Assignment 12

$\{X_t\} \sim (0,1)$ $\mu_t$: non constant function and $\sigma_t$: positive-non constant function

$y_t = \mu_t + \sigma_t X_t$

a)

- $E(y_t) = E(\mu_t + \sigma_t X_t) = E(\mu_t) + \cancel{E(X_t)}^{0} \cdot \sigma_t = E(\mu_t) = \mu_t$

- $Cov_{y_t}(t,h) = E\{(\mu_t + \sigma_t X_t - E(y_t)) \cdot (\mu_h + \sigma_h X_h - E(y_h))\} = E\{(\sigma_t X_t)(\sigma_h X_h)\} = \sigma_t \sigma_h \cdot E\{(X_t - \overset{0}{E(X_t)}) \cdot (X_h - \overset{0}{E(X_h)})\}$

$= \sigma_t \sigma_h \cdot Cov_X(t,h)$

$\overset{(E.1)}{=} \sigma_t \sigma_h \rho_X(h) \cdot \sqrt{\underset{1}{\sigma X_t} \cdot \underset{1}{\sigma X_h}}$

$= \sigma_t \sigma_h \rho_X(h)$.

we know that:

$\rho_X(h) = \dfrac{Cov_X(X_t, X_h)}{\sqrt{\sigma^2_{X_t} \cdot \sigma^2_{X_h}}} \Rightarrow$

$Cov_X(X_t, X_h) = \rho_X(h) \cdot \sqrt{\sigma^2_{X_t} \cdot \sigma^2_{X_h}}$ (E.1)

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b) $Corr_{y_t}(t,h) = \dfrac{Cov_{y_t}(t,h)}{\sqrt{\sigma^2_{y_t} \sigma^2_{y_h}}} \underset{(E.3)}{\overset{(E.2)}{=}} \dfrac{\sigma_t \sigma_h \rho_X(h)}{\sqrt{\sigma_t^2 \sigma_h^2}} = \rho_X(h)$.

so the ACF depends only on lag $h$

we know that

$\sigma^2_{y_t} = E\{(y_t - E y_t)^2\}$

$= E\{(\cancel{\mu_t} + \sigma_t X_t - \cancel{\mu_t})^2\}$

$= E\{(\sigma_t X_t)^2\} = \sigma_t^2 \cdot \underbrace{E\{(X_t - E X_t)^2\}}_{\sigma^2_{X_t} = 1} = \sigma_t^2$ (E.2)

with the same steps

$\sigma^2_{y_h} = \sigma_h^2$ (E.3).

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following from this assignment if we assume that $y_t = \sigma_t X_t$ then:

- $E y_t = 0$
- $Corr(y_t) = \rho_X(h)$
- $Cov(y_t, y_{t+h}) = \sigma_t \sigma_{t+h} \rho_X(h)$ $\rightsquigarrow$ depends on $t$ and $h$ so not stationary.

Assignment 18

c) $x_t - 3x_{t-1} = w_t + 2w_{t-1} - 8w_{t-2} \Rightarrow (1-3B)x_t = (1+2B+8B^2)w_t$

- $1 - 3z = 0 \Rightarrow$

$z = \frac{1}{3} < 1$

not casual

- $\overset{c}{1} + \overset{b}{2z} + \overset{a}{8z^2} = 0$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{4-4\cdot1(-8)}}{-16} = \frac{-2\pm\sqrt{4+32}}{-16} = \frac{-2\pm 6}{-16} \to \frac{1}{2} = z_1, \; -\frac{1}{4} = z_2$

$|z_1| = \frac{1}{2} < 1$

$|z_2| = \frac{1}{4} < 1$

not invertible

b) $x_t - 2x_{t-1} + 2x_{t-2} = w_t - \frac{8}{9}w_{t-1} \Rightarrow (1-2B+2B^2)x_t = (1-\frac{8}{9}B)w_t$

- $1 - 2z + 2z^2 = 0$

$\frac{2\pm\sqrt{4-4\cdot1\cdot2}}{2\cdot2} = \frac{2\pm\sqrt{4-8}}{2} \to \frac{1+i}{2}, \; \frac{1-i}{2}$

$|z_1| = \sqrt{(\frac{1}{2})^2 + (\frac{1}{2})^2} = \sqrt{\frac{1}{2}} = 0.7 < 1$

$|z_2| = \sqrt{(\frac{1}{2})^2 + (-\frac{1}{2})^2} = \sqrt{\frac{1}{2}} = 0.7 < 1$

not casual

- $1 - \frac{8}{9}z = 0 \Rightarrow$

$z = \frac{9}{8} = 1{,}125$

invertible

g) $x_t - 4x_{t-2} = w_t - w_{t-1} + 0.5w_{t-2} \Rightarrow (1-4B^2)x_t = (1-B+0.5B^2)w_t$

- $1 - 4z^2 = 0 \Rightarrow$

$z^2 = \frac{1}{4}$

$z = \frac{1}{2} < 1$ not casual

- $1 - z + 0.5z^2 = 0 \Rightarrow$

$\frac{1\pm\sqrt{1-4\cdot1\cdot 1/2}}{2\cdot 1/2} = \frac{1\pm\sqrt{-1}}{1} = 1 \pm i$

$|z_1| = \sqrt{1^2+1^2} = \sqrt{2} = 1.41 > 1$

$|z_2| = \sqrt{1^2+(-1)^2} = \sqrt{2} = 1.41 > 1$

invertible

) $x_t - \frac{9}{4}x_{t-1} - \frac{9}{4}x_{t-2} = w_t \Rightarrow (1 - \frac{9}{4}B + \frac{9}{4}B^2)x_t = w_t$

- $1 - \frac{9}{4}z - \frac{9}{4}z^2 = 0 -$

$\frac{\frac{9}{4} \pm \sqrt{\frac{81}{16} - 4\cdot1\cdot(-\frac{9}{4})}}{2\cdot(-9/4)} = \frac{\frac{9}{4} \pm \sqrt{\frac{81}{16}+9}}{-\frac{9}{2}}$

$= \frac{\frac{9}{4} \pm \frac{15}{4}}{-\frac{9}{2}} \to -\frac{\frac{24}{2}}{-\frac{9}{2}} = \frac{2\cdot24}{4\cdot9} = \frac{24}{18} = 1{,}33 = z_1$

$\to -\frac{\frac{6}{4}}{-\frac{9}{2}} = \frac{12}{36} = 0.33 = z_2$

$|z_1| = 1{,}33 > 1$

$|z_2| = 0.33 < 1$ not casual

- invertible