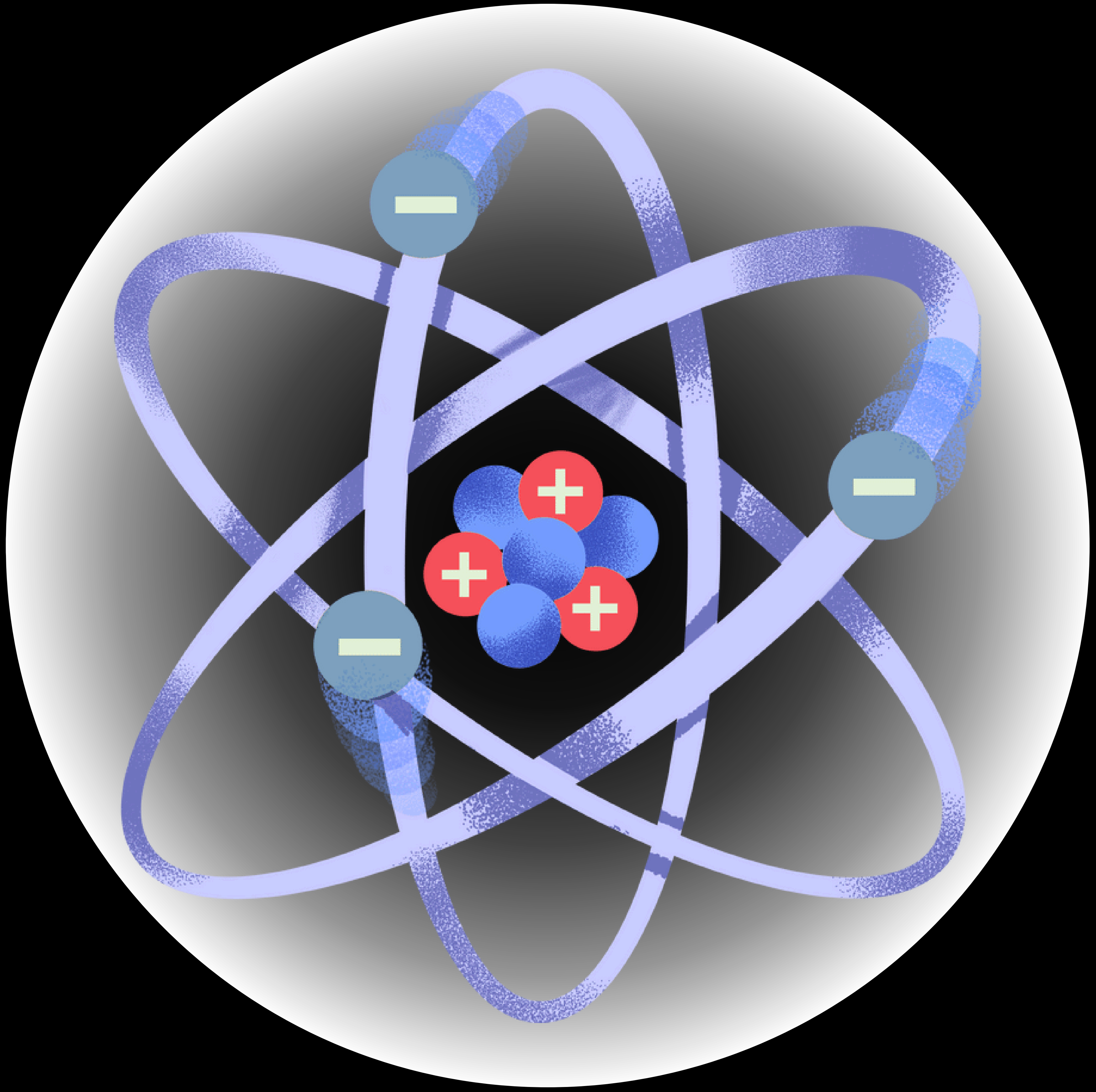
DEEP DIVE INTO QUANTUM GATES

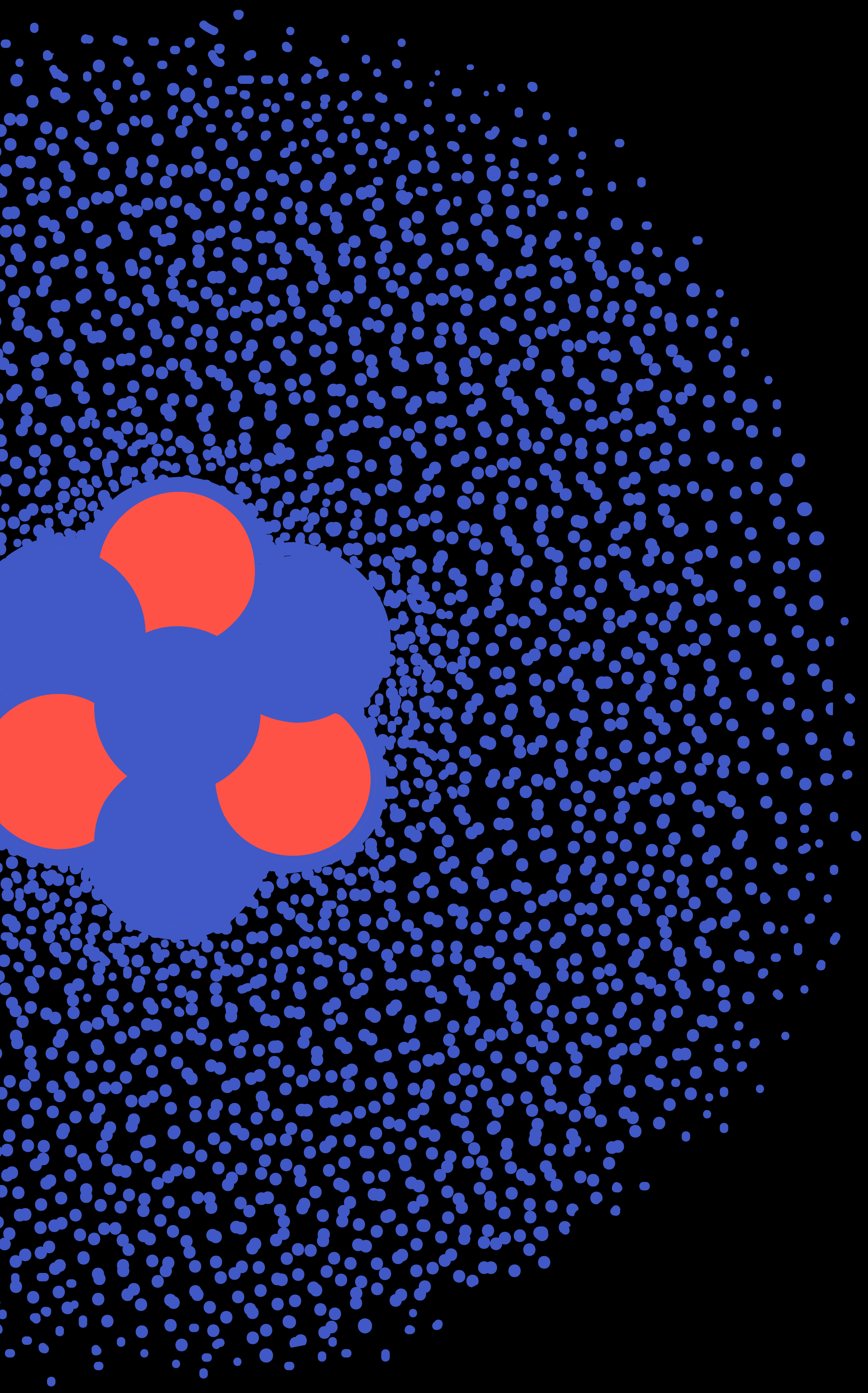




Recap:

Dirac Notation

Bloch Sphere



instead of using matrices we use

Dirac Notation

Writing the below qubit state in Dirac Notation

$$|\psi\rangle = \begin{pmatrix} \frac{1}{2} \\ \frac{2\sqrt{3}}{4} \end{pmatrix} =$$

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \underbrace{\alpha|0\rangle + \beta|1\rangle}_{\text{Dirac Notation}}$$

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \underbrace{\alpha|0\rangle + \beta|1\rangle}_{\text{Dirac Notation}}$$

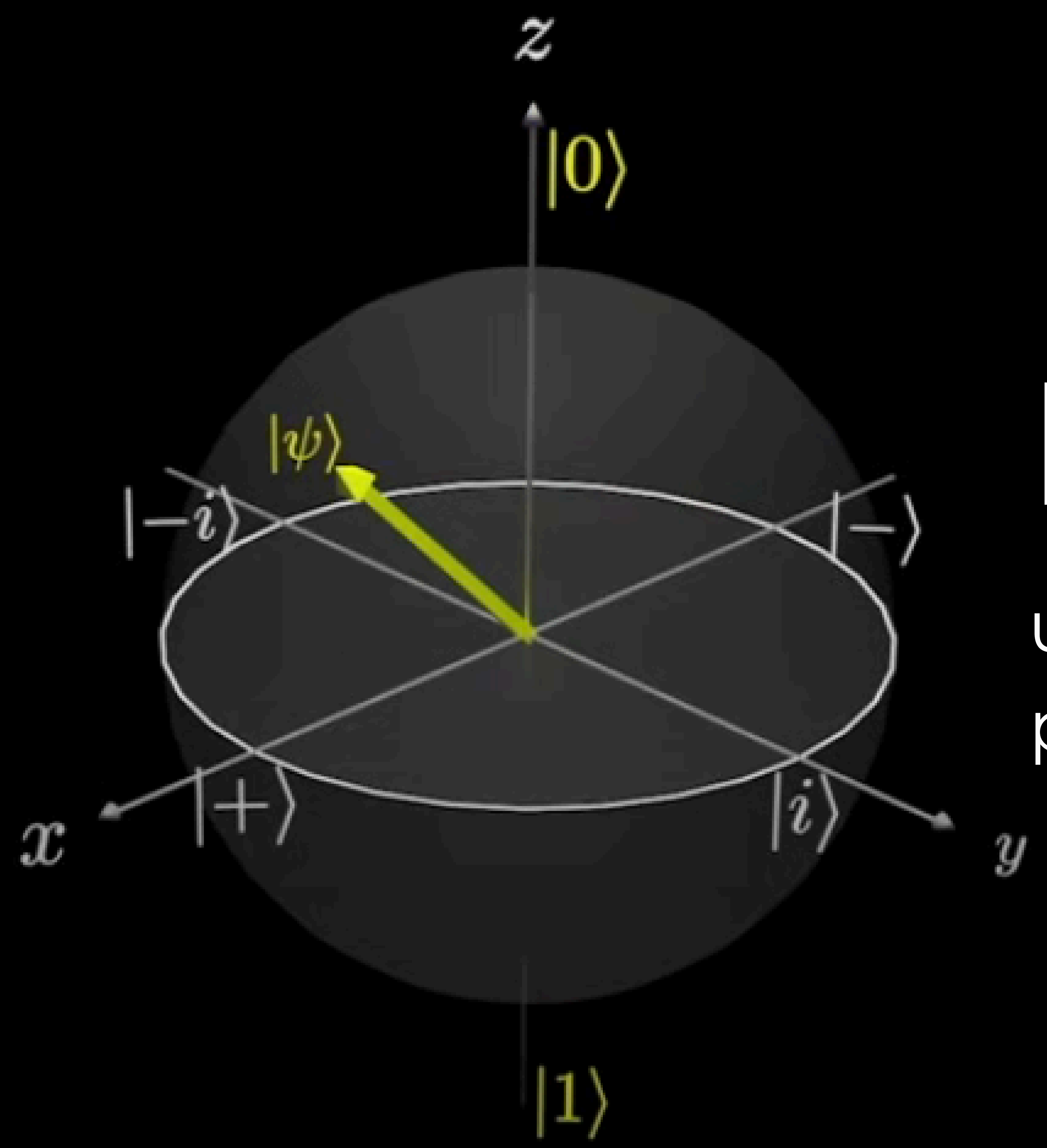
$$|0\rangle \, |1\rangle$$ called Ket Vectors

$$|\psi\rangle = \begin{pmatrix} \frac{1}{2} \\ \frac{2\sqrt{3}}{4} \end{pmatrix} = \frac{1}{2}|0\rangle + \frac{2\sqrt{3}}{4}|1\rangle$$

1. Convert the following qubit states from matrix to Dirac Notation

(a) $|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ (b) $|\psi\rangle = \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix}$ (c) $|\psi\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

2. If we measure a qubit in the state $|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ as 0, what would we measure if we were to measure the qubit again? Why?

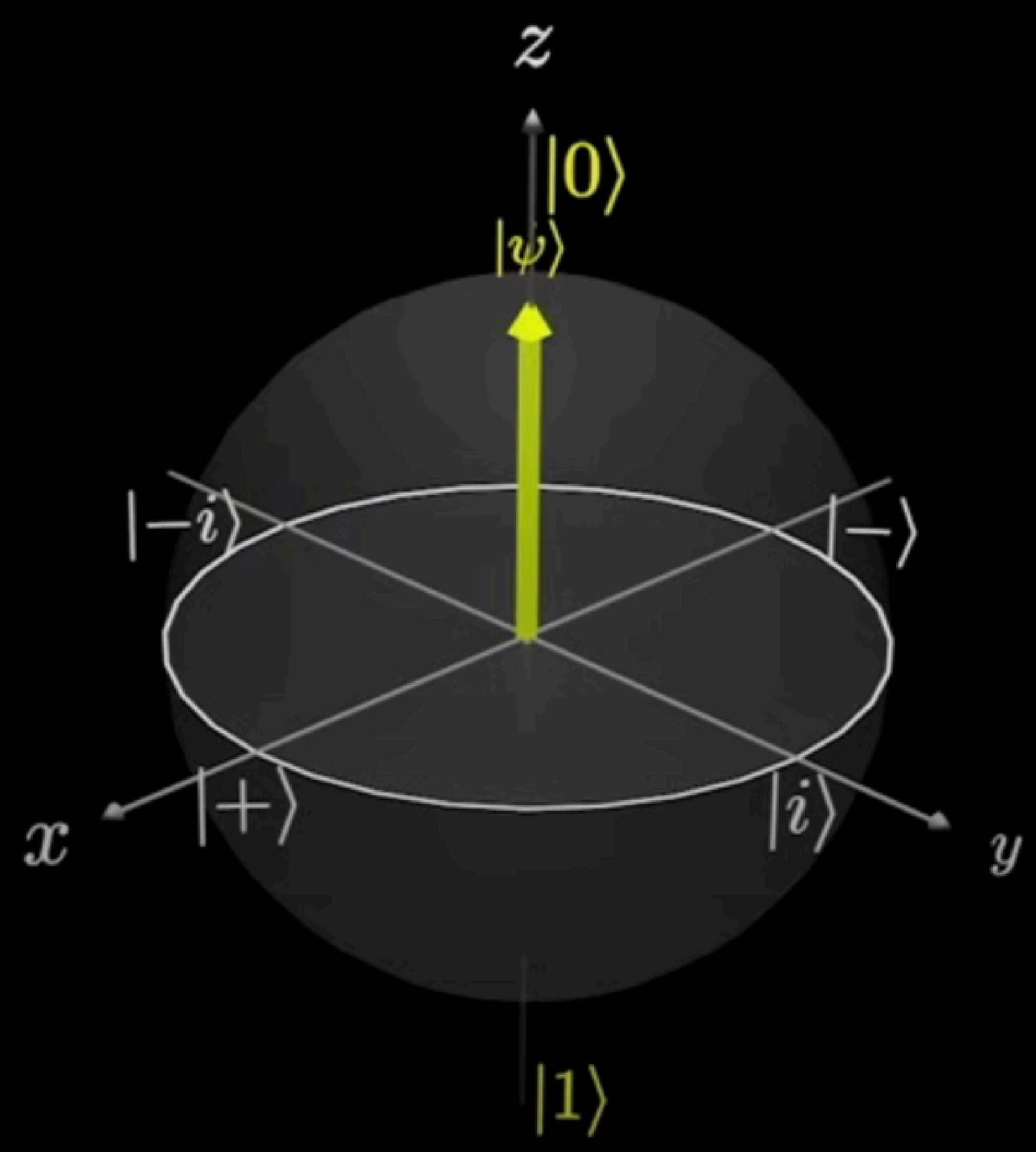


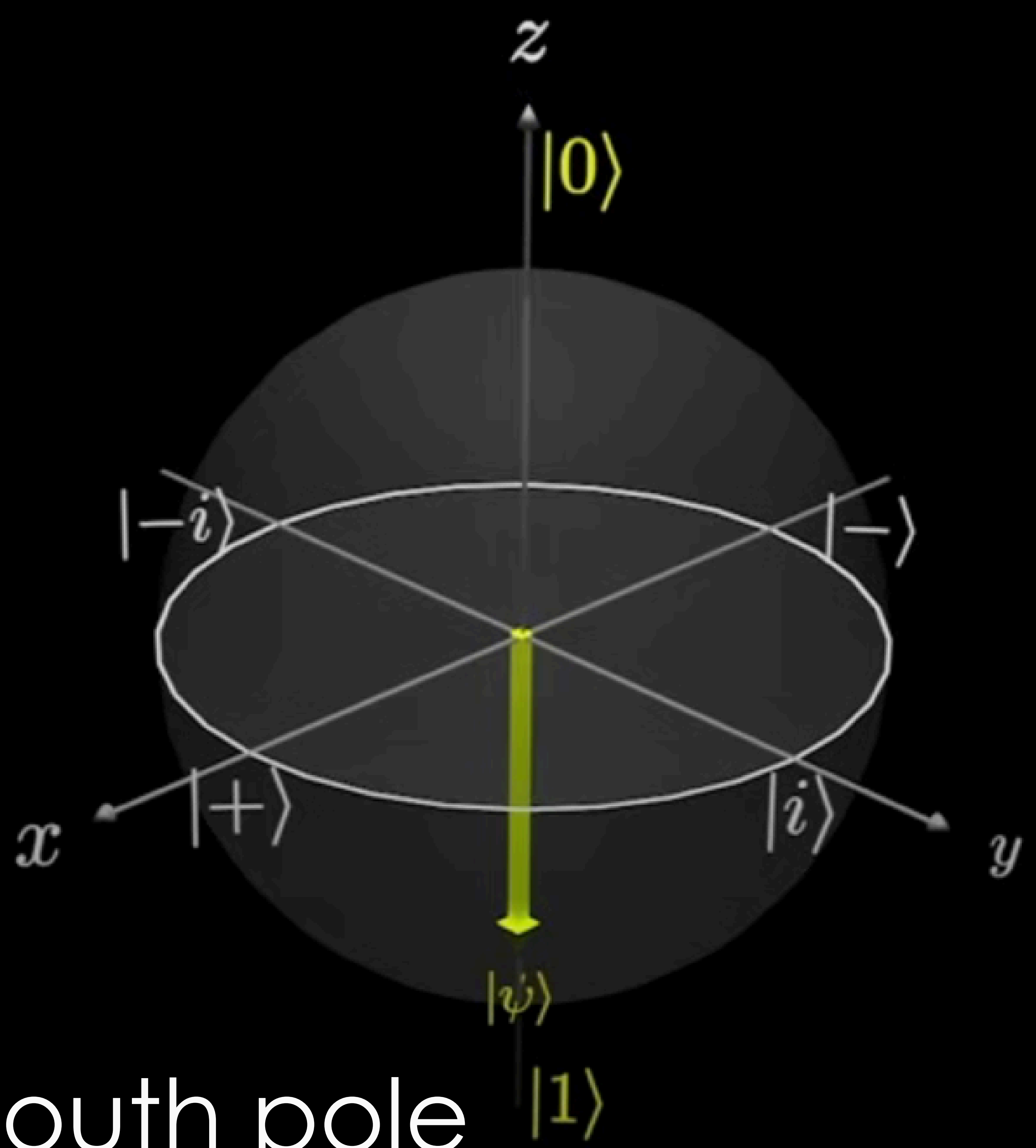
Bloch Sphere

used to represent a qubit as a point on its surface

Higher vertically = Higher probability of measuring $|\psi\rangle$ as $|0\rangle$

$$\implies \text{if } |\psi\rangle \text{ is on the north pole, } |\psi\rangle = |0\rangle$$

