Time Series Analysis

Teaching Session II: ARIMA models-3
Seasonal models

Tohid Ardeshiri

Linköping University
Division of Statistics and Machine Learning

September 18, 2019

LINKÖPING UNIVERSITY

Seasonal ARMA

- Seasonal patterns
  - Yearly (ocean temperature)
  - Daily, weekly (Server workload)
- Strong correlation of x_t and x_{t+s}
  - $s = 12, 24, \ldots$
- Applications
  - Physics, biology, economics, computer science

Seasonal ARMA

- Pure seasonal $ARMA(P,Q)_s$

$$\Phi_P(B^s)x_t = \theta_Q(B^s)w_t$$

- Seasonal autoregressive operator

$$\Phi_P(B^s) = 1 - \Phi_1(B^{(1\cdot s)}) - \ldots - \Phi_P B^{P\cdot s}$$

- Seasonal moving average operator

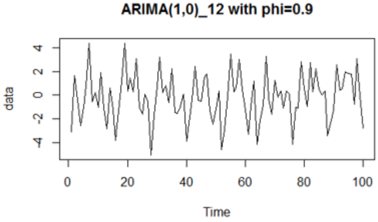
$$\Theta_Q(B^s) = 1 + \Theta_1(B^{(1\cdot s)}) + \ldots + \Theta_Q B^{Q\cdot s}$$

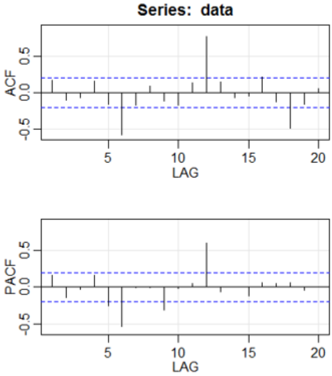
- Same principles for causality and invertibility

- Example: $ARMA(1,0)_{12}$ and $ARMA(0,1)_{12}$
  - Autocovariance

Seasonal ARMA

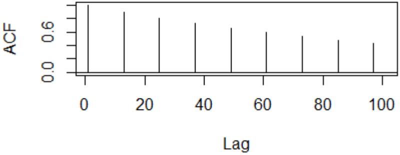
- Example: Simulated $ARMA(1,0)_{12}, \Phi = 0.9$

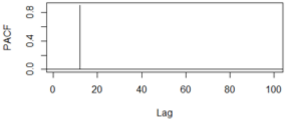




Seasonal ARMA

- Example: Simulated $ARMA(1,0)_{12}, \Phi = 0.9$
  - Theoretical ones





Seasonal ARMA

	$AR(P)_s$	$MA(Q)_s$	$ARMA(P,Q)_s$
ACF*	Tails off at lags ks, $k = 1, 2, \ldots,$	Cuts off after lag Qs	Tails off at lags ks
PACF*	Cuts off after lag Ps	Tails off at lags ks $k = 1, 2, \ldots,$	Tails off at lags ks

*The values at nonseasonal lags $h \neq ks$, for $k = 1, 2, \ldots,$ are zero.

Multiplicative seasonal ARMA

- Problem: in real data, it is hard to assume x_t is dependent on x_{t-kh} only...
  - Combinal seasonal and nonseasonal!

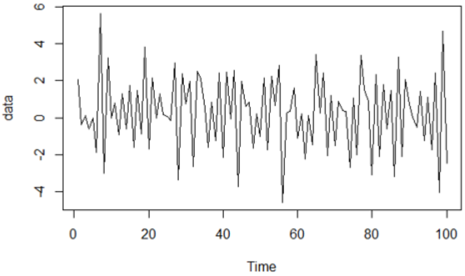
- Multiplicative Seasonal $ARMA(p, q)\ x\ (P, Q)_s$

