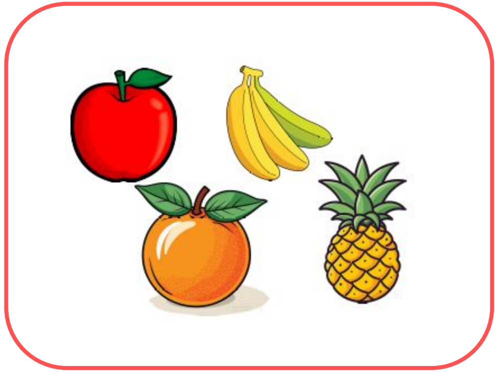
Week 8 Lecture: Semantics

Linguistics 201 – Fall 2025

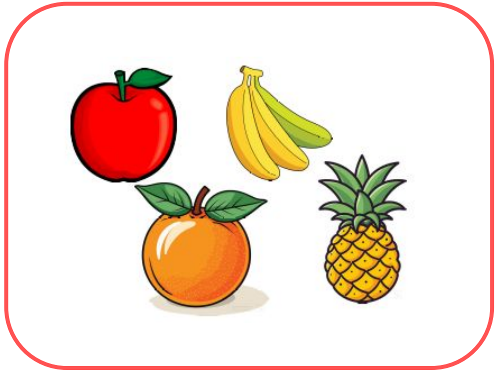
Sense and reference

- Last week we talked about defining the meaning of words by **sense** and by **reference**:
  - A word's **sense** is the mental concept of the word's meaning or a dictionary definition of the word's meaning.
  - A word's **reference** is the set of entities (**referents**) that the word refers to.
- For example, consider the word *university*:
  - **Sense**: an institution of higher education
  - **Reference**: Rutgers, NYU, CUNY, Harvard, MIT, Penn, Penn State, Maryland, Johns Hopkins, UVA, Virginia Tech…



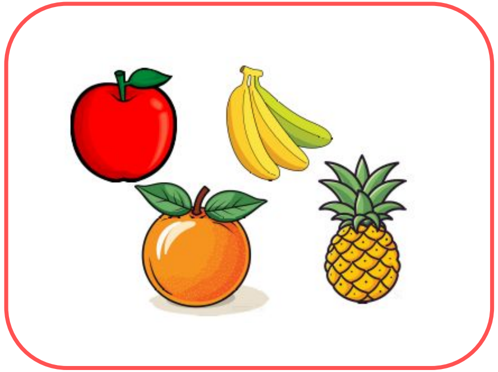
Reference

- In this class, we're going to focus on defining words by **reference**
- As we saw last week, reference helps us define relationships between words such as:
  - synonyms
  - hyponyms
  - hypernyms
- All of which have an impact on a sentence's truth values and entailment.



Set theory

- Each word has a given **set** of **referents**.
- In order to talk precisely about sets of items, we're going to use a field of mathematics called **set theory**.
- With set theory, we can more precisely:
  - Define which elements are in a set (*what referents a word has*)
  - Compare two sets to look for common elements (*whether two words have referents in common*)
  - Perform operations on sets to combine and manipulate them (*which will help us define the meaning of sentences*)



Set theory: Basic concepts

Set theory: Sets and elements

- First we need to define a **set** and what it contains.
- A **set** is a collection of items:
  - {Mon, Tue, Wed, Thu, Fri, Sat, Sun} is the **set** of days of the week
- The individual items in a set are called **members** or **elements**:
  - Thursday is a **member** or **element** of the set of days of the week
- The symbols $\in$ and $\notin$ are used to specify whether an element is a member of a given set:
  - Mon $\in$ {Mon, Tue, Wed, Thu, Fri, Sat, Sun}
  - Oct $\notin$ {Mon, Tue, Wed, Thu, Fri, Sat, Sun}

Set theory: Sets and elements

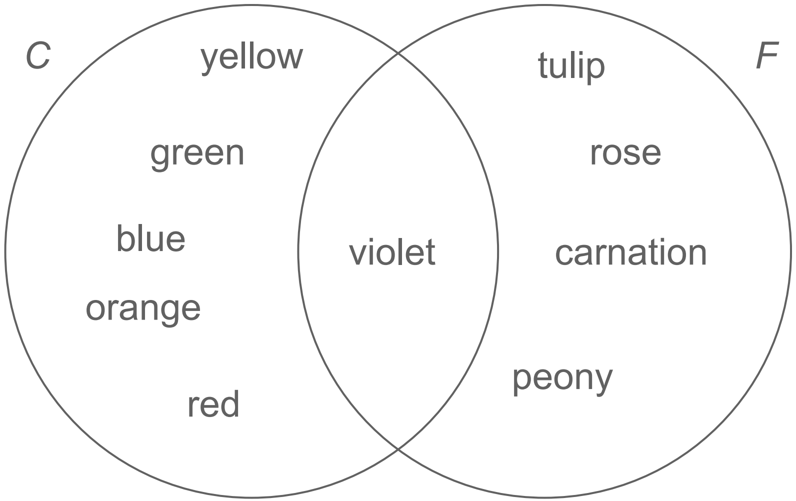
- As in other branches of mathematics, we can use variables to stand in for values:
  - A = {Mon, Tue, Wed, Thu, Fri, Sat, Sun} defines variable A as the set of days
- By convention, we use capital letters (A, B, C) for sets and lowercase letters (x, y, z) for elements.
- We can first assign various elements to variables and then define sets based on them:
  - a = Tom b = Olivia c = Tania d = Jin
  - F = {Tom, Olivia, Tania} G = {Olivia, Tania, Jin}
- We can also define sets in terms of their natural language meaning.
  - D = [[days of the week]] [[Monday]] $\in D$
  - We put [[ ]] around *natural language* expressions to show we're talking about their *meaning*, not their structure, morphology, or sounds. We don't need [[ ]] around set theory expressions.

