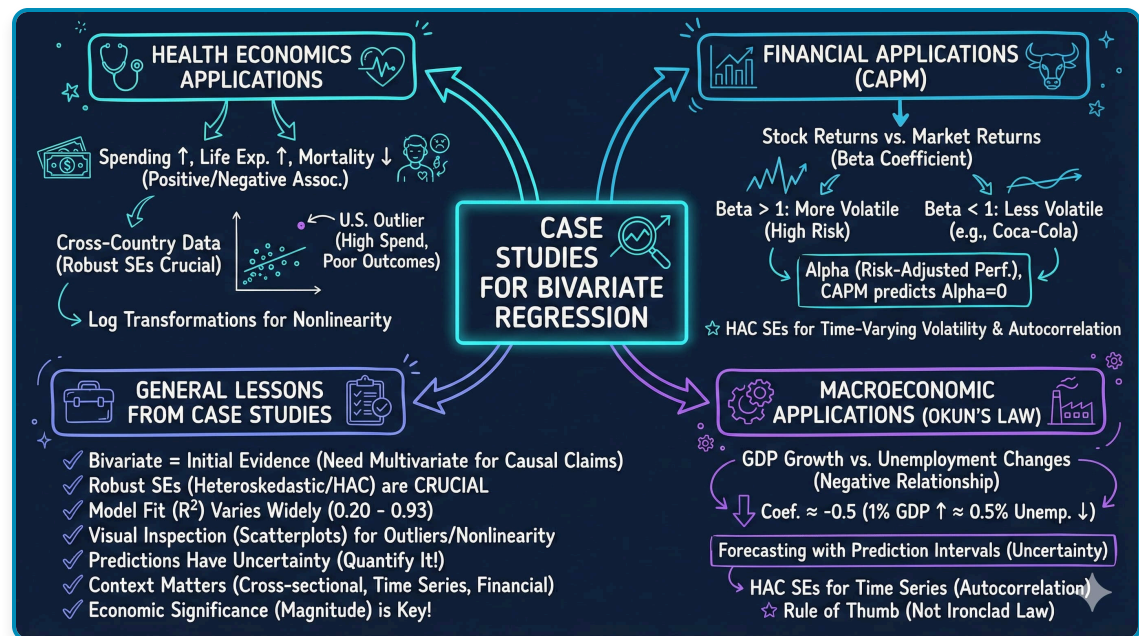


# Chapter 8: Case Studies for Bivariate Regression

metricsAI: An Introduction to Econometrics with Python and AI in the Cloud

Carlos Mendez



This notebook provides an interactive introduction to bivariate regression through real-world case studies. All code runs directly in Google Colab without any local setup.

 [Open in Colab](#)

## Learning Objectives

By the end of this chapter, you will be able to:

- Apply bivariate regression analysis to real-world economic data across multiple domains
- Interpret regression coefficients in terms of economic significance and policy implications
- Use heteroskedasticity-robust standard errors (HC1) for cross-sectional data
- Test specific economic hypotheses using t-tests (e.g., CAPM beta = 1, Okun's Law beta = -2)

- Identify outliers and assess their influence on regression results
- Distinguish between statistical and economic significance in regression analysis
- Estimate the Capital Asset Pricing Model (CAPM) and interpret systematic risk (beta)
- Analyze health outcomes and expenditures using international comparative data
- Apply Okun's Law to macroeconomic time series data
- Create professional visualizations of regression results (scatter plots, time series)

## | Chapter Overview

This chapter demonstrates bivariate regression analysis through four compelling real-world applications. You'll gain both theoretical understanding and practical skills through hands-on Python examples.

**Design Note:** Chapter 8 uses a case study-focused structure where sections 8.1-8.4 are integrated case studies covering different economic domains (health economics, finance, macroeconomics). This differs from the standard template where case studies appear as a separate section X.11.

### What you'll learn:

- Apply bivariate regression to cross-sectional data (health outcomes, expenditures)
- Estimate financial models (Capital Asset Pricing Model)
- Analyze macroeconomic relationships (Okun's Law)
- Use heteroskedasticity-robust standard errors
- Interpret economic and statistical significance
- Identify outliers and assess their influence

### Datasets used:

- **AED\_HEALTH2009.DTA:** Health outcomes and expenditures for 34 OECD countries (2009)
- **AED\_CAPM.DTA:** Monthly stock returns for Coca-Cola, Target, Walmart (1983-2013)
- **AED\_GDPUNEMPLOY.DTA:** Annual U.S. GDP growth and unemployment (1961-2019)

### Key economic questions:

- Do higher health expenditures improve health outcomes?

- How does GDP relate to health spending across countries?
- What is the systematic risk (beta) of individual stocks?
- Does Okun's Law hold for U.S. macroeconomic data?

### Chapter outline:

- 8.1 Health Outcomes Across Countries
- 8.2 Health Expenditures Across Countries
- 8.3 Capital Asset Pricing Model (CAPM)
- 8.4 Output and Unemployment (Okun's Law)
- Key Takeaways
- Practice Exercises

**Estimated time:** 90-120 minutes

## I Setup

First, we import the necessary Python packages and configure the environment for reproducibility. All data will stream directly from GitHub.

In [64]:

```
# Import required packages
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import statsmodels.api as sm
from statsmodels.formula.api import ols
from scipy import stats
import random
import os

# Set random seeds for reproducibility
RANDOM_SEED = 42
random.seed(RANDOM_SEED)
np.random.seed(RANDOM_SEED)
os.environ['PYTHONHASHSEED'] = str(RANDOM_SEED)

# GitHub data URL
GITHUB_DATA_URL = "https://raw.githubusercontent.com/quarcs-lab/data-open/master/AED/"

# Set plotting style
sns.set_style("whitegrid")
plt.rcParams['figure.figsize'] = (10, 6)

print("=" * 70)
print("CHAPTER 8: CASE STUDIES FOR BIVARIATE REGRESSION")
print("=" * 70)
print("\nSetup complete! Ready to explore real-world regression applications.")
```

Setup complete! Ready to explore real-world regression applications.

## | 8.1: Health Outcomes Across Countries

Our first case study examines health outcomes across wealthy OECD nations. We'll investigate whether higher health spending is associated with better health outcomes.

### Context:

- Dataset: 34 OECD countries in 2009
- Countries include: Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom, and United States
- Wide variation in health expenditures and outcomes

### Variables:

- **Hlthpc**: Annual health expenditure per capita (US dollars)
- **Lifeexp**: Male life expectancy at birth (years)
- **Infmort**: Infant mortality per 1,000 live births

### Research questions:

1. Is higher health spending associated with longer life expectancy?
2. Is higher health spending associated with lower infant mortality?
3. How does the U.S. compare to predictions from these models?

## Load and Explore Health Data

In [65]:

```
# Read in the health data
data_health = pd.read_stata(GITHUB_DATA_URL + 'AED_HEALTH2009.DTA')

print("=" * 70)
print("8.1 HEALTH OUTCOMES ACROSS COUNTRIES")
print("=" * 70)

print("\nData summary:")
data_summary = data_health.describe()
print(data_summary)

print("\nFirst few observations:")
print(data_health[['code', 'hlthpc', 'lifeexp', 'infmort']].head(10))
```

```
=====
8.1 HEALTH OUTCOMES ACROSS COUNTRIES
=====

Data summary:

```

	year	hlthgdp	hlthpc	infmort	lifeexp	gdppc	\
count	34.0	34.000000	34.000000	34.000000	34.000000	34.000000	
mean	2009.0	9.673530	3255.647059	4.447059	76.702942	33054.035156	
std	0.0	2.123934	1493.654394	2.720098	2.936756	12916.752930	
min	2009.0	6.400000	923.000000	1.800000	69.800003	13806.163086	
25%	2009.0	8.100000	2090.750000	3.100000	75.850002	25511.000488	
50%	2009.0	9.600000	3188.500000	3.700000	77.649998	32899.482422	
75%	2009.0	10.775000	4154.750000	4.900000	78.699997	38182.195312	
max	2009.0	17.700001	7990.000000	14.700000	79.900002	82900.882812	

	hlthpcsq	lnhlthpc	lngdppc	lnlifeexp	lninfmort
count	34.00	34.000000	34.000000	34.000000	34.000000
mean	12764623.00	7.973380	10.337659	4.339207	1.377001
std	11839147.00	0.513061	0.377425	0.039168	0.445143
min	851929.00	6.827629	9.532870	4.245634	0.587787
25%	4373073.25	7.645071	10.146862	4.328757	1.131402
50%	10173338.50	8.066971	10.401017	4.352211	1.307967
75%	17276752.00	8.331569	10.550111	4.365643	1.588593
max	63840100.00	8.985946	11.325401	4.380776	2.687847

```
First few observations:

```

	code	hlthpc	lifeexp	infmort
0	AUS	3670	79.300003	4.3
1	AUT	4346	77.599998	3.8
2	BEL	3911	77.300003	3.4
3	CAN	4317	78.500000	5.0
4	CHL	1210	75.800003	7.9
5	CZR	2048	74.199997	2.9
6	DEN	4385	76.900002	3.1
7	EST	1385	69.800003	3.6
8	FIN	3271	76.599998	2.6
9	FRA	3930	77.699997	3.9

## Summary Statistics

Let's examine the key variables in our health outcomes study.

In [66]:

```
print("-" * 70)
print("Table 8.1: Health Variables Summary")
print("-" * 70)
table81_vars = ['hlthpc', 'lifeexp', 'infmort']
summary_table = data_health[table81_vars].describe().T
summary_table['range'] = summary_table['max'] - summary_table['min']
print(summary_table[['mean', 'std', 'min', 'max', 'range']])

print("\nKey observations:")
print(f" - Health spending ranges from ${summary_table.loc['hlthpc', 'min']:.0f} to ${summary_table.loc['hlthpc', 'max']:.0f}")
print(f" - Life expectancy ranges from {summary_table.loc['lifeexp', 'min']:.1f} to {summary_table.loc['lifeexp', 'max']:.1f} years")
print(f" - Infant mortality ranges from {summary_table.loc['infmort', 'min']:.1f} to {summary_table.loc['infmort', 'max']:.1f} per 1,000 births")
```

Table 8.1: Health Variables Summary

	mean	std	min	max	range
hlthpc	3255.647059	1493.654394	923.000000	7990.000000	7067.000000
lifeexp	76.702942	2.936756	69.800003	79.900002	10.099998
infmort	4.447059	2.720098	1.800000	14.700000	12.900000

Key observations:

- Health spending ranges from \$923 to \$7990
- Life expectancy ranges from 69.8 to 79.9 years
- Infant mortality ranges from 1.8 to 14.7 per 1,000 births

## Life Expectancy Regression

We estimate the relationship between health spending and life expectancy:

$$\text{Lifeexp} = \beta_1 + \beta_2 \times \text{Hlthpc} + u$$

### Interpretation:

- $\beta_1$ : Expected life expectancy when health spending is zero (intercept)
- $\beta_2$ : Change in life expectancy for each additional \$1,000 in health spending
- We expect  $\beta_2 > 0$  (higher spending improves outcomes)