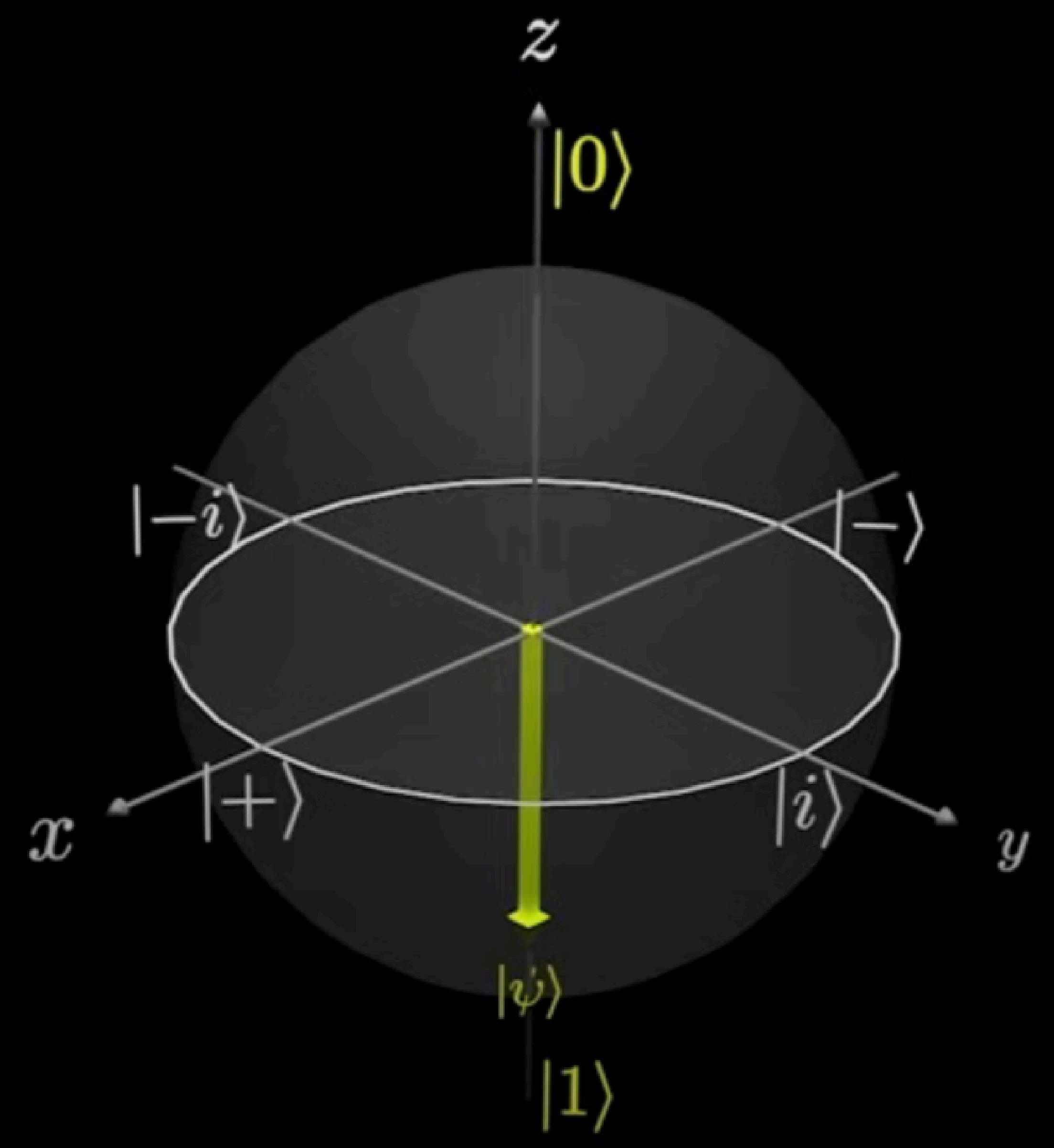




qubit on the south pole

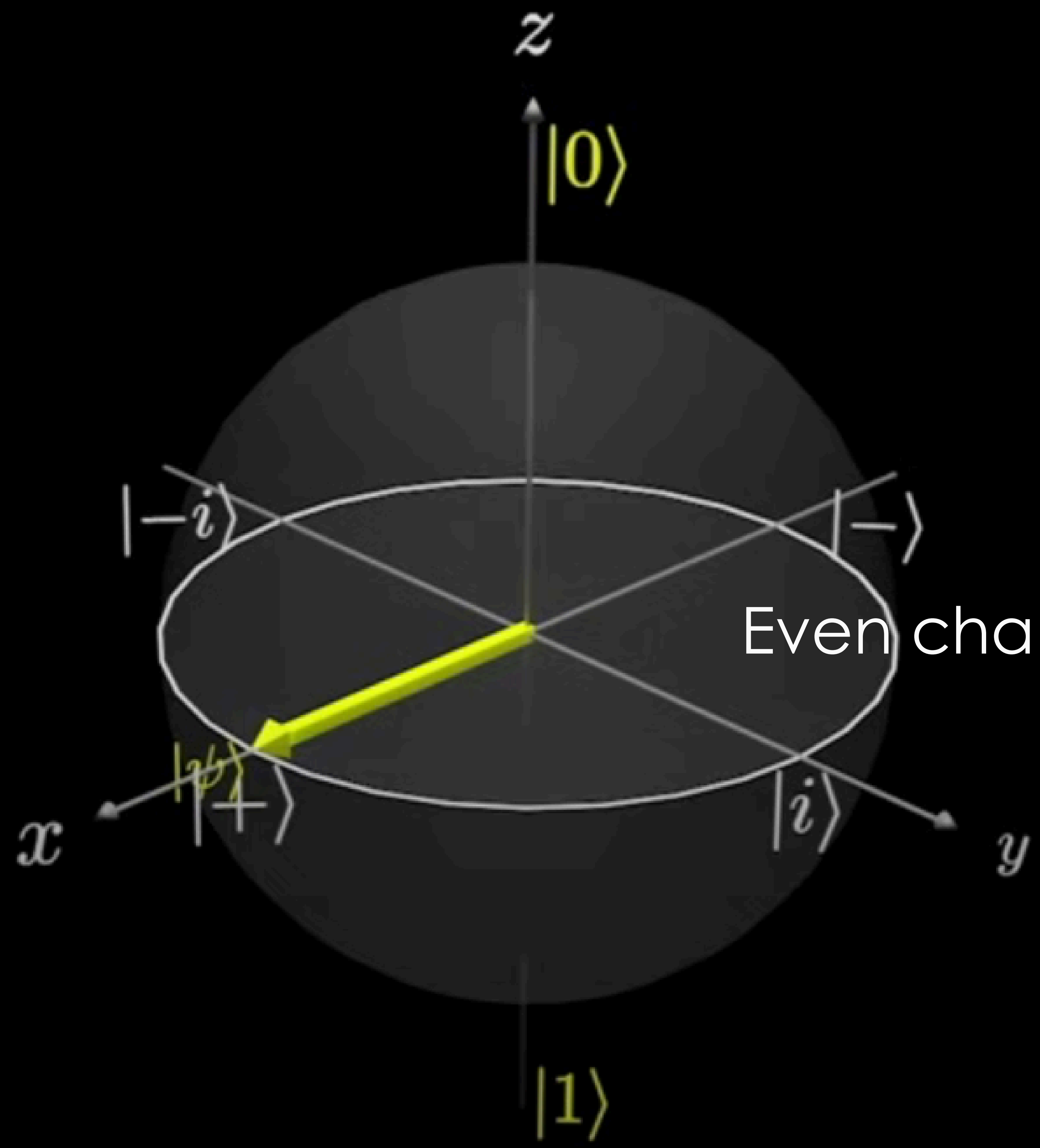
Lower vertically = Higher probability of measuring $|\psi\rangle$ as $|1\rangle$

$\implies$ if $|\psi\rangle$ is on the south pole, $|\psi\rangle = |1\rangle$



Where will this be represented ?

$$|\psi\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$



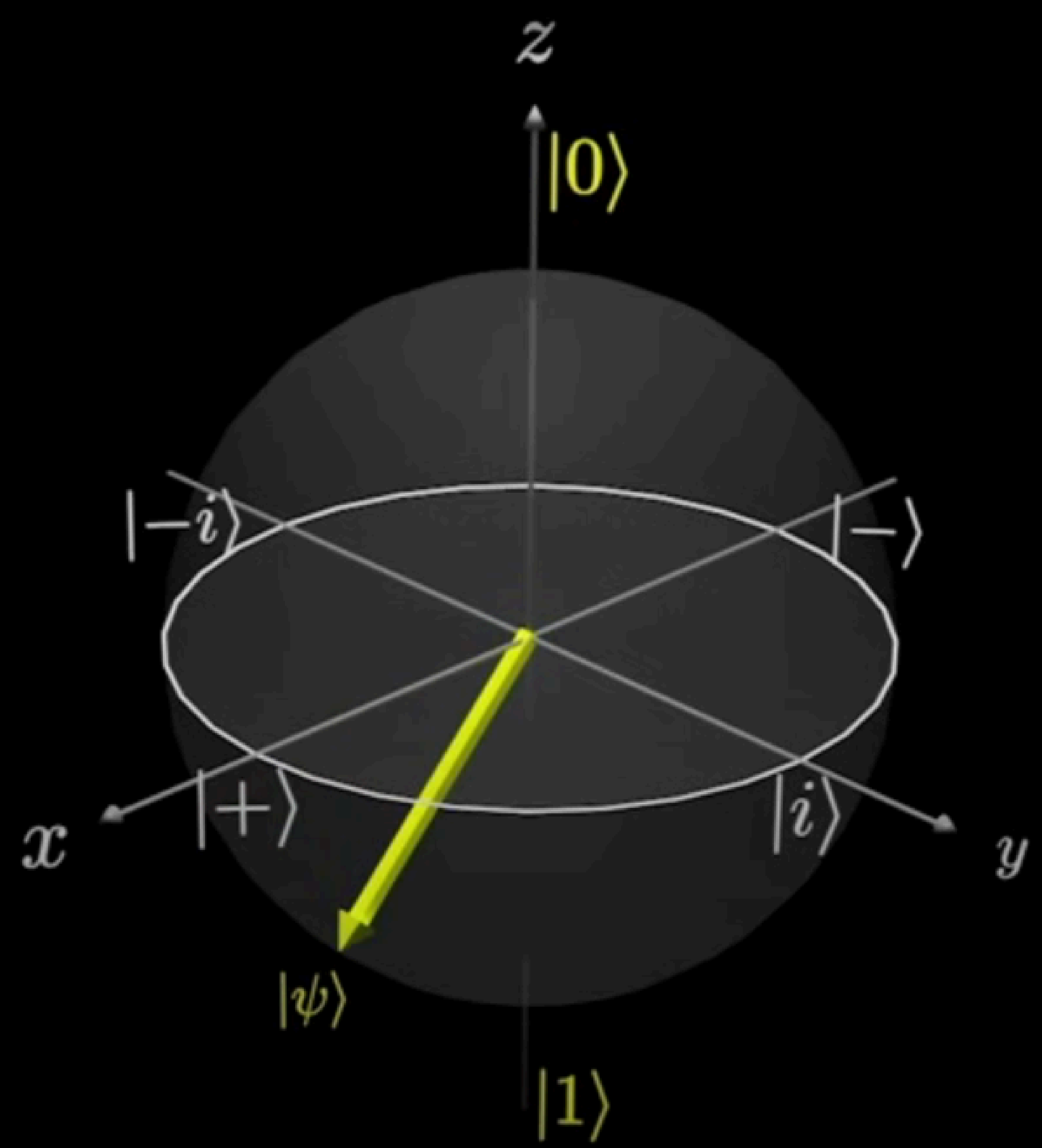
Even chance of being measured as 0 and 1 where will the below state be?

$$|\psi\rangle = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

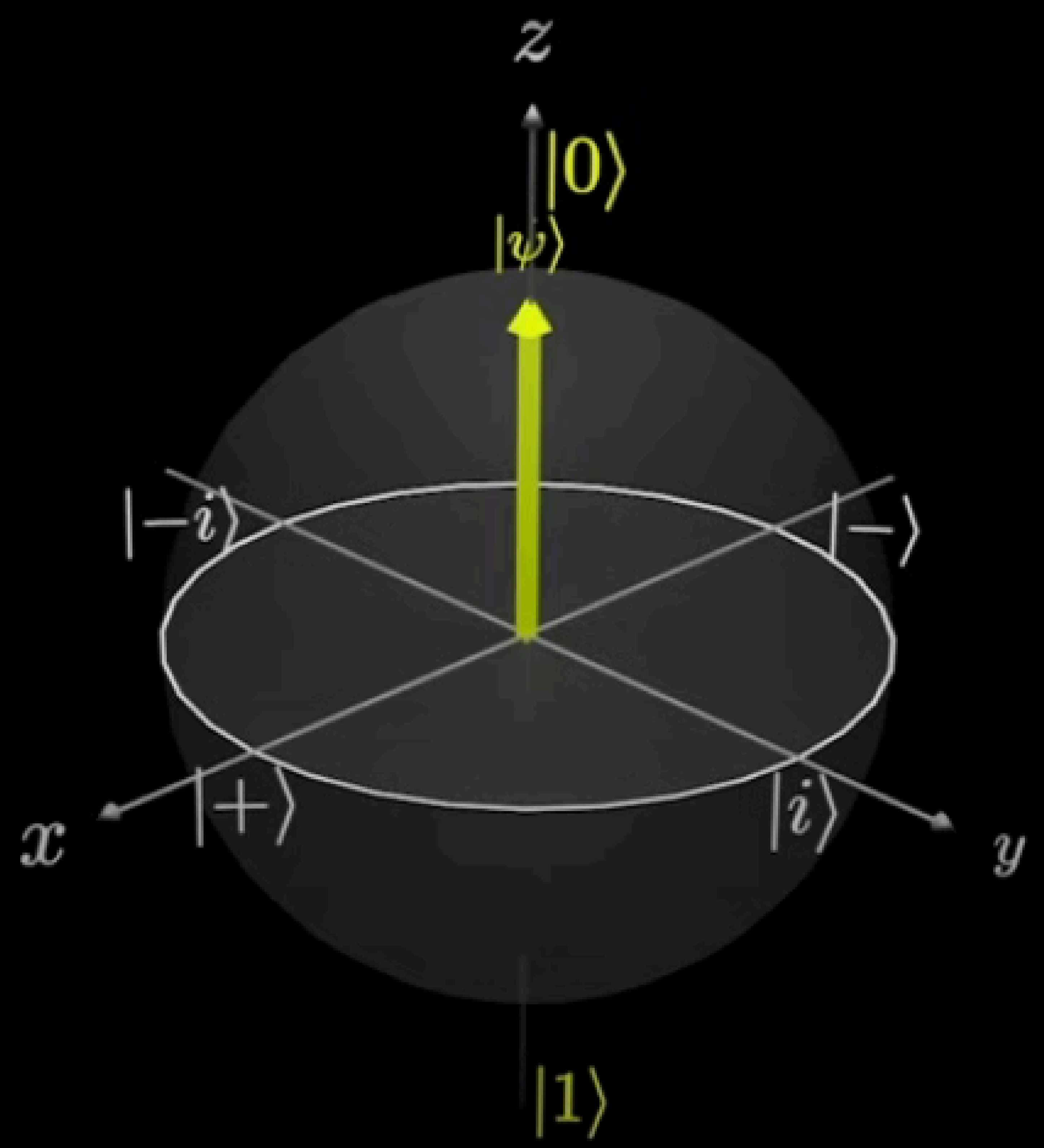
where will the below state be?

$$|\psi\rangle = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

$\implies$ has 1/4 chance of being measured as $|0\rangle$
and a 3/4 chance of being measured as $|1\rangle$

