

## University of Tartu ICPC Team Notebook (2017-2018)

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## gcc ordered set

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 #include <ext/pb_ds/tree_policy.hpp>
3 using namespace __gnu_pbds;
4 template <typename T>
5 using ordered_set = tree<T, null_type, less<T>, rb_tree_tag, tree_order_statistics_node_update>;
6 int main(){
7     ordered_set<int> cur;
8     cur.insert(1);
9     cur.insert(3);
10    cout << cur.order_of_key(2) << endl; // the number of elements in the set less than 2
11    cout << *cur.find_by_order(0) << endl; // the 0-th smallest number in the set(0-based)
12    cout << *cur.find_by_order(1) << endl; // the 1-th smallest number in the set(0-based)
13 }
```

## Triangle centers

```

1 const double min_delta = 1e-13;
2 const double coord_max = 1e6;
3 typedef complex < double > point;
4 point A, B, C; // vertices of the triangle
5 bool collinear(){
6     double min_diff = min(abs(A - B), min(abs(A - C), abs(B - C)));
7     if(min_diff < coord_max * min_delta)
8         return true;
9     point sp = (B - A) / (C - A);
10    double ang = M_PI/2 - abs(abs(arg(sp)) - M_PI/2); //positive angle with the real line
11    return ang < min_delta;
12 }
13 point circum_center(){
14     if(collinear())
15         return point(NAN, NAN);
16     //squared lengths of sides
17     double a2, b2, c2;
18     a2 = norm(B - C);
```

```

19  b2 = norm(A - C);
20  c2 = norm(A - B);
21  //barycentric coordinates of the circumcenter
22  double c_A, c_B, c_C;
23  c_A = a2 * (b2 + c2 - a2); //sin(2 * alpha) may be used as well
24  c_B = b2 * (a2 + c2 - b2);
25  c_C = c2 * (a2 + b2 - c2);
26  double sum = c_A + c_B + c_C;
27  c_A /= sum;
28  c_B /= sum;
29  c_C /= sum;
30  // cartesian coordinates of the circumcenter
31  return c_A * A + c_B * B + c_C * C;
32 }
33 point centroid(){ //center of mass
34     return (A + B + C) / 3.0;
35 }
36 point ortho_center(){ //euler line
37     point O = circum_center();
38     return O + 3.0 * (centroid() - O);
39 };
40 point nine_point_circle_center(){ //euler line
41     point O = circum_center();
42     return O + 1.5 * (centroid() - O);
43 };
44 point in_center(){
45     if(collinear())
46         return point(NAN,NAN);
47     double a, b, c; //side lengths
48     a = abs(B - C);
49     b = abs(A - C);
50     c = abs(A - B);
51     //trilinear coordinates are (1,1,1)
52     //barycentric coordinates
53     double c_A = a, c_B = b, c_C = c;
54     double sum = c_A + c_B + c_C;
55     c_A /= sum;
56     c_B /= sum;
57     c_C /= sum;
58     // cartesian coordinates of the incenter
59     return c_A * A + c_B * B + c_C * C;
60 }

```

## 2D line segment

### Dinic

```

1 struct MaxFlow{
2     typedef long long ll;
3     const ll INF = 1e18;
4     struct Edge{
5         int u,v;
6         ll c,rc;
7         shared_ptr<ll> flow;
8         pair<int,int> id() const {
9             return make_pair(min(u,v),max(u,v));
10        }
11        Edge(int _u, int _v, ll _c, ll _rc = 0):u(_u),v(_v),c(_c),rc(_rc){
12        }
13        void join(const Edge &t){
14            if(u == t.u){
15                c += t.c;
16                rc += t.rc;
17            }
18            else{
19                c += t.rc;
20                rc += t.c;
21            }
22        }
23    };
24    struct FlowTracker{
25        shared_ptr<ll> flow;
26        ll cap, rcap;

```

