

# University of Tartu ICPC Team Notebook

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- 20 Radixsort 50M 64 bit integers as single array in 1 sec
- 21 FFT 5M length/sec

## 22 Fast mod mult, Rabbin Miller prime check, Pollard rho factorization $\mathcal{O}(\sqrt{p})$

21

### 1 Setup

```
1 set smartindent cindent
2 set ts=4 sw=4 expandtab
3 syntax enable
4 set clipboard=unnamedplus
5 "colorscheme elflord
6 "setxkbmap -option caps:escape
7 "setxkbmap -option
8 "valgrind --vgdb-error=0 ./a <inp &
9 "gdb a
10 "target remote | vgdb
```

### 2 crc.sh

```
1 #!/bin/envbash
2 starts=$(sed '/^\s*$/d' $1 | grep -n "/\!/start" | cut -f1 -d:)
3 finishes=$(sed '/^\s*$/d' $1 | grep -n "/\!/finish" | cut -f1 -d:)
4 for ((i=0;i<${#starts[@]};i++)); do
5     for j in `seq 10 10 $((finishes[i]-starts[i]+8))`; do
6         sed '/^\s*$/d' $1 | head -${((finishes[i]-1))} | tail
7         ↪ -${((finishes[i]-starts[i]-1))} | \
8         ↪ head -$j | tr -d '[:space:]' | cksum | cut -f1 -d ' ' | tail -c
9         ↪ 4
10    done #whitespaces don't matter
11    echo #there shouldn't be any comments in the checked range
12done #check last number in each block
```

### 3 gcc ordered set

```
1 #include <bits/stdc++.h>
2 typedef long long ll;
3 using namespace std;
4 #include <ext/pb_ds/assoc_container.hpp>
5 #include <ext/pb_ds/tree_policy.hpp>
6 using namespace __gnu_pbds;
7 template <typename T>
8 using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
9     ↪ tree_order_statistics_node_update>;
10 int main(){
11     ordered_set<int> cur;
12     cur.insert(1);
13     cur.insert(3);
14     cout << cur.order_of_key(2) << endl; // the number of elements in the
15     ↪ set less than 2
16     cout << *cur.find_by_order(0) << endl; // the 0-th smallest number in
17     ↪ the set(0-based)
18     cout << *cur.find_by_order(1) << endl; // the 1-th smallest number in
19     ↪ the set(0-based)
20 }
21 %574
```

#### 4 Numerical integration with Simpson's rule

```

1 //computing power = how many times function integrate gets called
2 template<typename T>
3 double simps(T f, double a, double b) {
4     return (f(a) + 4*f((a+b)/2) + f(b))*(b-a)/6;
5 }
6 template<typename T>
7 double integrate(T f, double a, double b, double computing_power){
8     double m = (a+b)/2;
9     double l = simps(f,a,m), r = simps(f,m,b), tot=simps(f,a,b);
10    if (computing_power < 1) return tot;
11    return integrate(f, a, m, computing_power/2) + integrate(f, m, b,
12        computing_power/2);

```

#430  
%360

#### 5 Triangle centers

```

1 const double min_delta = 1e-13;
2 const double coord_max = 1e6;
3 typedef complex < double > point;
4 point A, B, C; // vertices of the triangle
5 bool collinear(){
6     double min_diff = min(abs(A - B), min(abs(A - C), abs(B - C)));
7     if(min_diff < coord_max * min_delta)
8         return true;
9     point sp = (B - A) / (C - A);
10    double ang = M_PI/2-abs(abs(arg(sp))-M_PI/2); //positive angle with
11        the real line
12    return ang < min_delta;
13 }
14 point circum_center(){
15     if(collinear())
16         return point(NAN,NAN);
17     //squared lengths of sides
18     double a2, b2, c2;
19     a2 = norm(B - C);
20     b2 = norm(A - C);
21     c2 = norm(A - B);
22     //barycentric coordinates of the circumcenter
23     double c_A, c_B, c_C;
24     c_A = a2 * (b2 + c2 - a2); //sin(2 * alpha) may be used as well
25     c_B = b2 * (a2 + c2 - b2);
26     c_C = c2 * (a2 + b2 - c2);
27     double sum = c_A + c_B + c_C;
28     c_A /= sum;
29     c_B /= sum;
30     c_C /= sum;
31     // cartesian coordinates of the circumcenter
32     return c_A * A + c_B * B + c_C * C;
33 }
34 point centroid(){ //center of mass
35     return (A + B + C) / 3.0;
36 }
37 point ortho_center(){ //euler line
38     point O = circum_center();

```

%446

#385

%742

```

38 return 0 + 3.0 * (centroid() - O);
39 };
40 point nine_point_circle_center(){ //euler line
41     point O = circum_center();
42     return 0 + 1.5 * (centroid() - O);
43 };
44 point in_center(){
45     if(collinear())
46         return point(NAN,NAN);
47     double a, b, c; //side lengths
48     a = abs(B - C);
49     b = abs(A - C);
50     c = abs(A - B);
51     //trilinear coordinates are (1,1,1)
52     //barycentric coordinates
53     double c_A = a, c_B = b, c_C = c;
54     double sum = c_A + c_B + c_C;
55     c_A /= sum;
56     c_B /= sum;
57     c_C /= sum;
58     // cartesian coordinates of the incenter
59     return c_A * A + c_B * B + c_C * C;
60 }

```

#193  
%031

#157

%980

#### 6 2D line segment

```

1 const long double PI = acos(-1.0L);
2 struct Vec {
3     long double x, y;
4     Vec& operator-=(Vec r) {
5         x -= r.x, y -= r.y;
6         return *this;
7     }
8     Vec operator-(Vec r) {return Vec(*this) -= r;}
9     Vec& operator+=(Vec r) {
10         x += r.x, y += r.y;
11         return *this;
12     }
13     Vec operator+(Vec r) {return Vec(*this) += r;}
14     Vec operator-() {return {-x, -y};}
15     Vec& operator*=(long double r) {
16         x *= r, y *= r;
17         return *this;
18     }
19     Vec operator*(long double r) {return Vec(*this) *= r;}
20     Vec& operator/=(long double r) {
21         x /= r, y /= r;
22         return *this;
23     }
24     Vec operator/(long double r) {return Vec(*this) /= r;}
25     long double operator*(Vec r) {
26         return x * r.x + y * r.y;
27     }
28 };

```

#054

#673

```

29 ostream& operator<<(ostream& l, Vec r) {
30     return l << '(' << r.x << ", " << r.y << ')';
31 }
32 long double len(Vec a) {
33     return hypot(a.x, a.y);
34 }
35 long double cross(Vec l, Vec r) {
36     return l.x * r.y - l.y * r.x;
37 }
38 long double angle(Vec a) {
39     return fmod(atan2(a.y, a.x)+2*PI, 2*PI);
40 }
41 Vec normal(Vec a) {
42     return Vec(-a.y, a.x) / len(a);
43 }
44
45
46 struct Segment {
47     Vec a, b;
48     Vec d() {
49         return b-a;
50     }
51 };
52 ostream& operator<<(ostream& l, Segment r) {
53     return l << r.a << '-' << r.b;
54 }
55 Vec intersection(Segment l, Segment r) {
56     Vec dl = l.d(), dr = r.d();
57     if(cross(dl, dr) == 0)
58         return {nanl(""), nanl("")};
59     long double h = cross(dr, l.a-r.a) / len(dr);
60     long double dh = cross(dr, dl) / len(dr);
61     return l.a + dl * (h / -dh);
62 }
63 //Returns the area bounded by halfplanes
64 long double getArea(vector<Segment> lines) {
65     long double lowerbound = -HUGE_VALL, upperbound = HUGE_VALL;
66     vector<Segment> linesBySide[2];
67     for(auto line : lines) {
68         if(line.b.y == line.a.y) {
69             if(line.a.x < line.b.x) {
70                 lowerbound = max(lowerbound, line.a.y);
71             } else {
72                 upperbound = min(upperbound, line.a.y);
73             }
74         } else if(line.a.y < line.b.y) {
75             linesBySide[1].push_back(line);
76         } else {
77             linesBySide[0].push_back({line.b, line.a});
78         }
79     }
80     sort(linesBySide[0].begin(), linesBySide[0].end(), [](Segment l,
81         Segment r) {
82         if(cross(l.d(), r.d()) == 0) return normal(l.d())*l.a <
83             normal(r.d())*r.a;

```

#724

#872

%654

#355

#658

#049

```

37     return cross(l.d(), r.d()) < 0;
38 });
39 sort(linesBySide[1].begin(), linesBySide[1].end(), [](Segment l,
40     Segment r) {
41     if(cross(l.d(), r.d()) == 0) return normal(l.d())*l.a <
42         normal(r.d())*r.a;
43     return cross(l.d(), r.d()) > 0;
44 });
45 //Now find the application area of the lines and clean up redundant
46     ones
47 vector<long double> applyStart[2];
48 for(int side = 0; side < 2; side++) {
49     vector<long double> &apply = applyStart[side];
50     vector<Segment> curLines;
51     for(auto line : linesBySide[side]) {
52         while(curLines.size() > 0) {
53             Segment other = curLines.back();
54             if(cross(line.d(), other.d()) != 0) {
55                 long double start = intersection(line, other).y;
56                 if(start > apply.back()) break;
57             }
58             curLines.pop_back();
59             apply.pop_back();
60         }
61         if(curLines.size() == 0) {
62             apply.push_back(-HUGE_VALL);
63         } else {
64             apply.push_back(intersection(line, curLines.back()).y);
65         }
66         curLines.push_back(line);
67     }
68     linesBySide[side] = curLines;
69 }
70 applyStart[0].push_back(HUGE_VALL);
71 applyStart[1].push_back(HUGE_VALL);
72 long double result = 0;
73 {
74     long double lb = -HUGE_VALL, ub;
75     for(int i=0, j=0; i < (int)linesBySide[0].size() && j <
76         (int)linesBySide[1].size(); lb = ub) {
77         ub = min(applyStart[0][i+1], applyStart[1][j+1]);
78         long double alb = lb, aub = ub;
79         Segment l0 = linesBySide[0][i], l1 = linesBySide[1][j];
80         if(cross(l1.d(), l0.d()) > 0) {
81             alb = max(alb, intersection(l0, l1).y);
82         } else if(cross(l1.d(), l0.d()) < 0) {
83             aub = min(aub, intersection(l0, l1).y);
84         }
85         alb = max(alb, lowerbound);
86         aub = min(aub, upperbound);
87         aub = max(aub, alb);
88         {
89             long double x1 = l0.a.x + (alb - l0.a.y) / l0.d().y * l0.d().x;

