for the general case (E.1). following the same steps.

$$r = \max\left((P \cdot s) + p + (s \cdot D) + d,\ (Q \cdot s + q) + 1 \right).$$

$$z_t = \vartheta^r(B)\, z_t \qquad \text{and} \qquad \varphi^r(B)\, z_t = w_t$$

$$z_t = \begin{bmatrix} z_t \\ z_{t-1} \\ \vdots \\ \vdots \\ z_{t-(r-1)} \end{bmatrix}_{(r \times 1)} \qquad \text{and} \quad z_t = \underbrace{\begin{bmatrix} \varphi_1 & \varphi_2 & \cdots & \varphi_r \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}}_{(r \times r)} z_{t-1} + \begin{bmatrix} w_t \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}_{(r \times 1)}.$$

$$x_t = [1\ \vartheta_1\ \vartheta_2\ \cdots\ \vartheta_{r-1}]\, z_t$$

we manage to decompose the $ARIMA(p,d,q) \times (P,D,Q)_s$ to an <u>AR</u> process.