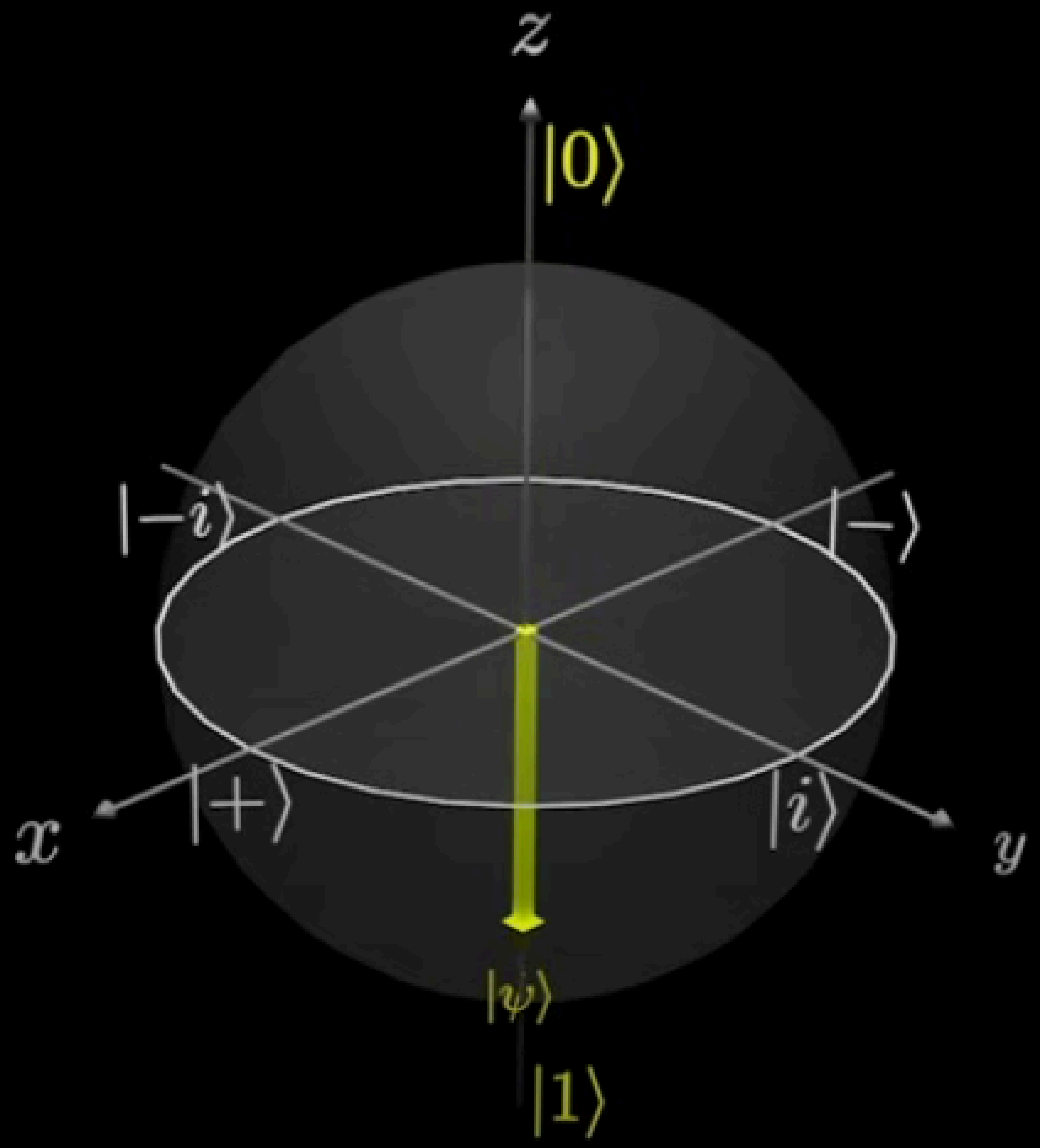

z
|0⟩
|−i⟩
|−⟩
|+⟩
|i⟩
x
y
|ψ⟩
|1⟩

Manipulating a Qubit: The X, Y, Z Gates

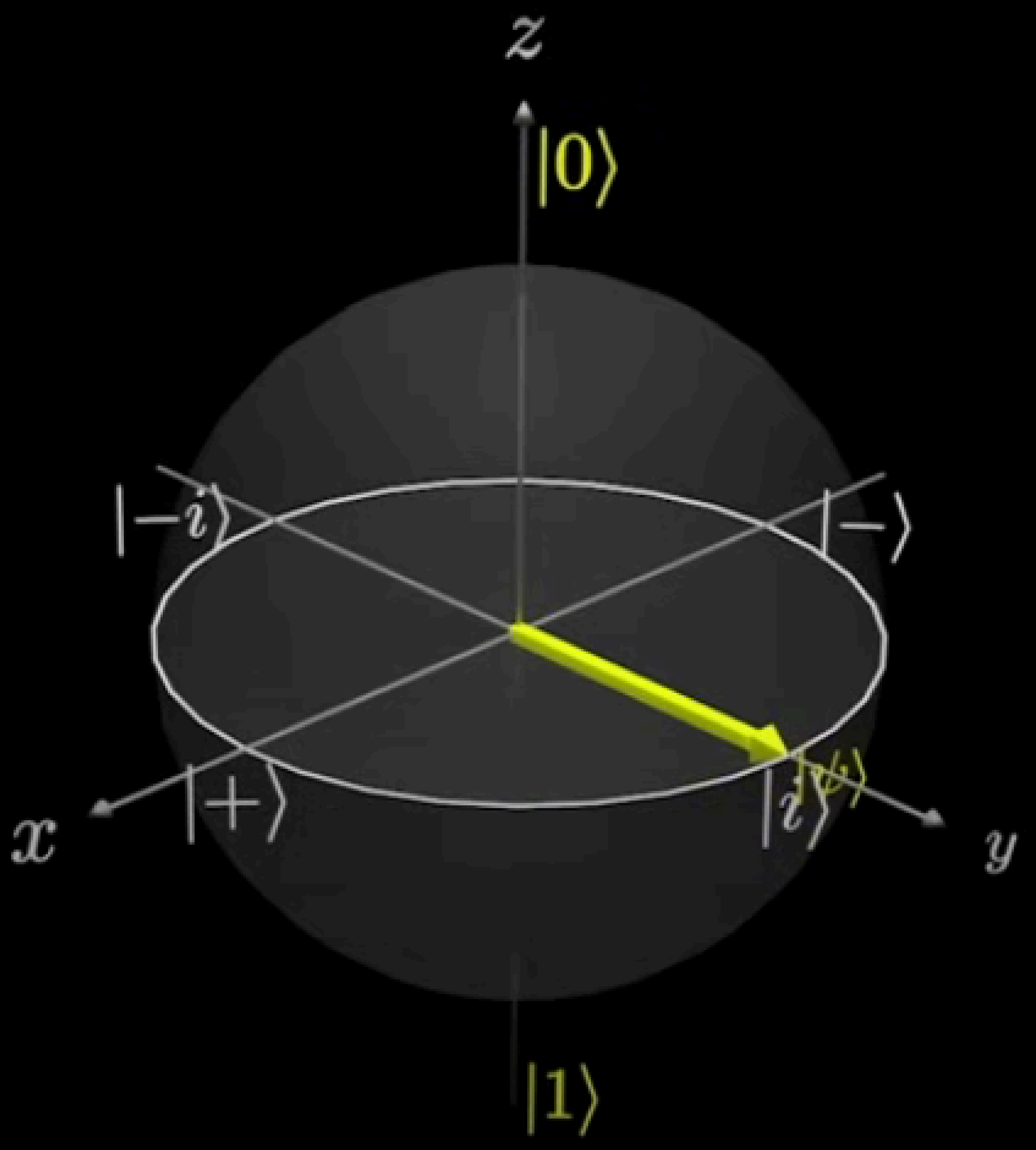


z
$|0\rangle$
$|\psi\rangle$
$|-i\rangle$
$|-\rangle$
$|+\rangle$
$|i\rangle$
x
y
$|1\rangle$

The X Gate

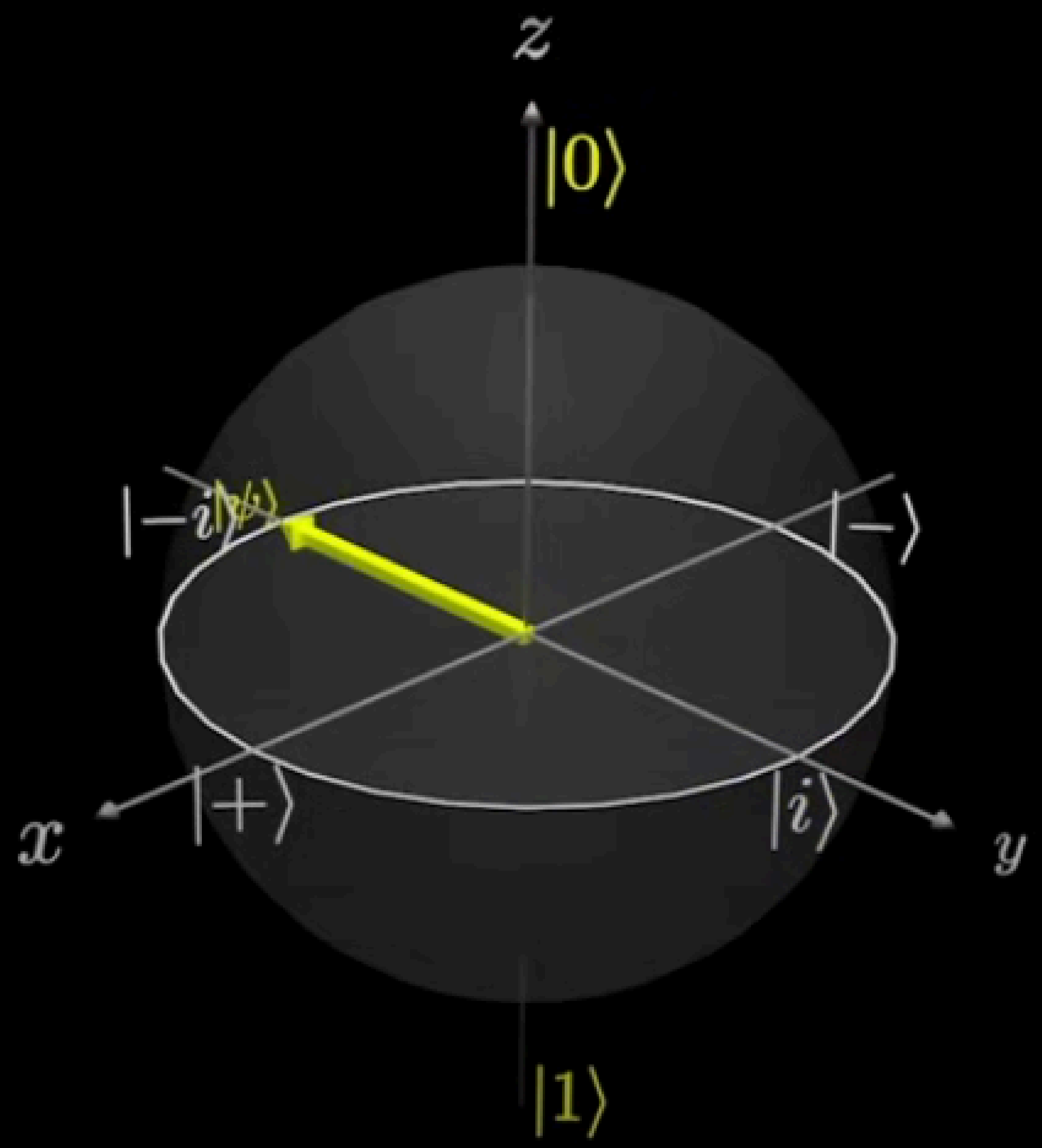


z
$|0\rangle$
$|-i\rangle$
$|-\rangle$
x
$|+\rangle$
$|i\rangle$
y
$|\psi\rangle$
$|1\rangle$



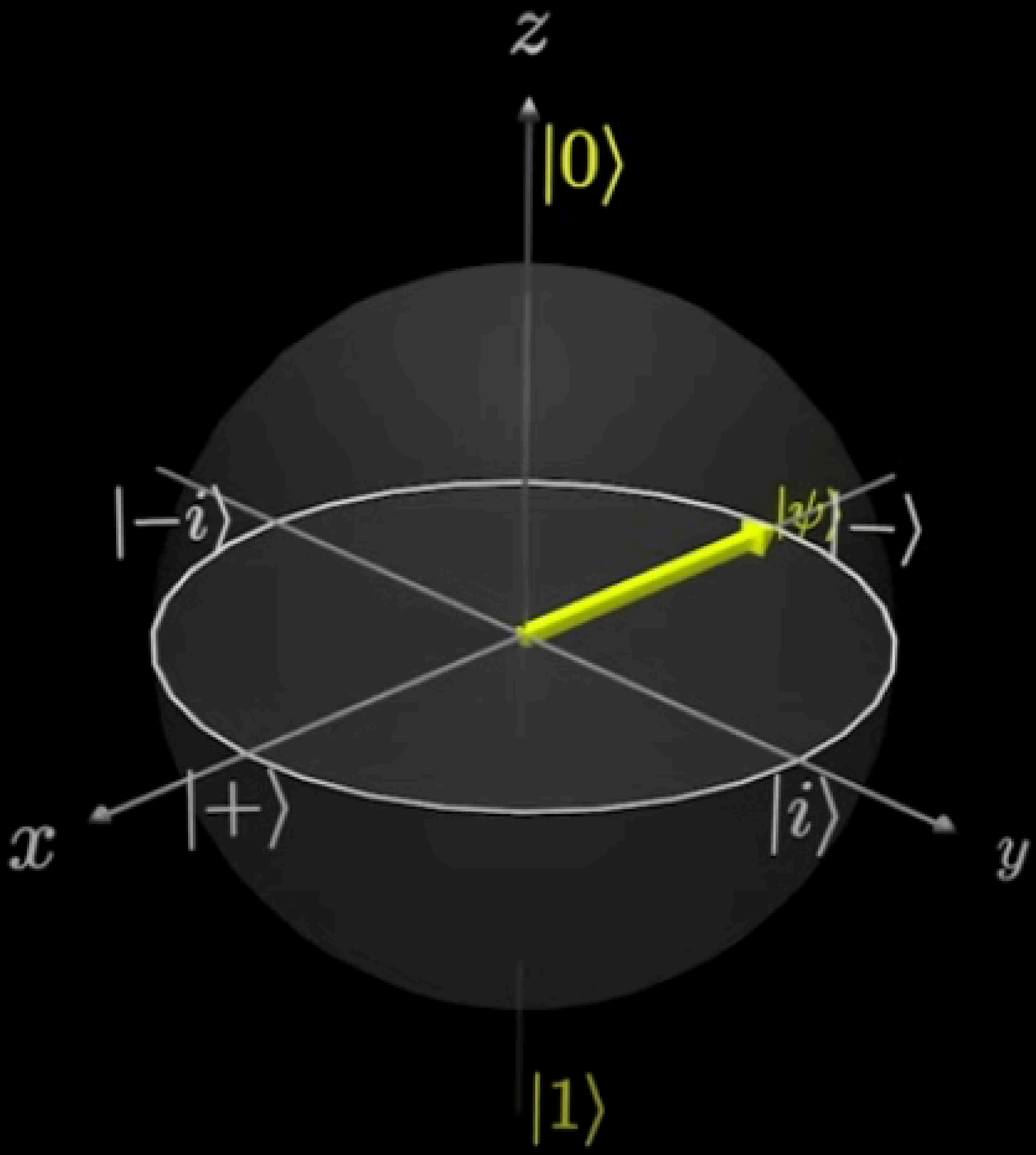
z
$|0\rangle$
$|-i\rangle$
$|-\rangle$
$|+\rangle$
$|i\rangle$
$|\psi\rangle$
x
y
$|1\rangle$

The X Gate



z
$|0\rangle$
$|-i\rangle$
$|\psi\rangle$
$|-\rangle$
$|+\rangle$
$|i\rangle$
x
y
$|1\rangle$

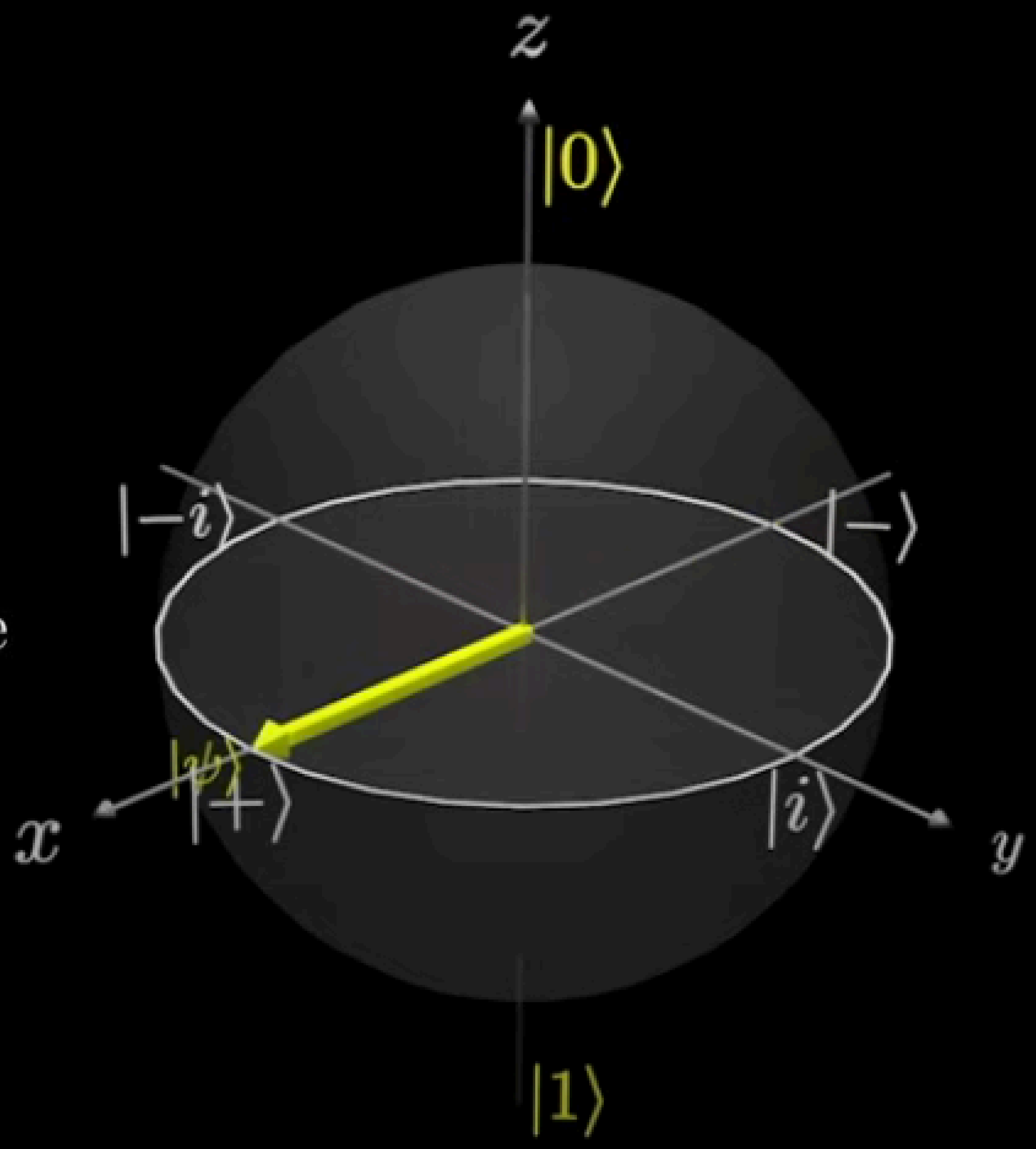
The X-gate flips the qubit π radians around the x-axis on the Bloch Sphere



z
$|0\rangle$
$|-i\rangle$
$|\psi\rangle$
$|-\rangle$
$|+\rangle$
$|i\rangle$
x
y
$|1\rangle$