Set theory: Properties of sets

- Some important **properties** of sets are:
  - Order of elements doesn't matter:
    - {a, e, i, o, u} = {u, o, i, e, a}
  - Repeating doesn't matter – an element is either in the set or not, it can't be in it twice:
    - {a, e, i, o, u} = {a, a, a, e, e, i, o, o, o, o, u, u}
  - Sets can be finite or infinite:
    - infinite: set of positive integers = {1, 2, 3…}
    - finite: set of days of the week = {Mon, Tue, Wed, Thu, Fri, Sat, Sun}
  - Members don't have to be relevant to each other – though in Semantics, they usually will be:
    - {u, 3, Wed, blue} is a legitimate set

Set theory: Specifying a set

- **List notation**: we can specify a set by listing its members between curly brackets:
  - C = {red, orange, yellow, green, blue, violet}
  - F = {tulip, rose, violet, carnation, peony}
- **Venn diagram**: we can specify a set by placing its members in a circle:



Venn diagrams are especially useful for illustrating where sets overlap (what members they have in common):

violet $\in C$ <u>and</u> violet $\in F$

Set theory: Specifying a set

- Sometimes we can't list all the members (we don't know them, there are too many)
- Can you list all the stars in the universe? *Sun, Polaris, Proxima Centauri, Betelgeuse, Rigel…*
- We can use **predicate notation** to define such a set:
  - Set of all stars: $S = \{ \underbrace{x}_{\text{variable standing in for the set's members}} \; \underbrace{|}_{\text{"such that"}} \; \underbrace{x \text{ is a star}}_{\text{condition elements have to fulfill in order to be members}} \}$

We read this out loud as: *S equals the set of elements x, such that x is a star*

This is more or less the same as saying *the set of all stars* or *the set of all referents of the word* "star"

Set theory: Cardinality

- **Cardinality** refers to the number of elements in a set (think **cardinal** numbers).
- We use vertical bars $|A|$ to indicate cardinality. (Same symbol as absolute value, but in set theory it means cardinality, not absolute value. $|A|$ is pronounced: *cardinality of A*, not ~~*absolute value of A*~~)
  - Suppose C = { red, orange, yellow, green, blue, violet }.
  - What is $|C|$?
- A set with only one element (cardinality = 1) is called a **singleton set**:
  - {red}, {Wednesday}, {Neptune}, {3}, {Snoopy}
- A set with no elements (cardinality = 0) is called the **null set** or the **empty set**:
  - We can write this as either {} or Ø.
  - We don't write a null set as {Ø}. Why not?

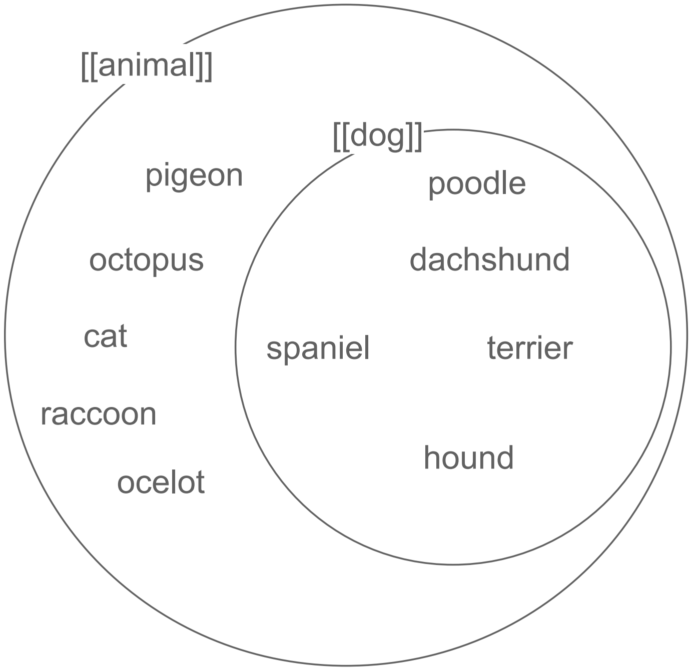
Words and sets

- Now that we know how to define a set, we can use it to define the meaning of a word.
- Recall that we're trying to define words in terms of their **reference**: the set of things they refer to.
- We can define the reference of a word by:
  - list notation: [[bird]] = {goose, duck, sparrow, eagle…}
  - predicate notation: [[bird]] = {*x* | *x* is a bird}
- Remember that we put natural language expressions like words in [[ ]]:
  - This shows we're talking about the *meaning* of the word, not its structure.

Set relations

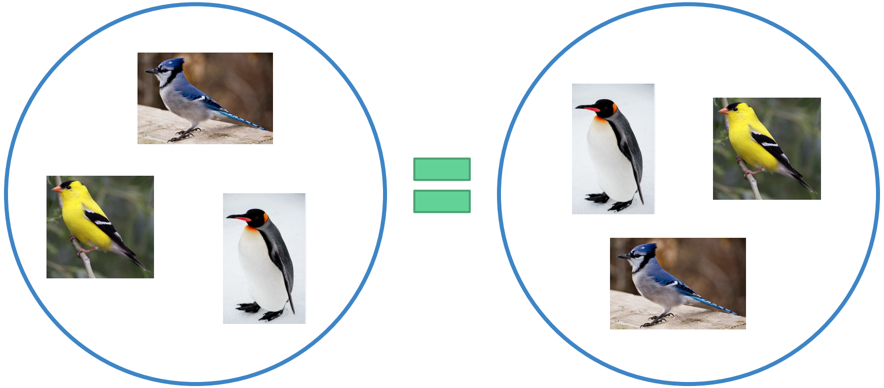
Sets

- The reference of words can **overlap** in various ways.
- In order to compare two sets, we need **set relations**:
    - *equality (identity)*
    - *subset*
    - *proper subset*