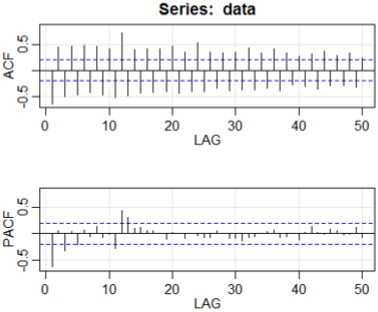
$$\Phi_P(B^s)\phi(B)x_t = \Theta_Q(B^s)\theta(B)w_t$$

- Example Expression for $ARMA(1, 1)\ x\ (1, 0)_s$

Multiplicative seasonal ARMA

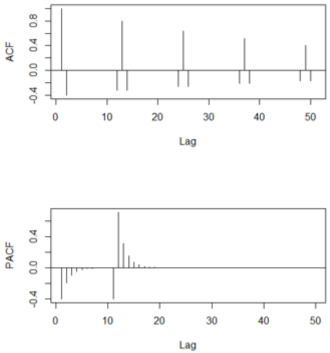
- Example $x_t = 0.8x_{t-12} + w_t - 0.5w_{t-1}$





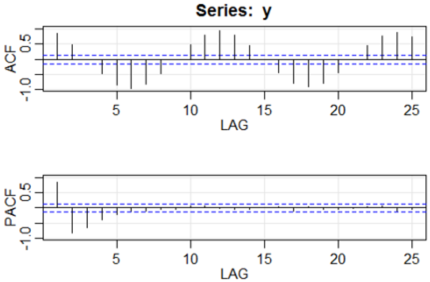
Multiplicative seasonal ARMA

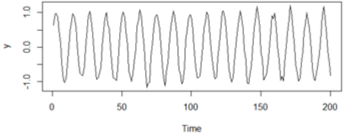
- Example $x_t = 0.8x_{t-12} + w_t - 0.5w_{t-1}$
  - Theoretical



SARIMA

- What if there is a seasonal pattern which differs a little between the series





Note: ACF almost decays very slowly at peaks 12h

SARIMA

- Multiplicative seasonal autoregressive integrated moving average model $ARIMA(p, d, q)$ x $(P, D, Q)_s$

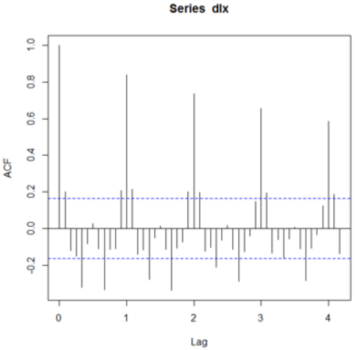
$$\Phi_p(B^s)\phi(B)\nabla_s^D\nabla^d x_t = \delta + \Theta_Q(B^s)\theta(B)w_t$$

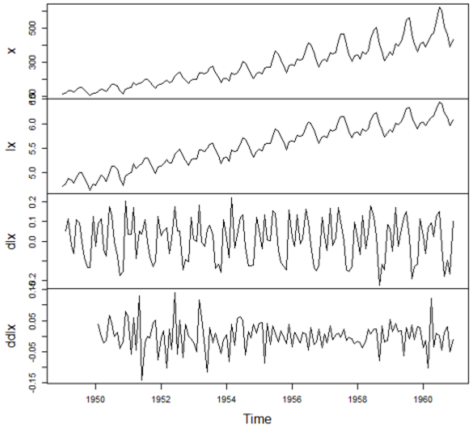
$$\nabla_s^D = (1 - B^s)^D$$

- How to identify SARIMA?
  1. Perform differencing first (trend)
  2. Investigate ACF $\rightarrow$ slowly decays at peaks?
     1. Yes $\rightarrow$ Additional differencing by ∇_s^D
  3. Model non-seasonal part
  4. Model seasonal part (check peaks), check ACF and PACF of residuals

