732A62 Time Series Analysis
Division of Statistics and Machine Learning
Department of Computer and Information Science

Teaching session I

Instructions

The assignments in the first section will be solved by the teacher during the teaching session. You are welcome to ask questions if you cannot follow the derivations. The problems in the second section are take home exercises and the key is given in section 3. The following assignments are hand-in and no solution is given in the key.

Assignment 12
Assignment 18

The hand-in assignment should be solved individually and should be submitted via LISAM in pdf format before the deadline also specified in LISAM.
The solutions are graded pass / insufficient. An insufficient solution can be completed and resubmitted.

1. Assignments solved by the teacher

Assignment 1

Suppose $E(X) = 2$, $var(X) = 9$, $E(Y) = 0$, $var(Y) = 4$, and $corr(X,Y) = 0.25$. Find:
(a) $var(X + Y)$.
(b) $cov(X, X + Y)$.

Assignment 2

Suppose $y_t = 5 + 2t + x_t$, where $\{x_t\}$ is a zero-mean stationary series with autocovariance function γ_k.
(a) Find the mean function for $\{y_t\}$.
(b) Find the autocovariance function for $\{y_t\}$.
(c) Is $\{y_t\}$ stationary? Why or why not?

Assignment 3

Suppose that $\{x_t\}$ is stationary with autocovariance function γ_k. Show that for any fixed positive integer n and any constants $c_1, c_2, \ldots, c_n$, the process $\{y_t\}$ defined by $y_t = \sum_{i=1}^{n} c_i x_{t-i+1}$ is stationary.

Assignment 4

Suppose that $x_t = w_t - w_{t-12}$. Show that $\{x_t\}$ is stationary and that, for $k > 0$, its autocorrelation function is nonzero only for lag $k = 12$.

Assignment 5

Suppose $x_t = \mu + w_t - w_{t-1}$. Find $var(\bar{x})$. Note any unusual results. In particular, compare your answer to what would have been obtained if $x_t = \mu + w_t$.

Assignment 6

Calculate and sketch the autocorrelation functions for AR(1) model with $\phi = 0.6$. Plot for sufficient lags that the autocorrelation function has nearly died out.

Assignment 7

Let $\{x_t\}$ be an AR(2) process $x_t = \phi x_{t-2} + w_t$ Find the range of values of ϕ for which the process is causal.

Assignment 8

For each of the following ARMA models, find the roots of the AR and MA polynomials, identify the values of p and q for which they are ARMA(p,q) (be careful of parameter redundancy), determine whether they are causal, and determine whether they are invertible. In each case, $w_t \sim wn(0,1)$.
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a) $x_t + 0.81x_{t-2} = w_t + 1/3w_{t-1}$
b) $x_t - x_{t-1} = w_t - 0.5w_{t-1} - 0.5w_{t-2}$

Assignment 9

For those the following model, compute the first four coefficients $\psi_0, \dots \psi_3$ in the causal linear process representation $x_t = \sum_{j=0}^{\infty} \psi_j w_{t-j}$

a) $x_t + 0.81x_{t-2} = w_t + 1/3w_{t-1}$

2. Take home assignments

Assignment 10

Suppose $E(X) = 2$, $Var(X) = 9$, $E(Y) = 0$, $Var(Y) = 4$, and $Corr(X,Y) = 0.25$. Find:
(a) $Corr(X + Y, X - Y)$.

Assignment 11

Let $\{w_t\}$ be a zero mean white noise process. Suppose that the observed process is $x_t = w_t + \theta w_{t-1}$, where θ is either 3 or 1/3.

(a) Find the autocorrelation function for $\{x_t\}$ both when θ = 3 and when θ = 1/3.
(b) You should have discovered that the time series is stationary regardless of the value of θ and that the autocorrelation functions are the same for θ = 3 and θ =1/3. For simplicity, suppose that the process mean is known to be zero and the variance of y_t is known to be 1. You observe the series $\{y_t\}$ for $t = 1, 2,..., n$ and suppose that you can produce good estimates of the autocorrelations ρ_k. Do you think that you could determine which valueθ is correct (3 or 1/3) based on the estimate of ρ_k? Why or why not?