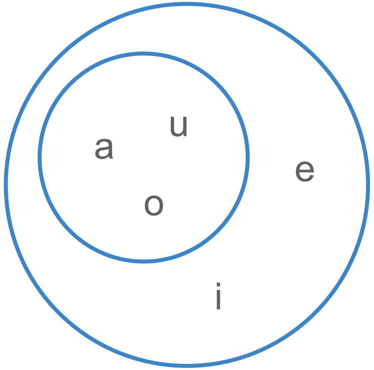
Set relations: Equality/Identity

- **Equality/Identity symbol**: $A = B$ (not equal/identical symbol: $A \neq B$)
  - Two sets are **equal** or **identical** if they have exactly the same elements:
    - { a, e, i, o, u } = { i, e, a, u, o }
    - { x | x is a bird } = { x | x is a winged, feathered animal that lays eggs }
  - Formally: $A = B$ iff for every x: $x \in A$ iff $x \in B$



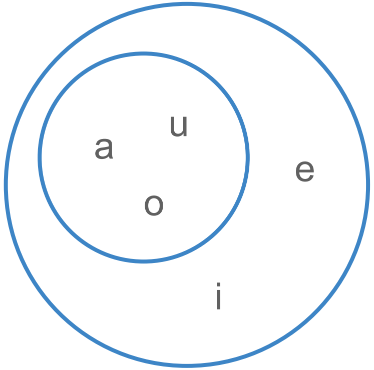
Set relations: Subset

- **Subset symbol**: $A \subseteq B$ (not a subset symbol: $A \nsubseteq B$)
  - A is a subset of B if every element of A is also an element of B:
    - { a, o, u } $\subseteq$ { a, e, i, o, u }
    - Formally: $A \subseteq B$ iff for every x: if $x \in A$ then $x \in B$
  - If A contains anything that's not in B, it's not a subset of B:
    - { a, o, y } $\nsubseteq$ { a, e, i, o, u }
  - Note: If A and B are identical, A is technically still a subset of B, because every element in A is also in B:
    - { a, e, i, o, u } $\subseteq$ { a, e, i, o, u }



Set relations: Proper subset

- **Proper subset symbol**: $A \subset B$ (not a proper subset symbol: $A \not\subset B$)
  - A is a proper subset of B if A is a subset of B and they are not identical:
    - $\{ a, o, u \} \subset \{ a, e, i, o, u\}$
    - $\{ a, e, i, o, u \} \not\subset \{ a, e, i, o, u\}$
  - Formally: $A \subset B$ iff $A \subseteq B$ and $A \neq B$
- In natural language, we typically use subsets ($\subseteq$) rather than proper subsets ($\subset$) because they're more flexible.



Comprehension check

Are the following statements true or false?

1. $B \subset A$
2. $B = C$
3. $B \subseteq C$
4. $B \subset C$
5. $B \subseteq D$

$A = \{$Mary, Peter, Ivan, Josefina, Antonio$\}$

$B = \{$Peter, Ivan, Antonio$\}$

$C = \{$Antonio, Peter, Ivan$\}$

$D = \{$Peter, Mary, Ivan, Josefina$\}$

Comprehension check

Are the following statements true or false?

1. $B \subset A$
2. $B = C$
3. $B \subseteq C$
4. $B \subset C$
5. $B \subseteq D$

1. True. All elements in B are also in A.
2. True. Order does not matter.
3. True. If B = C, B is also a subset of C.
4. False. To be a proper subset, there must be elements in C but not in B.
5. False. There are elements in B that are not in D.

A = {Mary, Peter, Ivan, Josefina, Antonio}

B = {Peter, Ivan, Antonio}

C = {Antonio, Peter, Ivan}

D = {Peter, Mary, Ivan, Josefina}

Set relations in natural language

Set relations in natural language

- Now that we know how to talk about **set relations** (equality, subset), we can return to our discussion of **synonyms**, **hyponyms**, and **hypernyms**.
- Are the following pairs of words synonyms or hypernyms/hyponyms? What is the set relation between them? How would you write that out using set theory notation (=, ⊆)
    - For example: *cats* vs. *felines*   [[cats]] = [[felines]]
    - *drink* vs. *beverage*
    - *squirrel* vs. *mammal*
    - *sparrow* vs. *bird*
    - *soft drink* vs. *soda*
    - *beverage* vs. *juice*

Set relations in natural language

- **Synonyms**
  - Have the same set of referents: *drink* and *beverage* refer to the same set of things
  - So the reference of **synonyms** is **equal/identical**: [[cat]] = [[feline]], [[drink]] = [[beverage]]
  - So the reference of a **hyponym** is a **proper subset** of the reference of its **hypernym**
- For a given **hypernym** and **hyponym** pair:
  - Every referent of the hyponym will also be a referent of the hypernym:
    [[Snoopy]] is a member of [[dog]] and also [[animal]]
  - But not every referent of the hypernym will also be a referent of the hyponym:
    [[Big Bird]] is a member of [[animal]] but not [[dog]]
- Note that the "regular" **subset** includes both relationships, synonymy and hypo/hypernymy.

Principle of Semantic Compositionality

- Recall that we can talk about two levels of semantic meaning:
  - **Lexical semantics**: the meaning of words
  - **Compositional semantics**: the meaning of sentences
- We can do the same with set theory:
  - We've seen how to use set theory to define words
  - But what about sentences?
- **Principle of Semantic Compositionality** states that the meaning of sentences derives from the meaning of their words *plus* the meaning of their structures
  - This means we need to find a way to use set theory to show a sentence's structure

Declarative sentences

- So how can we describe a sentence in set theory?
- First, we have to talk about what a sentence is.
- **Declarative sentences** are sentences that state some proposition:
  - *Snoopy is a dog. Birds are animals.*
  - A **proposition** is a statement that can be **true** or **false**.
  - Declarative statements contrast with **interrogative** (questions) and **imperative** (commands) sentences, neither of which states a proposition.
  - We'll ignore these for now and only focus on declarative sentences.