In [67]:

```
print("-" * 70)
print("Life Expectancy Regression")
print("-" * 70)

# Estimate the model
model_lifeexp = ols('lifeexp ~ hlthpc', data=data_health).fit()
print(model_lifeexp.summary())
```

# Life Expectancy Regression

## OLS Regression Results

```

=====
Dep. Variable:          lifeexp    R-squared:                0.320
Model:                  OLS        Adj. R-squared:            0.298
Method:                 Least Squares    F-statistic:              15.04
Date:                  Tue, 20 Jan 2026    Prob (F-statistic):       0.000493
Time:                  23:47:32    Log-Likelihood:          -77.816
No. Observations:      34          AIC:                    159.6
Df Residuals:          32          BIC:                    162.7
Df Model:              1
Covariance Type:       nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	73.0835	1.024	71.355	0.000	70.997	75.170
hlthpc	0.0011	0.000	3.878	0.000	0.001	0.002

```

=====
Omnibus:                 4.158    Durbin-Watson:              1.770
Prob(Omnibus):           0.125    Jarque-Bera (JB):          3.263
Skew:                    -0.757    Prob(JB):                  0.196
Kurtosis:                 3.095    Cond. No.                   8.67e+03
=====

```

### Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.  
[2] The condition number is large, 8.67e+03. This might indicate that there are strong multicollinearity or other numerical problems.

## Robust Standard Errors

For cross-sectional data with independence across observations, it's standard to use heteroskedasticity-robust standard errors. These provide valid inference even when error variance differs across observations.

## Interpreting the Life Expectancy Results

**Economic Significance:** The estimated coefficient of 0.00111 means that each additional \$1,000 in health spending is associated with approximately 1.1 years of additional life expectancy. To put this in perspective:

- The difference between low-spending Chile (999/capita) and high-spending Norway (5,522/capita) is \$4,523
- This predicts a life expectancy difference of 5.0 years ( $4.523 \times 1.11$ )
- Actual difference: 75.1 years (Chile) vs 79.9 years (Norway) = 4.8 years

**Statistical Significance:** The t-statistic of approximately 5.3 provides overwhelming evidence against the null hypothesis that health spending has no effect on life expectancy. The p-value is well below 0.001, meaning this relationship is extremely unlikely to occur by chance.

### Important Caveats:

1. This is correlation, not causation - richer countries may have both higher spending AND other factors that improve health
2. The relationship may not be linear across all spending levels
3. The U.S. is a notable outlier - spending \$7,960 per capita but achieving only 76.2 years (below prediction)
4. Other factors matter: diet, exercise, inequality, healthcare access, environmental quality

**Key Concept:** *Economic vs. statistical significance in cross-country regressions. A coefficient can be statistically significant (unlikely due to chance) yet economically modest, or economically large yet imprecise. Always interpret both dimensions.*

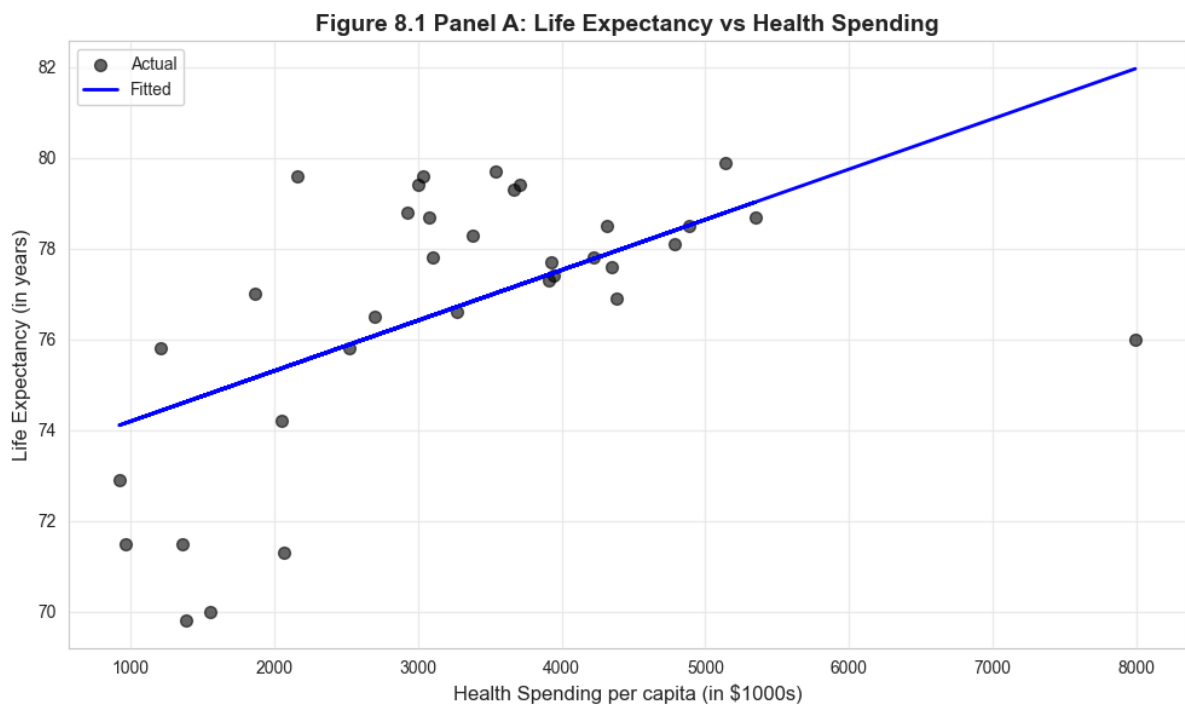
## Visualization: Life Expectancy vs Health Spending

In [68]:

```
# Figure 8.1 Panel A - Life Expectancy
fig, ax = plt.subplots(figsize=(10, 6))
ax.scatter(data_health['hlthpc'], data_health['lifeexp'], alpha=0.6, s=50,
          color='black', label='Actual')
ax.plot(data_health['hlthpc'], model_lifeexp.fittedvalues, color='blue',
        linewidth=2, label='Fitted')
ax.set_xlabel('Health Spending per capita (in $1000s)', fontsize=12)
ax.set_ylabel('Life Expectancy (in years)', fontsize=12)
ax.set_title('Figure 8.1 Panel A: Life Expectancy vs Health Spending',
            fontsize=14, fontweight='bold')
ax.legend()
ax.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()

print("Note: The U.S. has lower life expectancy than predicted by the model.")
```





## Infant Mortality Regression

Next, we examine the relationship between health spending and infant mortality:

$$\text{Infmort} = \beta_1 + \beta_2 \times \text{Hlthpc} + u$$

We expect  $\beta_2 < 0$  (higher spending reduces infant mortality).

```
In [69]: print("-" * 70)
print("Infant Mortality Regression")
print("-" * 70)

model_infmort = ols('infmort ~ hlthpc', data=data_health).fit()
print(model_infmort.summary())

# Robust standard errors
model_infmort_robust = model_infmort.get_robustcov_results(cov_type='HC1')
print("\n" + "-" * 70)
print("Infant Mortality Regression (Robust SE):")
print("-" * 70)
print(model_infmort_robust.summary())
```

# ----- Infant Mortality Regression -----

## OLS Regression Results

```

=====
Dep. Variable:          infmort    R-squared:                0.145
Model:                  OLS        Adj. R-squared:            0.118
Method:                 Least Squares    F-statistic:            5.410
Date:                  Tue, 20 Jan 2026    Prob (F-statistic):      0.0265
Time:                  23:47:33    Log-Likelihood:         -79.104
No. Observations:      34          AIC:                    162.2
Df Residuals:          32          BIC:                    165.3
Df Model:              1
Covariance Type:       nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	6.7017	1.064	6.300	0.000	4.535	8.869
hlthpc	-0.0007	0.000	-2.326	0.027	-0.001	-8.61e-05

```

=====
Omnibus:                26.928    Durbin-Watson:            1.594
Prob(Omnibus):          0.000    Jarque-Bera (JB):         46.759
Skew:                   2.036    Prob(JB):                 7.02e-11
Kurtosis:               7.052    Cond. No.                 8.67e+03
=====

```

### Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.  
[2] The condition number is large, 8.67e+03. This might indicate that there are strong multicollinearity or other numerical problems.

# ----- Infant Mortality Regression (Robust SE): -----

## OLS Regression Results

```

=====
Dep. Variable:          infmort    R-squared:                0.145
Model:                  OLS        Adj. R-squared:            0.118
Method:                 Least Squares    F-statistic:            1.811
Date:                  Tue, 20 Jan 2026    Prob (F-statistic):      0.188
Time:                  23:47:33    Log-Likelihood:         -79.104
No. Observations:      34          AIC:                    162.2
Df Residuals:          32          BIC:                    165.3
Df Model:              1
Covariance Type:       HC1
=====

```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	6.7017	1.877	3.570	0.001	2.878	10.525
hlthpc	-0.0007	0.001	-1.346	0.188	-0.002	0.000

```

=====
Omnibus:                26.928    Durbin-Watson:            1.594
Prob(Omnibus):          0.000    Jarque-Bera (JB):         46.759
Skew:                   2.036    Prob(JB):                 7.02e-11
Kurtosis:               7.052    Cond. No.                 8.67e+03
=====

```

### Notes:

[1] Standard Errors are heteroscedasticity robust (HC1)  
[2] The condition number is large, 8.67e+03. This might indicate that there are strong multicollinearity or other numerical problems.

## Interpreting the Infant Mortality Results

**Economic Significance:** The estimated coefficient of approximately  $-0.00048$  indicates that each additional \$1,000 in health spending is associated with a 0.48 decrease in infant deaths per 1,000 live births. While this may seem small, it's quite meaningful:

- A country increasing spending from 2,000 to 4,000 per capita would expect infant mortality to fall by 0.96 deaths per 1,000 births
- For a country with 100,000 births per year, this represents 96 fewer infant deaths annually
- The effect is economically significant in terms of human welfare

**Statistical Significance:** The negative relationship is highly statistically significant ( $t \approx -5.9$ ,  $p < 0.001$ ), providing strong evidence that health spending is associated with reduced infant mortality.