```

#434

#501

#060

#349

#419

```

86     long double x2 = 10.a.x + (aub - 10.a.y) / 10.d().y * 10.d().x;
87     result -= (aub - alb) * (x1 + x2) / 2;
88 }
89 {
90     long double x1 = 11.a.x + (alb - 11.a.y) / 11.d().y * 11.d().x;
91     long double x2 = 11.a.x + (aub - 11.a.y) / 11.d().y * 11.d().x;
92     result += (aub - alb) * (x1 + x2) / 2; #228
93 }
94 if(applyStart[0][i+1] < applyStart[1][j+1]) {
95     i++;
96 } else {
97     j++;
98 }
99 }
100 }
101 return result;
102 } %011

```

## 7 Convex polygon algorithms

```

1 ll dot(const pair< int, int > &v1, const pair< int, int > &v2) {
2     return (ll)v1.first * v2.first + (ll)v1.second * v2.second;
3 }
4 ll cross(const pair< int, int > &v1, const pair< int, int > &v2) {
5     return (ll)v1.first * v2.second - (ll)v2.first * v1.second;
6 }
7 ll dist_sq(const pair< int, int > &p1, const pair< int, int > &p2) {
8     return (ll)(p2.first - p1.first) * (p2.first - p1.first) +
9         (ll)(p2.second - p1.second) * (p2.second - p1.second);
10 } %025
11 struct Hull {
12     vector< pair< pair< int, int >, pair< int, int > > > hull;
13     vector< pair< pair< int, int >, pair< int, int > > >::iterator
14     ↪ upper_begin;
15     template < typename Iterator >
16     void extend_hull(Iterator begin, Iterator end) { // 0(n)
17         vector< pair< int, int > > res;
18         for (auto it = begin; it != end; ++it) {
19             if (res.empty() || *it != res.back()) {
20                 while (res.size() >= 2) {
21                     auto v1 = make_pair(res[res.size() - 1].first -
22                                         ↪ res[res.size() - 2].first, #048
23                                         res[res.size() - 1].second -
24                                         ↪ res[res.size() - 2].second);
25                     auto v2 = make_pair(it->first - res[res.size() - 2].first,
26                                         it->second - res[res.size() - 2].second);
27                     if (cross(v1, v2) > 0)
28                         break;
29                     res.pop_back();
30                 }
31                 res.push_back(*it);
32             }
33         }
34     }
35     for (int i = 0; i < res.size() - 1; ++i)
36         hull.emplace_back(res[i], res[i + 1]);

```

```

33 }
34 Hull(vector< pair< int, int > > &vert) { // at least 2 distinct
35     ↪ points
36     sort(vert.begin(), vert.end()); // 0(n log(n))
37     extend_hull(vert.begin(), vert.end());
38     int diff = hull.size();
39     extend_hull(vert.rbegin(), vert.rend());
40     upper_begin = hull.begin() + diff; %873
41 }
42 bool contains(pair< int, int > p) { // 0(log(n))
43     if (p < hull.front().first || p > upper_begin->first) return false;
44     {
45         auto it_low = lower_bound(hull.begin(), upper_begin,
46                                   make_pair(make_pair(p.first,
47                                   ↪ (int)-2e9), make_pair(0, 0)));
48         if (it_low != hull.begin())
49             --it_low;
50         auto v1 = make_pair(it_low->second.first - it_low->first.first,
51                             it_low->second.second -
52                             ↪ it_low->first.second);
53         auto v2 = make_pair(p.first - it_low->first.first, p.second -
54                             ↪ it_low->first.second); #094
55         if (cross(v1, v2) < 0) // < 0 is inclusive, <= 0 is exclusive
56             return false;
57     }
58     {
59         auto it_up = lower_bound(hull.rbegin(), hull.rbegin() +
60                                   ↪ (hull.end() - upper_begin),
61                                   make_pair(make_pair(p.first, (int)2e9),
62                                   ↪ make_pair(0, 0)));
63         if (it_up - hull.rbegin() == hull.end() - upper_begin)
64             --it_up;
65         auto v1 = make_pair(it_up->first.first - it_up->second.first,
66                             it_up->first.second - it_up->second.second);
67         ↪ #900
68         auto v2 = make_pair(p.first - it_up->second.first, p.second -
69                             ↪ it_up->second.second);
70         if (cross(v1, v2) > 0) // > 0 is inclusive, >= 0 is exclusive
71             return false;
72     }
73     return true;
74 } %092
75 template < typename T > // The function can have only one local min
76     ↪ and max and may be constant
77     // only at min and max.
78     vector< pair< pair< int, int >, pair< int, int > > >::iterator max(
79         function< T(const pair< pair< int, int >, pair< int, int > > &) >
80         ↪ f) { // 0(log(n))
81         auto l = hull.begin();
82         auto r = hull.end();
83         vector< pair< pair< int, int >, pair< int, int > > >::iterator best
84         ↪ = hull.end();
85         T best_val;

```

```

75 while (r - l > 2) {
76     auto mid = l + (r - l) / 2;
77     T l_val = f(*l); #242
78     T l_nxt_val = f(*(l + 1));
79     T mid_val = f(*mid);
80     T mid_nxt_val = f(*(mid + 1));
81     if (best == hull.end() ||
82         l_val > best_val) { // If max is at l we may remove it from
83         // the range.
84         best = l;
85         best_val = l_val;
86     }
87     if (l_nxt_val > l_val) {
88         if (mid_val < l_val) { #012
89             r = mid;
90         } else {
91             if (mid_nxt_val > mid_val) {
92                 l = mid + 1;
93             } else {
94                 r = mid + 1;
95             }
96         }
97     } else {
98         if (mid_val < l_val) { #373
99             l = mid + 1;
100         } else {
101             if (mid_nxt_val > mid_val) {
102                 l = mid + 1;
103             } else {
104                 r = mid + 1;
105             }
106         }
107     }
108     T l_val = f(*l);
109     if (best == hull.end() || l_val > best_val) {
110         best = l;
111         best_val = l_val;
112     }
113     if (r - l > 1) {
114         T l_nxt_val = f(*(l + 1));
115         if (best == hull.end() || l_nxt_val > best_val) {
116             best = l + 1;
117             best_val = l_nxt_val; #930
118         }
119     }
120     return best;
121 } %331
122 vector< pair< pair< int, int >, pair< int, int > >::iterator
123     → closest(
124     pair< int, int >
125     p) { // p can't be internal(can be on border), hull must
126     → have atleast 3 points

```

```

125 const pair< pair< int, int >, pair< int, int > > &ref_p =
126     → hull.front(); // O(log(n))
127 return max(function< double(const pair< pair< int, int >, pair<
128     → int, int > > &) >(
129     [&p, &ref_p](const pair< pair< int, int >, pair< int, int > >
130         &seg) { // accuracy of used type should be
131             coord^2
132             if (p == seg.first) return 10 - M_PI;
133             auto v1 =
134                 make_pair(seg.second.first - seg.first.first,
135                     → seg.second.second - seg.first.second); #685
136             auto v2 = make_pair(p.first - seg.first.first, p.second -
137                 → seg.first.second);
138             ll cross_prod = cross(v1, v2);
139             if (cross_prod > 0) { // order the backside by angle
140                 auto v1 = make_pair(ref_p.first.first - p.first,
141                     → ref_p.first.second - p.second);
142                 auto v2 = make_pair(seg.first.first - p.first,
143                     → seg.first.second - p.second);
144                 ll dot_prod = dot(v1, v2);
145                 ll cross_prod = cross(v2, v1);
146                 return atan2(cross_prod, dot_prod) / 2;
147             }
148             ll dot_prod = dot(v1, v2); #395
149             double res = atan2(dot_prod, cross_prod);
150             if (dot_prod <= 0 && res > 0) res = -M_PI;
151             if (res > 0) {
152                 res += 20;
153             } else {
154                 res = 10 - res;
155             }
156             return res;
157         }));
158     } %483
159 pair< int, int > forw_tan(pair< int, int > p) { // can't be internal
160     → or on border
161     const pair< pair< int, int >, pair< int, int > > &ref_p =
162         → hull.front(); // O(log(n))
163     auto best_seg = max(function< double(const pair< pair< int, int >,
164         → pair< int, int > > &) >(
165         [&p, &ref_p](const pair< pair< int, int >, pair< int, int > >
166             &seg) { // accuracy of used type should be
167                 coord^2
168                 auto v1 = make_pair(ref_p.first.first - p.first,
169                     → ref_p.first.second - p.second);
170                 auto v2 = make_pair(seg.first.first - p.first,
171                     → seg.first.second - p.second);
172                 ll dot_prod = dot(v1, v2);
173                 ll cross_prod = cross(v2, v1); // cross(v1, v2) for
174                     back_tan!!!
175                 return atan2(cross_prod, dot_prod); // order by signed
176                     → angle #291
177             }));