```

27     bool dir;
28     FlowTracker(ll _cap, ll _rcap, shared_ptr<ll> _flow, int
→   _dir): cap(_cap), rcap(_rcap), flow(_flow), dir(_dir){ }
29     ll rem() const {
30         if(dir == 0){
31             return cap-*flow;
32         }
33         else{
34             return rcap+*flow;
35         }
36     }
37     void add_flow(ll f){
38         if(dir == 0)
39             *flow += f;
40         else
41             *flow -= f;
42         assert(*flow <= cap);
43         assert(-*flow <= rcap);
44     }
45     operator ll() const { return rem(); }
46     void operator--=(ll x){ add_flow(x); }
47     void operator+=(ll x){ add_flow(-x); }
48 };
49 int source,sink;
50 vector<vector<int>> > adj;
51 vector<vector<FlowTracker>> > cap;
52 vector<Edge> edges;
53 MaxFlow(int _source, int _sink):source(_source),sink(_sink){
54     assert(source != sink);
55 }
56 int add_edge(Edge e){
57     edges.push_back(e);
58     return edges.size()-1;
59 }
60 }
61 int add_edge(int u, int v, ll c, ll rc = 0){
62     return add_edge(Edge(u,v,c,rc));
63 }
64 void group_edges(){
65     map<pair<int,int>,vector<Edge>> > edge_groups;
66     for(auto edge: edges)
67         if(edge.u != edge.v)
68             edge_groups[edge.id()].push_back(edge);
69     vector<Edge> grouped_edges;
70     for(auto group: edge_groups){
71         Edge main_edge = group.second[0];
72         for(int i = 1; i < group.second.size(); ++i)
73             main_edge.join(group.second[i]);
74         grouped_edges.push_back(main_edge);
75     }
76     edges = grouped_edges;
77 }
78 vector<int> now,lvl;
79 void prep(){
80     int max_id = max(source,sink);
81     for(auto edge : edges)
82         max_id = max(max_id,max(edge.u,edge.v));
83     adj.resize(max_id+1);
84     cap.resize(max_id+1);
85     now.resize(max_id+1);
86     lvl.resize(max_id+1);
87     for(auto &edge : edges){
88         auto flow = make_shared<ll>(0);
89         adj[edge.u].push_back(edge.v);
90         cap[edge.u].push_back(FlowTracker(edge.c,edge.rc,flow,0));
91         adj[edge.v].push_back(edge.u);
92         cap[edge.v].push_back(FlowTracker(edge.c,edge.rc,flow,1));
93         assert(cap[edge.u].back() == edge.c);
94         edge.flow = flow;
95     }
96 }
97 bool dinic_bfs(){
98     fill(now.begin(),now.end(),0);

```

```

99     fill(lvl.begin(),lvl.end(),0);
100    lvl[source] = 1;
101    vector<int> bfs(1,source);
102    for(int i = 0; i < bfs.size(); ++i){
103        int u = bfs[i];
104        for(int j = 0; j < adj[u].size(); ++j){
105            int v = adj[u][j];
106            if(cap[u][j] > 0 && lvl[v] == 0){
107                lvl[v] = lvl[u]+1;
108                bfs.push_back(v);
109            }
110        }
111    }
112    return lvl[sink] > 0;
113 }
114 ll dinic_dfs(int u, ll flow){
115     if(u == sink)
116         return flow;
117     while(now[u] < adj[u].size()){
118         int v = adj[u][now[u]];
119         if(lvl[v] == lvl[u] + 1 && cap[u][now[u]] != 0){
120             ll res = dinic_dfs(v,min(flow,(ll)cap[u][now[u]]));
121             if(res > 0){
122                 cap[u][now[u]] -= res;
123                 return res;
124             }
125         }
126         ++now[u];
127     }
128     return 0;
129 }
130 ll calc(){
131     prep();
132     ll ans = 0;
133     while(dinic_bfs()){
134         ll cur = 0;
135         do{
136             cur = dinic_dfs(source,INF);
137             ans += cur;
138         }while(cur > 0);
139     }
140     return ans;
141 }
142 };
143 int main(){
144     int n,m;
145     cin >> n >> m;
146     auto mf = MaxFlow(1,n); // arguments source and sink, memory usage O(largest node index), sink doesn't need
    ↪ to be last
147     int edge_index;
148     for(int i = 0; i < m; ++i){
149         int a,b,c;
150         cin >> a >> b >> c;
151         //undirected edge is a pair of edges (a,b,c,0) and (a,b,0,c)
152         edge_index = mf.add_edge(a,b,c,c); //store edge index if care about flow value
153     }
154     mf.group_edges(); // small auxillary to remove multiple edges, only use this if we need to know TOTAL FLOW
    ↪ ONLY
155     cout << mf.calc() << '\n';
156     //cout << *mf.edges[edge_index].flow << '\n'; // ONLY if group_edges() WAS NOT CALLED
157 }

```

## Min Cost Max Flow with successive dijkstra $\mathcal{O}(\text{flow} \cdot n^2)$