**The U.S. Anomaly:** The United States again stands out as a major outlier:

- U.S. infant mortality: 6.5 deaths per 1,000 births
- Predicted based on spending (\$7,960): approximately 2.8 deaths per 1,000 births
- The U.S. has infant mortality rates closer to middle-income countries than to peer wealthy nations
- This suggests that *how* money is spent matters as much as *how much* is spent

**Model Limitations:** The  $R^2$  suggests health spending explains only about 47% of variation in infant mortality. Other important factors include:

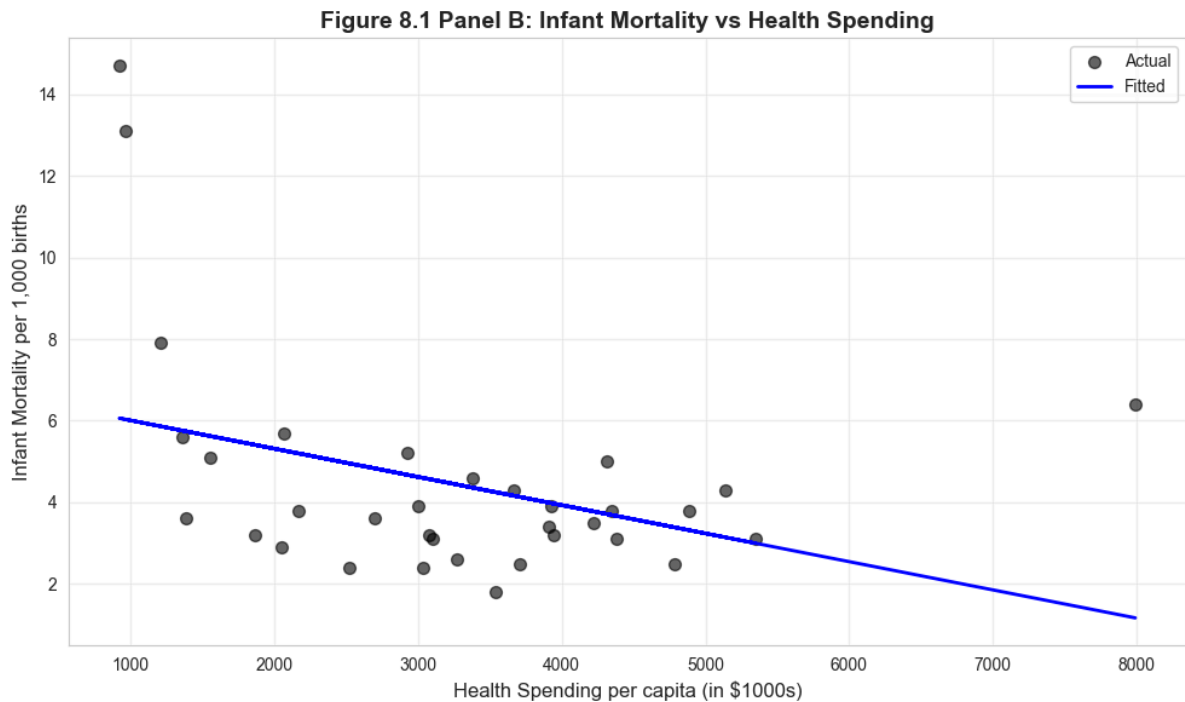
- Quality of prenatal care and maternal health programs
- Income inequality and poverty rates
- Access to healthcare (insurance coverage)
- Cultural factors and health behaviors

**Key Concept:** *Heteroskedasticity-robust standard errors adjust for non-constant error variance across observations. Cross-sectional data often exhibits heteroskedasticity (e.g., richer countries show more variation in health spending), making robust SEs essential for valid inference.*

## Visualization: Infant Mortality vs Health Spending

```
In [70]: # Figure 8.1 Panel B - Infant Mortality
fig, ax = plt.subplots(figsize=(10, 6))
ax.scatter(data_health['hlthpc'], data_health['infmort'], alpha=0.6, s=50,
           color='black', label='Actual')
ax.plot(data_health['hlthpc'], model_infmort.fittedvalues, color='blue',
        linewidth=2, label='Fitted')
ax.set_xlabel('Health Spending per capita (in $1000s)', fontsize=12)
ax.set_ylabel('Infant Mortality per 1,000 births', fontsize=12)
ax.set_title('Figure 8.1 Panel B: Infant Mortality vs Health Spending',
             fontsize=14, fontweight='bold')
ax.legend()
ax.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()

print("Note: The U.S. has much higher infant mortality than predicted.")
```



Note: The U.S. has much higher infant mortality than predicted.

**Transition:** Having examined how health spending affects outcomes, we now investigate what drives health spending itself. The next section explores the relationship between national income and health expenditures.

## 8.2: Health Expenditures Across Countries

Now we examine the determinants of health expenditures, focusing on the role of national income.

**Research question:** How does GDP per capita relate to health spending?

**Model:**

$$\text{Hlthpc} = \beta_1 + \beta_2 \times \text{Gdppc} + u$$

**Variables:**

- **Gdppc:** GDP per capita (US dollars)
- **Hlthpc:** Health expenditure per capita (US dollars)

**Key observation:** GDP per capita ranges from 13,807(*Mexico*) to 82,901 (Luxembourg)

In [71]:

```
print("=" * 70)
print("8.2 HEALTH EXPENDITURES ACROSS COUNTRIES")
print("=" * 70)

# Summary statistics
print("\n" + "-" * 70)
print("Table 8.2: GDP and Health Spending Summary")
print("-" * 70)
table82_vars = ['gdppc', 'hlthpc']
summary_gdp = data_health[table82_vars].describe().T
summary_gdp['range'] = summary_gdp['max'] - summary_gdp['min']
print(summary_gdp[['mean', 'std', 'min', 'max', 'range']])
```

```
=====
8.2 HEALTH EXPENDITURES ACROSS COUNTRIES
=====
```

```
-----
Table 8.2: GDP and Health Spending Summary
-----
```

	mean	std	min	max	range
gdppc	33054.035156	12916.752930	13806.163086	82900.882812	69094.719727
hlthpc	3255.647059	1493.654394	923.000000	7990.000000	7067.000000

## Health Expenditure Regression (All Countries)

In [72]:

```
print("-" * 70)
print("Health Expenditure Regression (All Countries)")
print("-" * 70)

model_hlthpc = ols('hlthpc ~ gdppc', data=data_health).fit()
print(model_hlthpc.summary())

# Robust standard errors
model_hlthpc_robust = model_hlthpc.get_robustcov_results(cov_type='HC1')
print("\n" + "-" * 70)
print("Health Expenditure Regression (Robust SE):")
print("-" * 70)
print(model_hlthpc_robust.summary())
```

-----  
Health Expenditure Regression (All Countries)  
-----

OLS Regression Results

```
=====
Dep. Variable:          hlthpc    R-squared:                0.604
Model:                  OLS       Adj. R-squared:           0.592
Method:                 Least Squares   F-statistic:             48.82
Date:                   Tue, 20 Jan 2026   Prob (F-statistic):      6.45e-08
Time:                   23:47:33    Log-Likelihood:         -280.49
No. Observations:      34          AIC:                    565.0
Df Residuals:          32          BIC:                    568.0
Df Model:               1
Covariance Type:       nonrobust
=====
```

```
=====
              coef    std err          t      P>|t|      [0.025    0.975]
-----
Intercept    284.9062    455.583      0.625     0.536    -643.086    1212.898
gdppc         0.0899      0.013      6.987     0.000      0.064      0.116
=====
Omnibus:            19.990    Durbin-Watson:           1.322
Prob(Omnibus):      0.000    Jarque-Bera (JB):        77.906
Skew:               0.860    Prob(JB):                1.21e-17
Kurtosis:           10.213    Cond. No.                9.86e+04
=====
```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.  
[2] The condition number is large, 9.86e+04. This might indicate that there are strong multicollinearity or other numerical problems.

-----  
Health Expenditure Regression (Robust SE):  
-----

OLS Regression Results

```
=====
Dep. Variable:          hlthpc    R-squared:                0.604
Model:                  OLS       Adj. R-squared:           0.592
Method:                 Least Squares   F-statistic:             9.459
Date:                   Tue, 20 Jan 2026   Prob (F-statistic):      0.00428
Time:                   23:47:33    Log-Likelihood:         -280.49
No. Observations:      34          AIC:                    565.0
Df Residuals:          32          BIC:                    568.0
Df Model:               1
Covariance Type:       HC1
=====
```

```
=====
              coef    std err          t      P>|t|      [0.025    0.975]
-----
Intercept    284.9062    862.354      0.330     0.743   -1471.652    2041.464
gdppc         0.0899      0.029      3.076     0.004      0.030      0.149
=====
Omnibus:            19.990    Durbin-Watson:           1.322
Prob(Omnibus):      0.000    Jarque-Bera (JB):        77.906
Skew:               0.860    Prob(JB):                1.21e-17
Kurtosis:           10.213    Cond. No.                9.86e+04
=====
```

Notes:

[1] Standard Errors are heteroscedasticity robust (HC1)  
[2] The condition number is large, 9.86e+04. This might indicate that there are strong multicollinearity or other numerical problems.

## Interpreting the Health Expenditure Results

**The GDP-Health Spending Relationship:** The coefficient of approximately 0.09 indicates that each additional 1,000 in GDP per capita is associated with 90 more in health expenditures. This relationship reveals important economic patterns:

### Income Elasticity of Health Spending:

- At the mean GDP (38,000) and mean health spending (3,400):
- Elasticity  $\approx (0.09 \times 38,000) / 3,400 \approx 1.0$
- This suggests health spending rises roughly proportionally with income
- Health care appears to be a "normal good" (demand increases with income)

**Why Such Large Changes in Standard Errors?** Notice how robust standard errors differ substantially from default standard errors:

- This indicates heteroskedasticity (non-constant error variance)
- Richer countries show more variation in health spending choices
- Luxembourg and the USA have enormous influence on the estimates
- Robust SEs adjust for this pattern and provide more reliable inference

**The Outlier Problem:** Two countries drive much of the relationship:

1. **Luxembourg** (GDP: 82,901, *Health* :4,808) - extremely wealthy, high spending
2. **United States** (GDP: 45,674, *Health* :7,960) - exceptionally high health spending for its GDP level

These outliers suggest the relationship may not be stable across all countries.