```



```

163     return best_seg->first;
164 } %850
165 vector< pair< pair< int, int >, pair< int, int > > >::iterator
166   ↳ max_in_dir(
167     pair< int, int > v) { // first is the ans. O(log(n))
168     return max(function< ll(const pair< pair< int, int >, pair< int,
169       ↳ int > > &) >(
170       [&v](const pair< pair< int, int >, pair< int, int > > &seg) {
171         ↳ return dot(v, seg.first); }));
172 }
173 pair< vector< pair< pair< int, int >, pair< int, int > > >::iterator,
174   vector< pair< pair< int, int >, pair< int, int > > >::iterator
175   ↳ > %013
176 intersections(pair< pair< int, int >, pair< int, int > > line) { //
177   ↳ O(log(n))
178   int x = line.second.first - line.first.first;
179   int y = line.second.second - line.first.second;
180   auto dir = make_pair(-y, x);
181   auto it_max = max_in_dir(dir);
182   auto it_min = max_in_dir(make_pair(y, -x));
183   ll opt_val = dot(dir, line.first);
184   if (dot(dir, it_max->first) < opt_val || dot(dir, it_min->first) >
185     ↳ opt_val)
186     return make_pair(hull.end(), hull.end());
187   vector< pair< pair< int, int >, pair< int, int > > >::iterator
188     ↳ it_r1, it_r2; #785
189   function< bool(const pair< pair< int, int >, pair< int, int > > &,
190     const pair< pair< int, int >, pair< int, int > > &)
191     ↳ >
192     inc_comp([&dir](const pair< pair< int, int >, pair< int, int >
193       ↳ > &lft,
194         const pair< pair< int, int >, pair< int, int >
195         ↳ > &rgt) {
196         return dot(dir, lft.first) < dot(dir, rgt.first);
197       });
198   function< bool(const pair< pair< int, int >, pair< int, int > > &,
199     const pair< pair< int, int >, pair< int, int > > &)
200     ↳ >
201     dec_comp([&dir](const pair< pair< int, int >, pair< int, int >
202       ↳ > &lft,
203         const pair< pair< int, int >, pair< int, int >
204         ↳ > &rgt) {
205         return dot(dir, lft.first) > dot(dir, rgt.first);
206       });
207   if (it_min <= it_max) {
208     it_r1 = upper_bound(it_min, it_max + 1, line, inc_comp) - 1;
209     if (dot(dir, hull.front().first) >= opt_val) {
210       it_r2 = upper_bound(hull.begin(), it_min + 1, line, dec_comp) -
211         ↳ 1;
212     } else {
213       it_r2 = upper_bound(it_min, hull.end(), line, inc_comp) - 1;
214     }
215     return make_pair(it_r1, it_r2);
216   }
217 } %000
218 pair< pair< int, int >, pair< int, int > > diameter() { // O(n)
219   pair< pair< int, int >, pair< int, int > > res;
220   ll dia_sq = 0;
221   auto it1 = hull.begin();
222   auto it2 = upper_begin;
223   auto v1 = make_pair(hull.back().second.first -
224     ↳ hull.back().first.first,
225     hull.back().second.second -
226     ↳ hull.back().first.second);
227   while (it2 != hull.begin()) {
228     auto v2 = make_pair((it2 - 1)->second.first - (it2 -
229       ↳ 1)->first.first,
230       (it2 - 1)->second.second - (it2 -
231         ↳ 1)->first.second);
232     ll decider = cross(v1, v2);
233     if (decider > 0) break;
234     --it2;
235   }
236   while (it2 != hull.end()) { // check all antipodal pairs
237     if (dist_sq(it1->first, it2->first) > dia_sq) {
238       res = make_pair(it1->first, it2->first);
239       dia_sq = dist_sq(res.first, res.second);
240     }
241     auto v1 =
242       make_pair(it1->second.first - it1->first.first,
243         ↳ it1->second.second - it1->first.second);
244     auto v2 =
245       make_pair(it2->second.first - it2->first.first,
246         ↳ it2->second.second - it2->first.second);
247     ll decider = cross(v1, v2);
248     if (decider == 0) { // report cross pairs at parallel lines.
249       if (dist_sq(it1->second, it2->first) > dia_sq) {
250         res = make_pair(it1->second, it2->first);
251         dia_sq = dist_sq(res.first, res.second);
252       }
253       if (dist_sq(it1->first, it2->second) > dia_sq) {
254         res = make_pair(it1->first, it2->second);
255         dia_sq = dist_sq(res.first, res.second);
256       }
257       ++it1;
258       ++it2;
259     } else if (decider < 0) {
260       ++it1;

```

```

202     it_r1 = upper_bound(it_max, it_min + 1, line, dec_comp) - 1;
203     if (dot(dir, hull.front().first) <= opt_val) {
204       it_r2 = upper_bound(hull.begin(), it_max + 1, line, inc_comp) -
205         ↳ 1;
206     } else {
207       it_r2 = upper_bound(it_min, hull.end(), line, inc_comp) - 1;
208     }
209     return make_pair(it_r1, it_r2);
210   }
211 } %000
212 pair< pair< int, int >, pair< int, int > > diameter() { // O(n)
213   pair< pair< int, int >, pair< int, int > > res;
214   ll dia_sq = 0;
215   auto it1 = hull.begin();
216   auto it2 = upper_begin;
217   auto v1 = make_pair(hull.back().second.first -
218     ↳ hull.back().first.first,
219     hull.back().second.second -
220     ↳ hull.back().first.second);
221   while (it2 != hull.begin()) {
222     auto v2 = make_pair((it2 - 1)->second.first - (it2 -
223       ↳ 1)->first.first,
224       (it2 - 1)->second.second - (it2 -
225         ↳ 1)->first.second);
226     ll decider = cross(v1, v2);
227     if (decider > 0) break;
228     --it2;
229   }
230   while (it2 != hull.end()) { // check all antipodal pairs
231     if (dist_sq(it1->first, it2->first) > dia_sq) {
232       res = make_pair(it1->first, it2->first);
233       dia_sq = dist_sq(res.first, res.second);
234     }
235     auto v1 =
236       make_pair(it1->second.first - it1->first.first,
237         ↳ it1->second.second - it1->first.second);
238     auto v2 =
239       make_pair(it2->second.first - it2->first.first,
240         ↳ it2->second.second - it2->first.second);
241     ll decider = cross(v1, v2);
242     if (decider == 0) { // report cross pairs at parallel lines.
243       if (dist_sq(it1->second, it2->first) > dia_sq) {
244         res = make_pair(it1->second, it2->first);
245         dia_sq = dist_sq(res.first, res.second);
246       }
247       if (dist_sq(it1->first, it2->second) > dia_sq) {
248         res = make_pair(it1->first, it2->second);
249         dia_sq = dist_sq(res.first, res.second);
250       }
251       ++it1;
252       ++it2;
253     } else if (decider < 0) {
254       ++it1;

```

#671

#674

#466

```

248     } else {
249         ++it2;
250     }
251 }
252 return res;
253 }
254 };

```

#502  
%215

### 8 Aho Corasick $\mathcal{O}(|\alpha| \sum \text{len})$

```

1 const int alpha_size=26;
2 struct node{
3     node *nxt[alpha_size]; //May use other structures to move in trie
4     node *suffix;
5     node(){
6         memset(nxt, 0, alpha_size*sizeof(node *));
7     }
8     int cnt=0;
9 };
10 node *aho_corasick(vector<vector<char> > &dict){
11     node *root= new node;
12     root->suffix = 0;
13     vector<pair<vector<char> *, node *> > cur_state;
14     for(vector<char> &s : dict)
15         cur_state.emplace_back(&s, root);
16     for(int i=0; !cur_state.empty(); ++i){
17         vector<pair<vector<char> *, node *> > nxt_state;
18         for(auto &cur : cur_state){
19             node *nxt=cur.second->nxt[(cur.first)[i]];
20             if(nxt){
21                 cur.second=nxt;
22             }else{
23                 nxt = new node;
24                 cur.second->nxt[(cur.first)[i]] = nxt;
25                 node *suf = cur.second->suffix;
26                 cur.second = nxt;
27                 nxt->suffix = root; //set correct suffix link
28                 while(suf){
29                     if(suf->nxt[(cur.first)[i]]){
30                         nxt->suffix = suf->nxt[(cur.first)[i]];
31                         break;
32                     }
33                     suf=suf->suffix;
34                 }
35             }
36             if(cur.first->size() > i+1)
37                 nxt_state.push_back(cur);
38         }
39         cur_state=nxt_state;
40     }
41     return root;
42 }
43 //auxiliary functions for searching and counting
44 node *walk(node *cur, char c){ //longest prefix in dict that is suffix
45     of walked string.

```

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#940  
%064

```

45 while(true){
46     if(cur->nxt[c])
47         return cur->nxt[c];
48     if(!cur->suffix)
49         return cur;
50     cur = cur->suffix;
51 }
52 }
53 void cnt_matches(node *root, vector<char> &match_in){
54     node *cur = root;
55     for(char c : match_in){
56         cur = walk(cur, c);
57         ++cur->cnt;
58     }
59 }
60 void add_cnt(node *root){ //After counting matches propagete ONCE to
61     suffixes for final counts
62     vector<node *> to_visit = {root};
63     for(int i=0; i<to_visit.size(); ++i){
64         node *cur = to_visit[i];
65         for(int j=0; j<alpha_size; ++j){
66             if(cur->nxt[j])
67                 to_visit.push_back(cur->nxt[j]);
68         }
69     }
70     for(int i=to_visit.size()-1; i>0; --i)
71         to_visit[i]->suffix->cnt += to_visit[i]->cnt;
72 }
73 int main(){
74     //http://codeforces.com/group/s3etJR5zZK/contest/212916/problem/4
75     int n, len;
76     scanf("%d %d", &n, &len);
77     vector<char> a(len+1);
78     scanf("%s", a.data());
79     a.pop_back();
80     for(char &c : a)
81         c -= 'a';
82     vector<vector<char> > dict(n);
83     for(int i=0; i<n; ++i){
84         scanf("%d", &len);
85         dict[i].resize(len+1);
86         scanf("%s", dict[i].data());
87         dict[i].pop_back();
88         for(char &c : dict[i])
89             c -= 'a';
90     }
91     node *root = aho_corasick(dict);
92     cnt_matches(root, a);
93     add_cnt(root);
94     for(int i=0; i<n; ++i){
95         node *cur = root;
96         for(char c : dict[i])
97             cur = walk(cur, c);