Assignment 12

Let $\{x_t\}$ be a zero-mean, unit-variance stationary process with autocorrelation function ρ_h. Suppose that μ_t is a nonconstant function and that σ_t is a positive-valued nonconstant function. The observed series is formed as $y_t = \mu_t + \sigma_t X_t$.
(a) Find the mean and covariance function for the $\{y_t\}$ process.
(b) Show that the autocorrelation function for the $\{y_t\}$ process depends only on the time lag. Is the $\{y_t\}$ process stationary?
(c) Is it possible to have a time series with a constant mean and with $Corr(y_t, y_{t+h})$ free of t but with $\{y_t\}$ *not* stationary?

Assignment 13

Suppose that x is a random variable with zero mean. Define a time series by
$y_t = (-1)^t x$
(a) Find the mean function for $\{y_t\}$.
(b) Find the autocovariance function for $\{y_t\}$.
(c) Is $\{y_t\}$ stationary?

Assignment 14

Suppose $x_t = \mu + w_t + w_{t-1}$. Find $var(\bar{x})$. Note any unusual results. In particular, compare your answer to what would have been obtained if $x_t = \mu + w_t$.

Assignment 15

Calculate and sketch the autocorrelation function for MA(2) model with $\theta_1 = 0.5$. and $\theta_2 = 0.4$

Assignment 16

Describe the important characteristics of the autocorrelation function for the following models: (a) MA(1), (b) MA(2), (c) AR(1), (d) AR(2), and (e) ARMA(1,1).

Assignment 17

Suppose that $\{x_t\}$ is an AR(1) process with $-1 < \varphi < +1$.
(a) Find the autocovariance function for $y_t = \nabla x_t = x_t - x_{t-1}$ in terms of φ and σ_w^2
(b) In particular, show that $var(y_t) = \frac{2\sigma_w^2}{1+\phi}$

Assignment 18

For each of the following ARMA models, find the roots of the AR and MA polynomials, identify the values of *p* and *q* for which they are ARMA(p,q) (be careful of parameter redundancy), determine whether they are causal, and determine whether they are invertible. In each case, $w_t \sim wn(0,1)$.
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c) $x_t - 3x_{t-1} = w_t + 2w_{t-1} - 8w_{t-2}$
d) $x_t - 2x_{t-1} + 2x_{t-2} = w_t - \frac{8}{9}w_{t-1}$
e) $x_t - 4x_{t-2} = w_t - w_{t-1} + 0.5w_{t-2}$
f) $x_t - \frac{9}{4}x_{t-1} - \frac{9}{4}x_{t-2} = w_t$

Assignment 19

For the following models, compute the first four coefficients $\psi_0, \ldots \psi_3$ in the causal linear process representation $x_t = \sum_{j=0}^{\infty} \psi_j w_{t-j}$

a) $x_t - 2x_{t-1} + 2x_{t-2} = w_t - \frac{8}{9}w_{t-1}$
b) $x_t - \frac{9}{4}x_{t-1} - \frac{9}{4}x_{t-2} = w_t - 3w_{t-1} + \frac{1}{9}w_{t-2} - \frac{1}{3}w_{t-3}$

3. Key

Assignment 10

Approximately 0.39

Assignment 11

$\rho(0) = 1, \rho(1) = 0.3, \rho(h) = 0 \ otherwise$

Assignment 13

a) 0
b) $(-1)^h \sigma_x^2$
c) Yes

Assignment 14

$var(\bar{x}) = \frac{2(2n-1)}{n^2}\sigma_w^2$

Assignment 15

$\rho_1 \approx 0.5, \rho_2 \approx 0.28, \rho_i = 0, i > 2$

Assignment 16

a) $-\frac{1-\phi}{1+\phi}\phi^{h-1}\sigma_w^2$

Assignment 17

a) p=1,q=2, neither causal or invertible
b) p=2,q=1, invertible, but not causal
c) p=2,q=2, invertible, but not causal
d) p=2, q=0, invertible, not causal

Assignment 19

a)1, 10/9, 2/9, -16/9

b) 1, -3/4, 1/9+9/16, -1/12-27/64