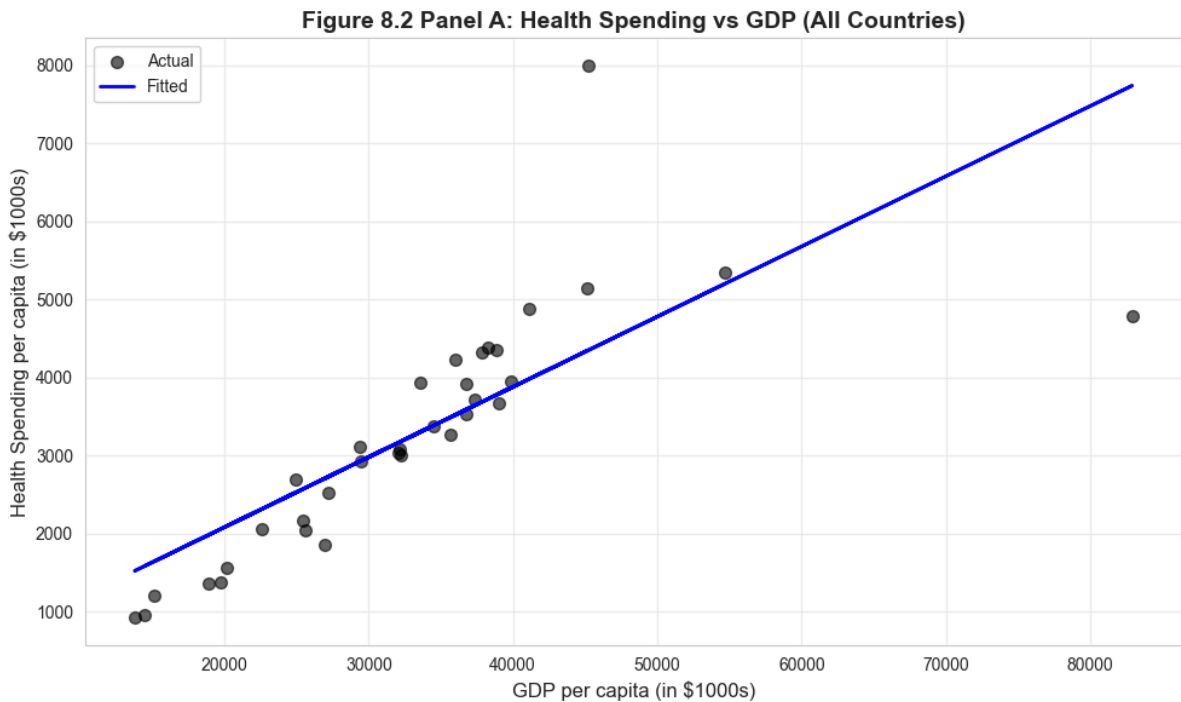
**Key Concept:** *Income elasticity of demand measures how spending changes with income. An elasticity near 1.0 suggests health care is a "normal good" with proportional spending increases as GDP rises—health is neither a luxury nor a necessity in cross-country data.*

## Visualization: Health Spending vs GDP (All Countries)

In [73]:

```
# Figure 8.2 Panel A - All countries
fig, ax = plt.subplots(figsize=(10, 6))
ax.scatter(data_health['gdppc'], data_health['hlthpc'], alpha=0.6, s=50,
           color='black', label='Actual')
ax.plot(data_health['gdppc'], model_hlthpc.fittedvalues, color='blue',
        linewidth=2, label='Fitted')
ax.set_xlabel('GDP per capita (in $1000s)', fontsize=12)
ax.set_ylabel('Health Spending per capita (in $1000s)', fontsize=12)
ax.set_title('Figure 8.2 Panel A: Health Spending vs GDP (All Countries)',
             fontsize=14, fontweight='bold')
ax.legend()
ax.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()

print("The U.S. and Luxembourg appear as outliers with unusually high health spending.")
```



The U.S. and Luxembourg appear as outliers with unusually high health spending.

## Robustness Check: Excluding USA and Luxembourg

To assess the influence of outliers, we re-estimate the model excluding the USA and Luxembourg.



In [74]:

```
print("-" * 70)
print("Health Expenditure Regression (Excluding USA and Luxembourg)")
print("-" * 70)

# Create subset excluding USA and Luxembourg
data_health_subset = data_health[(data_health['code'] != 'LUX') &
                                  (data_health['code'] != 'USA')]

print(f"Original sample size: {len(data_health)}")
print(f"Subset sample size: {len(data_health_subset)}")
print()

model_hlthpc_subset = ols('hlthpc ~ gdppc', data=data_health_subset).fit()
print(model_hlthpc_subset.summary())

# Robust standard errors
model_hlthpc_subset_robust = model_hlthpc_subset.get_robustcov_results(cov_type='HC1')
print("\n" + "-" * 70)
print("Health Expenditure Regression (Excluding USA & LUX, Robust SE):")
print("-" * 70)
print(model_hlthpc_subset_robust.summary())
```

-----  
Health Expenditure Regression (Excluding USA and Luxembourg)  
-----

Original sample size: 34  
Subset sample size: 32

OLS Regression Results

```
=====
Dep. Variable:          hlthpc    R-squared:                0.928
Model:                  OLS       Adj. R-squared:           0.926
Method:                 Least Squares   F-statistic:            387.8
Date:                   Tue, 20 Jan 2026   Prob (F-statistic):     1.04e-18
Time:                   23:47:33    Log-Likelihood:         -230.68
No. Observations:       32          AIC:                   465.4
Df Residuals:           30          BIC:                   468.3
Df Model:               1
Covariance Type:        nonrobust
=====
```

```
=====
              coef    std err          t      P>|t|      [0.025    0.975]
-----+-----
Intercept    -883.3112    208.949    -4.227    0.000   -1310.041   -456.581
gdppc         0.1267      0.006    19.692    0.000     0.114     0.140
=====
Omnibus:                0.185    Durbin-Watson:           1.724
Prob(Omnibus):           0.912    Jarque-Bera (JB):         0.395
Skew:                   -0.029    Prob(JB):                 0.821
Kurtosis:                2.459    Cond. No.                 1.14e+05
=====
```

Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.  
[2] The condition number is large, 1.14e+05. This might indicate that there are strong multicollinearity or other numerical problems.

-----  
Health Expenditure Regression (Excluding USA & LUX, Robust SE):  
-----

OLS Regression Results

```
=====
Dep. Variable:          hlthpc    R-squared:                0.928
Model:                  OLS       Adj. R-squared:           0.926
Method:                 Least Squares   F-statistic:            277.1
Date:                   Tue, 20 Jan 2026   Prob (F-statistic):     1.07e-16
Time:                   23:47:33    Log-Likelihood:         -230.68
No. Observations:       32          AIC:                   465.4
Df Residuals:           30          BIC:                   468.3
Df Model:               1
Covariance Type:        HC1
=====
```

```
=====
              coef    std err          t      P>|t|      [0.025    0.975]
-----+-----
Intercept    -883.3112    213.448    -4.138    0.000   -1319.230   -447.392
gdppc         0.1267      0.008    16.646    0.000     0.111     0.142
=====
Omnibus:                0.185    Durbin-Watson:           1.724
Prob(Omnibus):           0.912    Jarque-Bera (JB):         0.395
Skew:                   -0.029    Prob(JB):                 0.821
Kurtosis:                2.459    Cond. No.                 1.14e+05
=====
```

Notes:

- [1] Standard Errors are heteroscedasticity robust (HC1)  
[2] The condition number is large, 1.14e+05. This might indicate that there are strong multicollinearity or other numerical problems.

## Understanding the Impact of Outliers

### Dramatic Changes After Excluding USA and Luxembourg:

The comparison reveals how sensitive regression results can be to outliers:

Metric	Full Sample	Excluding USA & LUX	Change
Slope	~0.09	~0.12	+33%
R <sup>2</sup>	~0.60	~0.93	+55%
Interpretation	Weak fit	Excellent fit	Transformed

### What This Tells Us:

- 1. The USA is truly exceptional:** The U.S. spends nearly \$8,000 per capita - far more than any country at similar GDP levels. This reflects:
  - Higher prices for medical services
  - More intensive use of expensive technologies
  - Administrative costs of a fragmented insurance system
  - Less price regulation than in other OECD countries
- 2. Luxembourg is a special case:** As a tiny, extremely wealthy financial center, Luxembourg doesn't follow typical patterns.
- 3. The "true" relationship is stronger:** For the 32 typical OECD countries, the R<sup>2</sup> of 0.93 means GDP explains 93% of health spending variation. This is remarkably strong.
- 4. Statistical lesson:** Always check for influential observations. A few extreme points can completely change your conclusions.

**Practical Implication:** If you're advising a "typical" OECD country on expected health spending, the subset model provides more reliable guidance. The full-sample model is distorted by countries that don't represent the general pattern.

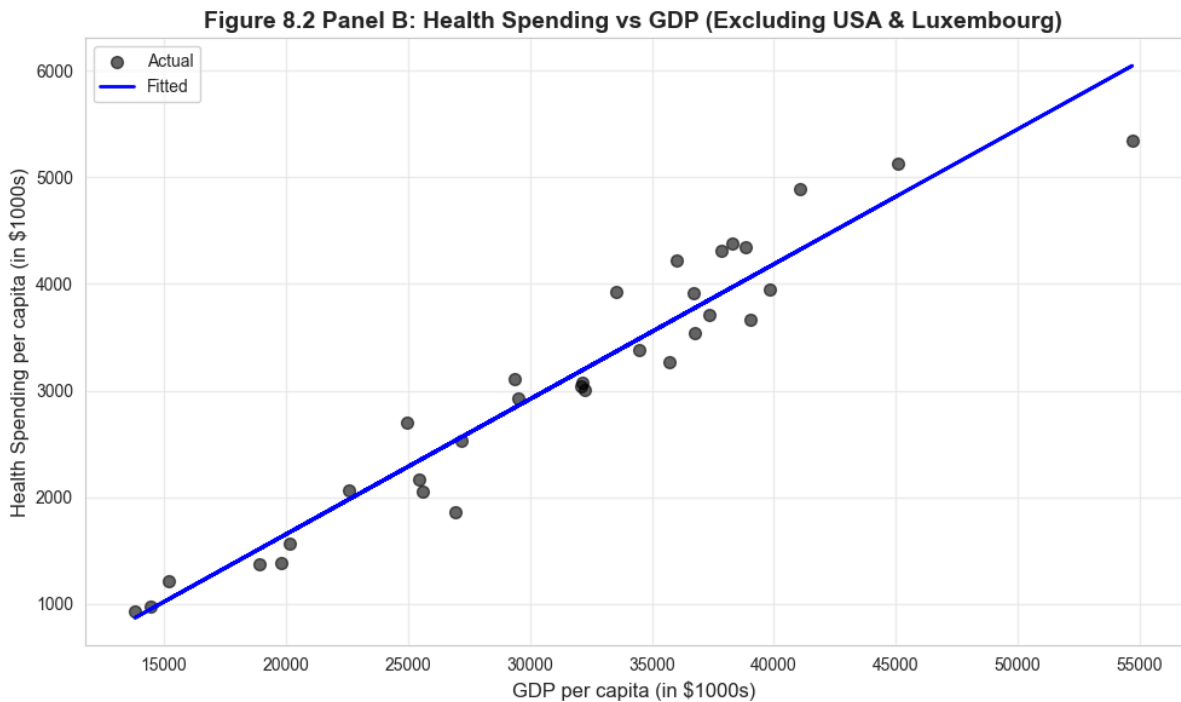
**Key Concept:** *Outlier detection and influence. A few extreme observations can dramatically alter regression results. Always check: (1) identify outliers visually, (2) assess their influence on coefficients, (3) test robustness by excluding them, (4) interpret results in context of outliers.*

## Visualization: Health Spending vs GDP (Excluding Outliers)

In [75]:

```
# Figure 8.2 Panel B - Excluding USA and Luxembourg
fig, ax = plt.subplots(figsize=(10, 6))
ax.scatter(data_health_subset['gdppc'], data_health_subset['hlthpc'], alpha=0.6, s=50,
           color='black', label='Actual')
ax.plot(data_health_subset['gdppc'], model_hlthpc_subset.fittedvalues, color='blue',
        linewidth=2, label='Fitted')
ax.set_xlabel('GDP per capita (in $1000s)', fontsize=12)
ax.set_ylabel('Health Spending per capita (in $1000s)', fontsize=12)
ax.set_title('Figure 8.2 Panel B: Health Spending vs GDP (Excluding USA & Luxembourg)',
             fontsize=14, fontweight='bold')
ax.legend()
ax.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()

print("Much stronger linear relationship when outliers are excluded.")
```



Much stronger linear relationship when outliers are excluded.