Declarative sentences and claims

- **Declarative sentences** typically state propositions in two steps:
    - First, they introduce some **topic** in the subject of the sentence:
        - *<u>Snoopy</u> is a dog. <u>Birds</u> are animals.*
    - Then, they make some **comment** about that topic in the predicate:
        - *Snoopy <u>is a dog</u>. Birds <u>are animals</u>.*
- So to translate into a sentence into set theory, we need to figure out how to:
    - Identify a sentence's topic and state it in set theory
    - Identify a sentence's comment and state it in set theory

Compositional semantics: Proper noun subjects

- Let's start by examining sentences with a **proper noun** (i.e. **name**) as a subject, and with the structure **Name is NOUN**:
  - *Snoopy is a dog*.
- What is the subject (topic) of this sentence?
- What is the predicate (comment) of this sentence? What is it saying about the topic?
- Is there anything in this sentence that could be represented as a set or element?

Compositional semantics: Name is NOUN

- In a sentence with the structure **Name is NOUN** like *Snoopy is a dog*:
  - The subject is a proper noun – that is, an element, not a set
  - The predicate:
    - Introduces some noun referring to a set: [[dog]] = set of all dogs
    - Makes a claim about the relationship between the subject and this set:
      Snoopy is a member of the set of all dogs.
  - What symbol does set theory use to indicate that an element is a member of a set?
  - We can write this sentence in set theory as: [[Snoopy]] ∈ [[dog]]
  - In general, sentences of the structure **Name is NOUN** can be written as [[name]] ∈ [[NOUN]]

Compositional semantics: NAME is ADJ

- We can use a similar analysis to talk about sentences with other types of predicates.
- Consider the sentence *Snoopy is happy*.
  - This sentence has the form ***NAME is ADJ***.
  - So far we've only been using set theory to define nouns: [[bird]] = {*x* | *x* is a bird}
  - But we can use set theory to define adjectives, too:
    happy refers to the set of all happy things in the world, so [[happy]] = {*x* | *x* is happy}
  - So what does the sentence *Snoopy is happy* do? It mentions an individual *Snoopy* and a set [[happy]], and it makes the claim that *Snoopy* is a member of [[happy]].
  - The meaning of *Snoopy is happy* can be written in set theory as [[Snoopy]] ∈ [[happy]]
  - Sentences of the form **NAME is ADJ** can be encoded in set theory as: **[[name]] ∈ [[ADJ]]**

Compositional semantics: NAME VERBs

- Consider the sentence *Snoopy runs*.
  - Here the sentence has the form *NAME VERBs* – the predicate is a verb.
  - Suppose we define the verb *run* as referring to the set of all things in the world that run: [[run]] = {x | x runs}
  - Then we can see that *Snoopy runs* introduces an element *Snoopy*, introduces a set [[run]], and again makes the claim that the element is a member of the set: [[Snoopy]] ∈ [[run]]
  - Sentences of the type ***NAME VERBs*** can be encoded: **[[name]] ∈ [[VERB]]**
- That's three different sentence types (*NAME is NOUN, NAME is ADJ, NAME VERBs*) that can all be encoded in set theory in similar ways – through **set membership**:
  - [[name]] ∈ [[NOUN]]   [[name]] ∈ [[ADJ]]   [[name]] ∈ [[VERB]]

Compositional semantics: Common noun subjects

- But what if the subject is a **common noun** (a noun that's not a name)? *Cats are animals*.
  - Now the subject doesn't refer to an individual, it refers to a set of things: [[cat]]
  - Note that this sentence type ***NOUNs are NOUNs*** still has a predicate that:
    - introduces another set [[animal]]
    - and makes a claim about the relationship between the subject [[cat]] and [[animal]]
  - But we can no longer use membership ($x \in A$), because [[cat]] is a set, not an element.
  - What symbol do we use to state that all members of [[cat]] are also members of [[animal]]?
    - **Subset**: [[cat]] $\subseteq$ [[animal]]
  - Sentences of the type ***NOUNs are NOUNs*** can be encoded: **[[NOUN]] $\subseteq$ [[NOUN]]**

Compositional semantics: Common noun subjects

- We can use a similar formulation for sentences like *Cats are fuzzy* and *Cats meow*.
- *Cats are fuzzy* has the form ***NOUNs are ADJ***
  - [[fuzzy]] = set of all fuzzy things = {*x* | *x* is fuzzy}
  - *Cats are fuzzy* = [[cat]] ⊆ [[fuzzy]]
- *Cats meow* has the form ***NOUNs VERB***
  - [[meow]] = set of all things that meow = {*x* | *x* meows}
  - *Cats meow* = [[cat]] ⊆ [[meow]]

Compositional semantics: Subset vs. proper subset

- Note that we use **subset** (⊆) rather than proper subset (⊂), because it works even when the sets in the subject and predicate are synonyms.
  - [[cat]] ⊆ [[meow]] still true even if nothing else in the universe meows besides cats
  - [[cat]] ⊆ [[meow]] still true even if cats are just one of many beings that meow
- So using subsets gives us more flexibility and helps us deal with ambiguity in language.

Compositional semantics: Recap

- So according to the **Principle of Semantic Compositionality**, the meaning of sentences derives from both lexical meaning and syntactic structure
- We've seen six simple syntactic structures and how their meaning can be described in set theory:
  - NAME is NOUN — [[NAME]] ∈ [[NOUN]]
  - NAME is ADJ — [[NAME]] ∈ [[ADJ]]
  - NAME VERBs — [[NAME]] ∈ [[VERB]]
  - NOUNs are NOUNs — [[NOUN]] ⊆ [[NOUN]]
  - NOUNs are ADJ — [[NOUN]] ⊆ [[ADJ]]
  - NOUNs VERB — [[NOUN]] ⊆ [[VERB]]
- In each case, the full meaning depends on the sets/elements as well as the structure.