SARIMA

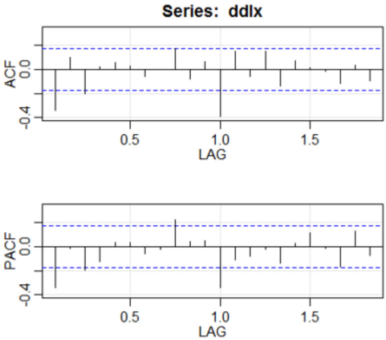
- Example: Air passangers





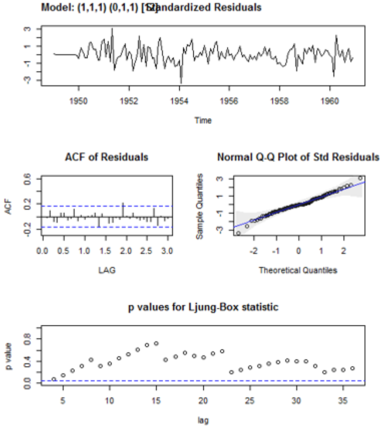
SARIMA

- Example: Air passangers

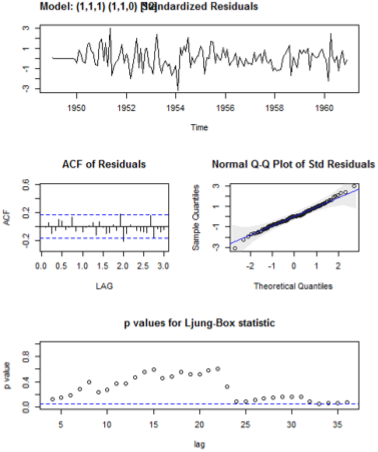


$$(0,1,1)_{12} \text{ or } (1,1,0)_{12}$$

SARIMA



Model: (1,1,1) (0,1,1) [12] Standardized Residuals
Time
ACF of Residuals
ACF
LAG
Normal Q-Q Plot of Std Residuals
Sample Quantiles
Theoretical Quantiles
p values for Ljung-Box statistic
p value
lag



Model: (1,1,1) (1,1,0) [12] Standardized Residuals
Time
ACF of Residuals
ACF
LAG
Normal Q-Q Plot of Std Residuals
Sample Quantiles
Theoretical Quantiles
p values for Ljung-Box statistic
p value
lag

SARIMA

- Remove AR term!

Is one model much better the other one?

$(1,1,1)\ x\ (1,1,0)_{12}$

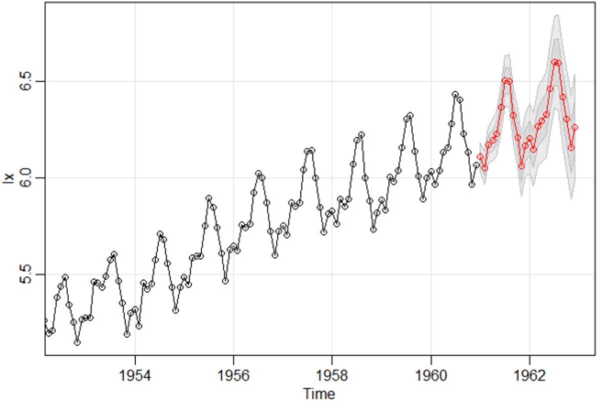
```
> m1$fit

Call:
stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
    Q), period = S), include.mean = !no.constant, optim.control = list(trace = trc,
    REPORT = 1, reltol = tol))

Coefficients:
         ar1      ma1     sar1
      0.0547  -0.4886  -0.4731
s.e.  0.2161   0.1933   0.0800

sigma^2 estimated as 0.001425:  log likelihood = 241.73,  aic = -475.47
```

$(1,1,1)\ x\ (0,1,1)_{12}$

```
> m2$fit

Call:
stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
    Q), period = S), include.mean = !no.constant, optim.control = list(trace = trc,
    REPORT = 1, reltol = tol))

Coefficients:
         ar1      ma1     sma1
      0.1960  -0.5784  -0.5643
s.e.  0.2475   0.2132   0.0747

sigma^2 estimated as 0.001341:  log likelihood = 244.95,  aic = -481.9
```

SARIMA

- Forecasting



Read home

- Shumway and Stoffer, section 3.9

- R code: sarima, sarima.for, runs