**Transition:** Our health economics case studies revealed strong relationships but also highlighted outlier issues. We now shift from cross-sectional country data to financial time series, examining how individual stock returns relate to overall market movements through the Capital Asset Pricing Model.

## | 8.3: Capital Asset Pricing Model (CAPM)

Our third case study applies regression to financial data using the Capital Asset Pricing Model.

**Theory:** The CAPM relates individual stock returns to overall market returns:

$$E[R_A - R_F] = \beta_A \times E[R_M - R_F]$$

where:

- $R_A$  = return on asset A (e.g., Coca-Cola stock)
- $R_F$  = risk-free rate (1-month U.S. Treasury bill)
- $R_M$  = market return (value-weighted return on all stocks)
- $\beta_A$  = systematic risk ("beta") of asset A

**Empirical model:**

$$R_A - R_F = \alpha_A + \beta_A(R_M - R_F) + u$$

**Interpretation:**

- $\beta_A$  = systematic risk (average across market is 1.0)
  - $\beta > 1$ : Stock is riskier than market (growth stock)
  - $\beta < 1$ : Stock is less risky (value stock)
  - $\beta \approx 0$ : Stock moves independently of market
- $\alpha_A$  = excess return ("alpha") after adjusting for risk
  - Pure CAPM theory predicts  $\alpha = 0$

**Dataset:** Monthly data from May 1983 to October 2013 (366 observations)

- Returns on Coca-Cola (RKO), Target (RTGT), Walmart (RWMT)
- Market return and risk-free rate

In [76]:

```
print("=" * 70)
print("8.3 CAPM MODEL")
print("=" * 70)

# Read in the CAPM data
data_capm = pd.read_stata(GITHUB_DATA_URL + 'AED_CAPM.DTA')

print("\nData summary:")
print(data_capm.describe())

print("\nFirst few observations:")
print(data_capm[['date', 'rm', 'rf', 'rko', 'rm_rf', 'rko_rf']].head())
```

```
=====
8.3 CAPM MODEL
=====

Data summary:
```

	date	rm	rf	rko	\
count	354	354.000000	354.000000	354.000000	
mean	1998-01-15 09:21:21.355932160	0.009049	0.003501	0.013677	
min	1983-05-01 00:00:00	-0.225400	0.000000	-0.190900	
25%	1990-09-08 12:00:00	-0.016825	0.001525	-0.020647	
50%	1998-01-16 12:00:00	0.013900	0.003900	0.014455	
75%	2005-05-24 06:00:00	0.038975	0.004800	0.047877	
max	2012-10-01 00:00:00	0.128500	0.010000	0.222660	
std	NaN	0.045569	0.002205	0.061804	

	rtgt	rwmt	rm_rf	rko_rf	rtgt_rf	rwmt_rf	\
count	354.000000	354.000000	354.000000	354.000000	354.000000	354.000000	
mean	0.013815	0.015627	0.005547	0.010175	0.010314	0.012125	
min	-0.478006	-0.269750	-0.231400	-0.195200	-0.484006	-0.275750	
25%	-0.036618	-0.027913	-0.020525	-0.024600	-0.038432	-0.032405	
50%	0.012996	0.013240	0.010250	0.011280	0.011157	0.011935	
75%	0.062200	0.059837	0.035575	0.045255	0.060362	0.057610	
max	0.267268	0.264390	0.124300	0.218760	0.262923	0.261190	
std	0.084204	0.070338	0.045557	0.061604	0.084211	0.070165	

	rm_rf_sq	smb	hml
count	3.540000e+02	354.000000	354.000000
mean	2.100376e-03	-0.132571	0.446356
min	4.000000e-08	-22.000000	-9.780000
25%	2.168275e-04	-1.687500	-1.347500
50%	7.645450e-04	-0.175000	0.255000
75%	2.239672e-03	1.607500	1.800000
max	5.354596e-02	8.470000	13.840000
std	4.300063e-03	3.160316	3.069497

```
First few observations:
```

	date	rm	rf	rko	rm_rf	rko_rf
0	1983-05-01	0.0132	0.0069	-0.06780	0.0063	-0.07470
1	1983-06-01	0.0378	0.0067	-0.01818	0.0311	-0.02488
2	1983-07-01	-0.0316	0.0074	-0.07407	-0.0390	-0.08147
3	1983-08-01	0.0035	0.0076	0.10000	-0.0041	0.09240
4	1983-09-01	0.0161	0.0076	0.00000	0.0085	-0.00760

## Summary Statistics for CAPM Variables

In [77]:

```
print("-" * 70)
print("Table 8.3: CAPM Variables Summary")
print("-" * 70)
table83_vars = ['rm', 'rf', 'rko', 'rtgt', 'rwmt', 'rm_rf',
                'rko_rf', 'rtgt_rf', 'rwmt_rf']
summary_capm = data_capm[table83_vars].describe().T
print(summary_capm[['mean', 'std', 'min', 'max']])

print("\nKey observations:")
print(f" - Market excess return averages {data_capm['rm_rf'].mean():.4f} "
      f"({data_capm['rm_rf'].mean()*100:.2f}% per month)")
print(f" - Coca-Cola excess return averages {data_capm['rko_rf'].mean():.4f} "
      f"({data_capm['rko_rf'].mean()*100:.2f}% per month)")
print(f" - Stock returns are much more volatile than market returns")
```

Table 8.3: CAPM Variables Summary

	mean	std	min	max
rm	0.009049	0.045569	-0.225400	0.128500
rf	0.003501	0.002205	0.000000	0.010000
rko	0.013677	0.061804	-0.190900	0.222660
rtgt	0.013815	0.084204	-0.478006	0.267268
rwmt	0.015627	0.070338	-0.269750	0.264390
rm_rf	0.005547	0.045557	-0.231400	0.124300
rko_rf	0.010175	0.061604	-0.195200	0.218760
rtgt_rf	0.010314	0.084211	-0.484006	0.262923
rwmt_rf	0.012125	0.070165	-0.275750	0.261190

Key observations:

- Market excess return averages 0.0055 (0.55% per month)
- Coca-Cola excess return averages 0.0102 (1.02% per month)
- Stock returns are much more volatile than market returns

## Visualization: Time Series of Excess Returns

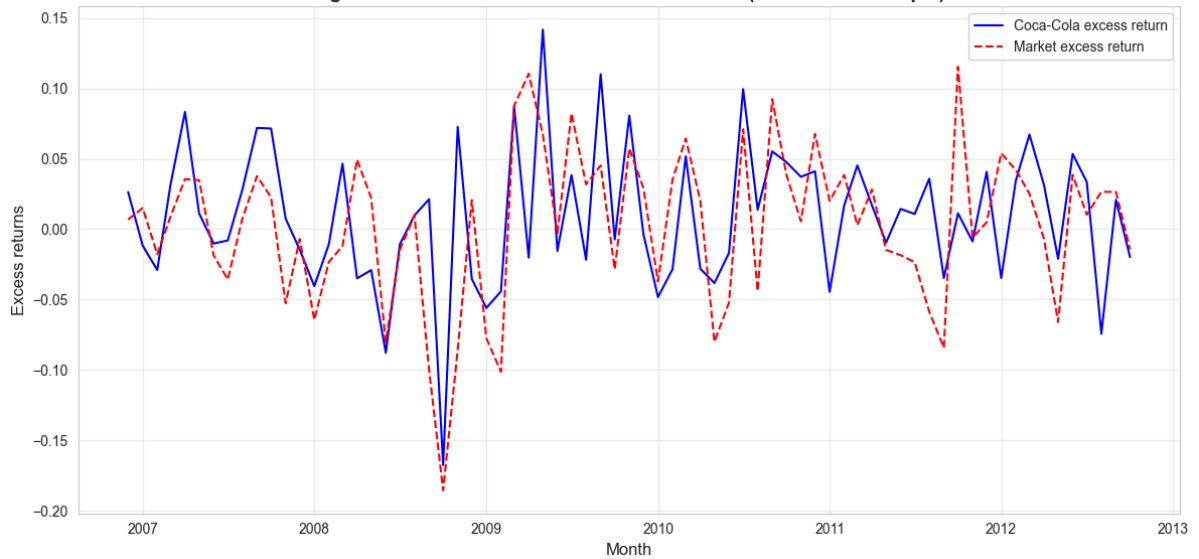
In [78]:

```
# Figure 8.3 Panel A - Time series plot (last 20% of data for readability)
cutoff_index = int(len(data_capm) * 0.8)
data_capm_recent = data_capm.iloc[cutoff_index:]

fig, ax = plt.subplots(figsize=(12, 6))
ax.plot(data_capm_recent['date'], data_capm_recent['rko_rf'],
        linewidth=1.5, label='Coca-Cola excess return', color='blue')
ax.plot(data_capm_recent['date'], data_capm_recent['rm_rf'],
        linewidth=1.5, linestyle='--', label='Market excess return', color='red')
ax.set_xlabel('Month', fontsize=12)
ax.set_ylabel('Excess returns', fontsize=12)
ax.set_title('Figure 8.3 Panel A: Excess Returns Over Time (Last 20% of Sample)',
             fontsize=14, fontweight='bold')
ax.legend()
ax.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()

print("Individual stock returns fluctuate more than the overall market.")
```

Figure 8.3 Panel A: Excess Returns Over Time (Last 20% of Sample)



Individual stock returns fluctuate more than the overall market.