Compositional semantics: Truth values

- As we've seen, declarative sentences state some **proposition**, which can be encoded in set theory.
- But just because you state a proposition, that doesn't mean it's true – you can make a **false claim**! You can lie, you can be wrong, or you can say things without knowing if they're true.
- So when we think of what a sentence means, we need to separate these two concepts:
  - What is its **proposition**? (and how do we encode that proposition in set theory?)
  - Is the proposition **true** or **false**? (that is, what is its **truth value**?)
- Defining sentences in set theory and determining whether they're true are two separate processes:
  - *Snoopy is a cat* — $[[\text{Snoopy}]] \in [[\text{cat}]]$ — FALSE!
  - *Cats bark*. — $[[\text{cat}]] \subseteq [[\text{bark}]]$ — FALSE!
  - *Cats are mammals*. — $[[\text{cat}]] \subseteq [[\text{mammal}]]$ — TRUE!

Set operations

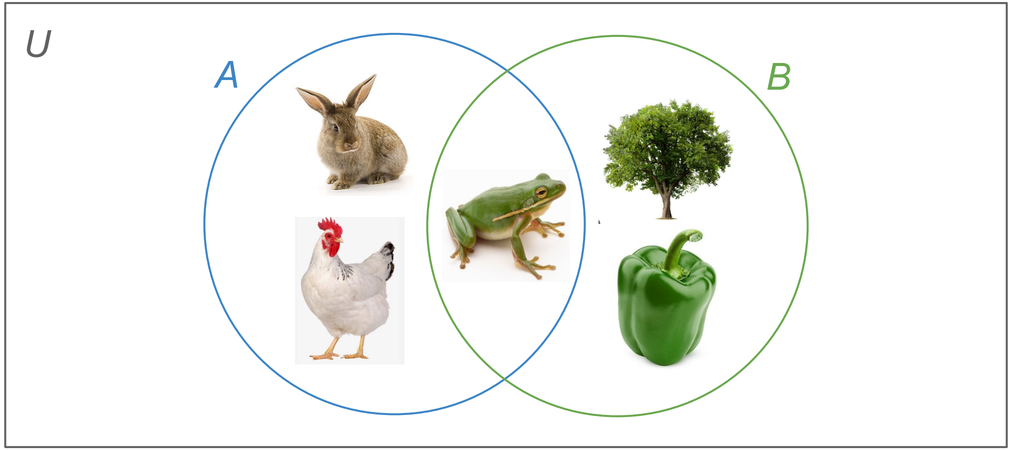
Compositional semantics: NPs

- Suppose I wanted to use an NP like *yellow birds*.
    - Maybe: *What kind of birds do you like? Yellow birds!*
- Notice that by using this NP, I'm not making a claim – I didn't say *birds are yellow* – I'm just trying to refer to all elements that are simultaneously in both [[bird]] and [[yellow]].
- To define this in set theory, we need **set operations**.

[[bird]] [[yellow bird]] [[yellow]]

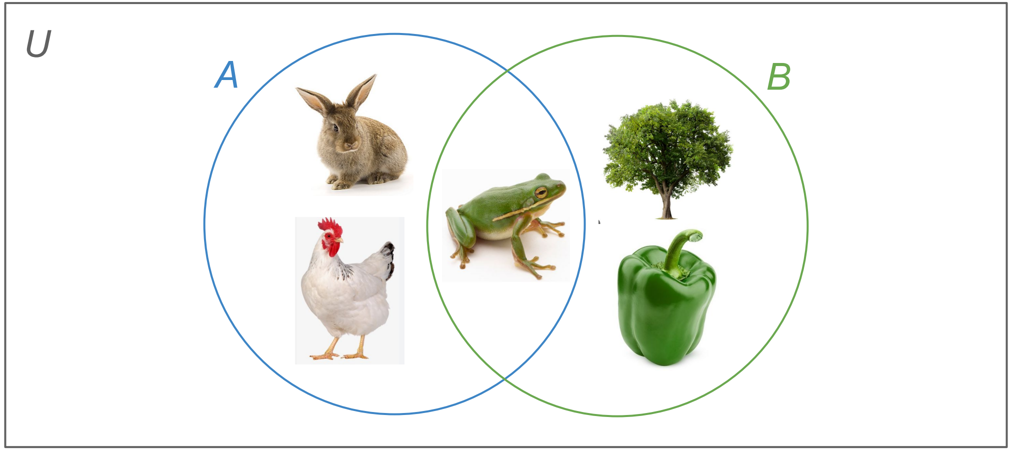
Set operations

- **Set operations** are ways of taking existing sets and creating a new set out of them.
  - This is analogous to how addition and subtraction take two numbers and make a third number.
- There are four set operations: **union**, **intersection**, **difference**, **complement**