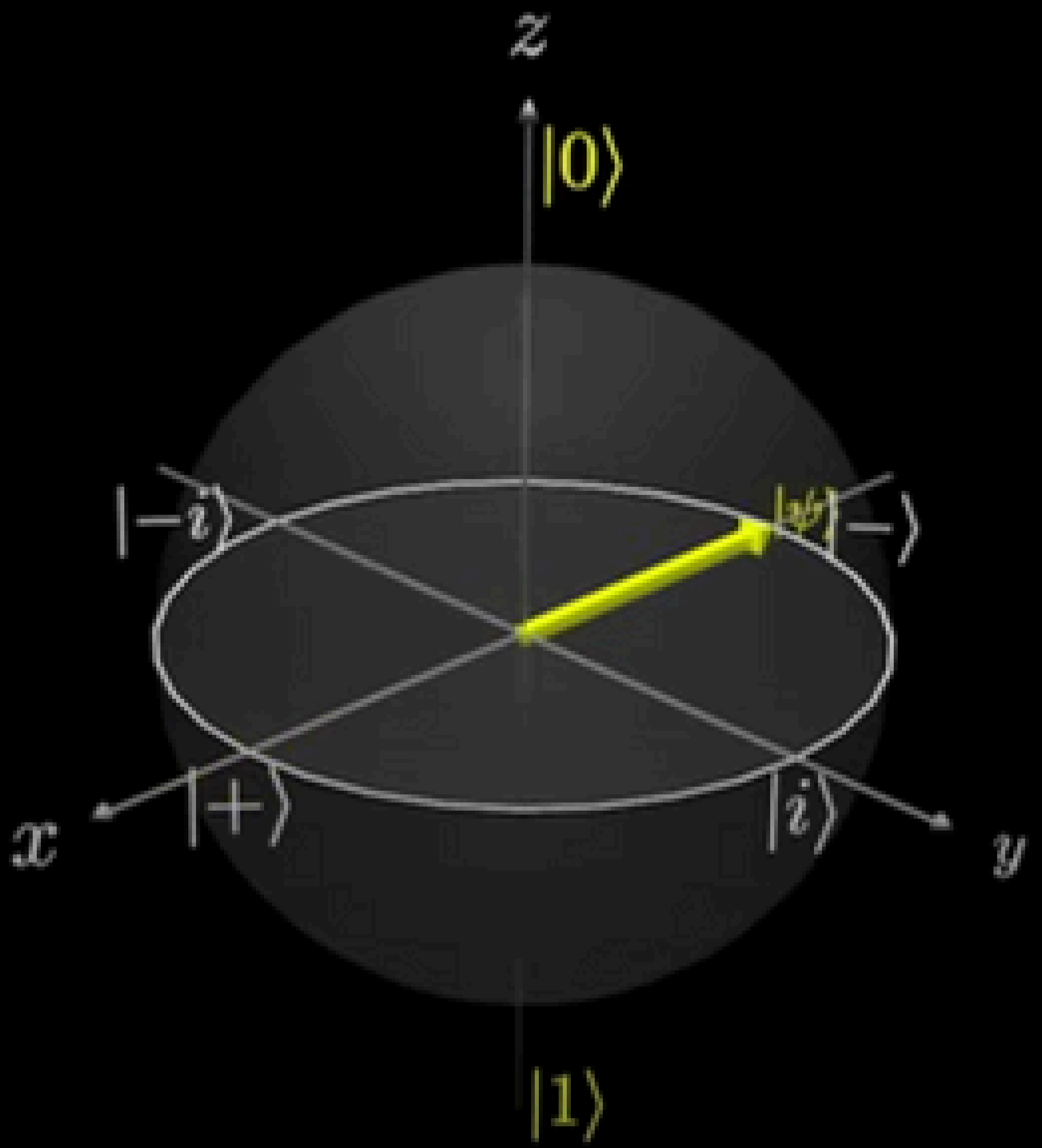
The Y Gate

The Y-gate flips the qubit π radians around the y-axis on the Bloch Sphere



The Z Gate

The Z-gate flips the qubit π radians around the z-axis on the Bloch Sphere



the X, Y and Z gates are their own inverses

Since the X, Y and Z gates rotate around the specified axis π radians

If we apply the same gate twice we rotate around 2π radians, meaning the qubit will end up in the original state

Matrix Representation of the Gates:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Example: Applying X gate to an arbitrary qubit $|\psi\rangle = \binom{\alpha}{\beta}$

$$X|\psi\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$$

applying an X gate to $|0\rangle$ gives

applying an X gate to $|0\rangle$ gives $|1\rangle$

$$X|0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

Important Property

Let $U = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, be an arbitrary gate

$$U|0\rangle = \begin{pmatrix} a \\ c \end{pmatrix} = a|0\rangle + c|1\rangle$$

$$U|1\rangle = \begin{pmatrix} b \\ d \end{pmatrix} = b|0\rangle + d|1\rangle$$

linear property of Quantum Gates

$$\text{Let } |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$U|\psi\rangle = U(\alpha|0\rangle + \beta|1\rangle)$$

$$U|\psi\rangle = \alpha\boxed{U|0\rangle} + \beta\boxed{U|1\rangle}$$

Applying the Y gate:

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Y|\psi\rangle = Y\left(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle\right)$$

Y gets Distributed

$$Y|\psi\rangle = \frac{\sqrt{3}}{2}Y|0\rangle + \frac{1}{2}Y|1\rangle$$

Y gets Distributed

$$Y|\psi\rangle = \frac{\sqrt{3}}{2}Y|0\rangle + \frac{1}{2}Y|1\rangle$$

$$Y|\psi\rangle = \frac{\sqrt{3}}{2}\begin{pmatrix} 0 \\ i \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -i \\ 0 \end{pmatrix}$$

$$Y|\psi\rangle = \frac{\sqrt{3}}{2}\begin{pmatrix} 0 \\ i \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -i \\ 0 \end{pmatrix}$$

$$Y|\psi\rangle = \frac{\sqrt{3}}{2}\, i \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \frac{1}{2}\, i \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$Y|\psi\rangle = \frac{\sqrt{3}}{2}\, i\, |1\rangle - \frac{1}{2}\, i\, |0\rangle$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

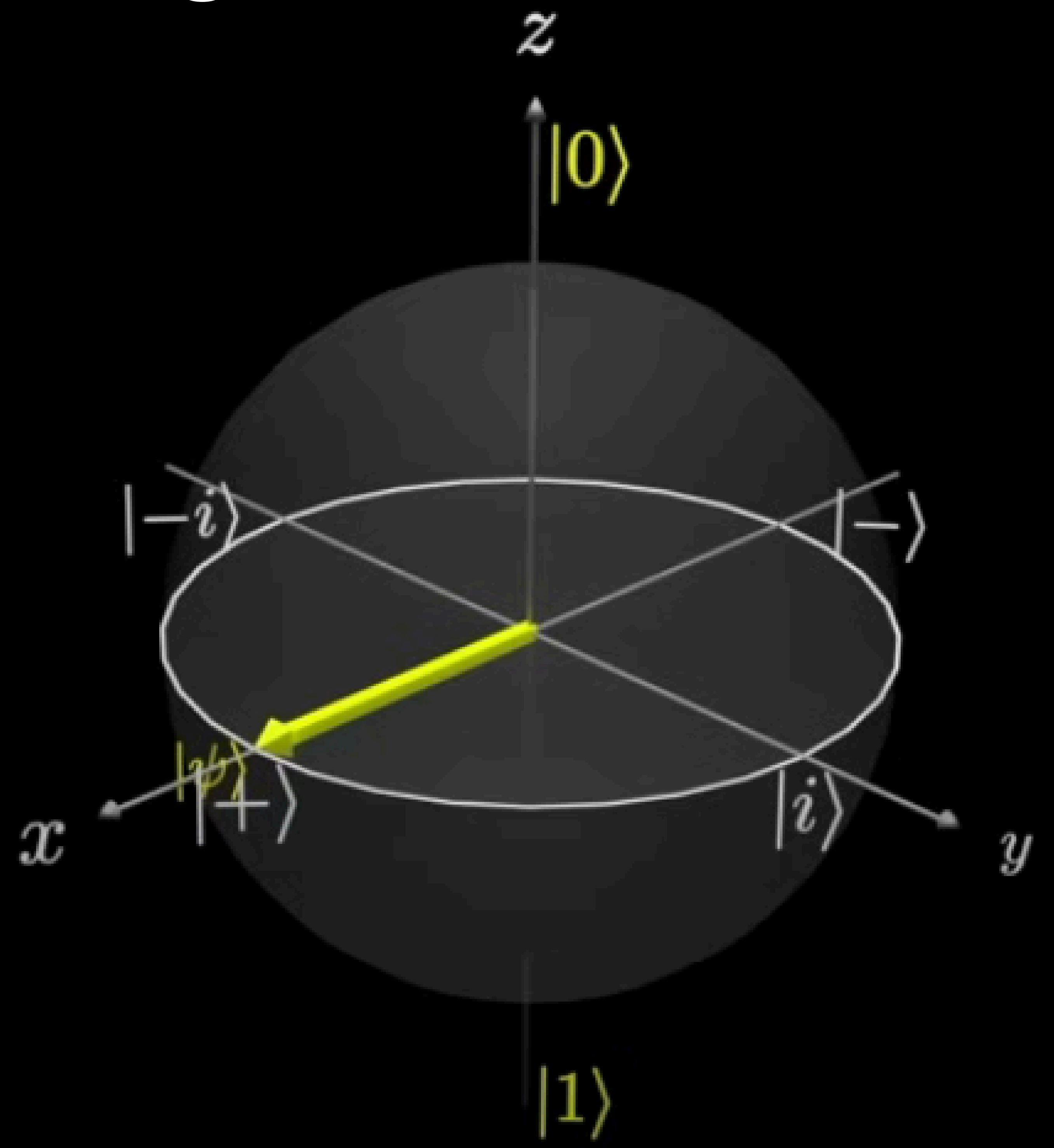
$$Z(\alpha|0\rangle + \beta|1\rangle) = \alpha|0\rangle - \beta|1\rangle$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{aligned} & Z(\alpha|0\rangle + \beta|1\rangle) \\ &= \alpha Z|0\rangle + \beta Z|1\rangle \\ &= \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ -1 \end{pmatrix} \\ &= \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta(-1) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= \alpha|0\rangle - \beta|1\rangle \end{aligned}$$

Global and Relative Phase

Phase might not seem useful as probability of measuring 0 or 1 is still the same.



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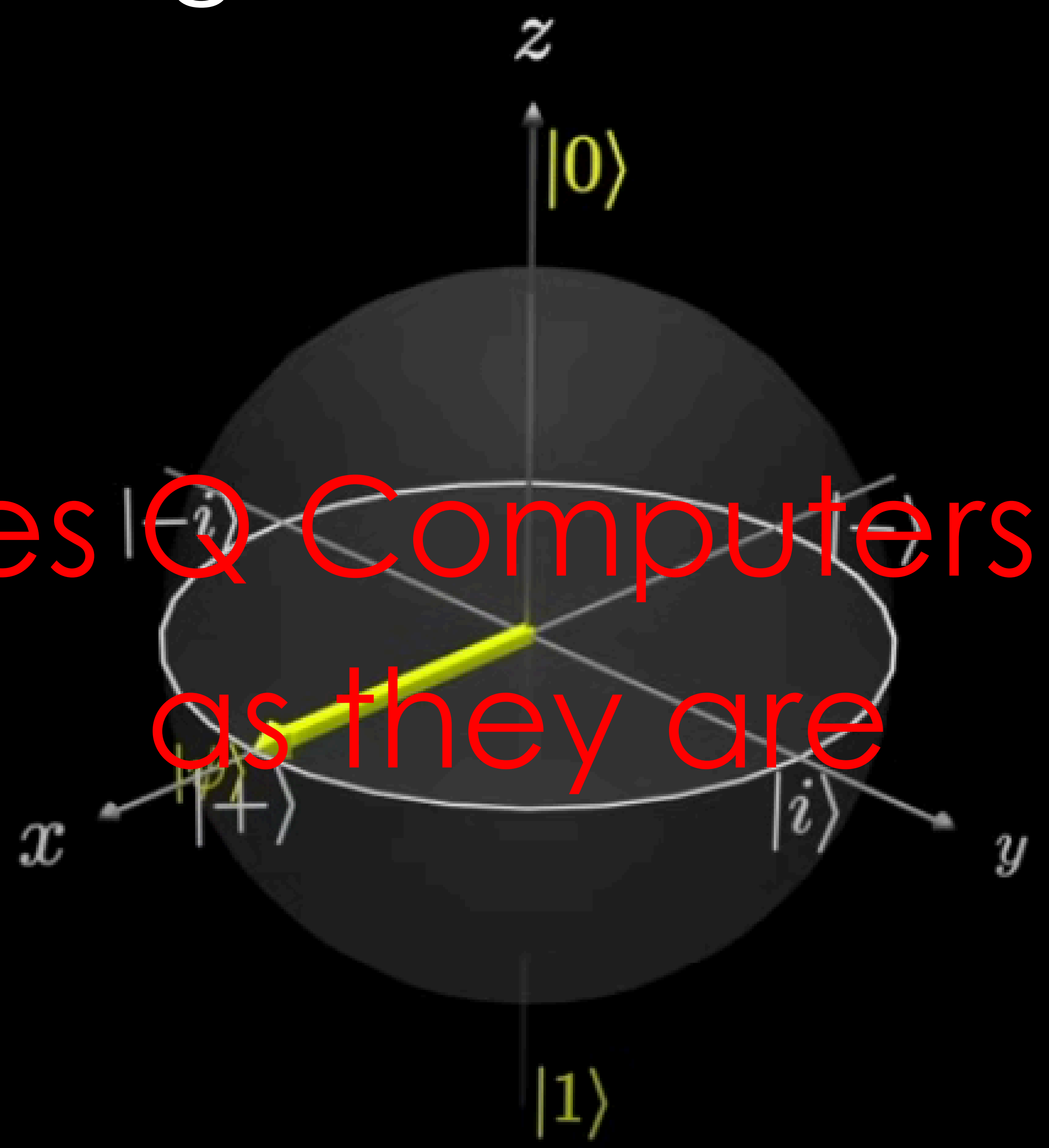
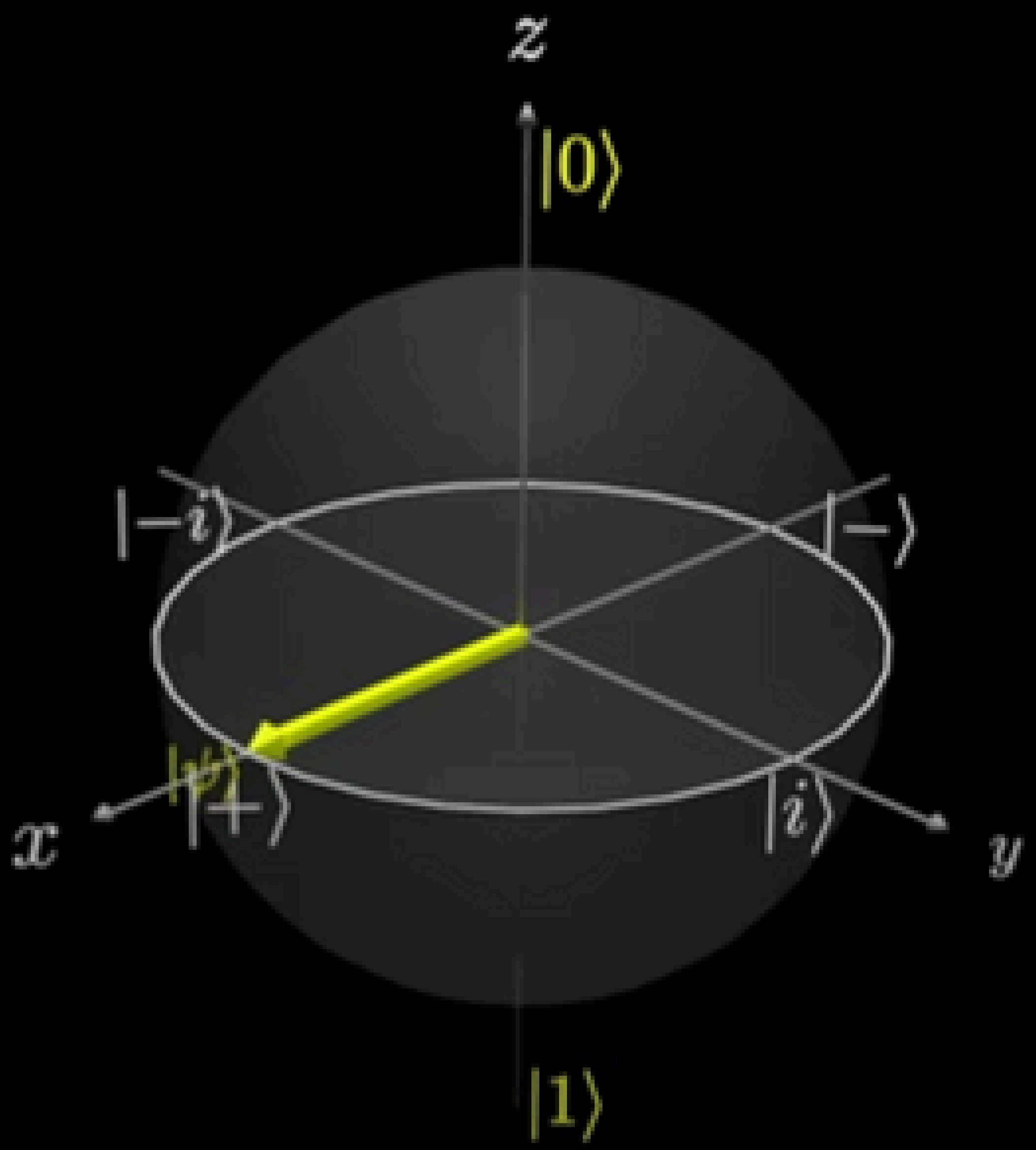