```

1 const int nmax=1055;
2 const ll inf=1e14;
3 int t, n, v; //0 is source, v-1 sink
4 ll rem_flow[nmax][nmax]; //set [x][y] for directed capacity from x to y.
5 ll cost[nmax][nmax]; //set [x][y] for directed cost from x to y. SET TO inf IF NOT USED
6 ll min_dist[nmax];
7 int prev_node[nmax];
8 ll node_flow[nmax];
9 bool visited[nmax];

```

```

10 ll tot_cost, tot_flow; //output
11 void min_cost_max_flow(){ //incase of negative edges have to add Bellman-Ford that is run once.
12     tot_cost=0; //Does not work with negative cycles.
13     tot_flow=0;
14     ll sink_pot=0;
15     while(true){
16         for(int i=0; i<=v; ++i){
17             min_dist[i]=inf;
18             visited[i]=false;
19         }
20         min_dist[0]=0;
21         node_flow[0]=inf;
22         int min_node;
23         while(true){ //Use Dijkstra to calculate potentials
24             int min_node=v;
25             for(int i=0; i<v; ++i){
26                 if((!visited[i]) && min_dist[i]<min_dist[min_node]){
27                     min_node=i;
28                 }
29             }
30             if(min_node==v){
31                 break;
32             }
33             visited[min_node]=true;
34             for(int i=0; i<v; ++i){
35                 if((!visited[i]) && min_dist[min_node]+cost[min_node][i] < min_dist[i]){
36                     min_dist[i]=min_dist[min_node]+cost[min_node][i];
37                     prev_node[i]=min_node;
38                     node_flow[i]=min(node_flow[min_node], rem_flow[min_node][i]);
39                 }
40             }
41         }
42         if(min_dist[v-1]==inf){
43             break;
44         }
45         for(int i=0; i<v; ++i){ //Apply potentials to edge costs.
46             for(int j=0; j<v; ++j){ //Found path from source to sink becomes 0 cost.
47                 if(cost[i][j]!=inf){
48                     cost[i][j]+=min_dist[i];
49                     cost[i][j]-=min_dist[j];
50                 }
51             }
52         }
53         sink_pot+=min_dist[v-1];
54         tot_flow+=node_flow[v-1];
55         tot_cost+=sink_pot*node_flow[v-1];
56         int cur=v-1;
57         while(cur!=0){ //Backtrack along found path that now has 0 cost.
58             rem_flow[prev_node[cur]][cur]-=node_flow[v-1];
59             rem_flow[cur][prev_node[cur]]+=node_flow[v-1];
60             cost[cur][prev_node[cur]]=0;
61             if(rem_flow[prev_node[cur]][cur]==0){
62                 cost[prev_node[cur]][cur]=inf;
63             }
64             cur=prev_node[cur];
65         }
66     }
67 }

```

## Min Cost Max Flow with Cycle Cancellation $\mathcal{O}(\text{flow} \cdot nm)$

### Aho Corasick $\mathcal{O}(|\alpha| \sum \text{len})$