Set operations

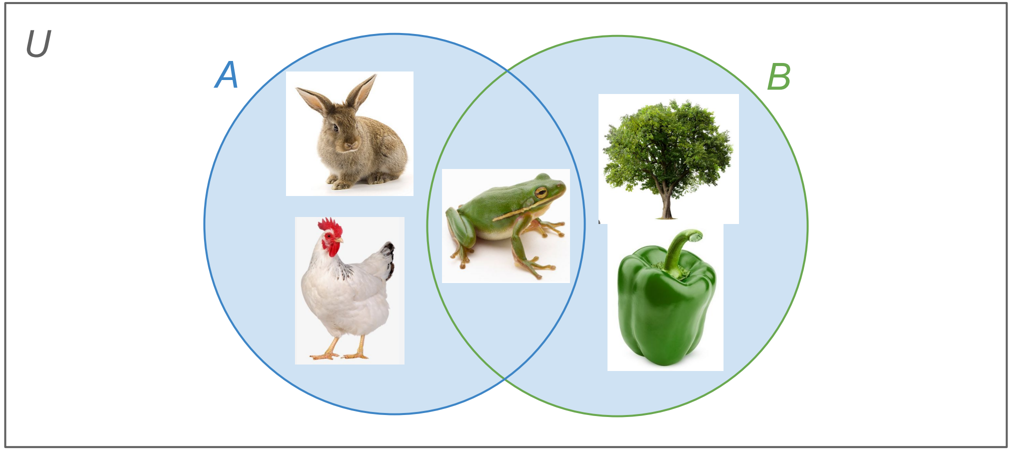
- Let's imagine a universe, U. In that universe there are five elements divided into two sets:
  - Let $A = \{ x \in U \mid x \text{ is an animal} \} = \{ \text{rabbit, chicken, frog} \}$
  - Let $B = \{ x \in U \mid x \text{ is green} \} = \{ \text{tree, frog, pepper} \}$ (Notice that *frog* is in both A and B.)



Set operations: Union

- **Union** of A and B produces a new set, which contains all elements that belong to A **<u>or</u>** B (or both).
  - $A \cup B$ = { rabbit, chicken, frog, tree, pepper }

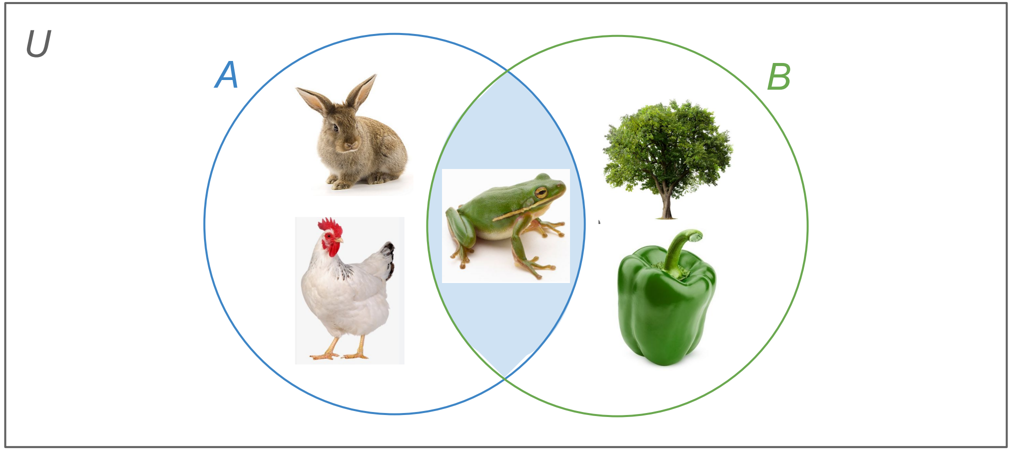
Mnemonic tip:
the symbol for *union*
(∪) looks like the
letter U



Set operations: Intersection

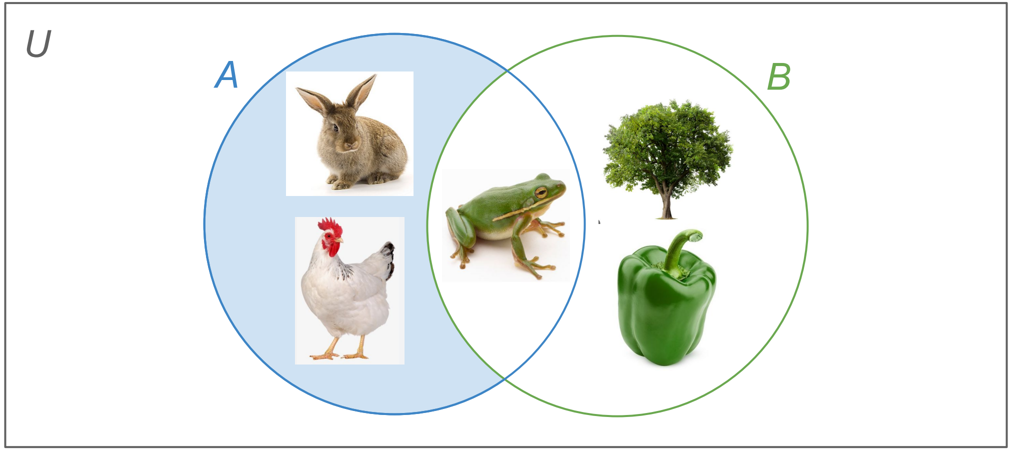
- **Intersection** of *A* and *B* produces a new set that contains all elements that belong to **both** *A* **and** *B.*
    - $A \cap B = \{ \text{frog} \}$
- Intersection is the part that overlaps between two sets in a Venn diagram.

Mnemonic tip:
the symbol for
intersection (⋂) looks
like a lowercase *n*
(kind of)



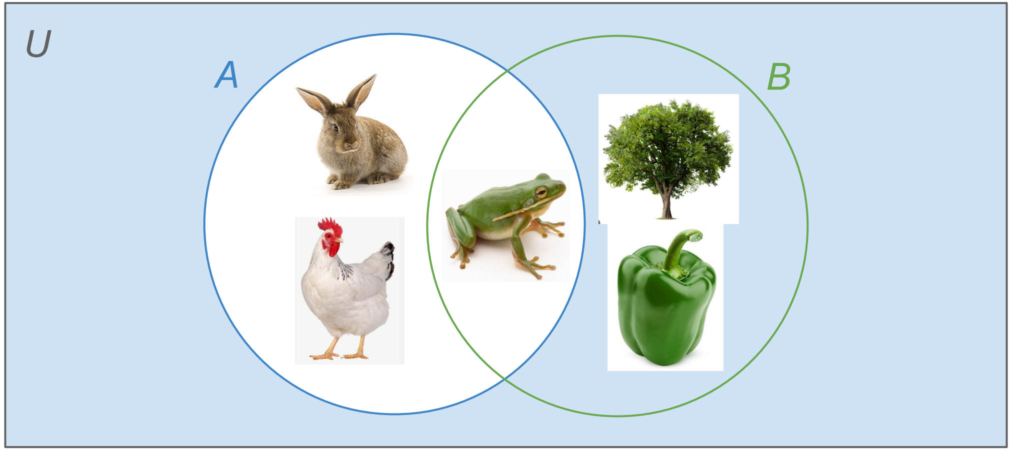
Set operations: Difference

- **Difference** of *A* and *B* produces a new set, which contains all elements that belong to *A* **<u>but not</u>** *B*
    - $A - B = \{$ rabbit, chicken $\}$