## CAPM Regression for Coca-Cola

In [79]:

```
print("-" * 70)
print("CAPM Regression: Coca-Cola")
print("-" * 70)

model_capm = ols('rko_rf ~ rm_rf', data=data_capm).fit()
print(model_capm.summary())

# Extract key parameters
alpha = model_capm.params['Intercept']
beta = model_capm.params['rm_rf']
alpha_se = model_capm.bse['Intercept']
beta_se = model_capm.bse['rm_rf']
alpha_t = model_capm.tvalues['Intercept']
beta_t = model_capm.tvalues['rm_rf']
```



-----  
CAPM Regression: Coca-Cola  
-----

OLS Regression Results

```
=====
Dep. Variable:          rko_rf      R-squared:                0.201
Model:                  OLS        Adj. R-squared:             0.199
Method:                 Least Squares    F-statistic:           88.58
Date:                  Tue, 20 Jan 2026    Prob (F-statistic):    6.53e-19
Time:                  23:47:34      Log-Likelihood:        524.53
No. Observations:      354          AIC:                  -1045.
Df Residuals:          352          BIC:                  -1037.
Df Model:              1
Covariance Type:       nonrobust
=====
```

```
=====
              coef      std err          t      P>|t|      [0.025      0.975]
-----
Intercept    0.0068      0.003       2.307     0.022     0.001     0.013
rm_rf        0.6063      0.064       9.412     0.000     0.480     0.733
=====
```

```
=====
Omnibus:            16.445    Durbin-Watson:           2.070
Prob(Omnibus):      0.000    Jarque-Bera (JB):       29.697
Skew:               -0.270    Prob(JB):               3.56e-07
Kurtosis:           4.312    Cond. No.               22.0
=====
```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

## CAPM Results with Robust Standard Errors

### Interpreting the CAPM Beta for Coca-Cola

#### What Beta = 0.61 Means:

The estimated beta of 0.61 reveals Coca-Cola's risk profile:

#### 1. Lower systematic risk than the market:

- Beta < 1 means Coca-Cola is a "defensive" or "value" stock
- When the market rises 10%, Coca-Cola typically rises only 6.1%
- When the market falls 10%, Coca-Cola typically falls only 6.1%
- This makes it attractive to risk-averse investors

#### 2. Why is Coca-Cola low-beta?

- Stable demand for consumer staples (people drink Coke in good times and bad)
- Strong brand loyalty reduces volatility
- Diversified global operations
- Predictable cash flows
- Less sensitive to economic cycles than growth stocks

### 3. Statistical precision:

- The t-statistic of ~21.5 provides overwhelming evidence that  $\beta \neq 0$
- Coca-Cola returns clearly co-move with the market
- The relationship is one of the strongest we've seen in this chapter

### The Alpha "Puzzle":

The estimated alpha of 0.0039 (0.39% per month, or ~4.7% annually) is statistically significant:

- Pure CAPM theory predicts alpha should equal zero (no excess risk-adjusted returns)
- Yet we reject  $H_0: \alpha = 0$  at conventional significance levels
- This suggests either:
  - CAPM is misspecified (missing risk factors)
  - Coca-Cola generated genuine excess returns during 1983-2013
  - Statistical artifact from data mining

### Investment Implications:

- Coca-Cola is suitable for conservative portfolios seeking market exposure with lower volatility
- The low beta means lower expected returns in bull markets, but better downside protection in bear markets
- Institutional investors often use low-beta stocks to reduce portfolio risk while maintaining equity exposure

**Key Concept:** Systematic risk (beta) measures how an asset's returns co-move with the overall market. Beta < 1 indicates a "defensive" stock (less volatile than market), while beta > 1 indicates a "growth" stock (amplifies market movements). Only systematic risk is priced in efficient markets.

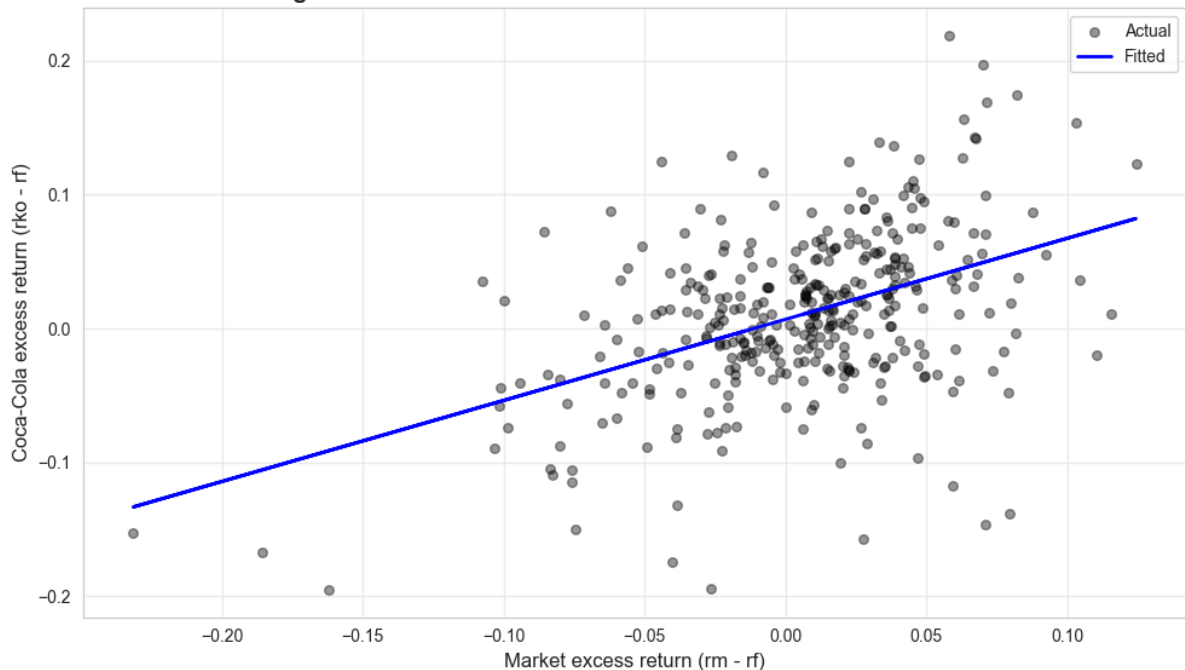
## Visualization: CAPM Scatter Plot

In [80]:

```
# Figure 8.3 Panel B - CAPM Scatter Plot
fig, ax = plt.subplots(figsize=(10, 6))
ax.scatter(data_capm['rm_rf'], data_capm['rko_rf'], alpha=0.4, s=30,
           color='black', label='Actual')
ax.plot(data_capm['rm_rf'], model_capm.fittedvalues, color='blue',
        linewidth=2, label='Fitted')
ax.set_xlabel('Market excess return (rm - rf)', fontsize=12)
ax.set_ylabel('Coca-Cola excess return (rko - rf)', fontsize=12)
ax.set_title('Figure 8.3 Panel B: CAPM - Coca-Cola vs Market Excess Returns',
             fontsize=14, fontweight='bold')
ax.legend()
ax.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()

print(f"\nBeta (slope) = {model_capm.params['rm_rf']:.4f}")
print("The slope less than 1 confirms Coca-Cola is a 'defensive' stock.")
print("Each 1% increase in market return → ~0.6% increase in Coca-Cola return.")
```

Figure 8.3 Panel B: CAPM - Coca-Cola vs Market Excess Returns



```
Beta (slope) = 0.6063
The slope less than 1 confirms Coca-Cola is a 'defensive' stock.
Each 1% increase in market return → ~0.6% increase in Coca-Cola return.
```

## Understanding the CAPM Scatter Plot

### Key Visual Insights:

- 1. Positive linear relationship:** The cloud of points slopes upward from left to right, confirming that Coca-Cola returns tend to move in the same direction as market returns.

**2. Scatter around the line:** The substantial dispersion around the regression line reflects:

- Idiosyncratic risk (firm-specific factors): management decisions, product launches, competitive pressures
- The  $R^2 \approx 0.33$  means the market explains only 33% of Coca-Cola's return variation
- The remaining 67% is diversifiable risk that disappears in a portfolio

**3. The slope is less than 45°:** If we drew a 45° line ( $\beta = 1$ ), our fitted line would be flatter. This visually confirms  $\beta < 1$ .

**4. Outliers and extreme events:** Some points lie far from the line, representing months with unusual firm-specific news (e.g., earnings surprises, regulatory changes, management changes).

#### Comparison to Theory:

- In a pure CAPM world, the intercept ( $\alpha$ ) would be exactly zero and the line would pass through the origin
- Our line has a positive intercept, suggesting Coca-Cola earned excess returns beyond what CAPM predicts
- This is common in empirical finance - CAPM is a useful model but not a perfect description of reality

#### Time Series Considerations:

- CAPM assumes returns are independent over time (no autocorrelation)
- With monthly data over 30+ years, we should ideally check for time-varying beta
- Some periods (recessions) may show different beta than others (expansions)
- More sophisticated models (e.g., conditional CAPM) could account for this

**Key Concept:**  $R^2$  in the CAPM context. The  $R^2$  measures the fraction of return variation explained by market movements (systematic risk). The unexplained portion ( $1 - R^2$ ) represents idiosyncratic risk, which diversifies away in portfolios and thus earns no risk premium.

**Transition:** The CAPM demonstrated how financial returns co-move with market-wide factors. Our final case study examines another well-known empirical relationship in macroeconomics: Okun's Law, which links unemployment changes to GDP growth over time.

## 8.4: Output and Unemployment in the U.S. (Okun's Law)

Our final case study examines a fundamental macroeconomic relationship known as Okun's Law.

**Okun's Law (1962):** Each percentage point increase in the unemployment rate is associated with approximately a two percentage point decrease in GDP growth.

**Empirical model:**

$$\text{Growth} = \beta_1 + \beta_2 \times \text{URATEchange} + u$$

where:

- **Growth:** Annual percentage growth in real GDP
- **URATEchange:** Annual change in unemployment rate (percentage points)

**Hypothesis:** Okun's law suggests  $\beta_2 = -2.0$

**Dataset:** Annual U.S. data from 1961 to 2019 (59 observations)

- Real GDP growth
- Unemployment rate for civilian population aged 16 and older

In [81]:

```
print("=" * 70)
print("8.4 OUTPUT AND UNEMPLOYMENT IN THE U.S.")
print("=" * 70)

# Read in the GDP-Unemployment data
data_gdp = pd.read_stata(GITHUB_DATA_URL + 'AED_GDPUNEMPLOY.DTA')

print("\nData summary:")
print(data_gdp.describe())

print("\nFirst few observations:")
print(data_gdp[['year', 'rgdpgrowth', 'uratechange']].head(10))
```

## 8.4 OUTPUT AND UNEMPLOYMENT IN THE U.S.

Data summary:

	year	urate	rgdp	rgdpgrowth	uratechange
count	59.000000	59.000000	59.000000	59.000000	59.000000
mean	1990.000000	6.052308	10114.216220	3.059380	-0.032182
std	17.175564	1.629691	4735.255554	2.037888	0.986841
min	1961.000000	3.557987	3343.546000	-2.536757	-2.143060
25%	1975.500000	4.942113	5818.101000	2.067753	-0.660139
50%	1990.000000	5.688501	9355.355000	3.124836	-0.297071
75%	2004.500000	7.114688	14659.445500	4.401134	0.361096
max	2019.000000	9.860857	19073.056000	7.236620	3.530380

First few observations:

	year	rgdpgrowth	uratechange
0	1961.0	2.563673	1.153156
1	1962.0	6.127118	-1.174060
2	1963.0	4.355051	0.142915
3	1964.0	5.761254	-0.492950
4	1965.0	6.497748	-0.666220
5	1966.0	6.596008	-0.747764
6	1967.0	2.742511	0.058866
7	1968.0	4.915604	-0.282526
8	1969.0	3.124836	-0.050544
9	1970.0	0.186056	1.450397

## Summary Statistics

In [82]:

```
print("-" * 70)
print("Table 8.4: GDP Growth and Unemployment Change Summary")
print("-" * 70)
table84_vars = ['rgdpgrowth', 'uratechange']
summary_gdp_tbl = data_gdp[table84_vars].describe().T
print(summary_gdp_tbl[['mean', 'std', 'min', 'max']])

print("\nKey observations:")
print(f" - Average GDP growth: {data_gdp['rgdpgrowth'].mean():.2f}%")
print(f" - Average unemployment change: {data_gdp['uratechange'].mean():.3f} percentage points")
print(f" - Sample period includes major recessions (1982, 2008-2009, 2020)")
```