```

1 const int alpha_size=26;
2 struct node{
3     node *nxt[alpha_size]; //May use other structures to move in trie
4     node *suffix;
5     node(){
6         memset(nxt, 0, alpha_size*sizeof(node *));
7     }
8     int cnt=0;
9 };
10 node *aho_corasick(vector<vector<char> > &dict){

```

```

11 node *root= new node;
12 root->suffix = 0;
13 vector<pair<vector<char> *, node *> > cur_state;
14 for(vector<char> &s : dict)
15     cur_state.emplace_back(&s, root);
16 for(int i=0; !cur_state.empty(); ++i){
17     vector<pair<vector<char> *, node *> > nxt_state;
18     for(auto &cur : cur_state){
19         node *nxt=cur.second->nxt[(*cur.first)[i]];
20         if(nxt){
21             cur.second=nxt;
22         }else{
23             nxt = new node;
24             cur.second->nxt[(*cur.first)[i]] = nxt;
25             node *suf = cur.second->suffix;
26             cur.second = nxt;
27             nxt->suffix = root; //set correct suffix link
28             while(suf){
29                 if(suf->nxt[(*cur.first)[i]]){
30                     nxt->suffix = suf->nxt[(*cur.first)[i]];
31                     break;
32                 }
33                 suf=suf->suffix;
34             }
35             if(cur.first->size() > i+1)
36                 nxt_state.push_back(cur);
37         }
38     }
39     cur_state=nxt_state;
40 }
41 return root;
42 }
43 //auxiliary functions for searching and counting
44 node *walk(node *cur, char c){ //longest prefix in dict that is suffix of walked string.
45     while(true){
46         if(cur->nxt[c])
47             return cur->nxt[c];
48         if(!cur->suffix){
49             return cur;
50         }
51         cur = cur->suffix;
52     }
53 }
54 void cnt_matches(node *root, vector<char> &match_in){
55     node *cur = root;
56     for(char c : match_in){
57         cur = walk(cur, c);
58         ++cur->cnt;
59     }
60 }
61 void add_cnt(node *root){ //After counting matches propagete ONCE to suffixes for final counts
62     vector<node *> to_visit = {root};
63     for(int i=0; i<to_visit.size(); ++i){
64         node *cur = to_visit[i];
65         for(int j=0; j<alpha_size; ++j){
66             if(cur->nxt[j]){
67                 to_visit.push_back(cur->nxt[j]);
68             }
69         }
70     }
71     for(int i=to_visit.size()-1; i>0; --i){
72         to_visit[i]->suffix->cnt += to_visit[i]->cnt;
73     }
74 }

```

Suffix automaton  $O((n + q) \log(|\alpha|))$

```

1 class AutoNode {
2     private:
3         map< char, AutoNode * > nxt_char; // Map is faster than hashtable and unsorted arrays
4     public:
5         int len; //Length of longest suffix in equivalence class.
6         AutoNode *suf;

```