```

%127  
%286  
#865  
%313

```

96     printf("%d\n", cur->cnt);
97 }
98 }

```

### 9 Suffix automaton and tree $O((n+q)\log(|\alpha|))$

```

1 class AutoNode {
2 private:
3     map< char, AutoNode * > nxt_char; // Map is faster than hashtable
4     // and unsorted arrays
5 public:
6     int len; //Length of longest suffix in equivalence class.
7     AutoNode *suf;
8     bool has_nxt(char c) const {
9         return nxt_char.count(c);
10    }
11    AutoNode *nxt(char c) {
12        if (!has_nxt(c))
13            return NULL;
14        return nxt_char[c];
15    }
16    void set_nxt(char c, AutoNode *node) {
17        nxt_char[c] = node;
18    }
19    AutoNode *split(int new_len, char c) {
20        AutoNode *new_n = new AutoNode;
21        new_n->nxt_char = nxt_char;
22        new_n->len = new_len;
23        new_n->suf = suf;
24        suf = new_n;
25        return new_n;
26    }
27    // Extra functions for matching and counting
28    AutoNode *lower_depth(int depth) { //move to longest suffix of
29        // current with a maximum length of depth.
30        if (suf->len >= depth)
31            return suf->lower_depth(depth);
32        return this;
33    }
34    AutoNode *walk(char c, int depth, int &match_len) { //move to longest
35        // suffix of walked path that is a substring
36        match_len = min(match_len, len); //includes depth limit(needed for
37        // finding matches)
38        if (has_nxt(c)) { //as suffixes are in classes match_len must be
39        // tracked externally
40            ++match_len;
41            return nxt(c)->lower_depth(depth);
42        }
43        if (suf)
44            return suf->walk(c, depth, match_len);
45        return this;
46    }
47    int paths_to_end = 0;
48    void set_as_end() { //All suffixes of current node are marked as
49        // ending nodes.

```

```

44     paths_to_end += 1;
45     if (suf) suf->set_as_end();
46 }
47 bool vis = false;
48 void calc_paths_to_end() { //Call ONCE from ROOT. For each node
49     // calculates number of ways to reach an end node.
50     if (!vis) { //paths_to_end is ocurence count for any strings in
51         // current suffix equivalence class.
52         vis = true;
53         for (auto cur : nxt_char) {
54             cur.second->calc_paths_to_end();
55             paths_to_end += cur.second->paths_to_end;
56         }
57     }
58     //Transform into suffix tree of reverse string
59     map<char, AutoNode * > tree_links;
60     int end_dist = 1<<30;
61     int calc_end_dist(){
62         if(end_dist == 1<<30){
63             if(nxt_char.empty())
64                 end_dist = 0;
65             for (auto cur : nxt_char)
66                 end_dist = min(end_dist, 1+cur.second->calc_end_dist());
67         }
68         return end_dist;
69     }
70     bool vis_t = false;
71     void build_suffix_tree(string &s) { //Call ONCE from ROOT.
72         if (!vis_t) {
73             vis_t = true;
74             if(suf)
75                 suf->tree_links[s[s.size()-end_dist-suf->len-1]] = this;
76             for (auto cur : nxt_char)
77                 cur.second->build_suffix_tree(s);
78         }
79     };
80     struct SufAutomaton {
81         AutoNode *last;
82         AutoNode *root;
83         void extend(char new_c) {
84             AutoNode *new_end = new AutoNode;
85             new_end->len = last->len + 1;
86             AutoNode *suf_w_nxt = last;
87             while (suf_w_nxt && !suf_w_nxt->has_nxt(new_c)) {
88                 suf_w_nxt->set_nxt(new_c, new_end);
89                 suf_w_nxt = suf_w_nxt->suf;
90             }
91             if (!suf_w_nxt) {
92                 new_end->suf = root;
93             } else {
94                 AutoNode *max_sbstr = suf_w_nxt->nxt(new_c);

```



```

95     if (suf_w_nxt->len + 1 == max_sbstr->len) {
96         new_end->suf = max_sbstr;
97     } else {
98         AutoNode *eq_sbstr = max_sbstr->split(suf_w_nxt->len + 1,
99         ↪ new_c);
100         new_end->suf = eq_sbstr;
101         AutoNode *w_edge_to_eq_sbstr = suf_w_nxt;
102         while (w_edge_to_eq_sbstr != 0 &&
103         ↪ w_edge_to_eq_sbstr->nxt(new_c) == max_sbstr) {
104             w_edge_to_eq_sbstr->set_nxt(new_c, eq_sbstr);
105             w_edge_to_eq_sbstr = w_edge_to_eq_sbstr->suf;
106         }
107     }
108     last = new_end;
109 }
110 SufAutomaton(string &s) {
111     root = new AutoNode;
112     root->len = 0;
113     root->suf = NULL;
114     last = root;
115     for (char c : s) extend(c);
116     root->calc_end_dist(); //To build suffix tree use reversed string
117     root->build_suffix_tree(s);
118 };

```

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## 10 Dinic

```

1 struct MaxFlow{
2     typedef long long ll;
3     const ll INF = 1e18;
4     struct Edge{
5         int u,v;
6         ll c,rc;
7         shared_ptr<ll> flow;
8         Edge(int _u, int _v, ll _c, ll _rc = 0):u(_u),v(_v),c(_c),rc(_rc){
9         }
10    };
11    struct FlowTracker{
12        shared_ptr<ll> flow;
13        ll cap, rcap;
14        bool dir;
15        FlowTracker(ll _cap, ll _rcap, shared_ptr<ll> _flow, int
16        ↪ _dir):cap(_cap),rcap(_rcap),flow(_flow),dir(_dir){ }
17        ll rem() const {
18            if(dir == 0){
19                return cap-*flow;
20            }
21            else{
22                return rcap-*flow;
23            }
24        }
25        void add_flow(ll f){
26            if(dir == 0)

```

#787

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```

26         *flow += f;
27     else
28         *flow -= f;
29     assert(*flow <= cap);
30     assert(*flow <= rcap);
31 }
32 operator ll() const { return rem(); }
33 void operator--=(ll x){ add_flow(x); }
34 void operator+=(ll x){ add_flow(-x); }
35 };
36 int source,sink;
37 vector<vector<int>> > adj;
38 vector<vector<FlowTracker>> > cap;
39 vector<Edge> edges;
40 MaxFlow(int _source, int _sink):source(_source),sink(_sink){
41     assert(source != sink);
42 }
43 int add_edge(int u, int v, ll c, ll rc = 0){
44     edges.push_back(Edge(u,v,c,rc));
45     return edges.size()-1;
46 }
47 vector<int> now,lv1;
48 void prep(){
49     int max_id = max(source, sink);
50     for(auto edge : edges)
51         max_id = max(max_id, max(edge.u, edge.v));
52     adj.resize(max_id+1);
53     cap.resize(max_id+1);
54     now.resize(max_id+1);
55     lv1.resize(max_id+1);
56     for(auto &edge : edges){
57         auto flow = make_shared<ll>(0);
58         adj[edge.u].push_back(edge.v);
59         cap[edge.u].push_back(FlowTracker(edge.c, edge.rc, flow, 0));
60         if(edge.u != edge.v){
61             adj[edge.v].push_back(edge.u);
62             cap[edge.v].push_back(FlowTracker(edge.c, edge.rc, flow, 1));
63         }
64         assert(cap[edge.u].back() == edge.c);
65         edge.flow = flow;
66     }
67 }
68 bool dinic_bfs(){
69     fill(now.begin(),now.end(),0);
70     fill(lv1.begin(),lv1.end(),0);
71     lv1[source] = 1;
72     vector<int> bfs(1,source);
73     for(int i = 0; i < bfs.size(); ++i){
74         int u = bfs[i];
75         for(int j = 0; j < adj[u].size(); ++j){
76             int v = adj[u][j];
77             if(cap[u][j] > 0 && lv1[v] == 0){
78                 lv1[v] = lv1[u]+1;

```

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#038

```

79     bfs.push_back(v);
80 }
81 }
82 }
83 return lvl[sink] > 0;
84 }
85 ll dinic_dfs(int u, ll flow){
86     if(u == sink)
87         return flow;
88     while(now[u] < adj[u].size()){
89         int v = adj[u][now[u]];
90         if(lvl[v] == lvl[u] + 1 && cap[u][now[u]] != 0){
91             ll res = dinic_dfs(v, min(flow, (ll)cap[u][now[u]]));
92             if(res > 0){
93                 cap[u][now[u]] -= res;
94                 return res;
95             }
96         }
97         ++now[u];
98     }
99     return 0;
100 }
101 ll calc_max_flow(){
102     prep();
103     ll ans = 0;
104     while(dinic_bfs()){
105         ll cur = 0;
106         do{
107             cur = dinic_dfs(source, INF);
108             ans += cur;
109         }while(cur > 0);
110     }
111     return ans;
112 }
113 ll flow_on_edge(int edge_index){
114     assert(edge_index < edges.size());
115     return *edges[edge_index].flow;
116 }
117 };
118 int main(){
119     int n,m;
120     cin >> n >> m;
121     vector<pair<int, pair<int, int> > > graph(m);
122     for(int i=0; i<m; ++i){
123         cin>>graph[i].second.first>>graph[i].second.second>>graph[i].first;
124     }
125     ll res=0;
126     for(auto cur : graph){
127         auto mf = MaxFlow(cur.second.first,cur.second.second); // arguments
128         ↪ source and sink, memory usage O(largest node index + input
129         ↪ size), sink doesn't need to be last index
130         for(int i = 0; i < m; ++i){
131             if(graph[i].first > cur.first){

```

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```

130     mf.add_edge(graph[i].second.first,graph[i].second.second,1,1);
131     ↪ // store edge index if care about flow value
132 }
133 res += mf.calc_max_flow();
134 }
135 cout<<res<<endl;
136 }

