Assignment 12

$\{X_t\} \sim (0,1)$ μ_t: non constant function and σ_t: positive-non constant function

$Y_t = \mu_t + \sigma_t X_t$

a)

- $E(y_t) = E(\mu_t + \sigma_t X_t) = E(\mu_t) + \cancel{E(X_t)}^{0} \cdot \sigma_t = E(\mu_t) = \mu_t$

- $Cov_{y_t}(t,h) = E\{(\mu_t + \sigma_t X_t - E(y_t)) \cdot (\mu_h + \sigma_h X_h - E(y_h))\} = E\{(\sigma_t X_t)(\sigma_h X_h)\} = \sigma_t \sigma_h \cdot E\{(X_t - \overset{0}{E(X_t)}) \cdot (X_h - \overset{0}{E(X_h)})\}$

$= \sigma_t \sigma_h \cdot Cov_X(t,h)$

$\overset{(E.1)}{=} \sigma_t \sigma_h \rho_X(h) \cdot \sqrt{\underbrace{\sigma_{X_t}}_{1} \cdot \underbrace{\sigma_{X_h}}_{1}}$

$= \sigma_t \sigma_h \rho_X(h).$

we know that:

$$\rho_X(h) = \frac{Cov_X(X_t, X_h)}{\sqrt{\sigma_{X_t}^2 \cdot \sigma_{X_h}^2}} \Rightarrow$$

$$Cov_X(X_t, X_h) = \rho_X(h) \cdot \sqrt{\sigma_{X_t}^2 \cdot \sigma_{X_h}^2} \; \underline{(E.1)}$$

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b) $$Corr_{y_t}(t,h) = \frac{Cov_{y_t}(t,h)}{\sqrt{\sigma_{y_t}^2 \sigma_{y_h}^2}} \overset{(E.2)}{\underset{(E.3)}{=}} \frac{\sigma_t \sigma_h \rho_X(h)}{\sqrt{\sigma_t^2 \sigma_h^2}} = \rho_X(h).$$

so the ACF depends only on lag h

we know that

$\sigma_{y_t}^2 = E\{(y_t - E y_t)^2\}$

$= E\{(\cancel{\mu_t} + \sigma_t X_t - \cancel{\mu_t})^2\}$

$= E\{(\sigma_t X_t)^2\} = \sigma_t^2 \cdot \underbrace{E\{(X_t - E X_t)^2\}}_{\sigma_{X_t}^2 = 1} = \sigma_t^2 \; (E.2)$

with the same steps

$\sigma_{y_h}^2 = \sigma_h^2 \; (E.3).$

<u>following from this assignment if we assume that $y_t = \sigma_t X_t$ then:</u>

- $E y_t = 0$
- $Corr(y_t) = \rho_X(h)$
- $Cov(y_t, y_{t+h}) = \sigma_t \sigma_{t+h} \rho_X(h) \rightsquigarrow$ depends on t and h so not stationary.

Assignment 18

a) $x_t - 3x_{t-1} = w_t + 2w_{t-1} - 8w_{t-2} \Rightarrow (1-3B)x_t = (1+2B+8B^2)w_t$

- $1-3z=0 \Rightarrow$

$z = \frac{1}{3} < 1$

not casual

- $\overset{\gamma}{1} + \overset{\beta}{2z} + \overset{\alpha}{8z^2} = 0$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{4-4\cdot1(-8)}}{-16} = \frac{-2\pm\sqrt{4+32}}{-16} = \frac{-2\pm 6}{-16}$ → $\frac{1}{2} = z_1$, $-\frac{1}{4} = z_2$

$|z_1| = \frac{1}{2} < 1$

$|z_2| = \frac{1}{4} < 1$

not invertible

b) $x_t - 2x_{t-1} + 2x_{t-2} = w_t - \frac{8}{9}w_{t-1} \Rightarrow (1-2B+2B^2)x_t = (1-\frac{8}{9}B)w_t$

- $1-2z+2z^2=0$

$\frac{2\pm\sqrt{4-4\cdot1\cdot2}}{2\cdot2} = \frac{2\pm\sqrt{4-8}}{2}$ → $\frac{1+i}{2}$, $\frac{1-i}{2}$

$|z_1| = \sqrt{(\frac{1}{2})^2+(\frac{1}{2})^2} = \sqrt{\frac{1}{2}} = 0.7 < 1$

$|z_2| = \sqrt{(\frac{1}{2})^2+(-\frac{1}{2})^2} = \sqrt{\frac{1}{2}} = 0.7 < 1$

not casual

- $1 - \frac{8}{9}z = 0 \Rightarrow$

$z = \frac{9}{8} = 1{,}125$

invertible

g) $x_t - 4x_{t-2} = w_t - w_{t-1} + 0.5w_{t-2} \Rightarrow (1-4B^2)x_t = (1-B+0.5B^2)w_t$

- $1-4z^2=0 \Rightarrow$

$z^2 = \frac{1}{4}$

$z = \frac{1}{2} < 1$ not casual

- $1 - z + 0.5z^2 = 0 \Rightarrow$

$\frac{1\pm\sqrt{1-4\cdot1\cdot\frac{1}{2}}}{2\cdot\frac{1}{2}} = \frac{1\pm\sqrt{-1}}{1} = 1\pm i$

$|z_1| = \sqrt{1^2+1^2} = \sqrt{2} = 1.41 > 1$

$|z_2| = \sqrt{1^2+(-1)^2} = \sqrt{2} = 1.41 > 1$

invertible

) $x_t - \frac{9}{4}x_{t-1} - \frac{9}{4}x_{t-2} = w_t \Rightarrow (1-\frac{9}{4}B + \frac{9}{4}B^2)x_t = w_t$

- $1 - \frac{9}{4}z - \frac{9}{4}z^2 = 0 -$

$\frac{\frac{9}{4} \pm \sqrt{\frac{81}{16} - 4\cdot1\cdot(-\frac{9}{4})}}{2\cdot(-\frac{9}{4})} = \frac{\frac{9}{4} \pm \sqrt{\frac{81}{16}+9}}{-\frac{9}{2}}$

$= \frac{\frac{9}{4}\pm\frac{15}{4}}{-\frac{9}{2}}$ → $\frac{\frac{24}{2}}{-\frac{9}{2}} = \frac{2\cdot24}{4\cdot9} = \frac{24}{18} = 1{,}33 = z_1$; $\frac{\frac{6}{4}}{-\frac{9}{2}} = \frac{12}{36} = 0.33 = z_2$

$|z_1| = 1{,}33 > 1$

$|z_2| = 0.33 < 1$ not casual

- invertible