```

7  bool has_nxt(char c) const {
8      return nxt_char.count(c);
9  }
10 AutoNode *nxt(char c) {
11     if (!has_nxt(c))
12         return NULL;
13     return nxt_char[c];
14 }
15 void set_nxt(char c, AutoNode *node) {
16     nxt_char[c] = node;
17 }
18 AutoNode *split(int new_len, char c) {
19     AutoNode *new_n = new AutoNode;
20     new_n->nxt_char = nxt_char;
21     new_n->len = new_len;
22     new_n->suf = suf;
23     suf = new_n;
24     return new_n;
25 }
26 // Extra functions for matching and counting
27 AutoNode *lower_depth(int depth) { //move to longest suffix of current with a maximum length of depth.
28     if (suf->len >= depth)
29         return suf->lower_depth(depth);
30     return this;
31 }
32 AutoNode *walk(char c, int depth, int &match_len) { //move to longest suffix of walked path that is a
↳ substring
33     match_len = min(match_len, len); //includes depth limit(needed for finding matches)
34     if (has_nxt(c)) { //as suffixes are in classes match_len must be tracte
↳ eternally
35         ++match_len;
36         return nxt(c)->lower_depth(depth);
37     }
38     if (suf)
39         return suf->walk(c, depth, match_len);
40     return this;
41 }
42 int paths_to_end = 0;
43 void set_as_end() { //All suffixes of current node are marked as ending nodes.
44     paths_to_end = 1;
45     if (suf) suf->set_as_end();
46 }
47 bool vis = false;
48 void calc_paths_to_end() { //Call ONCE from ROOT. For each node calculates number of ways to reach an end
↳ node.
49     if (!vis) { //paths_to_end is ocurence count for any strings in current suffix equivalence
↳ class.
50         vis = true;
51         for (auto cur : nxt_char) {
52             cur.second->calc_paths_to_end();
53             paths_to_end += cur.second->paths_to_end;
54         }
55     }
56 }
57 };
58 struct SufAutomaton {
59     AutoNode *last;
60     AutoNode *root;
61     void extend(char new_c) {
62         AutoNode *new_end = new AutoNode; // The equivalence class containing the whole new string
63         new_end->len = last->len + 1;
64         AutoNode *suf_w_nxt = last; // The whole old string class
65         while (suf_w_nxt && !suf_w_nxt->has_nxt(new_c)) { // is turned into the longest suffix which
66 // can be turned into a substring of old state
67 // by appending new_c
68             suf_w_nxt->set_nxt(new_c, new_end);
69             suf_w_nxt = suf_w_nxt->suf;
70         }
71         if (!suf_w_nxt) { // The new character isn't part of the old string
72             new_end->suf = root;
73         } else {
74             AutoNode *max_sbstr = suf_w_nxt->nxt(new_c); // Equivalence class containing longest
75 // substring which is a suffix of the new state.

```

```

76     if (suf_w_nxt->len + 1 == max_sbstr->len) { // Check whether splitting is needed
77         new_end->suf = max_sbstr;
78     } else {
79         AutoNode *eq_sbstr = max_sbstr->split(suf_w_nxt->len + 1, new_c);
80         new_end->suf = eq_sbstr;
81         // Make suffixes of suf_w_nxt point to eq_sbstr instead of max_sbstr
82         AutoNode *w_edge_to_eq_sbstr = suf_w_nxt;
83         while (w_edge_to_eq_sbstr != 0 && w_edge_to_eq_sbstr->nxt(new_c) == max_sbstr) {
84             w_edge_to_eq_sbstr->set_nxt(new_c, eq_sbstr);
85             w_edge_to_eq_sbstr = w_edge_to_eq_sbstr->suf;
86         }
87     }
88 }
89 last = new_end;
90 }
91 SufAutomaton(string to_suffix) {
92     root = new AutoNode;
93     root->len = 0;
94     root->suf = NULL;
95     last = root;
96     for (char c : to_suffix) extend(c);
97 }
98 };

```

## Templated multi dimensional BIT $\mathcal{O}(\log(n)^{\dim})$

```

1 // Fully overloaded any dimensional BIT, use any type for coordinates, elements, return_value.
2 // Includes coordinate compression.
3 template < typename elem_t, typename coord_t, coord_t n_inf, typename ret_t >
4 class BIT {
5     vector< coord_t > positions;
6     vector< elem_t > elems;
7     bool initiated = false;
8
9 public:
10    BIT() {
11        positions.push_back(n_inf);
12    }
13    void initiate() {
14        if (initiated) {
15            for (elem_t &c_elem : elems)
16                c_elem.initiate();
17        } else {
18            initiated = true;
19            sort(positions.begin(), positions.end());
20            positions.resize(unique(positions.begin(), positions.end()) - positions.begin());
21            elems.resize(positions.size());
22        }
23    }
24    template < typename... loc_form >
25    void update(coord_t cord, loc_form... args) {
26        if (initiated) {
27            int pos = lower_bound(positions.begin(), positions.end(), cord) - positions.begin();
28            for (; pos < positions.size(); pos += pos & -pos)
29                elems[pos].update(args...);
30        } else {
31            positions.push_back(cord);
32        }
33    }
34    template < typename... loc_form >
35    ret_t query(coord_t cord, loc_form... args) { //sum in open interval (-inf, cord)
36        ret_t res = 0;
37        int pos = (lower_bound(positions.begin(), positions.end(), cord) - positions.begin())-1;
38        for (; pos > 0; pos -= pos & -pos)
39            res += elems[pos].query(args...);
40        return res;
41    }
42 };
43 template < typename internal_type >
44 struct wrapped {
45     internal_type a = 0;
46     void update(internal_type b) {
47         a += b;

```