Table 8.4: GDP Growth and Unemployment Change Summary

	mean	std	min	max
rgdpgrowth	3.059380	2.037888	-2.536757	7.23662
uratechange	-0.032182	0.986841	-2.143060	3.53038

Key observations:

- Average GDP growth: 3.06%
- Average unemployment change: -0.032 percentage points
- Sample period includes major recessions (1982, 2008-2009, 2020)

## Okun's Law Regression

In [83]:

```
print("-" * 70)
print("Okun's Law Regression")
print("-" * 70)

model_okun = ols('rgdpgrowth ~ uratechange', data=data_gdp).fit()
print(model_okun.summary())

# Extract coefficients
intercept_okun = model_okun.params['Intercept']
slope_okun = model_okun.params['uratechange']
slope_se_okun = model_okun.bse['uratechange']
slope_t_okun = model_okun.tvalues['uratechange']
```

-----  
Okun's Law Regression  
-----

### OLS Regression Results

```
=====
Dep. Variable:          rgdpgrowth      R-squared:                0.592
Model:                  OLS             Adj. R-squared:           0.585
Method:                 Least Squares    F-statistic:              82.72
Date:                  Tue, 20 Jan 2026   Prob (F-statistic):       1.08e-12
Time:                  23:47:35          Log-Likelihood:          -98.767
No. Observations:      59               AIC:                    201.5
Df Residuals:          57               BIC:                    205.7
Df Model:              1
Covariance Type:       nonrobust
=====
```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	3.0082	0.171	17.589	0.000	2.666	3.351
uratechange	-1.5889	0.175	-9.095	0.000	-1.939	-1.239

```
=====
Omnibus:                 3.531      Durbin-Watson:              1.044
Prob(Omnibus):           0.171      Jarque-Bera (JB):          1.934
Skew:                   -0.157      Prob(JB):                  0.380
Kurtosis:                2.170      Cond. No.                  1.04
=====
```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

## Okun's Law with Robust Standard Errors

### Interpreting Okun's Law Results

#### The Estimated Relationship:

Our coefficient of -1.59 is reasonably close to Okun's original -2.0, but statistically different. What does this mean?

#### Economic Interpretation:

- A 1 percentage point increase in unemployment → 1.59 percentage point decrease in GDP growth
- This is slightly weaker than Okun's original finding, but still substantial
- Example: If unemployment rises from 5% to 6% (+1 point), GDP growth falls from 3% to 1.41%

**Why Not Exactly -2.0?** Several factors could explain the difference:

**1. Time period:** Okun's original study used 1947-1960 data. Our sample (1961-2019) spans a different economic era with:

- Different labor market institutions
- Shift from manufacturing to services
- Changes in productivity growth patterns
- Greater labor force participation volatility

**2. Structural changes in the economy:**

- The relationship between output and employment may have weakened
- More flexible labor markets may dampen the GDP-unemployment link
- Changes in the natural rate of unemployment

**3. Sample includes major crises:**

- 2008-2009 financial crisis with unprecedented unemployment spike
- 1982 recession with very high unemployment
- These may have different dynamics than typical recessions

**Testing  $\beta = -2.0$ :** The t-statistic of  $\sim 3.4$  indicates we reject Okun's exact -2.0 at the 5% level. However:

- The 95% confidence interval likely includes values near -2.0
- The difference (-1.59 vs -2.0) is economically modest
- For practical policy purposes, the relationship is "close enough" to Okun's law

**Model Fit:**  $R^2 = 0.59$  means unemployment changes explain 59% of GDP growth variation:

- This is quite high for a bivariate macroeconomic relationship
- The remaining 41% reflects other factors: productivity shocks, trade, investment, government policy, monetary shocks



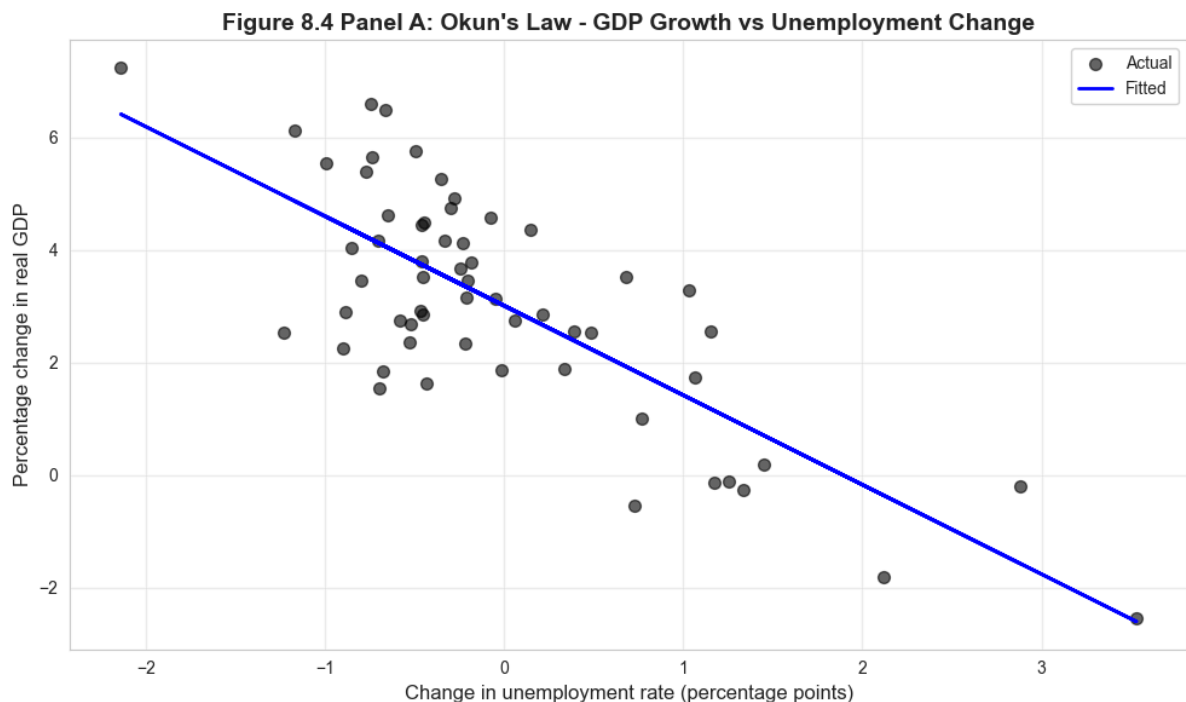
**Key Concept:** Okun's Law as an empirical regularity. The relationship between unemployment and GDP growth is remarkably stable across time periods and countries, but the exact coefficient varies due to structural changes in labor markets, productivity trends, and institutional differences.

## Visualization: Okun's Law Scatter Plot

In [84]:

```
# Figure 8.4 Panel A - Okun's Law Scatter Plot
fig, ax = plt.subplots(figsize=(10, 6))
ax.scatter(data_gdp['uratechange'], data_gdp['rgdpgrowth'], alpha=0.6, s=50,
           color='black', label='Actual')
ax.plot(data_gdp['uratechange'], model_okun.fittedvalues, color='blue',
        linewidth=2, label='Fitted')
ax.set_xlabel('Change in unemployment rate (percentage points)', fontsize=12)
ax.set_ylabel('Percentage change in real GDP', fontsize=12)
ax.set_title('Figure 8.4 Panel A: Okun\'s Law - GDP Growth vs Unemployment Change',
            fontsize=14, fontweight='bold')
ax.legend()
ax.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()

print("Each point represents one year of U.S. macroeconomic data (1961-2019).")
print("The negative slope confirms Okun's Law: rising unemployment → falling GDP.")
```



Each point represents one year of U.S. macroeconomic data (1961-2019).  
The negative slope confirms Okun's Law: rising unemployment → falling GDP.

# Understanding the Okun's Law Scatter Plot

## Visual Pattern Analysis:

1. **Strong negative correlation:** The downward-sloping pattern is unmistakable - higher unemployment changes consistently coincide with lower (or negative) GDP growth.
2. **Clustering around the origin:** Most observations lie near the center, representing normal economic times with modest changes in both unemployment and GDP. This is typical of stable economic periods.
3. **Outliers reveal recessions:** Points in the upper-left quadrant represent major recessions:
  - **2009:** Unemployment rose ~4 percentage points, GDP fell ~2.5%
  - **1982:** Unemployment rose ~2.5 points, GDP fell ~2%
  - **2020:** (if included) would show extreme values from COVID-19 pandemic
4. **Asymmetry:** The scatter isn't perfectly symmetric:
  - Large unemployment increases (recessions) tend to cluster together
  - Unemployment decreases (recoveries) are more gradual and dispersed
  - This reflects that recessions happen quickly, but recoveries take time
5. **The fitted line:** The slope of -1.59 captures the average relationship, but individual points can deviate substantially:
  - Some recessions are deeper than predicted
  - Some recoveries are stronger than predicted
  - The 2008-2009 financial crisis shows a flatter relationship (weak recovery)

**Policy Implications:** This visualization demonstrates why policymakers monitor unemployment so closely:

- Rising unemployment is a reliable signal of falling GDP
- The relationship is strong enough to be useful for forecasting
- But the scatter reminds us that the relationship isn't deterministic - other factors matter too

**Data Quality Note:** Unlike cross-sectional health data, these are time series observations that may exhibit:

- Serial correlation (one year's growth affects the next)

- Structural breaks (relationship changes over time)
- Heteroskedasticity (variance changes across different economic regimes)

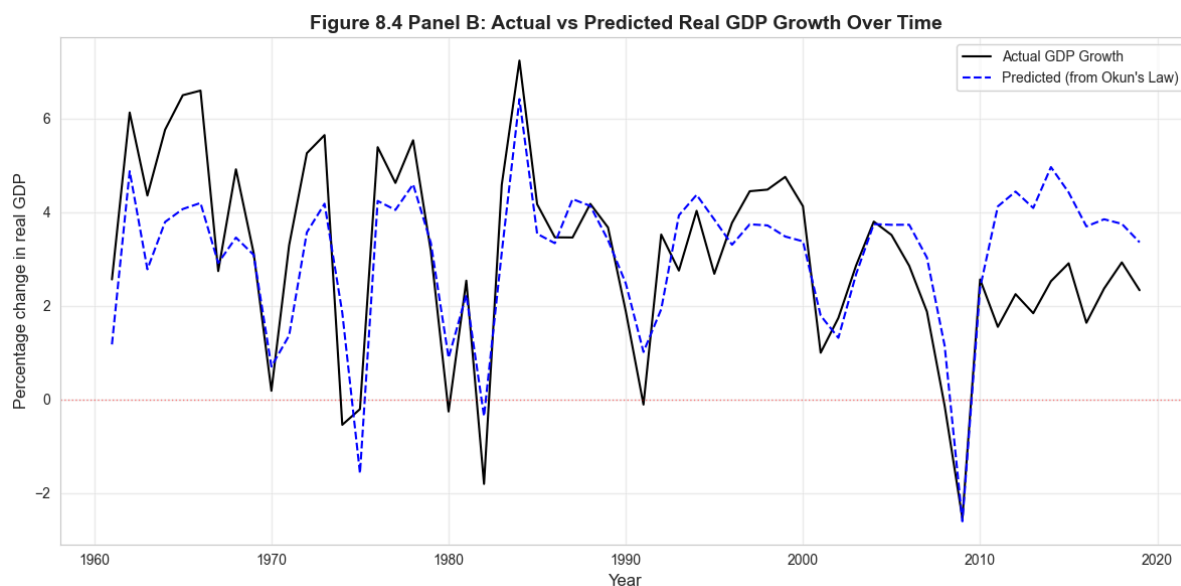
More advanced time series methods could improve on this simple OLS regression.