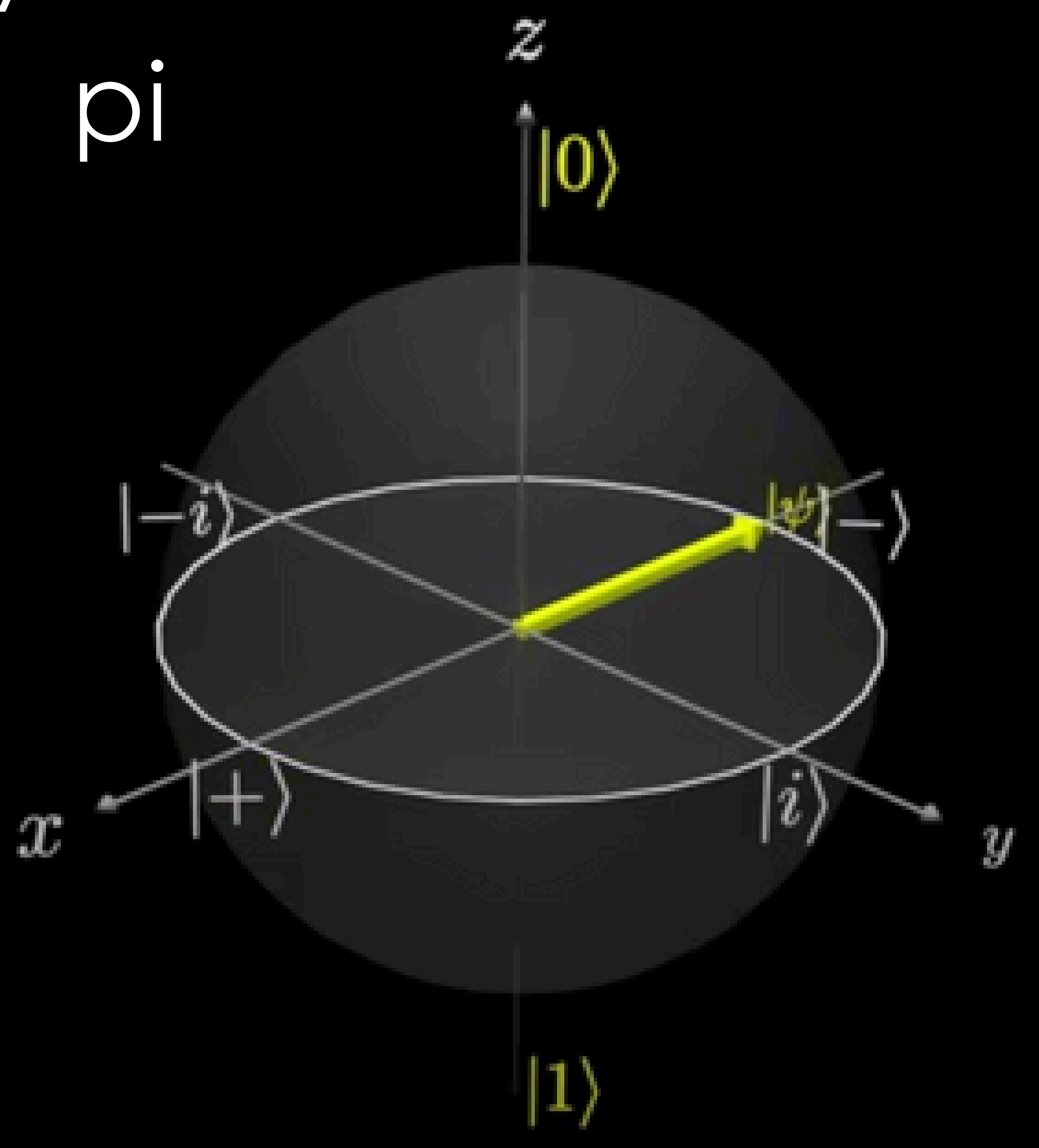
Phase makes Q Computers as powerful as they are consider

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$



the 1 state was multiplied by a factor of -1 and roated pi radians

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \xrightarrow{Z} \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

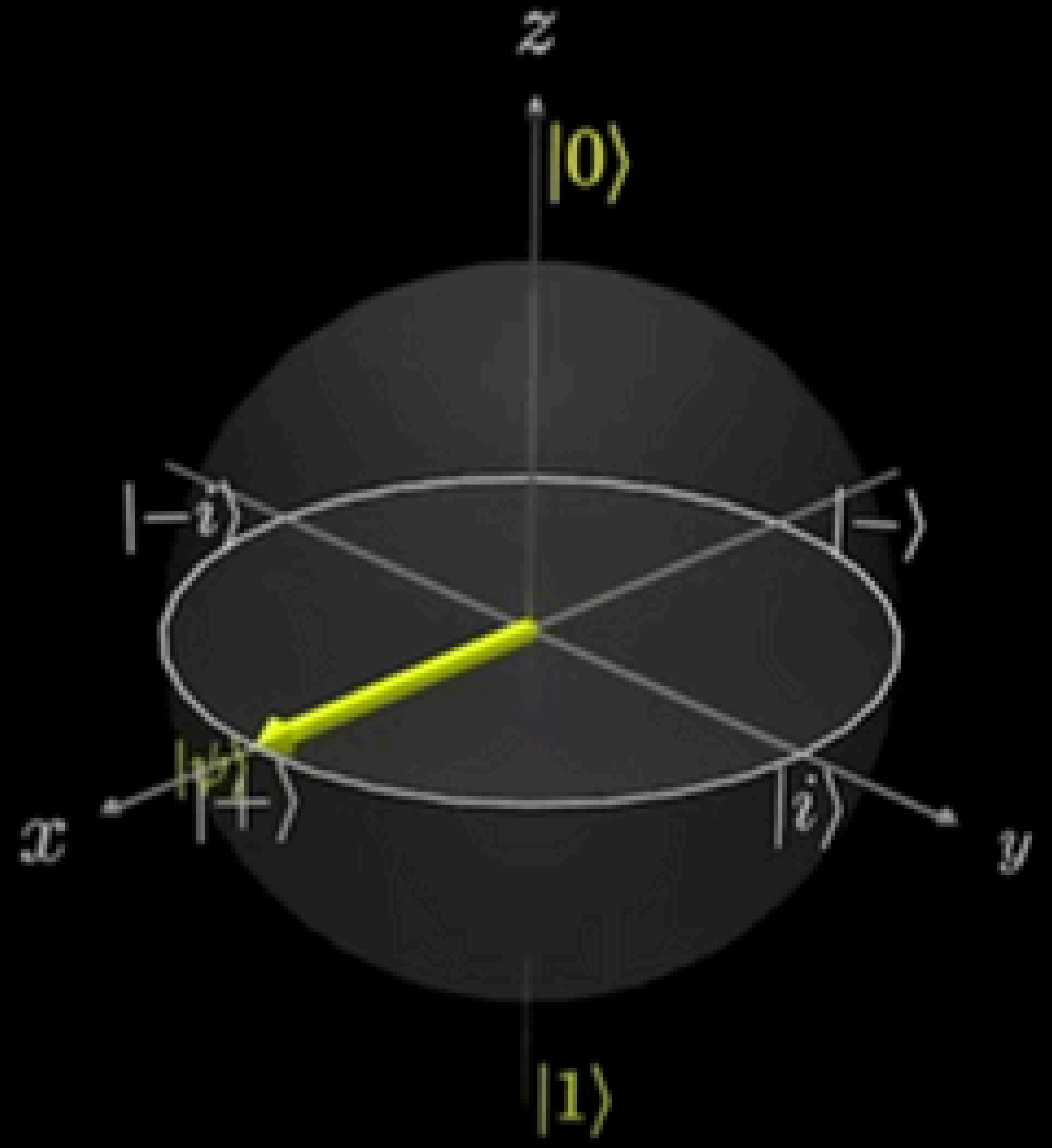


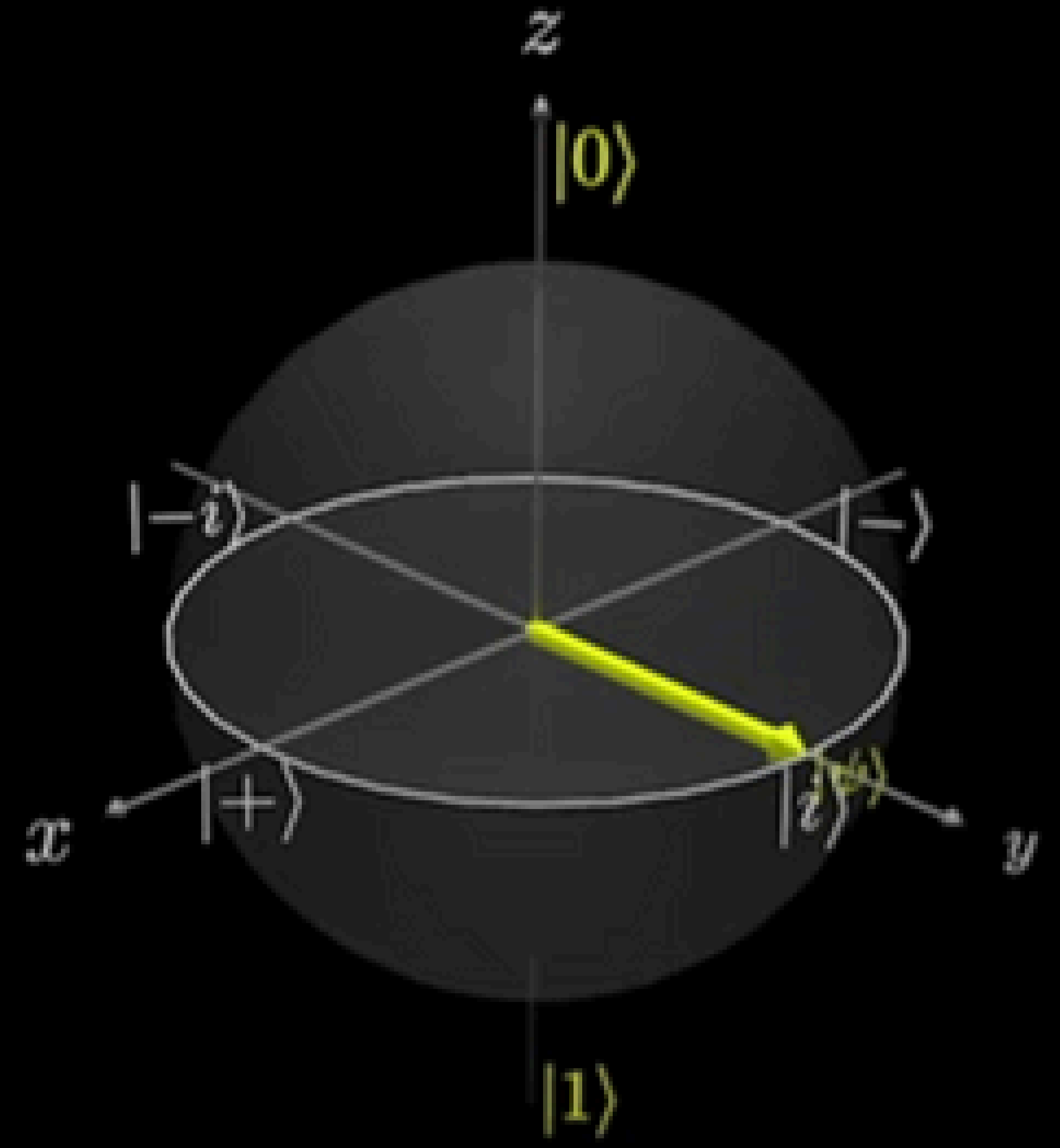
$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \xrightarrow{Z} \frac{1}{\sqrt{2}}|0\rangle + e^{i\pi}\frac{1}{\sqrt{2}}|1\rangle$$

$$(-1 = e^{i\pi})$$

We use complex numbers in exponential form in QC as it gives a mathematical way of rotating around a circle by changing the value of phi

$$e^{i\varphi}$$

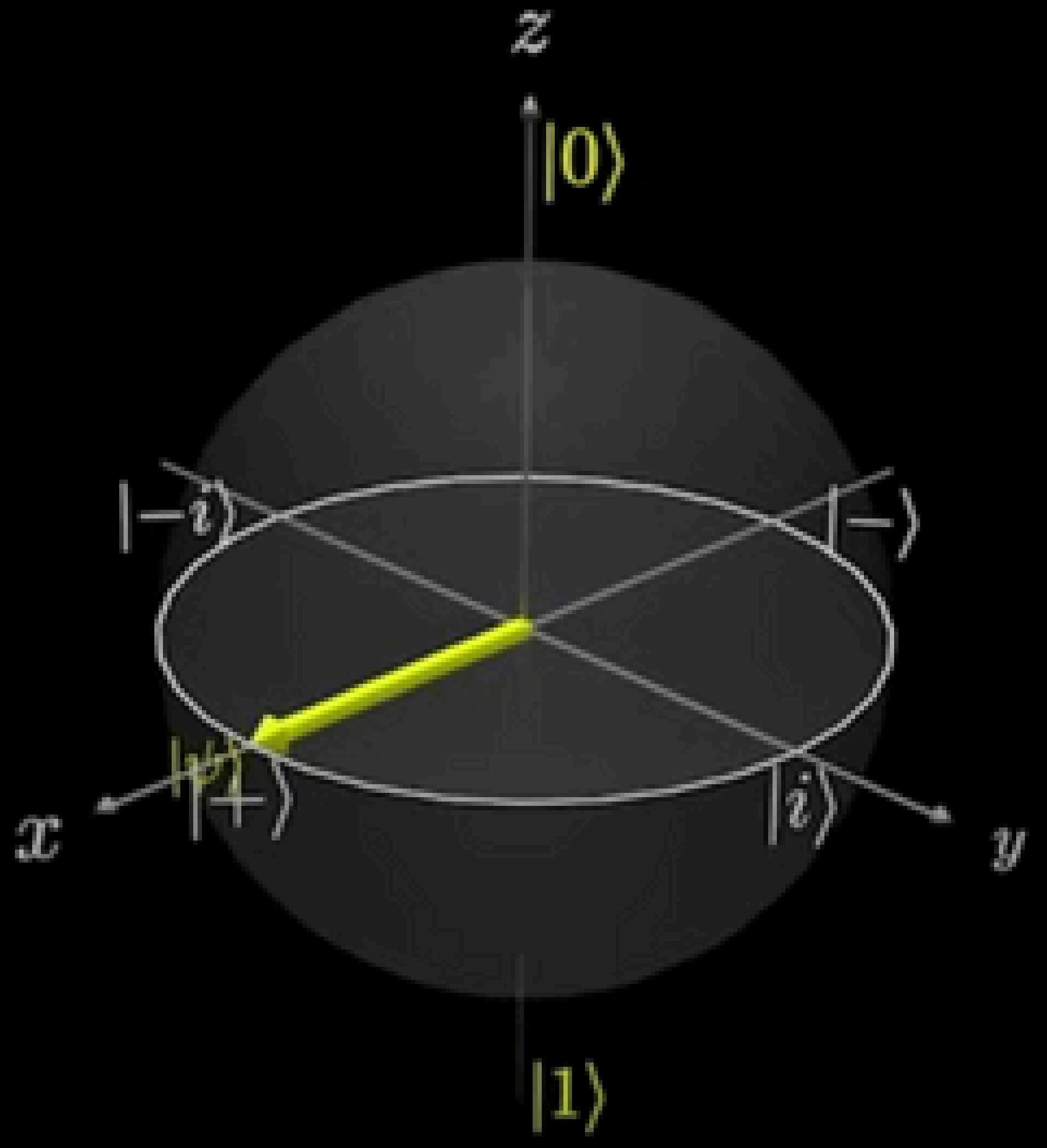


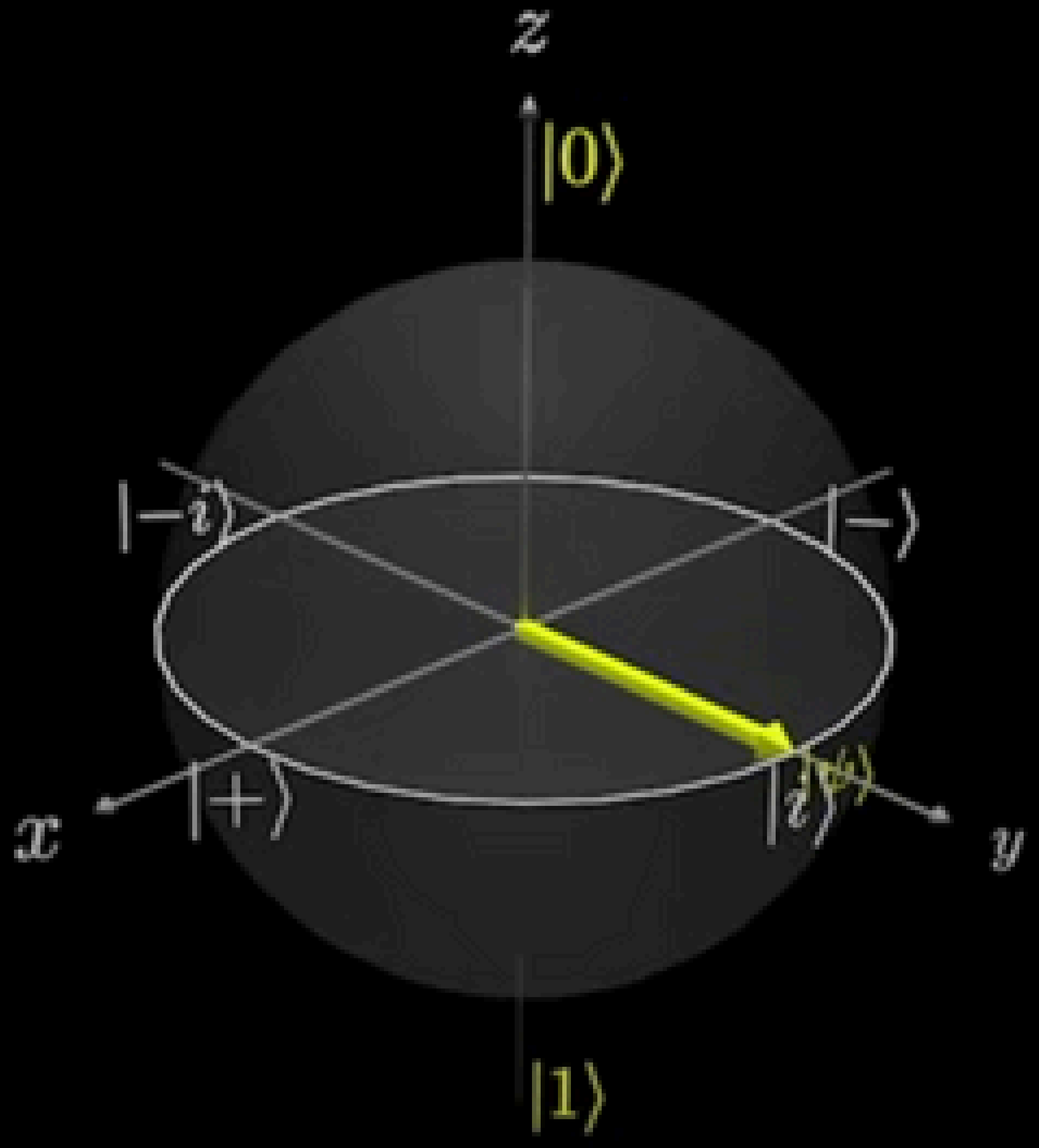


$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\frac{1}{\sqrt{2}}|0\rangle + i\frac{1}{\sqrt{2}}|1\rangle$$

$$(i = e^{i\frac{\pi}{2}})$$





$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\frac{1}{\sqrt{2}}|0\rangle + e^{i\frac{\pi}{2}}\frac{1}{\sqrt{2}}|1\rangle$$

$$(i = e^{i\frac{\pi}{2}})$$

$$|\psi\rangle = \alpha|0\rangle + e^{i\varphi}\beta|1\rangle$$

By mulitplying the $|1\rangle$ by $e^{i\varphi}$, we rotate around the z-axis (on the Bloch Sphere) by φ radians