```

48 }
49 internal_type query() {
50     return a;
51 }
52 // Should never be called, needed for compilation
53 void initiate() {
54     cerr << 'i' << endl;
55 }
56 void update() {
57     cerr << 'u' << endl;
58 }
59 };
60 int main() {
61     // return type should be same as type inside wrapped
62     BIT< BIT< wrapped< ll >, int, INT_MIN, ll >, int, INT_MIN, ll > fenwick;
63     int dim = 2;
64     vector< tuple< int, int, ll > > to_insert;
65     to_insert.emplace_back(1, 1, 1);
66     // set up all positions that are to be used for update
67     for (int i = 0; i < dim; ++i) {
68         for (auto &cur : to_insert)
69             fenwick.update(get< 0 >(cur), get< 1 >(cur)); // May include value which won't be used
70         fenwick.initiate();
71     }
72     // actual use
73     for (auto &cur : to_insert)
74         fenwick.update(get< 0 >(cur), get< 1 >(cur), get< 2 >(cur));
75     cout << fenwick.query(2, 2)<<'\\n';
76 }

```

## Templated HLD $\mathcal{O}(M(n) \log n)$ per query

```

1 class dummy {
2 public:
3     dummy () {
4     }
5
6     dummy (int, int) {
7     }
8
9     void set (int, int) {
10    }
11
12    int query (int left, int right) {
13        cout << this << ' ' << left << ' ' << right << endl;
14    }
15 };
16
17 /* T should be the type of the data stored in each vertex;
18  * DS should be the underlying data structure that is used to perform the
19  * group operation. It should have the following methods:
20  * * DS () - empty constructor
21  * * DS (int size, T initial) - constructs the structure with the given size,
22  *   initially filled with initial.
23  * * void set (int index, T value) - set the value at index 'index' to 'value'
24  * * T query (int left, int right) - return the "sum" of elements between left and right, inclusive.
25  */
26 template<typename T, class DS>
27 class HLD {
28     int vertexc;
29     vector<int> *adj;
30     vector<int> subtree_size;
31     DS structure;
32     DS aux;
33
34     void build_sizes (int vertex, int parent) {
35         subtree_size[vertex] = 1;
36         for (int child : adj[vertex]) {
37             if (child != parent) {
38                 build_sizes(child, vertex);
39                 subtree_size[vertex] += subtree_size[child];
40             }
41         }

```

