Untitled

Andreas C Charitos

20 2019

Subquestion 3-Posterior up to a constant proportionality and conditional probabilities

Subquestion 2-Posterior Prior Bayesian Model

we are given that $Y_i \sim N(\mu_i, \sigma^2)$ $i = 1, 2, ..., n$ $\overrightarrow{\mu} = (\mu_1, \mu_2, ..., \mu_n)$ $p(\mu_1) = 1$ $p(\mu_{i+1}, \mu_i \sim N(\mu_i, \sigma^2) \quad i = 1, 2, 3, ..n-1$

The pdf of the Y_i is given by the formula :

$$f(Y_i/\mu_i, \sigma^2) = \frac{1}{\sqrt{2}\pi\sigma^2} e^{-(\frac{(Y_i-\mu_i)^2}{2\sigma^2})}$$

Then the Likekihood is given by :

$$L(Y_1, Y_2, ..., Y_n/\mu_i, \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2}\pi\sigma^2} e^{-(\frac{(Y_i-\mu_i)^2}{\sigma^2})}$$

$$p(\overrightarrow{Y}/\overrightarrow{\mu}) \propto e^{-(\sum_{i=1}^{n} \frac{(Y_i-\mu_i)^2}{\sigma^2})}(1)$$

We now need to derive the prior which is given by

$$p(\overrightarrow{\mu}) = p(\mu_1)p(\mu_2/\mu_1)...p(\mu_n/\mu_{n-1}) = \prod_{i=1}^{n-1} \frac{1}{\sqrt{2}\pi\sigma^2} e^{(-\frac{(\mu_{i+1}-\mu_i)^2}{\sigma^2})}$$

$$p(\overrightarrow{\mu}) \propto e^{-(\sum_{i=1}^{n-1} \frac{(\mu_{i+1}-\mu_i)^2}{\sigma^2})}(2)$$

we know that

$$Posterior \propto Prior \;\; Likehihood$$

or

$$p(\overrightarrow{\mu}/\overrightarrow{Y}) \propto p(\overrightarrow{Y}/\overrightarrow{\mu}) \;\; p(\overrightarrow{\mu})$$

or

$$(1) * (2) = e^{(-\sum_{i=1}^{n-1} \frac{(\mu_{i+1}-\mu_i)^2}{2\sigma^2} - \sum_{i=1}^{n} \frac{(Y_i-\mu_i)^2}{2\sigma^2})}$$

Subquestion 3-Posterior up to a constant proportionality and conditional probabilities

We know that

$$p(x_j/x_1, x_2, x_3, ..., x_n) \propto \frac{p(x_1, x_2, x_3..., x_n)}{p(x_1, x_2, ..., x_{j-1}, x_{j+1}, ...x_n)}$$

using the previous formula we are going to find :

$$p(\mu_n/\overrightarrow{\mu_{-n}}/\overrightarrow{Y}) \propto \frac{p(\overrightarrow{\mu}/\overrightarrow{Y})}{p(\overrightarrow{\mu_{-n}}/\overrightarrow{Y})}$$

so we have

$$\frac{e^{(-\sum_{i=1}^{n-1}\frac{(\mu_{i+1}-\mu_i)^2}{2\sigma^2}-\sum_{i=1}^{n}\frac{(Y_i-\mu_i)^2}{2\sigma^2})}}{e^{(-\sum_{i=1}^{n-2}\frac{(\mu_{i+1}-\mu_i)^2}{2\sigma^2}-\sum_{i=1}^{n-1}\frac{(Y_i-\mu_i)^2}{2\sigma^2})}} =$$

$$e^{-\frac{1}{2\sigma^2}[(\mu_n-\mu_{n-1})^2+(Y_n-\mu_n)^2]} =^{(HINTB)}$$

$$e - \frac{(\mu_n - (\mu_{n-1} + Y_n)/2)^2}{2\sigma^2/2} \sim N(\frac{\mu_{n-1+Y_n}}{2}, \frac{\sigma^2}{2})$$

Next we need to find

$$p(\mu_i/\overrightarrow{\mu_{-i}}/\overrightarrow{Y}) \propto \frac{p(\overrightarrow{\mu}/\overrightarrow{Y})}{p(\overrightarrow{\mu_{-i}}/\overrightarrow{Y})}$$

so now we have

$$\frac{e^{(-\sum_{i=1}^{n-1}\frac{(\mu_{i+1}-\mu_i)^2}{2\sigma^2}-\sum_{i=1}^{n}\frac{(Y_i-\mu_i)^2}{2\sigma^2})}}{e^{(-\sum_{j=i}^{i-2}\frac{(\mu_{j+1}-\mu_j)^2}{2\sigma^2}-\sum_{j=i+1}^{n-1}\frac{(\mu_{j+1}-\mu_j)^2}{2\sigma^2}\sum_{j=1}^{i-1}\frac{(Y_j-\mu_j)^2}{2\sigma^2})-\sum_{j=i+1}^{n}\frac{(Y_j-\mu_j)^2}{2\sigma^2})}} =$$

$$e^{-\frac{1}{2\sigma^2}[(\mu_{i+1}-\mu_i)^2+(\mu_i-\mu_{i-1})^2+(Y_i-\mu_i)^2]} =^{(HINTC)}$$

$$e^{-\frac{(\mu_i-(\mu_{i-1}+Y_i+\mu_{i+1})/3)^2}{2\sigma^2/3}}$$

Using the same steps we obtain also that

$$p(\mu_1/\overrightarrow{\mu_{-1}}/\overrightarrow{Y}) \propto e^{-\frac{1}{2\sigma^2}[(\mu_2-\mu_1)^2+(Y_1-\mu_1)^2]} \propto e^{-\frac{(\mu_1-(\mu_2+Y_2)/2)^2}{2\sigma^2/2}}$$