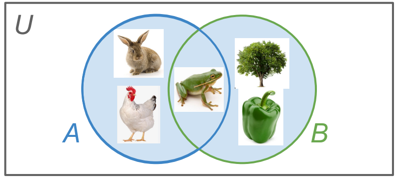
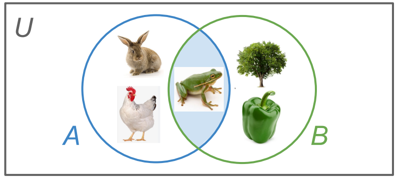
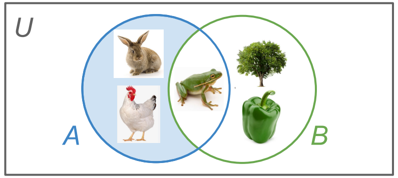
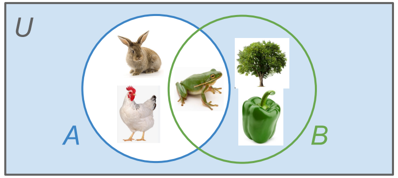
Set operations: Complement

- **Complement** of *A* produces a new set, which contains all elements in the specified universe (*U*) that **<u>are not in</u>** *A*
    - A' = { tree, pepper }



Note:
compl**<u>e</u>**ment, not
compl**<u>i</u>**ment

Set operations: Summary

Union	$A \cup B = \{ x \in U \mid x \in A \text{ or } x \in B \}$ (elements in A or B *or both*)	
Intersection	$A \cap B = \{ x \in U \mid x \in A \text{ and } x \in B \}$ (elements in both A and B)	
Difference	$A - B = \{ x \in U \mid x \in A \text{ and } x \notin B \}$ (elements in A but not B)	
Complement	$A' = \{ x \in U \mid x \notin A \}$ (elements in U but not A)	

Comprehension check: Set operations

Calculate the following:

1. $A \cup B$
2. $A - B$
3. $B \cap C$
4. $B \cap D$
5. C'
6. $(A \cap B)'$

A = {apple, pear, plum}

B = {orange, pear}

C = {plum, apricot, orange, nectarine}

D = {nectarine, apple}

Universe of fruit:

apple, pear, plum, orange, apricot, nectarine

Comprehension check: Set operations

Calculate the following, and name the set operation(s) involved:

1. $A \cup B$
2. $A - B$
3. $B \cap C$
4. $B \cap D$
5. C'
6. $(A \cap B)'$

1. union: {apple, pear, plum, orange}
   (don't add pear twice)
2. difference: {apple, plum}
   (B also has orange, doesn't matter)
3. intersection: {orange}
4. intersection: Ø or {}
   (no elements in common)
5. complement: {apple, pear}
6. calculate part in parentheses first
   intersection of A and B is {pear}
   complement of {pear} is
   {apple, plum, orange, apricot, nect}

A = {apple, pear, plum}

B = {orange, pear}

C = {plum, apricot, orange, nectarine}

D = {nectarine, apple}

Universe of fruit:

apple, pear, plum, orange, apricot, nectarine

Set operations in natural language

Set operations in natural language

- Let's return to our NP *yellow birds*:
    - [[yellow birds]] refers to all elements that are in both [[yellow]] and [[bird]]
    - That is, we're talking about elements that are simultaneously in two sets: [[yellow]] and [[bird]]
- What **set operation** do we use to specify elements that are simultaneously in two sets?
    - **Intersection** (∩): $A \cap B$ = all elements that are in both A and B at the same time
    - [[yellow]] ∩ [[bird]] = all elements that are in both [[yellow]] and [[bird]] = [[yellow bird]]
- Generally speaking, any NP of type ***ADJ NOUN*** can be specified in set theory via intersection:
    - **[[ADJ]] ∩ [[NOUN]]**

Set operations in natural language

- Notice that we can then insert this NP [[yellow]] ∩ [[bird]] into structures we've already seen:
  - *Tweety is a bird.* — [[Tweety]] ∈ [[bird]] — NAME is NOUN
  - *Tweety is a yellow bird*. — [[Tweety]] ∈ [[yellow]] ∩ [[bird]] — NAME is ADJ NOUN
  - *Birds are pretty.* — [[bird]] ⊆ [[pretty]] — NOUNs are ADJs
  - *Yellow birds are pretty.* — [[yellow]] ∩ [[bird]] ⊆ [[pretty]] — ADJ NOUNs are ADJs
- Note that set operators (∪ , ∩, −, ′) take precedence over set relation symbols (∈ , ⊆ , =) – compare this to arithmetic operators (+, –, ×, ÷) and (in)equality symbols (=, <, >) in algebra.
- Again, this demonstrates the **Principle of Semantic Compositionality** – as we build ever more complicated sentences, we need to think about the relationship between the different sets and how our sentence structure affects that relationship.