```

42 }
43
44 int cur;
45 vector<int> ord;
46 vector<int> chain_root;
47 vector<int> par;
48 void build_hld (int vertex, int parent, int chain_source) {
49     cur++;
50     ord[vertex] = cur;
51     chain_root[vertex] = chain_source;
52     par[vertex] = parent;
53
54     if (adj[vertex].size() > 1) {
55         int big_child, big_size = -1;
56         for (int child : adj[vertex]) {
57             if ((child != parent) &&
58                 (subtree_size[child] > big_size)) {
59                 big_child = child;
60                 big_size = subtree_size[child];
61             }
62         }
63
64         build_hld(big_child, vertex, chain_source);
65         for (int child : adj[vertex]) {
66             if ((child != parent) && (child != big_child)) {
67                 build_hld(child, vertex, child);
68             }
69         }
70     }
71 }
72
73 public:
74 HLD (int _vertexc) {
75     vertexc = _vertexc;
76     adj = new vector<int> [vertexc + 5];
77 }
78
79 void add_edge (int u, int v) {
80     adj[u].push_back(v);
81     adj[v].push_back(u);
82 }
83
84 void build (T initial) {
85     subtree_size = vector<int> (vertexc + 5);
86     ord = vector<int> (vertexc + 5);
87     chain_root = vector<int> (vertexc + 5);
88     par = vector<int> (vertexc + 5);
89     cur = 0;
90     build_sizes(1, -1);
91     build_hld(1, -1, 1);
92     structure = DS (vertexc + 5, initial);
93     aux = DS (50, initial);
94 }
95
96 void set (int vertex, int value) {
97     structure.set(ord[vertex], value);
98 }
99
100 T query_path (int u, int v) { /* returns the "sum" of the path u->v */
101     int cur_id = 0;
102     while (chain_root[u] != chain_root[v]) {
103         if (ord[u] > ord[v]) {
104             cur_id++;
105             aux.set(cur_id, structure.query(ord[chain_root[u]], ord[u]));
106             u = par[chain_root[u]];
107         } else {
108             cur_id++;
109             aux.set(cur_id, structure.query(ord[chain_root[v]], ord[v]));
110             v = par[chain_root[v]];
111         }
112     }
113
114     cur_id++;

```

150 }

Templated Persistent Segment Tree  $\mathcal{O}(\log n)$  per query

34

