Teaching Session II

Assignment 1

we are asked to find the state space representation of the $ARIMA(p,d,q)\times(P,D,Q)_s$.

$$\phi^{P}(B^s)\varphi^{p}(B)(1-B^s)^D(1-B)^d x_t = \theta^{Q}(B^s)\vartheta^{q}(B) w_t. \quad (E.1)$$

we start from the simple case with $p=3, d=2, q=1, P=2, D=1, Q=1, s=5$.

the model is then:

$$\phi^2(B^5)\ \varphi^3(B)\,(1-B^5)^1\,(1-B)^2 x_t = \theta^1(B^5)\,\vartheta^1(B)\,w_t$$

where :

- $\phi^2(B^5) = 1 - \phi_1 B^5 - \phi_2 B^{10}$
- $\varphi^3(B) = 1 - \varphi_1 B - \varphi_2 B^2 - \varphi_3 B^3$
- $\theta^1(B^5) = 1 - \theta_1 B^5$
- $\vartheta^1(B) = 1 - \vartheta_1 B$

the higher order of the : $\phi^2(B^5)\cdot\varphi^3(B)\cdot(1-B^5)^1\cdot(1-B)^2$ or $(1-\phi_1 B^5 - \phi_2 B^{10})\cdot(1-\varphi_1 B - \varphi_2 B^2 - \varphi_3 B^3)\cdot(1-B^5)^1(1-B)^2$

is $\tilde{p} = 20$.

the higher order of the : $\theta^1(B^5)\cdot\vartheta^1(B)$ or $(1-\theta_1 B^5)\cdot(1-\vartheta_1 B)$

is $\tilde{q} = 6$.

so $r = \max(\tilde{p}, \tilde{q}+1) = \max(20, 7) = 20$.

Hence :

$x_t = \vartheta^r(B)\cdot z_t$ and $\varphi^r(B) z_t = w_t$.

$\varphi^{20}(B) = 1 - \varphi_1 B - \dots - \varphi_{20} B^{20}$

$\vartheta^{r}(B) = 1 + \vartheta_1 B + \dots + \vartheta_{19} B^{19}$

so $$Z_t = \begin{bmatrix} z_t \\ z_{t-1} \\ z_{t-2} \\ \vdots \\ z_{t-19} \end{bmatrix}_{(20\times 1)} \quad \text{and} \quad Z_t = \begin{bmatrix} \varphi_1 & \varphi_2 & \cdots & \varphi_{20} \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 1 & 0 \end{bmatrix}_{(20\times 20)} \cdot Z_{t-1} + \begin{bmatrix} w_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{(20\times 1)}$$

$$x_t = [1\ \vartheta_1\ \vartheta_2\ \cdots\ \vartheta_{19}]\, Z_t$$