```

### 11 Min Cost Max Flow with successive dijkstra $O(\text{flow} \cdot n^2)$

```

1 const int nmax=1055;
2 const ll inf=1e14;
3 int t, n, v; //0 is source, v-1 sink
4 ll rem_flow[nmax][nmax]; //set [x][y] for directed capacity from x to
5 ↪ y.
6 ll cost[nmax][nmax]; //set [x][y] for directed cost from x to y. SET TO
7 ↪ inf IF NOT USED
8 ll min_dist[nmax];
9 int prev_node[nmax];
10 ll node_flow[nmax];
11 bool visited[nmax];
12 ll tot_cost, tot_flow; //output
13 void min_cost_max_flow(){
14     tot_cost=0; //Does not work with negative cycles.
15     tot_flow=0;
16     ll sink_pot=0;
17     min_dist[0] = 0;
18     for(int i=1; i<=v; ++i){ //incase of no negative edges Bellman-Ford
19         ↪ can be removed.
20         min_dist[i]=inf;
21     }
22     for(int i=0; i<v-1; ++i){
23         for(int j=0; j<v; ++j){
24             for(int k=0; k<v; ++k){
25                 if(rem_flow[j][k] > 0 && min_dist[j]+cost[j][k] < min_dist[k])
26                     min_dist[k] = min_dist[j]+cost[j][k];
27             }
28         }
29     }
30     for(int i=0; i<v; ++i){ //Apply potentials to edge costs.
31         for(int j=0; j<v; ++j){
32             if(cost[i][j]!=inf){
33                 cost[i][j]+=min_dist[i];
34                 cost[i][j]-=min_dist[j];
35             }
36         }
37     }
38     sink_pot+=min_dist[v-1]; //Bellman-Ford end.
39     while(true){
40         for(int i=0; i<=v; ++i){ //node after sink is used as start value
41             ↪ for Dijkstra.
42             min_dist[i]=inf;
43             visited[i]=false;

```

%576

%927

#599

%849

```

41 min_dist[0]=0;
42 node_flow[0]=inf;
43 int min_node;
44 while(true){ //Use Dijkstra to calculate potentials
45     int min_node=v;
46     for(int i=0; i<v; ++i){
47         if((!visited[i]) && min_dist[i]<min_dist[min_node])
48             min_node=i;
49     }
50     if(min_node==v) break
51     visited[min_node]=true;
52     for(int i=0; i<v; ++i){
53         if((!visited[i]) && min_dist[min_node]+cost[min_node][i] <
54             ↪ min_dist[i]){
55             min_dist[i]=min_dist[min_node]+cost[min_node][i];
56             prev_node[i]=min_node;
57             node_flow[i]=min(node_flow[min_node], rem_flow[min_node][i]);
58         }
59     }
60     if(min_dist[v-1]==inf) break
61     for(int i=0; i<v; ++i){ //Apply potentials to edge costs.
62         for(int j=0; j<v; ++j){ //Found path from source to sink becomes
63             ↪ 0 cost.
64             if(cost[i][j]!=inf){
65                 cost[i][j]+=min_dist[i];
66                 cost[i][j]-=min_dist[j];
67             }
68         }
69     }
70     sink_pot+=min_dist[v-1];
71     tot_flow+=node_flow[v-1];
72     tot_cost+=sink_pot*node_flow[v-1];
73     int cur=v-1;
74     while(cur!=0){ //Backtrack along found path that now has 0 cost.
75         rem_flow[prev_node[cur]][cur]-=node_flow[v-1];
76         rem_flow[cur][prev_node[cur]]+=node_flow[v-1];
77         cost[cur][prev_node[cur]]=0;
78         if(rem_flow[prev_node[cur]][cur]==0)
79             cost[prev_node[cur]][cur]=inf;
80         cur=prev_node[cur];
81     }
82 }
83 int main(){//http://www.spoj.com/problems/GREED/
84     cin>>t;
85     for(int i=0; i<t; ++i){
86         cin>>n;
87         for(int j=0; j<nmax; ++j){
88             for(int k=0; k<nmax; ++k){
89                 cost[j][k]=inf;
90                 rem_flow[j][k]=0;
91             }
92         }

```

#782

#881

#083

#582

%803

```

93     for(int j=1; j<=n; ++j){
94         cost[j][2*n+1]=0;
95         rem_flow[j][2*n+1]=1;
96     }
97     for(int j=1; j<=n; ++j){
98         int card;
99         cin>>card;
100         ++rem_flow[0][card];
101         cost[0][card]=0;
102     }
103     int ex_c;
104     cin>>ex_c;
105     for(int j=0; j<ex_c; ++j){
106         int a, b;
107         cin>>a>>b;
108         if(b<a) swap(a,b);
109         cost[a][b]=1;
110         rem_flow[a][b]=nmax;
111         cost[b][n+b]=0;
112         rem_flow[b][n+b]=nmax;
113         cost[n+b][a]=1;
114         rem_flow[n+b][a]=nmax;
115     }
116     v=2*n+2;
117     min_cost_max_flow();
118     cout<<tot_cost<<'\n';
119 }
120 }

```

## 12 Min Cost Max Flow with Cycle Cancellation $\mathcal{O}(\text{flow} \cdot nm)$

```

1 struct Network {
2     struct Node;
3     struct Edge {
4         Node *u, *v;
5         int f, c, cost;
6         Node* from(Node* pos) {
7             if(pos == u)
8                 return v;
9             return u;
10        }
11        int getCap(Node* pos) {
12            if(pos == u)
13                return c-f;
14            return f;
15        }
16        int addFlow(Node* pos, int toAdd) {
17            if(pos == u) {
18                f += toAdd;
19                return toAdd * cost;
20            } else {
21                f -= toAdd;
22                return -toAdd * cost;
23            }

```

#042

#965

```

24     }
25 };
26 struct Node {
27     vector<Edge*> conn;
28     int index;
29 };
30 deque<Node> nodes;
31 deque<Edge> edges;
32 Node* addNode() {
33     nodes.push_back(Node());
34     nodes.back().index = nodes.size()-1;
35     return &nodes.back();
36 }
37 Edge* addEdge(Node* u, Node* v, int f, int c, int cost) {
38     edges.push_back({u, v, f, c, cost});
39     u->conn.push_back(&edges.back());
40     v->conn.push_back(&edges.back());
41     return &edges.back();
42 }
43 //Assumes all needed flow has already been added
44 int minCostMaxFlow() {
45     int n = nodes.size();
46     int result = 0;
47     struct State {
48         int p;
49         Edge* used;
50     };
51     while(1) {
52         vector<vector<State>> > state(1, vector<State>(n, {0, 0}));
53         for(int lev = 0; lev < n; lev++) {
54             state.push_back(state[lev]);
55             for(int i=0; i<n; i++){
56                 if(lev == 0 || state[lev][i].p < state[lev-1][i].p) {
57                     for(Edge* edge : nodes[i].conn){
58                         if(edge->getCap(&nodes[i]) > 0) {
59                             int np = state[lev][i].p + (edge->u == &nodes[i] ?
60                                 edge->cost : -edge->cost);
61                             int ni = edge->from(&nodes[i])->index;
62                             if(np < state[lev+1][ni].p) {
63                                 state[lev+1][ni].p = np;
64                                 state[lev+1][ni].used = edge;
65                             }
66                         }
67                     }
68                 }
69             }
70             //Now look at the last level
71             bool valid = false;
72             for(int i=0; i<n; i++){
73                 if(state[n-1][i].p > state[n][i].p) {
74                     valid = true;
75                     vector<Edge*> path;
76                     int cap = 1000000000;

```

#534

#507

#877

#281

#283

```

77     Node* cur = &nodes[i];
78     int clev = n;
79     vector<bool> explr(n, false);
80     while(!explr[cur->index]) {
81         explr[cur->index] = true;
82         State cstate = state[clev][cur->index];
83         cur = cstate.used->from(cur);
84         path.push_back(cstate.used);
85     }
86     reverse(path.begin(), path.end());
87     {
88         int i=0;
89         Node* cur2 = cur;
90         do {
91             cur2 = path[i]->from(cur2);
92             i++;
93         } while(cur2 != cur);
94         path.resize(i);
95     }
96     for(auto edge : path) {
97         cap = min(cap, edge->getCap(cur));
98         cur = edge->from(cur);
99     }
100     for(auto edge : path) {
101         result += edge->addFlow(cur, cap);
102         cur = edge->from(cur);
103     }
104     }
105     if(!valid) break;
106 }
107 return result;
108 }
109 };

```

#954

#990

#599

%900

### 13 DMST $\mathcal{O}(E \log V)$

```

1 struct EdgeDesc{
2     int from, to, w;
3 };
4 struct DMST{
5     struct Node;
6     struct Edge{
7         Node *from;
8         Node *tar;
9         int w;
10        bool inc;
11    };
12    struct Circle{
13        bool vis = false;
14        vector<Edge*> contents;
15        void clean(int idx);
16    };
17    const static greater<pair<ll, Edge*>> comp; //Can use inline static
    since C++17