1. If we have a qubit in superposition that has a relative phase of $e^{5\pi i/4}$, on the Bloch Sphere how many radians has the qubit 'spun' around the z-axis

1. If we have a qubit in superposition that has a relative phase of $e^{5\pi i/4}$, on the Bloch Sphere how many radians has the qubit 'spun' around the z-axis

5π/4 radians

Global Phase

physically irrelevant

$$e^{i\varphi}(\alpha|0\rangle + \beta|1\rangle)$$

$$= e^{i\varphi}\alpha|0\rangle + e^{i\varphi}\beta|1\rangle$$

$$\equiv \alpha|0\rangle + \beta|1\rangle$$

We can discard Global phase

Relative Phase

$$\alpha|0\rangle + e^{i\varphi}\beta|1\rangle$$

Example:

$$e^{i\varphi}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \equiv \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\alpha|0\rangle + e^{i\varphi}\beta|1\rangle$$

$$\text{Probability of measuring } 0 = |\alpha|^2$$

$$\text{Probability of measuring } 1 = |e^{i\varphi}\beta|^2$$

$$\text{Probability of measuring } 1 = 1 \cdot |\beta|^2$$

: the magnitude of a complex number in exponential form $re^{i\varphi}$ is r

Why relative phase matters ?

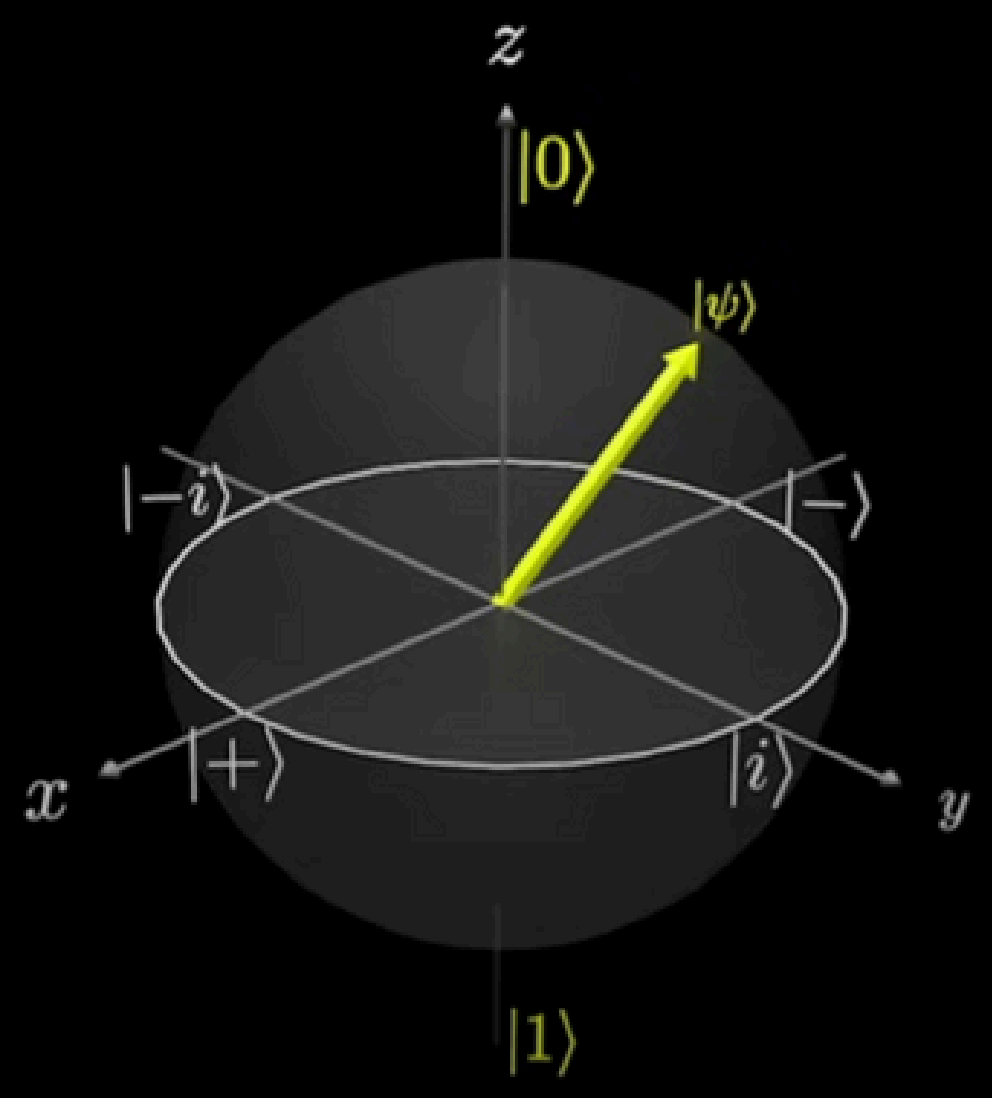
Hadamard Gate

$|+\rangle$

$|i\rangle$

$|-\rangle$

$|-i\rangle$

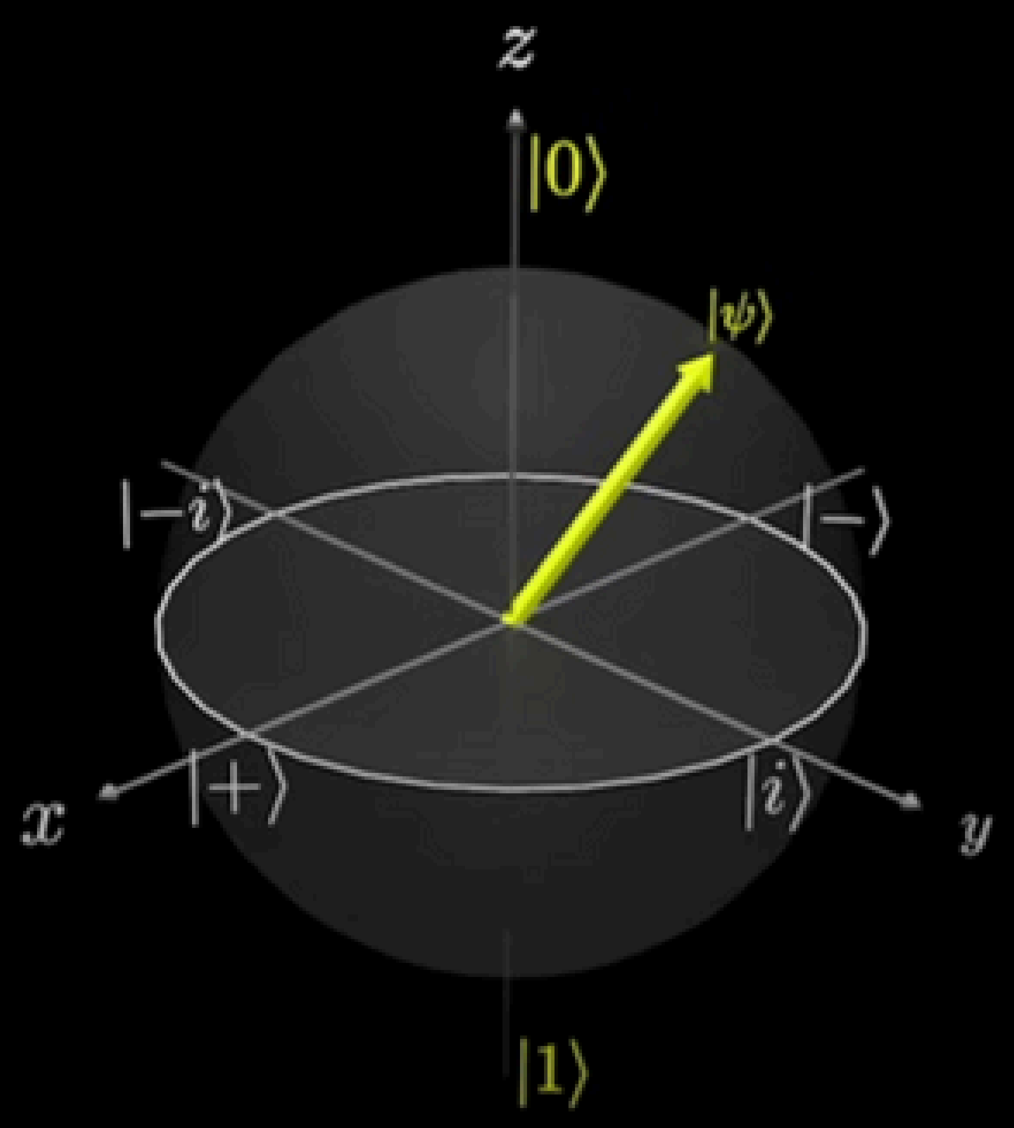


$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|i\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$$|-i\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$$



The Hadamard Gate

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$|0\rangle \xrightarrow{H} |+\rangle \qquad |+\rangle \xrightarrow{H} |0\rangle$$

$$|1\rangle \xrightarrow{H} |-\rangle \qquad |-\rangle \xrightarrow{H} |1\rangle$$

The Hadamard Gate is it's own inverse

$$H(\alpha|0\rangle + e^{i\varphi}\beta|1\rangle)$$

$$= \alpha H|0\rangle + e^{i\varphi}\beta H|1\rangle$$

$$= \alpha|+\rangle + e^{i\varphi}\beta|-\rangle$$

$$= \alpha(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle) + e^{i\varphi}\beta(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle)$$

$$= \left(\frac{\alpha + e^{i\varphi}\beta}{\sqrt{2}}\right)|0\rangle + \left(\frac{\alpha - e^{i\varphi}\beta}{\sqrt{2}}\right)|1\rangle$$

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \xrightarrow{H} |0\rangle$$

$$\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \xrightarrow{H} |1\rangle$$

Hadamard Gate and phase is extremely powerful

S and T gates

$$S = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$$

$$|0\rangle \xrightarrow{S} |0\rangle$$

$$|1\rangle \xrightarrow{S} e^{i\frac{\pi}{2}}|1\rangle$$

Adds a relative phase of $e^{i\frac{\pi}{2}}$

$$\alpha|0\rangle + \beta|1\rangle \xrightarrow{S} \alpha|0\rangle + e^{i\frac{\pi}{2}}\beta|1\rangle$$

$$|0\rangle \xrightarrow{T} |0\rangle$$

$$|1\rangle \xrightarrow{T} e^{i\frac{\pi}{4}}|1\rangle$$

Adds a relative phase of $e^{i\frac{\pi}{4}}$

$$\alpha|0\rangle + \beta|1\rangle \xrightarrow{T} \alpha|0\rangle + e^{i\frac{\pi}{4}}\beta|1\rangle$$

$$S^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & e^{i(-\frac{\pi}{2})} \end{pmatrix} \qquad T^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & e^{i(-\frac{\pi}{4})} \end{pmatrix}$$

Adds a relative phase of $e^{i(-\frac{\pi}{2})}$

$$S^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & e^{i(-\frac{\pi}{2})} \end{pmatrix}$$

$$T^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & e^{i(-\frac{\pi}{4})} \end{pmatrix}$$

Adds a relative phase of $e^{i(-\frac{\pi}{2})}$

$$S(\alpha|0\rangle + \beta|1\rangle)$$

$$= \alpha|0\rangle + e^{i\frac{\pi}{2}}\beta|1\rangle$$

$$S^\dagger(\alpha|0\rangle + e^{i\frac{\pi}{2}}\beta|1\rangle)$$

$$= (\alpha|0\rangle + e^{i(-\frac{\pi}{2})}e^{i\frac{\pi}{2}}\beta|1\rangle)$$

$$= (\alpha|0\rangle + e^{i(\frac{\pi}{2}-\frac{\pi}{2})}\beta|1\rangle)$$

$$= (\alpha|0\rangle + e^{i(0)}\beta|1\rangle)$$

$$= (\alpha|0\rangle + \beta|1\rangle)$$

Adds a relative phase of $e^{i(-\frac{\pi}{4})}$

Representing Multiple Qubits Mathematically

$$|0\rangle \otimes |0\rangle = |00\rangle$$

Tensor Product

$$\left(\alpha|0\rangle + \beta|1\rangle\right) \otimes \left(\gamma|0\rangle + \delta|1\rangle\right)$$

$$= \alpha|0\rangle \otimes \gamma|0\rangle + \alpha|0\rangle \otimes \delta|1\rangle + \beta|1\rangle \otimes \gamma|0\rangle + \beta|1\rangle \otimes \delta|1\rangle$$

$$= \alpha\gamma|0\rangle \otimes |0\rangle + \alpha\delta|0\rangle \otimes |1\rangle + \beta\gamma|1\rangle \otimes |0\rangle + \beta\delta|1\rangle \otimes |1\rangle$$

$$= \alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle$$

$$\alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle$$

$$\text{Prob}(\text{measuring } |00\rangle) = |\alpha\gamma|^2$$

$$\text{Prob}(\text{measuring } |01\rangle) = |\alpha\delta|^2$$

$$\text{Prob}(\text{measuring } |10\rangle) = |\beta\gamma|^2$$

$$\text{Prob}(\text{measuring } |11\rangle) = |\beta\delta|^2$$

$$\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \otimes \left(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle\right)$$

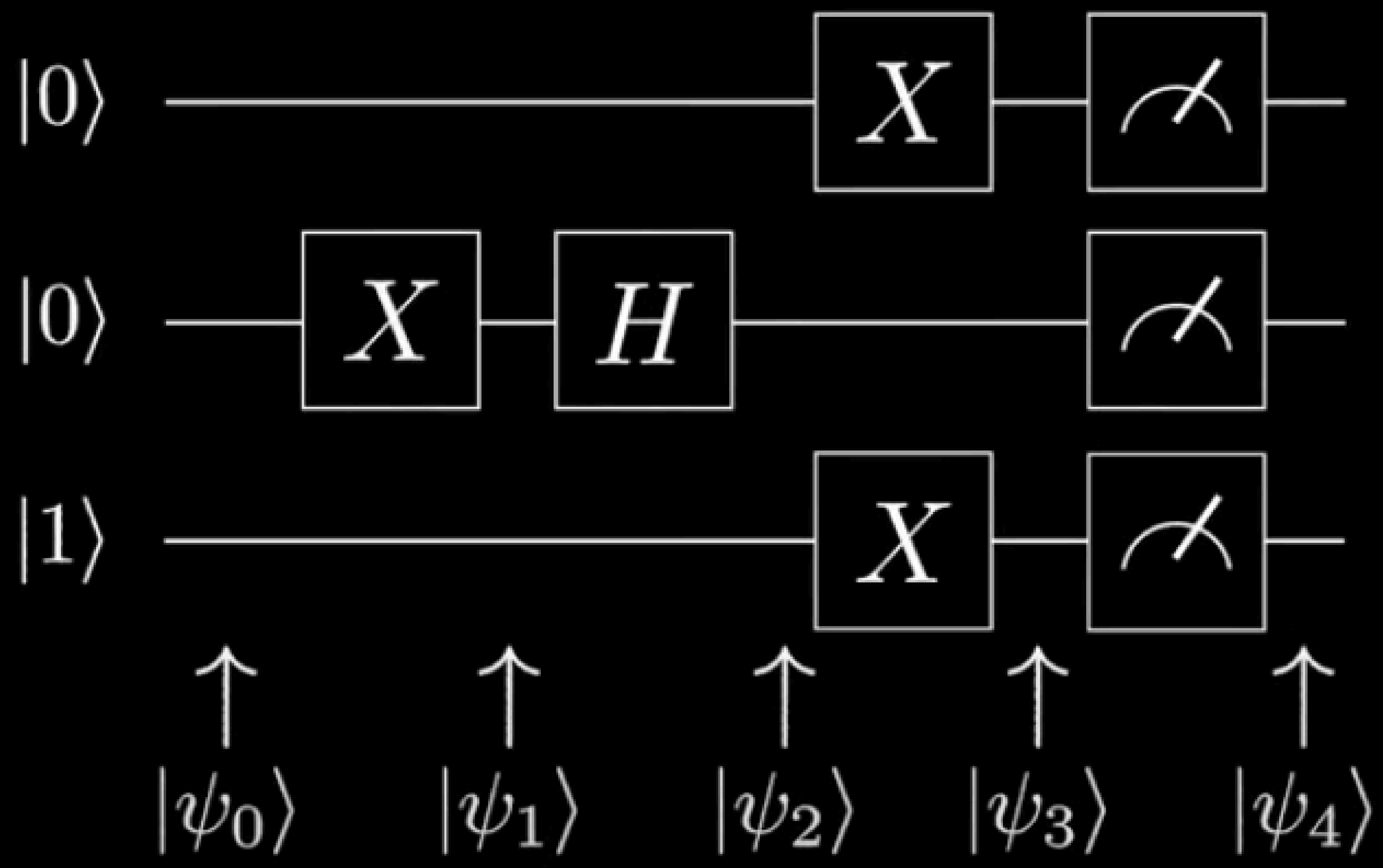
$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}|00\rangle + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}|01\rangle + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}|10\rangle + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}|11\rangle$$