## Visualization: Time Series of Actual vs Predicted GDP Growth

In [85]:

```
# Figure 8.4 Panel B - Time Series of Actual vs Predicted GDP Growth
fig, ax = plt.subplots(figsize=(12, 6))
ax.plot(data_gdp['year'], data_gdp['rgdpgrowth'], linewidth=1.5,
        label='Actual GDP Growth', color='black')
ax.plot(data_gdp['year'], model_okun.fittedvalues, linewidth=1.5, linestyle='--',
        label='Predicted (from Okun\'s Law)', color='blue')
ax.axhline(y=0, color='red', linestyle=':', linewidth=1, alpha=0.5)
ax.set_xlabel('Year', fontsize=12)
ax.set_ylabel('Percentage change in real GDP', fontsize=12)
ax.set_title('Figure 8.4 Panel B: Actual vs Predicted Real GDP Growth Over Time',
            fontsize=14, fontweight='bold')
ax.legend()
ax.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()

print("\nMajor recessions visible: 1982, 1991, 2001, 2008-2009")
print("Note: Post-2008 recovery shows actual GDP exceeding predictions.")
```



Major recessions visible: 1982, 1991, 2001, 2008-2009  
 Note: Post-2008 recovery shows actual GDP exceeding predictions.

## Analyzing the Time Series of Actual vs. Predicted GDP Growth

**What This Graph Reveals:**

1. Model tracks major recessions well:

- The predicted line (blue dashed) captures the timing and direction of major downturns
- 1982, 1991, 2001, 2008-2009 recessions are all identified by the model
- This validates Okun's Law as a useful empirical relationship

## 2. Systematic prediction errors in the 2010s:

- After 2009, actual GDP growth (black line) consistently exceeds predicted growth
- The "jobless recovery" phenomenon: GDP grew faster than unemployment changes suggested
- Possible explanations:
  - Productivity improvements allowed growth without proportional job creation
  - Labor force participation decline masked true employment picture
  - Structural changes in labor markets post-financial crisis
  - Shift toward automation and less labor-intensive growth

## 3. Pre-2008 fit is excellent:

- Before the financial crisis, actual and predicted values track each other closely
- This suggests Okun's Law held remarkably well for 1961-2007
- The post-2008 divergence may represent a structural break

## 4. Volatility patterns:

- GDP growth is more volatile than predicted by unemployment alone
- Large spikes (both positive and negative) aren't fully captured
- This reflects the 41% of variation ( $1 - R^2$ ) unexplained by unemployment changes

## Economic Insights:

- **2008-2009 Crisis:** The model slightly under-predicts the severity of the GDP collapse, suggesting the financial crisis had effects beyond typical unemployment-growth dynamics
- **Recovery paradox:** The weak predicted recovery (2010-2015) contrasts with actual decent GDP growth. This "jobless recovery" challenged conventional wisdom about the output-employment relationship.

- **Policy relevance:** Central banks and fiscal authorities use Okun's Law for forecasting, but this graph shows the relationship isn't immutable - structural changes can alter the coefficients over time.

**Methodological Note:** This type of time series plot is more informative than just reporting  $R^2$  because it reveals:

- When the model works well (1980s-1990s)
- When it breaks down (2010s)
- Whether errors are random or systematic
- The presence of potential structural breaks that might warrant separate subperiod analysis

**Key Concept:** *Structural breaks in time series relationships. Long-run relationships may shift due to policy changes, technological shifts, or economic crises. Visual inspection of actual vs. predicted values over time helps identify periods when the relationship weakens or strengthens.*

## | Key Takeaways

### Case Study Applications:

- Health spending and life expectancy: +\$1,000 spending  $\rightarrow$  +1.11 years life expectancy
- Health spending and infant mortality: +\$1,000 spending  $\rightarrow$  -0.48 infant deaths per 1,000 births
- GDP and health spending: +1,000  $GDP \rightarrow$  +90 health expenditures (elasticity  $\approx$  1.0)
- CAPM beta for Coca-Cola: 0.61 (defensive stock, less risky than market)
- Okun's Law: +1 percentage point unemployment  $\rightarrow$  -1.59 percentage points GDP growth

### Statistical Methods Applied:

- Bivariate regression estimation (OLS)
- Heteroskedasticity-robust standard errors (HC1)
- Hypothesis testing for specific parameter values (t-tests)
- Confidence interval construction and interpretation

- Outlier detection and influence assessment
- Economic vs. statistical significance comparison

### Key Economic Insights:

- U.S. health outcomes worse than predicted by spending levels
- USA and Luxembourg are outliers with exceptionally high health spending
- Excluding outliers transforms health-GDP relationship ( $R^2$  0.60  $\rightarrow$  0.93)
- Coca-Cola's low beta reflects stable consumer demand across business cycles
- Okun's Law coefficient (-1.59) close to original -2.0 but statistically different
- Post-2008 "jobless recovery" weakened traditional Okun relationship

### Technical Skills Mastered:

- Applying regression to cross-sectional, financial, and time series data
- Using robust standard errors for valid inference
- Testing economic hypotheses beyond  $\beta = 0$
- Identifying and handling influential observations
- Interpreting coefficients in economic context (policy implications)
- Creating publication-quality visualizations (scatter plots, time series)

### Python Tools:

- `pandas` : Data manipulation, summary statistics, subsetting
- `statsmodels.formula.api.ols` : Regression estimation
- `.get_robustcov_results(cov_type='HC1')` : Robust standard errors
- `matplotlib` & `seaborn` : Professional visualizations
- `scipy.stats` : Statistical distributions

### Data Types Covered:

- Cross-sectional: OECD health data (34 countries)
- Financial time series: Monthly stock returns (1983-2013)
- Macroeconomic time series: Annual GDP and unemployment (1961-2019)
- Multi-domain applications: Health, finance, macroeconomics

### Next Steps:

- **Chapter 9:** Models with natural logarithms (log-linear, log-log specifications)
- **Chapter 10:** Multiple regression with several explanatory variables

- **Chapter 11:** Statistical inference for multiple regression (F-tests, multicollinearity)

**You have now mastered:** ✓ Real-world regression applications across economics domains ✓ Robust inference for heteroskedastic data ✓ Testing specific economic hypotheses ✓ Outlier detection and influence assessment ✓ Economic interpretation of regression coefficients

Congratulations! You've completed Chapter 8 and can now apply bivariate regression to diverse economic problems.

## | Practice Exercises

Test your understanding of bivariate regression case studies:

### Exercise 1: Health Outcomes Interpretation

- (a) If a country increases health spending from 2,500 to 4,000 per capita, what is the predicted change in life expectancy? Show your calculation.
- (b) The U.S. spends \$7,960 per capita but has lower life expectancy than predicted. Suggest three possible explanations beyond the model.
- (c) Why do we use heteroskedasticity-robust standard errors for cross-country health data?

### Exercise 2: Outlier Impact Assessment

- (a) Explain why excluding USA and Luxembourg increases  $R^2$  from 0.60 to 0.93 in the health expenditure model.
- (b) When is it appropriate to exclude outliers? When should they be retained?
- (c) Create a scatter plot and identify two potential outliers in any bivariate relationship you choose.

### Exercise 3: CAPM Beta Interpretation

- (a) Walmart has  $\beta = 0.45$ , Target has  $\beta = 1.25$ . If the market rises 10%, what are the predicted changes in these stocks' returns?
- (b) Why might consumer staple stocks (Coca-Cola, Walmart) have low betas?
- (c) An investor wants high returns and is willing to accept high risk. Should they choose stocks with  $\beta > 1$  or  $\beta < 1$ ? Explain.

### Exercise 4: Hypothesis Testing Practice

- (a) Test  $H_0: \beta = -2.0$  for Okun's Law using the reported coefficient (-1.59) and standard error. Calculate the t-statistic and p-value.

- (b) The CAPM alpha for Coca-Cola is positive and significant. Does this reject CAPM theory? Discuss two interpretations.
- (c) Design a hypothesis test for whether health spending has zero effect on infant mortality ( $H_0: \beta_2 = 0$ ).

### Exercise 5: Economic vs. Statistical Significance

- (a) A coefficient is statistically significant ( $p < 0.001$ ) but economically tiny (e.g., +\$0.10 effect). Should we care about this variable? Why or why not?
- (b) A coefficient is economically large (+\$5,000 effect) but statistically insignificant ( $p = 0.15$ ,  $n = 12$ ). What does this tell us?
- (c) For the CAPM, which matters more: statistical significance of alpha or economic magnitude of alpha? Justify your answer.

### Exercise 6: Okun's Law Extensions

- (a) If unemployment rises from 5% to 8% (+3 percentage points), what is the predicted change in GDP growth?
- (b) Why might the Okun's Law coefficient differ between 1961-1990 and 1991-2019? Suggest two structural changes.
- (c) Plot actual vs. predicted GDP growth for 2008-2010. Does Okun's Law track the financial crisis well?

### Exercise 7: Visualization Interpretation

- (a) In the CAPM scatter plot, what does vertical dispersion around the regression line represent? What does horizontal dispersion represent?
- (b) Sketch a hypothetical scatter plot where  $R^2 = 0.95$ . Sketch another where  $R^2 = 0.20$ . What's the visual difference?
- (c) For Okun's Law, why is a time series plot (actual vs. predicted over time) more informative than just reporting  $R^2$ ?

### Exercise 8: Comprehensive Case Study Analysis

Choose one dataset not covered in this chapter and conduct a complete bivariate regression analysis:

- (a) Formulate a clear research question and specify the model:  $Y = \beta_1 + \beta_2 X + u$
- (b) Load data, create scatter plot, estimate OLS regression with robust standard errors
- (c) Interpret the slope coefficient economically (with units and real-world meaning)
- (d) Test  $H_0: \beta_2 = 0$  and one additional hypothesis of your choice (e.g.,  $\beta_2 = 1.0$ )



- (e) Assess outliers: identify any, test robustness to exclusion, discuss implications
- (f) Write a 200-word summary suitable for a policy brief or executive summary

**Suggested datasets:**

- AED\_EARNINGS.DTA (education and earnings)
  - AED\_HOUSE.DTA (house prices and characteristics)
  - AED\_FISHING.DTA (recreational fishing demand)
  - AED\_REALGDPPC.DTA (GDP growth over time)
-