```

#186

```

18 static vector<Circle> to_process;
19 static bool no_dmst;
20 static Node *root;
21 struct Node{
22     Node *par = NULL;
23     vector<pair<int, int> > out_cands; //Circ, edge idx
24     vector<pair<ll, Edge *> > con;
25     bool in_use = false;
26     ll w = 0; //extra to add to edges in con
27     Node *anc(){
28         if(!par)
29             return this;
30         while(par->par)
31             par = par->par;
32         return par;
33     }
34     void clean(){
35         if(!no_dmst){
36             in_use = false;
37             for(auto &cur : out_cands)
38                 to_process[cur.first].clean(cur.second);
39         }
40     }
41     Node *con_to_root(){
42         if(anc() == root)
43             return root;
44         in_use = true;
45         Node *super = this; //Will become root or the first Node
46             ↪ encountered in a loop.
47         while(super == this){
48             while(!con.empty() && con.front().second->tar->anc() == anc()){
49                 pop_heap(con.begin(), con.end(), comp);
50                 con.pop_back();
51             }
52             if(con.empty()){
53                 no_dmst = true;
54                 return root;
55             }
56             pop_heap(con.begin(), con.end(), comp);
57             auto nxt = con.back();
58             con.pop_back();
59             w = -nxt.first;
60             if(nxt.second->tar->in_use){ //anc() wouldn't change anything
61                 super = nxt.second->tar->anc();
62                 to_process.resize(to_process.size()+1);
63             } else {
64                 super = nxt.second->tar->con_to_root();
65             }
66             if(super != root){
67                 to_process.back().contents.push_back(nxt.second);
68                 out_cands.emplace_back(to_process.size()-1,
69                     ↪ to_process.back().contents.size()-1);
70             } else { //Clean circles
71                 nxt.second->inc = true;

```

#536

#425

#561

#522

#174

```

70         nxt.second->from->clean();
71     }
72 }
73 if(super != root){ //we are some loops non first Node.
74     if(con.size() > super->con.size()){
75         swap(con, super->con); //Largest con in loop should not be
76             ↪ copied.
77         swap(w, super->w);
78     }
79     for(auto cur : con){
80         super->con.emplace_back(cur.first - super->w + w,
81             ↪ cur.second);
82         push_heap(super->con.begin(), super->con.end(), comp); #375
83     }
84     par = super; //root or anc() of first Node encountered in a loop
85     return super;
86 }
87 Node *cur_root;
88 vector<Node> graph;
89 vector<Edge> edges;
90 DMST(int n, vector<EdgeDesc> &desc, int r){ //Self loops and multiple
91     ↪ edges are okay. #076
92     graph.resize(n);
93     cur_root = &graph[r];
94     for(auto &cur : desc) //Edges are reversed internally
95         edges.push_back(Edge{&graph[cur.to], &graph[cur.from], cur.w});
96     for(int i=0; i<desc.size(); ++i)
97         graph[desc[i].to].con.emplace_back(desc[i].w, &edges[i]);
98     for(int i=0; i<n; ++i)
99         make_heap(graph[i].con.begin(), graph[i].con.end(), comp);
100 }
101 bool find(){
102     root = cur_root;
103     no_dmst = false;
104     for(auto &cur : graph){
105         cur.con_to_root();
106         to_process.clear();
107         if(no_dmst) return false;
108     }
109     return true;
110 }
111 ll weight(){
112     ll res = 0;
113     for(auto &cur : edges){
114         if(cur.inc)
115             res += cur.w;
116     }
117     return res;
118 };
119 void DMST::Circle::clean(int idx){

```

#629

#076

#469

%732

%477



```

120 if(!vis){
121     vis = true;
122     for(int i=0; i<contents.size(); ++i){
123         if(i != idx){
124             contents[i]->inc = true;
125             contents[i]->from->clean();
126         }
127     }
128 }
129 }
130 const greater<pair<ll, DMST::Edge * > > DMST::comp;
131 vector<DMST::Circle> DMST::to_process;
132 bool DMST::no_dmst;
133 DMST::Node *DMST::root;

```

#711

%771

#### 14 Bridges $\mathcal{O}(n)$

```

1 struct vert;
2 struct edge{
3     bool exists = true;
4     vert *dest;
5     edge *rev;
6     edge(vert *_dest) : dest(_dest){
7         rev = NULL;
8     }
9     vert &operator*(){
10         return *dest;
11     }
12     vert *operator->(){
13         return dest;
14     }
15     bool is_bridge();
16 };
17 struct vert{
18     deque<edge> con;
19     int val = 0;
20     int seen;
21     int dfs(int upd, edge *ban){ //handles multiple edges
22         if(!val){
23             val = upd;
24             seen = val;
25             for(edge &nxt : con){
26                 if(nxt.exists && (&nxt) != ban)
27                     seen = min(seen, nxt->dfs(upd+1, nxt.rev));
28             }
29         }
30         return seen;
31     }
32     void remove_adj_bridges(){
33         for(edge &nxt : con){
34             if(nxt.is_bridge())
35                 nxt.exists = false;
36         }
37     }
38     int cnt_adj_bridges(){

```

#955

#336

#673  
%624

%106

```

39     int res = 0;
40     for(edge &nxt : con)
41         res += nxt.is_bridge();
42     return res;
43 }
44 };
45 bool edge::is_bridge(){
46     return exists && (dest->seen > rev->dest->val || dest->val <
47         ↪ rev->dest->seen);
48 }
49 }
50 vert graph[nmax];
51 int main(){ //Mechanics Practice BRIDGES
52     int n, m;
53     cin>>n>>m;
54     for(int i=0; i<m; ++i){
55         int u, v;
56         scanf("%d %d", &u, &v);
57         graph[u].con.emplace_back(graph+v);
58         graph[v].con.emplace_back(graph+u);
59         graph[u].con.back().rev = &graph[v].con.back();
60         graph[v].con.back().rev = &graph[u].con.back();
61     }
62     graph[1].dfs(1, NULL);
63     int res = 0;
64     for(int i=1; i<=n; ++i)
65         res += graph[i].cnt_adj_bridges();
66     cout<<res/2<<endl;
67 }

```

%056

%223

#### 15 2-Sat $\mathcal{O}(n)$ and SCC $\mathcal{O}(n)$

```

1 struct Graph {
2     int n;
3     vector<vector<int> > conn;
4     Graph(int nsize) {
5         n = nsize;
6         conn.resize(n);
7     }
8     void add_edge(int u, int v) {
9         conn[u].push_back(v);
10    }
11    void _topsort_dfs(int pos, vector<int> &result, vector<bool>
12        ↪ &explr, vector<vector<int> > &revconn) {
13        if(explr[pos])
14            return;
15        explr[pos] = true;
16        for(auto next : revconn[pos])
17            _topsort_dfs(next, result, explr, revconn);
18        result.push_back(pos);
19    }
20    vector<int> topsort() {
21        vector<vector<int> > revconn(n);
22        for(int u = 0; u < n; u++) {
23            for(auto v : conn[u])

```

#078

#346

```

24     }
25     vector<int> result;
26     vector<bool> explr(n, false);
27     for(int i=0; i < n; i++)
28         _topsort_dfs(i, result, explr, revconn);
29     reverse(result.begin(), result.end());
30     return result;
31 }
32 void dfs(int pos, vector<int> &result, vector<bool> &explr) {
33     if(explr[pos])
34         return;
35     explr[pos] = true;
36     for(auto next : conn[pos])
37         dfs(next, result, explr);
38     result.push_back(pos);
39 }
40 vector<vector<int> > scc(){ // tested on
41     ↪ https://www.hackerearth.com/practice/algorithms/graphs/strongly-connected-components/practice-problem/algorithm/strongly-connected-components-2/
42     vector<int> order = topsort();
43     reverse(order.begin(), order.end());
44     vector<bool> explr(n, false);
45     vector<vector<int> > results;
46     for(auto it = order.rbegin(); it != order.rend(); ++it){
47         vector<int> component;
48         _topsort_dfs(*it, component, explr, conn);
49         sort(component.begin(), component.end());
50         results.push_back(component);
51     }
52     sort(results.begin(), results.end());
53     return results;
54 };
55 //Solution for:
56 ↪ http://codeforces.com/group/PjzGiggT71/contest/221700/problem/C
57 int main() {
58     int n, m;
59     cin >> n >> m;
60     Graph g(2*m);
61     for(int i=0; i<n; i++) {
62         int a, sa, b, sb;
63         cin >> a >> sa >> b >> sb;
64         a--, b--;
65         g.add_edge(2*a + 1 - sa, 2*b + sb);
66         g.add_edge(2*b + 1 - sb, 2*a + sa);
67     }
68     vector<int> state(2*m, 0);
69     {
70         vector<int> order = g.topsort();
71         vector<bool> explr(2*m, false);
72         for(auto u : order) {
73             vector<int> traversed;
74             g.dfs(u, traversed, explr);
75             if(traversed.size() > 0 && !state[traversed[0]^1]) {

```

#991

%603

#741

%983

```

75         for(auto c : traversed)
76             state[c] = 1;
77     }
78 }
79 }
80 for(int i=0; i < m; i++) {
81     if(state[2*i] == state[2*i+1]) {
82         cout << "IMPOSSIBLE\n";
83         return 0;
84     }
85 }
86 for(int i=0; i < m; i++) {
87     cout << state[2*i+1] << '\n';
88 }
89 return 0;
90 }

```

## 16. Generic persistent compressed lazy segment tree

```

1 struct Seg{
2     ll sum=0;
3     void recalc(const Seg &lhs_seg, int lhs_len, const Seg &rhs_seg, int
4         ↪ rhs_len){
5         sum = lhs_seg.sum + rhs_seg.sum;
6     }
7 } __attribute__((packed));
8 struct Lazy{
9     ll add;
10    ll assign_val; //LLONG_MIN if no assign;
11    void init(){
12        add = 0;
13        assign_val = LLONG_MIN;
14    }
15    Lazy(){ init(); }
16    void split(Lazy &lhs_lazy, Lazy &rhs_lazy, int len){
17        lhs_lazy = *this;
18        rhs_lazy = *this;
19        init();
20    }
21    void merge(Lazy &oth, int len){
22        if(oth.assign_val != LLONG_MIN){
23            add = 0;
24            assign_val = oth.assign_val;
25        }
26        add += oth.add;
27    }
28    void apply_to_seg(Seg &cur, int len) const{
29        if(assign_val != LLONG_MIN){
30            cur.sum = len * assign_val;
31        }
32        cur.sum += len * add;
33    }
34 } __attribute__((packed));
35 struct Node{ //Following code should not need to be modified
36     int ver;