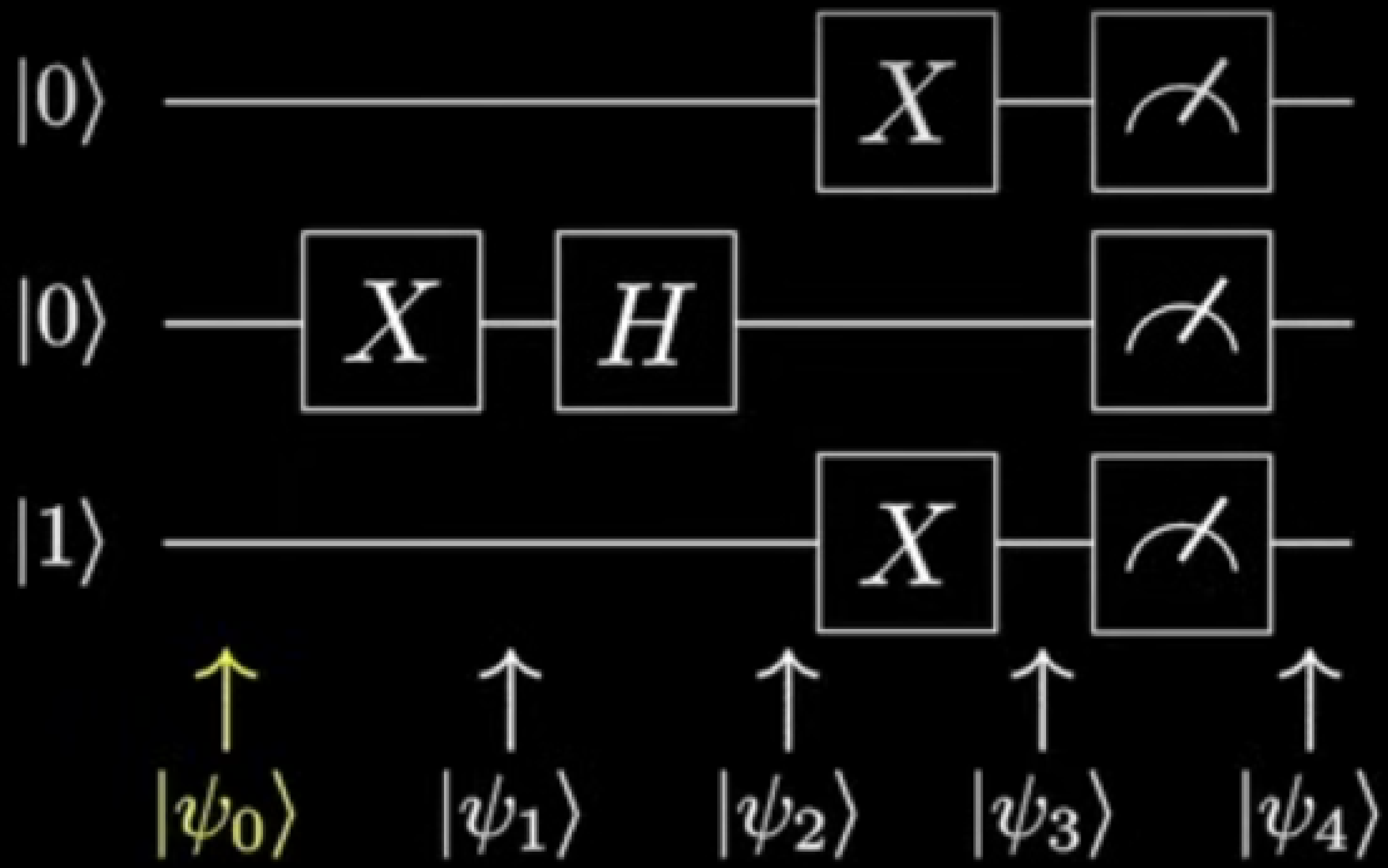
$$= \frac{\sqrt{3}}{2\sqrt{2}}|00\rangle + \frac{1}{2\sqrt{2}}|01\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|10\rangle + \frac{1}{2\sqrt{2}}|11\rangle$$

Short Hand Notation

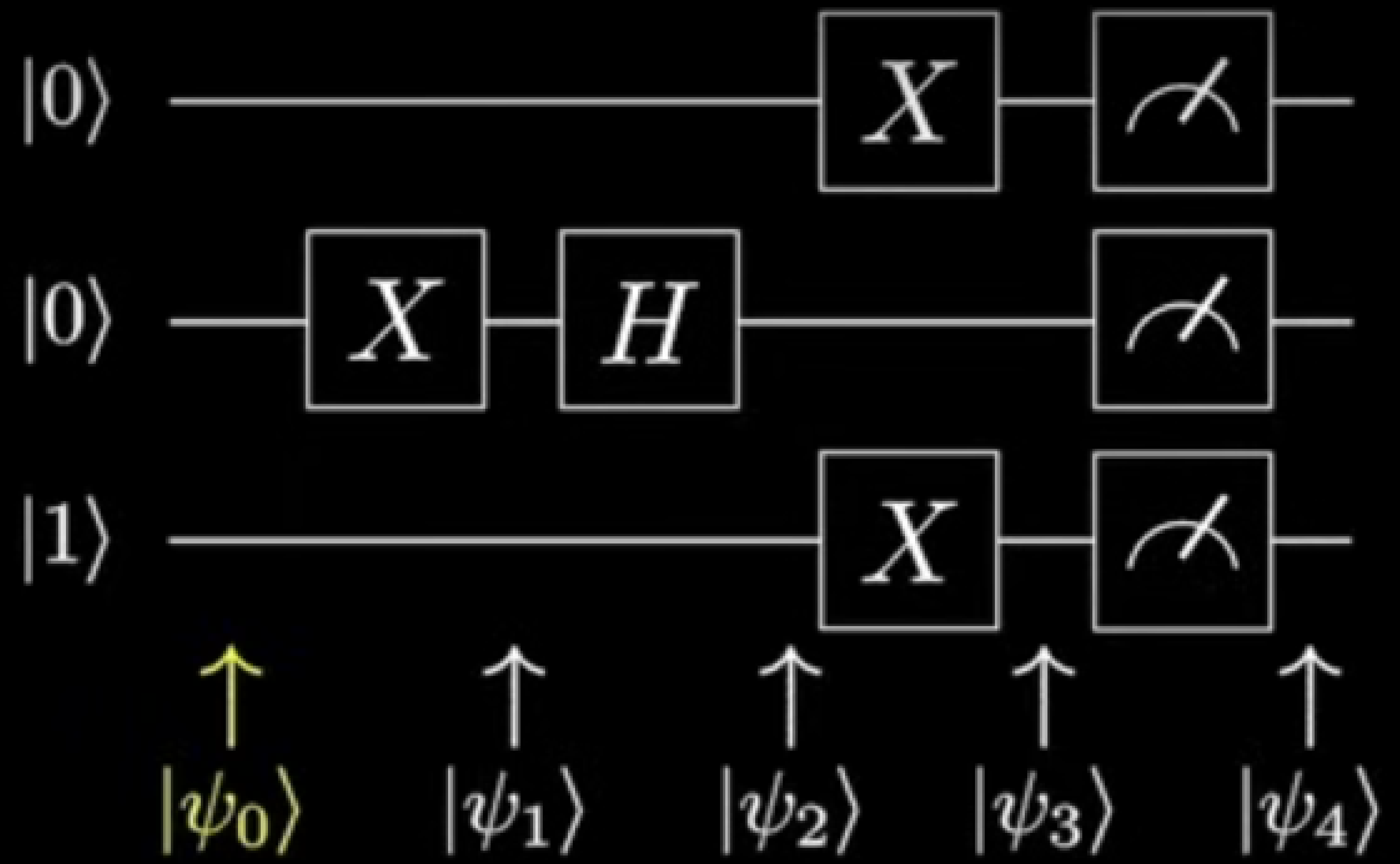
$$\underbrace{|0000\ldots0\rangle}_{n\ 0\text{'s}} = |0\rangle^{\otimes n}$$