```
35     }
36   }
37 };
38
39 int size;
40 Node **tree;
41 vector<Node*> roots;
42 public:
43 PersistentST () {
44 }
45
46 PersistentST (int _size, T initial) {
47   for (int i = 0; i < 32; i++) {
48     if ((1 << i) > _size) {
49       size = 1 << i;
50       break;
51     }
52   }
53
54   tree = new Node* [2 * size + 5];
55
56   for (int i = size; i < 2 * size; i++) {
57     tree[i] = new Node (i - size, initial);
58   }
59
60   for (int i = size - 1; i > 0; i--) {
61     tree[i] = new Node (tree[2 * i], tree[2 * i + 1]);
62   }
63
64   roots = vector<Node*> (1, tree[1]);
65 }
66
67 void set (int position, T _value) {
68   tree[size + position] = new Node (position, _value);
69   for (int i = (size + position) / 2; i >= 1; i /= 2) {
70     tree[i] = new Node (tree[2 * i], tree[2 * i + 1]);
71   }
72   roots.push_back(tree[1]);
73 }
74
75 int last_revision () {
76   return (int) roots.size() - 1;
77 }
78
79 T query (int qlleft, int qright, int revision) {
80   return roots[revision]->query(qlleft, qright);
81 }
82
83 T query (int qlleft, int qright) {
84   return roots[last_revision()]->query(qlleft, qright);
85 }
86 };
```

---

**FFT**  $\mathcal{O}(n \log(n))$ **MOD int, extended Euctclidean**

## Combinatorics Cheat Sheet

**Useful formulas**

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$  — number of ways to choose  $k$  objects out of  $n$

$\binom{n+k-1}{k-1}$  — number of ways to choose  $k$  objects out of  $n$  with repetitions

$\left[ \begin{smallmatrix} n \\ m \end{smallmatrix} \right]$  — Stirling numbers of the first kind; number of permutations of  $n$  elements with  $k$  cycles

$$\left[ \begin{smallmatrix} n+1 \\ m \end{smallmatrix} \right] = n \left[ \begin{smallmatrix} n \\ m \end{smallmatrix} \right] + \left[ \begin{smallmatrix} n \\ m-1 \end{smallmatrix} \right]$$

$$(x)_n = x(x-1)\dots x-n+1 = \sum_{k=0}^n (-1)^{n-k} \left[ \begin{smallmatrix} n \\ k \end{smallmatrix} \right] x^k$$

$\left\{ \begin{smallmatrix} n \\ m \end{smallmatrix} \right\}$  — Stirling numbers of the second kind; number of partitions of set  $1, \dots, n$  into  $k$  disjoint subsets.

$$\left\{ \begin{smallmatrix} n+1 \\ m \end{smallmatrix} \right\} = k \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\} + \left\{ \begin{smallmatrix} n \\ k-1 \end{smallmatrix} \right\}$$

$$\sum_{k=0}^n \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\} (x)_k = x^n$$

$$C_n = \frac{1}{n+1} \binom{2n}{n} \text{ — Catalan numbers}$$

$$C(x) = \frac{1-\sqrt{1-4x}}{2x}$$

**Binomial transform**

$$\text{If } a_n = \sum_{k=0}^n \binom{n}{k} b_k, \text{ then } b_n = \sum_{k=0}^n (-1)^{n-k} \binom{n}{k} a_k$$

$$\bullet a = (1, x, x^2, \dots), b = (1, (x+1), (x+1)^2, \dots)$$

$$\bullet a_i = i^k, b_i = \left\{ \begin{smallmatrix} n \\ i \end{smallmatrix} \right\} i!$$

**Burnside's lemma**

Let  $G$  be a group of *action* on set  $X$  (Ex.: cyclic shifts of array, rotations and symmetries of  $n \times n$  matrix, ...)

Call two objects  $x$  and  $y$  *equivalent* if there is an action  $f$  that transforms  $x$  to  $y$ :  $f(x) = y$ .

The number of equivalence classes then can be calculated as follows:  $C = \frac{1}{|G|} \sum_{f \in G} |X^f|$ , where  $X^f$

is the set of *fixed points* of  $f$ :  $X^f = \{x | f(x) = x\}$

**Generating functions**

Ordinary generating function (o.g.f.) for sequence  $a_0, a_1, \dots, a_n, \dots$  is  $A(x) = \sum_{n=0}^{\infty} a_n x^n$

Exponential generating function (e.g.f.) for sequence  $a_0, a_1, \dots, a_n, \dots$  is  $A(x) = \sum_{n=0}^{\infty} a_n x^n / n!$

$$B(x) = A'(x), b_{n-1} = n \cdot a_n$$

$$c_n = \sum_{k=0}^n a_k b_{n-k} \text{ (o.g.f. convolution)}$$

$$c_n = \sum_{k=0}^n \binom{n}{k} a_k b_{n-k} \text{ (e.g.f. convolution, compute with FFT using } \widetilde{a}_n = \frac{a_n}{n!})$$

**General linear recurrences**

If  $a_n = \sum_{k=1}^n b_k a_{n-k}$ , then  $A(x) = \frac{a_0}{1-B(x)}$ . We also can compute all  $a_n$  with Divide-and-Conquer algorithm in  $\mathcal{O}(n \log^2 n)$ .

**Inverse polynomial modulo  $x^l$** 

Given  $A(x)$ , find  $B(x)$  such that  $A(x)B(x) = 1 + x^l \cdot Q(x)$  for some  $Q(x)$

$$1. \text{ Start with } B_0(x) = \frac{1}{a_0}$$

$$2. \text{ Double the length of } B(x): B_{k+1}(x) = (-B_k(x)^2 A(x) + 2B_k(x)) \bmod x^{2^{k+1}}$$

**Fast subset convolution**

$$\text{Given array } a_i \text{ of size } 2^k, \text{ calculate } b_i = \sum_{j \& i = i} b_j$$

```
for b = 0..k-1
  for i = 0..2^k-1
    if (i & (1 << b)) != 0:
      a[i + (1 << b)] += a[i]
```

**Hadamard transform**

Treat array  $a$  of size  $2^k$  as  $k$ -dimensional array of size  $2 \times 2 \times \dots \times 2$ , calculate FFT of that array:

```
for b = 0..k-1
  for i = 0..2^k-1
    if (i & (1 << b)) != 0:
      u = a[i], v = a[i + (1 << b)]
      a[i] = u + v
      a[i + (1 << b)] = u - v
```