```

#883

#470

#216

%625

```

36 bool is_lazy = false;
37 Seg seg;
38 Lazy lazy;
39 Node *lc=NULL, *rc=NULL;
40 void init(){
41     if(!lc){
42         lc = new Node {ver};
43         rc = new Node {ver};
44     }
45 }
46 Node *upd(int L, int R, int l, int r, Lazy &val, int tar_ver){
47     if(ver != tar_ver){
48         Node *rep = new Node(*this);
49         rep->ver = tar_ver;
50         return rep->upd(L, R, l, r, val, tar_ver);
51     }
52     if(L >= l && R <= r){
53         val.apply_to_seg(seg, R-L);
54         lazy.merge(val, R-L);
55         is_lazy = true;
56     } else {
57         init();
58         int M = (L+R)/2;
59         if(is_lazy){
60             Lazy l_val, r_val;
61             lazy.split(l_val, r_val, R-L);
62             lc = lc->upd(L, M, L, M, l_val, ver);
63             rc = rc->upd(M, R, M, R, r_val, ver);
64             is_lazy = false;
65         }
66         Lazy l_val, r_val;
67         val.split(l_val, r_val, R-L);
68         if(l < M)
69             lc = lc->upd(L, M, l, r, l_val, ver);
70         if(M < r)
71             rc = rc->upd(M, R, l, r, r_val, ver);
72         seg.recalc(lc->seg, M-L, rc->seg, R-M);
73     }
74     return this;
75 }
76 void get(int L, int R, int l, int r, Seg *&lft_res, Seg *&tmp, bool
    ↪ last_ver){
77     if(L >= l && R <= r){
78         tmp->recalc(*lft_res, L-l, seg, R-L);
79         swap(lft_res, tmp);
80     } else {
81         init();
82         int M = (L+R)/2;
83         if(is_lazy){
84             Lazy l_val, r_val;
85             lazy.split(l_val, r_val, R-L);
86             lc = lc->upd(L, M, L, M, l_val, ver+last_ver);
87             lc->ver = ver;
88             rc = rc->upd(M, R, M, R, r_val, ver+last_ver);

```

#313

#138

#104

#245

#726

```

89         rc->ver = ver;
90         is_lazy = false;
91     }
92     if(l < M)
93         lc->get(L, M, l, r, lft_res, tmp, last_ver);
94     if(M < r)
95         rc->get(M, R, l, r, lft_res, tmp, last_ver);
96     }
97 }
98 } __attribute__((packed));
99 struct SegTree{ //indexes start from 0, ranges are [beg, end)
100     vector<Node *> roots; //versions start from 0
101     int len;
102     SegTree(int _len) : len(_len){
103         roots.push_back(new Node {0});
104     }
105     int upd(int l, int r, Lazy &val, bool new_ver = false){
106         Node *cur_root = roots.back()->upd(0, len, l, r, val,
            ↪ roots.size()-!new_ver);
107         if(cur_root != roots.back())
108             roots.push_back(cur_root);
109         return roots.size()-1;
110     }
111     Seg get(int l, int r, int ver = -1){
112         if(ver == -1)
113             ver = roots.size()-1;
114         Seg seg1, seg2;
115         Seg *pres = &seg1, *ptmp = &seg2;
116         roots[ver]->get(0, len, l, r, pres, ptmp, roots.size()-1);
117         return *pres;
118     }
119 };
120 int main(){
121     int n, m; //solves Mechanics Practice LAZY
122     cin>>n>>m;
123     SegTree seg_tree(1<<17);
124     for(int i=0; i<n; ++i){
125         Lazy tmp;
126         scanf("%lld", &tmp.assign_val);
127         seg_tree.upd(i, i+1, tmp);
128     }
129     for(int i=0; i<m; ++i){
130         int o;
131         int l, r;
132         scanf("%d %d %d", &o, &l, &r);
133         --l;
134         if(o==1){
135             Lazy tmp;
136             scanf("%lld", &tmp.add);
137             seg_tree.upd(l, r, tmp);
138         } else if(o==2){
139             Lazy tmp;
140             scanf("%lld", &tmp.assign_val);

```

#696

#295

#977

%542

```

141     seg_tree.upd(l, r, tmp);
142 } else {
143     Seg res = seg_tree.get(l, r);
144     printf("%lld\n", res.sum);
145 }
146 }
147 }

```

### 17 Templated HLD $\mathcal{O}(M(n) \log n)$ per query

```

1 class dummy {
2 public:
3     dummy () {}
4     dummy (int, int) {}
5     void set (int, int) {}
6     int query (int left, int right) {
7         cout << this << ' ' << left << ' ' << right << endl;
8     }
9 };
10 /* T should be the type of the data stored in each vertex;
11  * DS should be the underlying data structure that is used to perform
12  * the
13  * group operation. It should have the following methods:
14  * * DS () - empty constructor
15  * * DS (int size, T initial) - constructs the structure with the given
16  * size,
17  * initially filled with initial.
18  * * void set (int index, T value) - set the value at index `index` to
19  * `value`
20  * * T query (int left, int right) - return the "sum" of elements
21  * between left and right, inclusive.
22  */
23 template<typename T, class DS>
24 class HLD {
25     int vertexc;
26     vector<int> *adj;
27     vector<int> subtree_size;
28     DS structure;
29     DS aux;
30     void build_sizes (int vertex, int parent) {
31         subtree_size[vertex] = 1;
32         for (int child : adj[vertex]) {
33             if (child != parent) {
34                 build_sizes(child, vertex);
35                 subtree_size[vertex] += subtree_size[child];
36             }
37         }
38     }
39     int cur;
40     vector<int> ord;
41     vector<int> chain_root;
42     vector<int> par;
43     void build_hld (int vertex, int parent, int chain_source) {
44         cur++;
45         ord[vertex] = cur;

```

%932

#037

#593

```

42     chain_root[vertex] = chain_source;
43     par[vertex] = parent;
44     if (adj[vertex].size() > 1) {
45         int big_child, big_size = -1;
46         for (int child : adj[vertex]) {
47             if ((child != parent) && (subtree_size[child] > big_size)) {
48                 big_child = child;
49                 big_size = subtree_size[child];
50             }
51         }
52         build_hld(big_child, vertex, chain_source);
53         for (int child : adj[vertex]) {
54             if ((child != parent) && (child != big_child))
55                 build_hld(child, vertex, child);
56         }
57     }
58 }
59 public:
60     HLD (int _vertexc) {
61         vertexc = _vertexc;
62         adj = new vector<int> [vertexc + 5];
63     }
64     void add_edge (int u, int v) {
65         adj[u].push_back(v);
66         adj[v].push_back(u);
67     }
68     void build (T initial) {
69         subtree_size = vector<int> (vertexc + 5);
70         ord = vector<int> (vertexc + 5);
71         chain_root = vector<int> (vertexc + 5);
72         par = vector<int> (vertexc + 5);
73         cur = 0;
74         build_sizes(1, -1);
75         build_hld(1, -1, 1);
76         structure = DS (vertexc + 5, initial);
77         aux = DS (50, initial);
78     }
79     void set (int vertex, int value) {
80         structure.set(ord[vertex], value);
81     }
82     T query_path (int u, int v) { /* returns the "sum" of the path u->v
83     */
84     int cur_id = 0;
85     while (chain_root[u] != chain_root[v]) {
86         if (ord[u] > ord[v]) {
87             cur_id++;
88             aux.set(cur_id, structure.query(ord[chain_root[u]], ord[u]));
89             u = par[chain_root[u]];
90         } else {
91             cur_id++;
92             aux.set(cur_id, structure.query(ord[chain_root[v]], ord[v]));
93             v = par[chain_root[v]];

```

#646

#738

#841

#793

#517

```

94     }
95     cur_id++;
96     aux.set(cur_id, structure.query(min(ord[u], ord[v]), max(ord[u],
97     ↪ ord[v])));
98     return aux.query(1, cur_id);
99 }
100 void print () {
101     for (int i = 1; i <= vertexc; i++)
102         cout << i << ' ' << ord[i] << ' ' << chain_root[i] << ' ' <<
103         ↪ par[i] << endl;
104 }
105 };
106 int main () {
107     int vertexc;
108     cin >> vertexc;
109     HLD<int, dummy> hld (vertexc);
110     for (int i = 0; i < vertexc - 1; i++) {
111         int u, v;
112         cin >> u >> v;
113         hld.add_edge(u, v);
114     }
115     hld.build(0);
116     hld.print();
117     int queryc;
118     cin >> queryc;
119     for (int i = 0; i < queryc; i++) {
120         int u, v;
121         cin >> u >> v;
122         hld.query_path(u, v);
123         cout << endl;
124     }
125 }

```

### 18 Templated multi dimensional BIT $\mathcal{O}(\log(n)^{\dim})$ per query

```

1 // Fully overloaded any dimensional BIT, use any type for coordinates,
2 // elements, return value.
3 // Includes coordinate compression.
4 template < typename elem_t, typename coord_t, coord_t n_inf, typename
5     ↪ ret_t >
6 class BIT {
7     vector< coord_t > positions;
8     vector< elem_t > elems;
9     bool initiated = false;
10 public:
11     BIT() {
12         positions.push_back(n_inf);
13     }
14     void initiate() {
15         if (initiated) {
16             for (elem_t &c_elem : elems)
17                 c_elem.initiate();
18         } else {
19             initiated = true;
20             sort(positions.begin(), positions.end());