Quantum Circuits

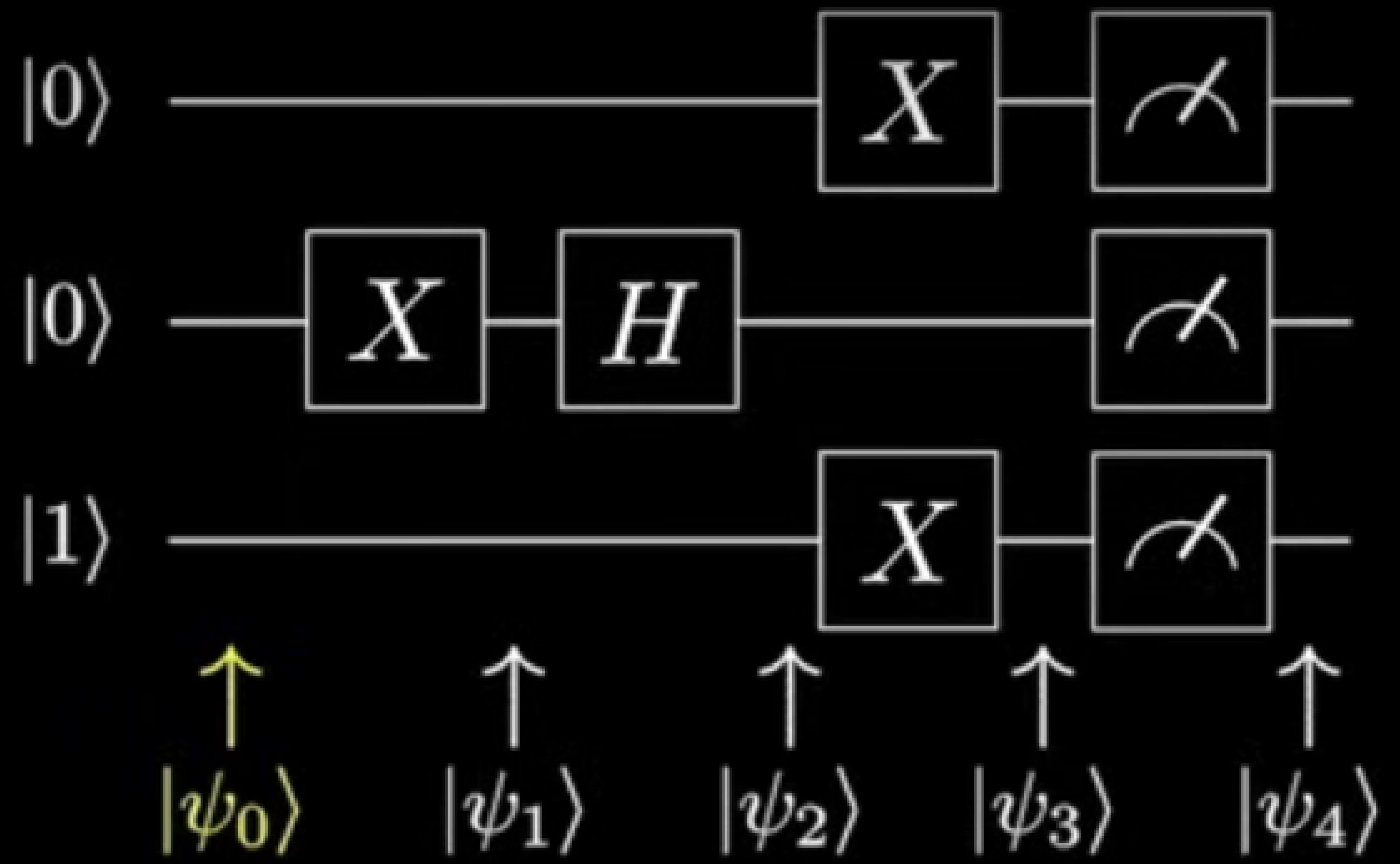




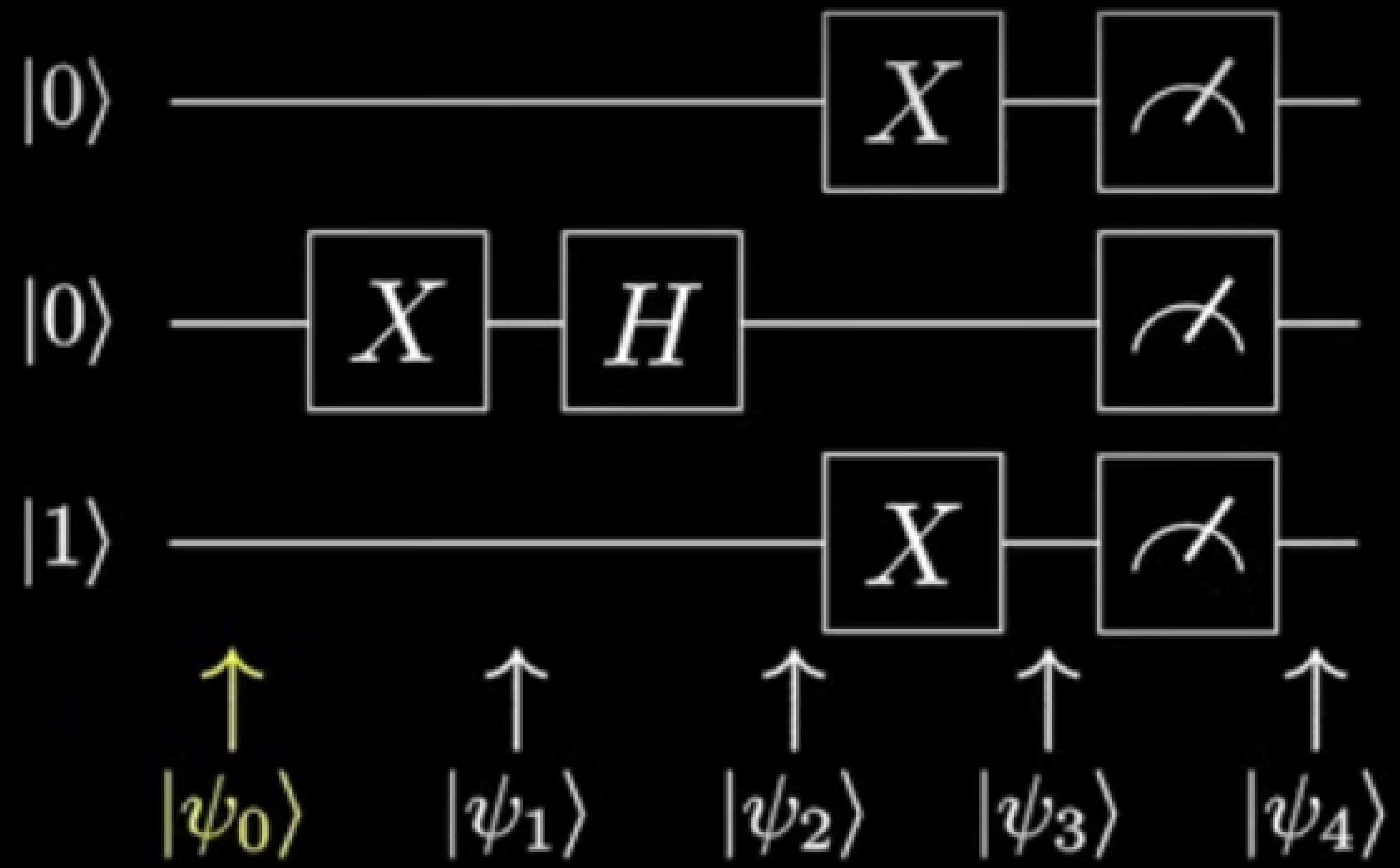
$$|\psi_0\rangle = |001\rangle$$



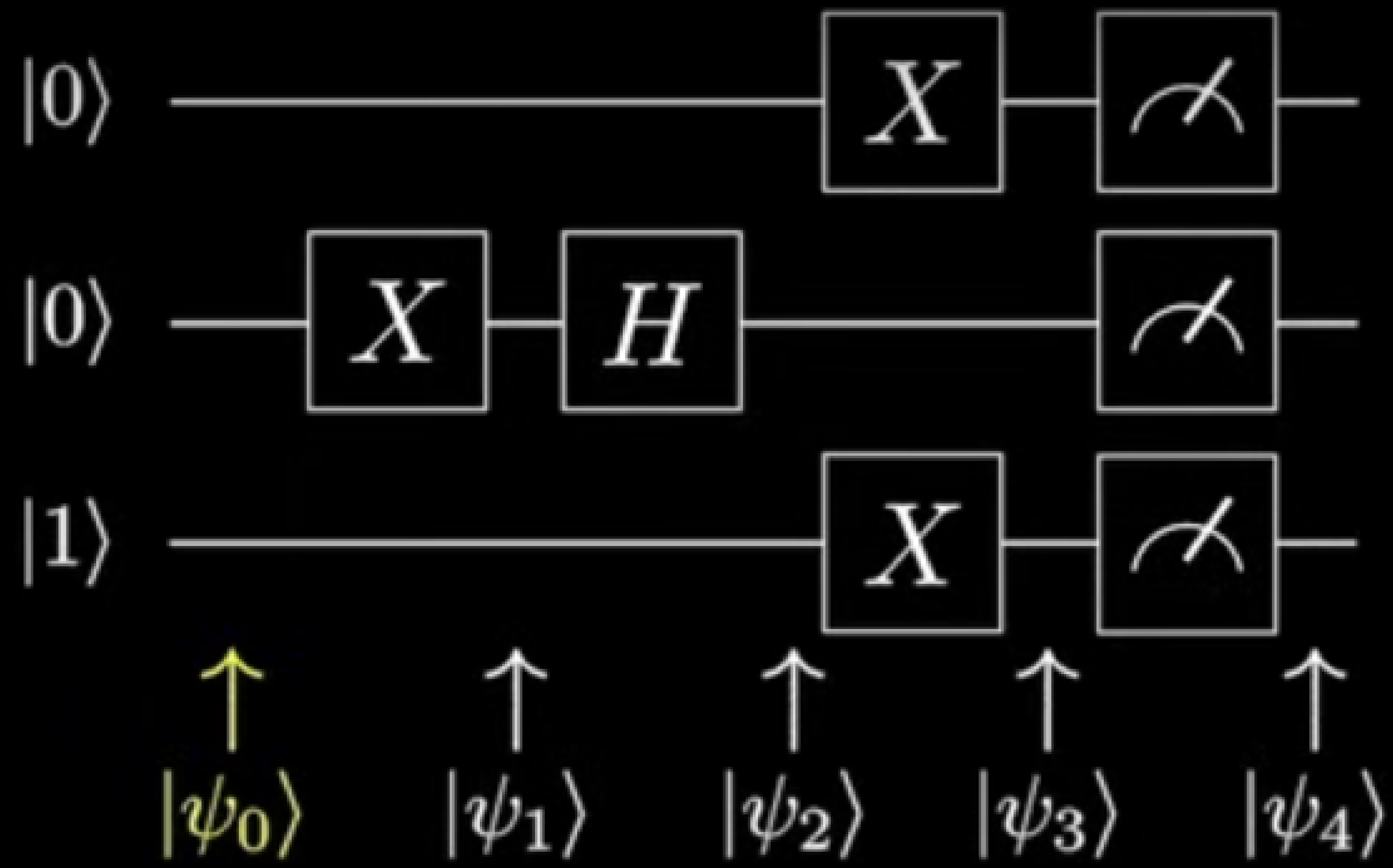
$$|\psi_1\rangle = |011\rangle$$



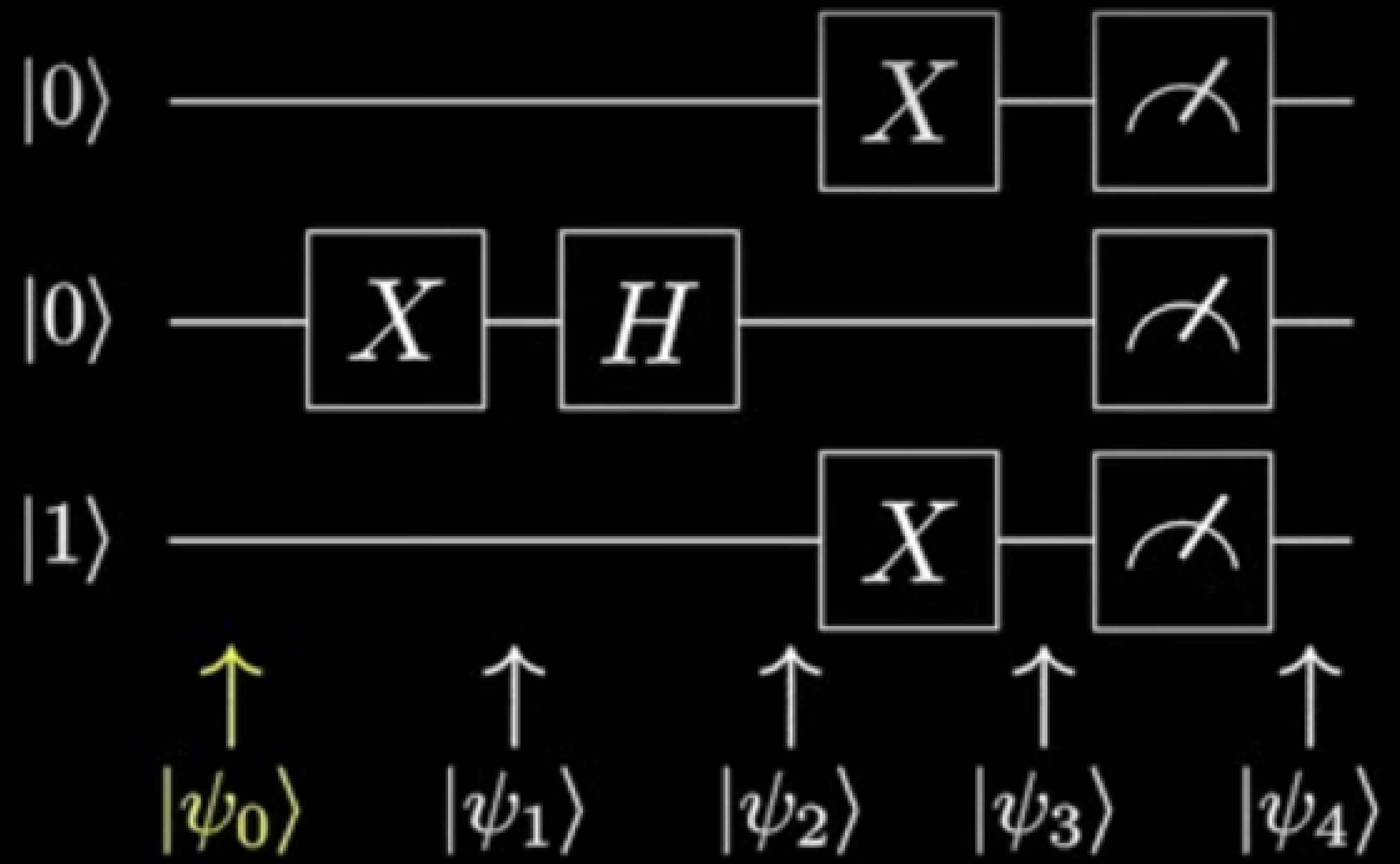
$$|\psi_2\rangle = |0-1\rangle$$



$$|\psi_2\rangle = |0\rangle \otimes \frac{1}{\sqrt{2}}\Big(|0\rangle - |1\rangle\Big) \otimes |1\rangle$$



$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\Big(|001\rangle - |011\rangle\Big)$$

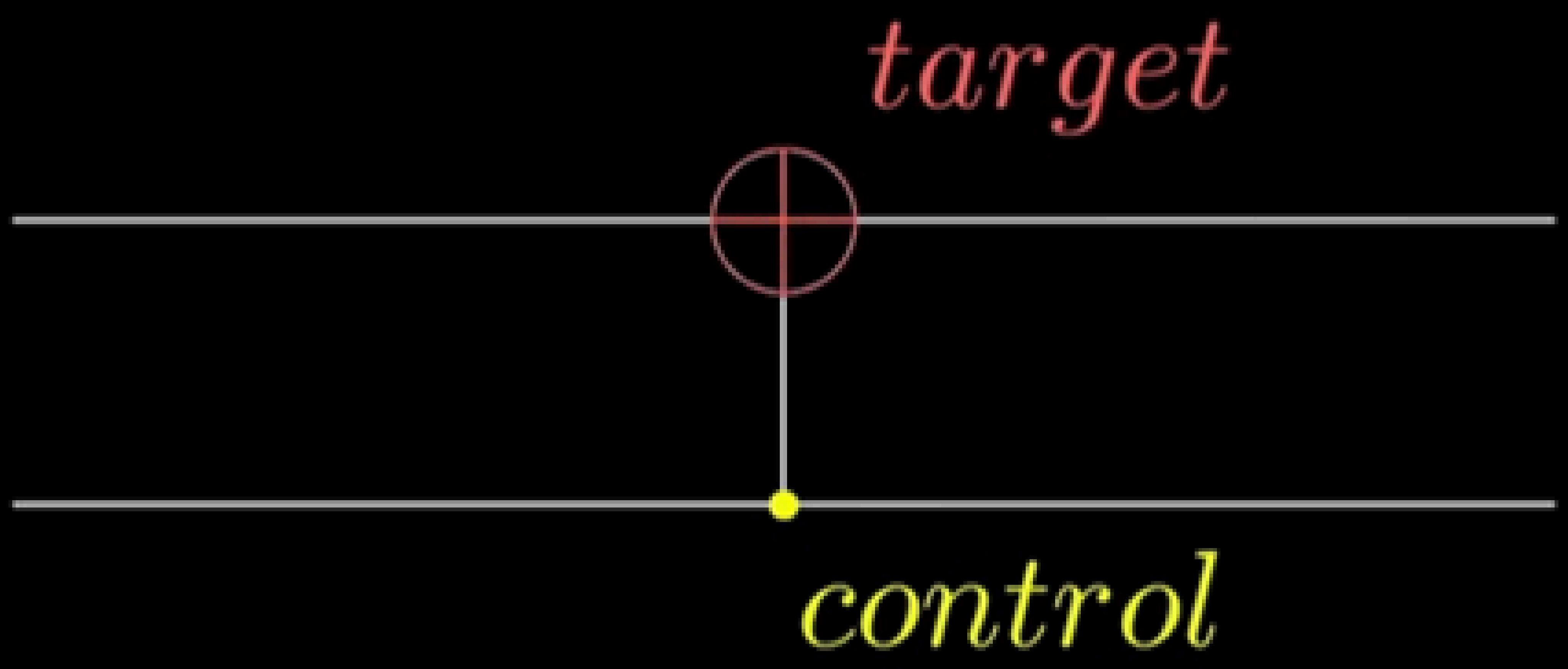


$$|\psi_3\rangle = \frac{1}{\sqrt{2}}\Big(|100\rangle - |110\rangle\Big)$$

$|\psi_4\rangle$ will be $|100\rangle$ $\frac{1}{2}$ of the time, and $|110\rangle$ $\frac{1}{2}$ of the time

Multi Qubit Gates

CNOT/Controlled X gate



The CNOT gate applies an X gate to the target qubit if the control qubit is a 1

$$CNOT\left(\frac{\sqrt{3}}{4}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle + \frac{1}{4}|11\rangle\right)$$

$$= \frac{\sqrt{3}}{4}CNOT|00\rangle + \frac{1}{2}CNOT|01\rangle + \frac{1}{\sqrt{2}}CNOT|10\rangle + \frac{1}{4}CNOT|11\rangle$$

$$= \frac{\sqrt{3}}{4}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{\sqrt{2}}|11\rangle + \frac{1}{4}|10\rangle$$

$$= \frac{\sqrt{3}}{4}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{4}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

1st qubit is the control

2nd qubit is the target

$$CNOT\left(\frac{\sqrt{3}}{2}|001\rangle + \frac{1}{2}|010\rangle\right)$$

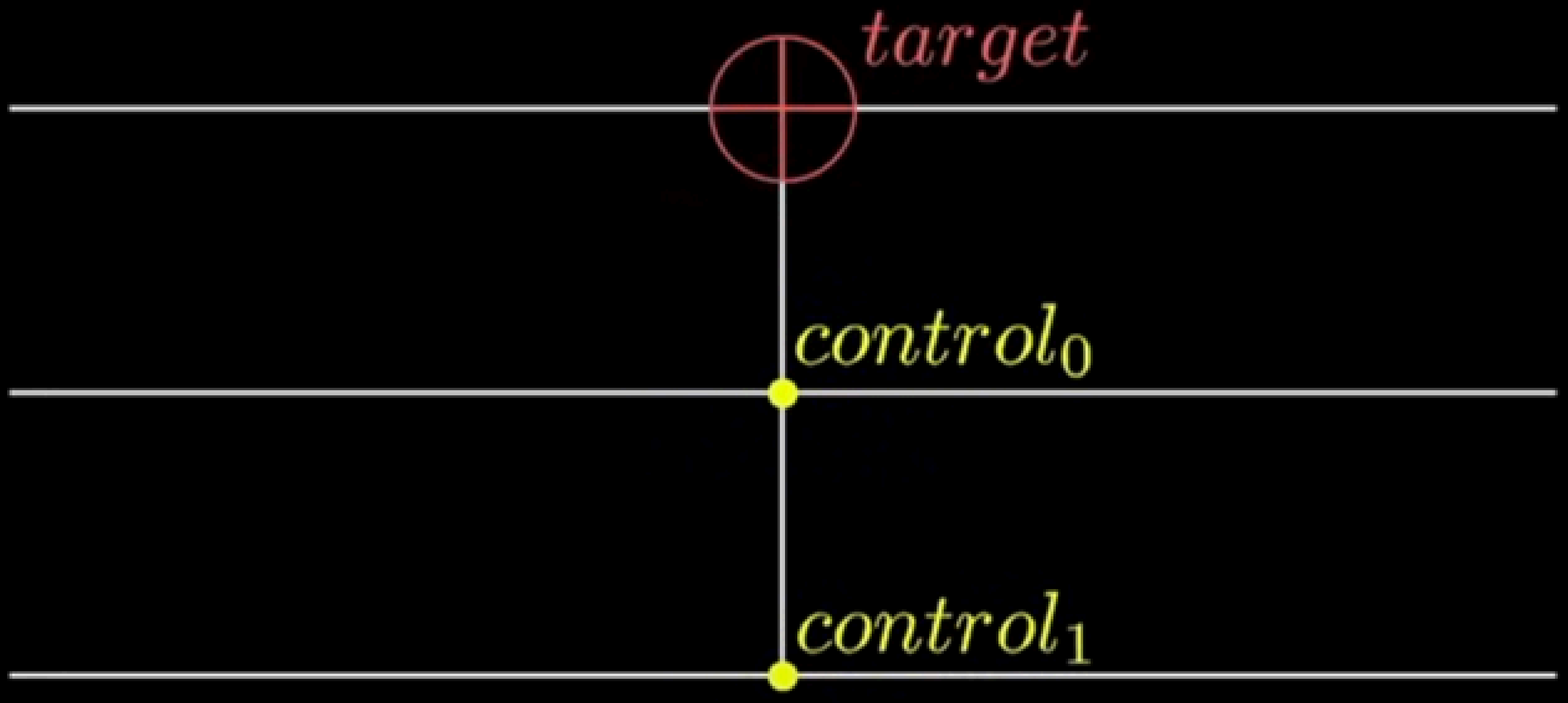
$$= \frac{\sqrt{3}}{2}CNOT|001\rangle + \frac{1}{2}CNOT|010\rangle$$

$$= \frac{\sqrt{3}}{2}|011\rangle + \frac{1}{2}|010\rangle$$

3rd qubit is the control

2nd qubit is the target

Toffoli Gate



Same as CNOT but has 2 control qubits

Toffoli Gate

$$TOFFOLI\left(\frac{1}{\sqrt{2}}|0011\rangle + \frac{1}{\sqrt{2}}|0110\rangle\right)$$

$$= \frac{1}{\sqrt{2}}TOFFOLI|0011\rangle + \frac{1}{\sqrt{2}}TOFFOLI|0110\rangle$$

$$= \frac{1}{\sqrt{2}}|0011\rangle + \frac{1}{\sqrt{2}}|0111\rangle$$

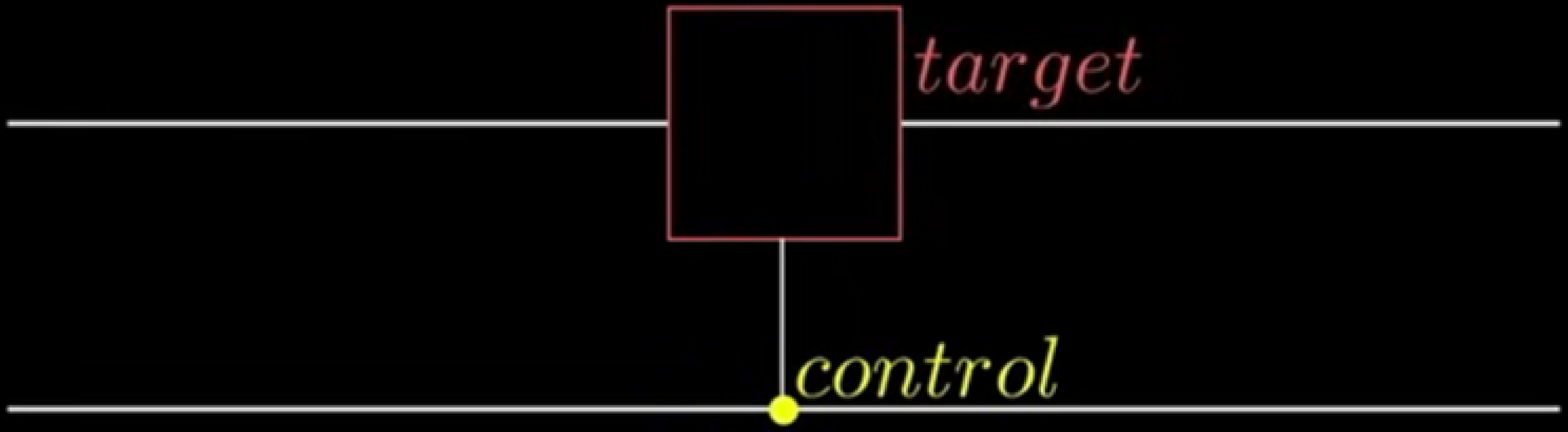
2nd, 3rd qubits are the control

4th qubit is the target

Using CNOT gates we can create controlled versions of our single qubit gates

$$CY, CZ, CS, CT, CH, \ldots$$

We write the gates as seen above. eg. Controlled Y = CY


target
control

When we measure a quantum system, it collapses into the measured state

If we have a photon that is in a superposition of both vertically and horizontally polarized, when we measure it we can only measure it as horizontally or vertically polarized, and once it has been measured it will collapse into (becomes) the measured state

For example if we have a photon in a superposition of both vertically and horizontally polarized and we measure it as horizontally polarized then it becomes horizontally polarized.

When we measure a qubit we only measure a 0 or a 1

If we were to measure the qubit $|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

we would not measure α or β, we still only measure a 0 or a 1

If we measured $|\psi\rangle$ as 0 then $|\psi\rangle$ would collapse into the 0 state, so $|\psi\rangle \implies |0\rangle$

or

if we measured $|\psi\rangle$ as 1 then $|\psi\rangle$ would collapse into the 1 state, so $|\psi\rangle \implies |1\rangle$

Measuring a qubit collapses it's superposition