Set theory in semantics so far

- We can define words by **reference** – the set of things they refer to.
- We can use **set theory** to precisely define the reference of a natural language expression:
    - Sets are collections of elements
    - **Set relations**: equality/identity, subset, proper subset
        - Useful for describing the relationship between synonyms or hypo/hypernyms and for specifying claims that are made in declarative sentences like *NOUN is ADJ*.
    - **Set operations**: union, intersection, difference, complement
        - Useful for describing the reference of NPs like *ADJ NOUN*.
- Now we're going to turn to **cardinality** and see how we can use it to define quantifiers.

Quantifiers

Semantics of Determiners

- We previously defined **determiners** as a part of speech that consists of the following subcategories:
    - articles – *the, a*
    - demonstratives – *this, that, these, those*
    - quantifiers – *every, some, many, most, no*
    - possessive pronouns – *my, your, her, his, their, our*
    - some question words – *which, what*
- Determiners serve to specify *which member* of a set we're referring to in the world:
    - *my dog* – a member of [[dog]] that belongs to me
    - *this dog* – a member of [[dog]] that is near me
    - *every dog* – the entire set of [[dog]]
    - *the dog* – a member of [[dog]] that is relevant to our current conversation
    - *a dog* – an unspecified member of [[dog]]

Semantics of Quantifiers

- **Quantifiers** (including **numerals**) are a specific type of determiner that describes **how many members** of a set we're talking about.
- How many members of the set [[cat]] are we talking about when we say the following?
  - *every cat* — *all members*
  - *some cats* — *more than zero members*
  - *three cats* — *exactly three members*
  - *many cats* — *some large number of members*
  - *most cats* — *more than half of members*
  - *no cats* — *zero members*

Semantics of Quantifiers

- We saw that declarative sentences like **NOUNs are NOUNs**, **NOUNs are ADJ**, or **NOUN VERBs** make some claim about the relationship between sets:
  - *Cats are animals.*
  - Claim: [[cat]] $\subseteq$ [[animal]] *the set of cats is a subset of the set of animals*
- Sentences with quantifiers like **QUANT NOUN VERBs** also make some claim about the relationship between sets – but now it involves some quantity.
  - If I say *Four cats are meowing*, how many elements are in both [[cat]] and [[meow]]?
  - Four!
  - Claim: | [[cat]] $\cap$ [[meow]] | = 4
    *the **cardinality** of the intersection of [[cat]] and [[meow]] is four*
    In other words, there are exactly four elements that are both in [[cats]] and [[meow]]

Semantics of Quantifiers

- Other declarative sentences with quantifiers will make similar claims:

Quantifier	Example	$\mid [[N]] \cap [[VP]] \mid = ?$
Numeral N VP	*Two students are happy.*	$\mid [[N]] \cap [[VP]] \mid = n$ (*n* specified by numeral)
Some N VP	*Some students are happy.*	
Every N VP	*Every student is happy.*	
Many N VP	*Many students are happy.*	
Most N VP	*Most students are happy.*	
No N VP	*No students are happy.*	

Semantics of Quantifiers

- Other declarative sentences with quantifiers will make similar claims:

Quantifier	Example	$\lvert [[N]] \cap [[VP]] \rvert = ?$
Numeral N VP	*Two students are happy.*	$\lvert [[N]] \cap [[VP]] \rvert = n$ (*n* specified by numeral)
Some N VP	*Some students are happy.*	$\lvert [[N]] \cap [[VP]] \rvert \neq 0$ Or: $[[N]] \cap [[VP]] \neq \varnothing$
Every N VP	*Every student is happy.*	$\lvert [[N]] \cap [[VP]] \rvert = \lvert [[N]] \rvert$ Or: $[[N]] \subseteq [[VP]]$ = True
Many N VP	*Many students are happy.*	$\lvert [[N]] \cap [[VP]] \rvert > n$ (*n* is some large number)
Most N VP	*Most students are happy.*	$\lvert [[N]] \cap [[VP]] \rvert > \frac{1}{2} \lvert [[N]] \rvert$
No N VP	*No students are happy.*	$\lvert [[N]] \cap [[VP]] \rvert = 0$ Or: $[[N]] \cap [[VP]] = \varnothing$

Summary

Uses of set theory in semantics

- We can define words by **reference** – the set of things they refer to – which means that set theory can be a useful way to precisely define the semantic meaning of an expression:
  - Sets are collections of elements
  - **Set relations**: equality/identity, subset, proper subset
    - Useful for describing the relationship between synonyms or hypo/hypernyms and for specifying claims that are made in declarative sentences like *NOUN is ADJ*.
  - **Set operations**: union, intersection, difference, complement
    - Useful for describing the reference of NPs like *ADJ NOUN*.
  - **Cardinality**: useful for describing expressions with quantifiers

Limitations of set theory

- So set theory is a useful way to be precise about the semantic meaning of language.
- But it's not perfect: there are a few things that can't be described in set theory
- An example is **non-intersective adjectives**:
  - An adjective like *yellow* in *yellow bird* is called **intersective** because [[yellow bird]] can be described as the **intersection** between [[yellow]] and [[bird]].
  - What about *biggest tree* or *matching shirts*? Can we describe these through intersection?
  - No, because we need more information – for example to know which tree is *biggest*, we need to know the size of all other trees we need to compare them.
- So set theory is useful, but it can't describe all aspects of semantics.

Summary

- Semantics is the study of meaning – specifically **literal meaning** – in language
- We've focused on a type of semantics that defines meaning in terms of **reference** rather than **sense**
    - We've used **set theory** to define, compare, and manipulate the reference of linguistic units
- We've focused on two levels of semantic meaning:
    - **Lexical semantics** – the meaning of words: reference, synonyms, hyper/hyponyms
    - **Compositional semantics** – the meaning of sentences = words + structures
        - NOUN is NOUN: $[[NOUN]] \subseteq [[NOUN]]$
        - ADJ NOUN: $[[ADJ]] \cap [[NOUN]]$
        - QUANT NOUN: $|\ [[NOUN]]\ | = [[QUANT]]$
- If you're interested in learning more, consider taking a semantics class!