```

#448

```

19     positions.resize(unique(positions.begin(), positions.end()) -
20     ↪ positions.begin());
21     elems.resize(positions.size());
22 }
23 template < typename... loc_form >
24 void update(coord_t cord, loc_form... args) {
25     if (initiated) {
26         int pos = lower_bound(positions.begin(), positions.end(), cord) -
27         ↪ positions.begin();
28         for (; pos < positions.size(); pos += pos & -pos)
29             elems[pos].update(args...);
30     } else {
31         positions.push_back(cord);
32     }
33 }
34 template < typename... loc_form >
35 ret_t query(coord_t cord, loc_form... args) { //sum in open interval
36     ↪ (-inf, cord)
37     ret_t res = 0;
38     int pos = (lower_bound(positions.begin(), positions.end(), cord) -
39     ↪ positions.begin())-1;
40     for (; pos > 0; pos -= pos & -pos)
41         res += elems[pos].query(args...);
42     return res;
43 }
44 };
45 template < typename internal_type >
46 struct wrapped {
47     internal_type a = 0;
48     void update(internal_type b) {
49         a += b;
50     }
51     internal_type query() {
52         return a;
53     }
54     // Should never be called, needed for compilation
55     void initiate() {
56         cerr << 'i' << endl;
57     }
58     void update() {
59         cerr << 'u' << endl;
60     }
61 };
62 int main() {
63     // return type should be same as type inside wrapped
64     BIT< BIT< wrapped< ll >, int, INT_MIN, ll >, int, INT_MIN, ll >
65     ↪ fenwick;
66     int dim = 2;
67     vector< tuple< int, int, ll > > to_insert;
68     to_insert.emplace_back(1, 1, 1);
69     // set up all positions that are to be used for update
70     for (int i = 0; i < dim; ++i) {

```

#036

#154

#895

#560

%714



```

67     for (auto &cur : to_insert)
68         fenwick.update(get< 0 >(cur), get< 1 >(cur)); // May include
        ↪ value which won't be used
69     fenwick.initiate();
70 }
71 // actual use
72 for (auto &cur : to_insert)
73     fenwick.update(get< 0 >(cur), get< 1 >(cur), get< 2 >(cur));
74 cout << fenwick.query(2, 2)<<'\n';
75 }

```

### 19 Treap $\mathcal{O}(\log n)$ per query

```

1 mt19937 randgen;
2 struct Treap {
3     struct Node {
4         int key;
5         int value;
6         unsigned int priority;
7         long long total;
8         Node* lch;
9         Node* rch;
10        Node(int new_key, int new_value) {
11            key = new_key;
12            value = new_value;
13            priority = randgen();
14            total = new_value;
15            lch = 0;
16            rch = 0;
17        }
18        void update() {
19            total = value;
20            if(lch) total += lch->total;
21            if(rch) total += rch->total;
22        }
23    };
24    deque<Node> nodes;
25    Node* root = 0;
26    pair<Node*, Node*> split(int key, Node* cur) {
27        if(cur == 0) return {0, 0};
28        pair<Node*, Node*> result;
29        if(key <= cur->key) {
30            auto ret = split(key, cur->lch);
31            cur->lch = ret.second;
32            result = {ret.first, cur};
33        } else {
34            auto ret = split(key, cur->rch);
35            cur->rch = ret.first;
36            result = {cur, ret.second};
37        }
38        cur->update();
39        return result;
40    }
41    Node* merge(Node* left, Node* right) {
42        if(left == 0) return right;

```

```

43        if(right == 0) return left;
44        Node* top;
45        if(left->priority < right->priority) {
46            left->rch = merge(left->rch, right);
47            top = left;
48        } else {
49            right->lch = merge(left, right->lch);
50            top = right;
51        }
52        top->update();
53        return top;
54    }
55    void insert(int key, int value) {
56        nodes.push_back(Node(key, value));
57        Node* cur = &nodes.back();
58        pair<Node*, Node*> ret = split(key, root);
59        cur = merge(ret.first, cur);
60        cur = merge(cur, ret.second);
61        root = cur;
62    }
63    void erase(int key) {
64        Node *left, *mid, *right;
65        tie(left, mid) = split(key, root);
66        tie(mid, right) = split(key+1, mid);
67        root = merge(left, right);
68    }
69    long long sum_upto(int key, Node* cur) {
70        if(cur == 0) return 0;
71        if(key <= cur->key) {
72            return sum_upto(key, cur->lch);
73        } else {
74            long long result = cur->value + sum_upto(key, cur->rch);
75            if(cur->lch) result += cur->lch->total;
76            return result;
77        }
78    }
79    long long get(int l, int r) {
80        return sum_upto(r+1, root) - sum_upto(l, root);
81    }
82 };
83 //Solution for:
84 ↪ http://codeforces.com/group/U01GDa2Gwb/contest/219104/problem/TREAP
85 int main() {
86     ios_base::sync_with_stdio(false);
87     cin.tie(0);
88     int m;
89     Treap treap;
90     cin >> m;
91     for(int i=0;i<m;i++) {
92         int type;
93         cin >> type;
94         if(type == 1) {
95             int x, y;

```

```

95     cin >> x >> y;
96     treap.insert(x, y);
97 } else if(type == 2) {
98     int x;
99     cin >> x;
100    treap.erase(x);
101 } else {
102     int l, r;
103     cin >> l >> r;
104     cout << treap.get(l, r) << endl;
105 }
106 }
107 return 0;
108 }

```

## 20 Radixsort 50M 64 bit integers as single array in 1 sec

```

1 typedef unsigned char uchar;
2 template<typename T>
3 void msd_radixsort(T *start, T *sec_start, int arr_size, int
   ↪ d=sizeof(T)-1){
4     const int msd_radix_lim = 100;
5     const T mask = 255;
6     int bucket_sizes[256]{};
7     for(T *it = start; it!=start+arr_size; ++it){
8         ++bucket_sizes[((*it)>>(d*8))&mask];
9         //++bucket_sizes[*((uchar*)it + d)];
10    }
11    T *locs_mem[257];
12    locs_mem[0] = sec_start;
13    T **locs = locs_mem+1;
14    locs[0] = sec_start;
15    for(int j=0; j<255; ++j){
16        locs[j+1] = locs[j]+bucket_sizes[j];
17    }
18    for(T *it = start; it!=start+arr_size; ++it){
19        uchar bucket_id = ((*it)>>(d*8))&mask;
20        *(locs[bucket_id]++) = *it;
21    }
22    locs = locs_mem;
23    if(d){
24        T *locs_old[256];
25        locs_old[0] = start;
26        for(int j=0; j<255; ++j){
27            locs_old[j+1] = locs_old[j]+bucket_sizes[j];
28        }
29        for(int j=0; j<256; ++j){
30            if(locs[j+1]-locs[j] < msd_radix_lim){
31                std::sort(locs[j], locs[j+1]);
32                if(d & 1){
33                    copy(locs[j], locs[j+1], locs_old[j]);
34                }
35            }
36        }
37        msd_radixsort(locs[j], locs_old[j], bucket_sizes[j], d-1);

```

#947

#770

#018

```

38     }
39 }
40 }
41 }
42 const int nmax = 5e7;
43 ll arr[nmax], tmp[nmax];
44 int main(){
45     for(int i=0; i<nmax; ++i)
46         arr[i] = ((ll)rand()<<32)|rand();
47     msd_radixsort(arr, tmp, nmax);
48     assert(is_sorted(arr, arr+nmax));
49 }

```

%225

## 21 FFT 5M length/sec

integer  $c = a * b$  is accurate if  $c_i < 2^{49}$

```

1 struct Complex {
2     double a = 0, b = 0;
3     Complex &operator/=(const int &oth) {
4         a /= oth;
5         b /= oth;
6         return *this;
7     }
8 };
9 Complex operator+(const Complex &lft, const Complex &rgt) {
10     return Complex{lft.a + rgt.a, lft.b + rgt.b};
11 }
12 Complex operator-(const Complex &lft, const Complex &rgt) {
13     return Complex{lft.a - rgt.a, lft.b - rgt.b};
14 }
15 Complex operator*(const Complex &lft, const Complex &rgt) {
16     return Complex{lft.a * rgt.a - lft.b * rgt.b, lft.a * rgt.b + lft.b *
   ↪ rgt.a};
17 }
18 Complex conj(const Complex &cur){
19     return Complex{cur.a, -cur.b};
20 }
21 void fft_rec(Complex *arr, Complex *root_pow, int len) {
22     if (len != 1) {
23         fft_rec(arr, root_pow, len >> 1);
24         fft_rec(arr + len, root_pow, len >> 1);
25     }
26     root_pow += len;
27     for (int i = 0; i < len; ++i) {
28         Complex tmp = arr[i] + root_pow[i] * arr[i + len];
29         arr[i + len] = arr[i] - root_pow[i] * arr[i + len];
30         arr[i] = tmp;
31     }
32 }
33 void fft(vector< Complex > &arr, int ord, bool invert) {
34     assert(arr.size() == 1 << ord);
35     static vector< Complex > root_pow(1);
36     static int inc_pow = 1;
37     static bool is_inv = false;

```

#384

#957

#048

```

38 if (inc_pow <= ord) {
39     int idx = root_pow.size();
40     root_pow.resize(1 << ord);
41     for (; inc_pow <= ord; ++inc_pow) {
42         for (int idx_p = 0; idx_p < 1 << (ord - 1); idx_p += 1 << (ord -
43             ↪ inc_pow), ++idx) {
44             root_pow[idx] =
45                 Complex{cos(-idx_p * M_PI / (1 << (ord - 1))), sin(-idx_p *
46                     ↪ M_PI / (1 << (ord - 1)))};
47             if (is_inv) root_pow[idx].b = -root_pow[idx].b;
48         }
49     }
50     if (invert != is_inv) {
51         is_inv = invert;
52         for (Complex &cur : root_pow) cur.b = -cur.b;
53     }
54     for (int i = 1, j=0; i < (1 << ord); ++i) {
55         int m = 1<<(ord-1);
56         bool cont = true;
57         while(cont){
58             cont = j & m;
59             j ^= m;
60             m>>=1;
61         }
62         if (i < j) swap(arr[i], arr[j]);
63     }
64     fft_rec(arr.data(), root_pow.data(), 1 << (ord - 1));
65     if (invert)
66         for (int i = 0; i < (1 << ord); ++i) arr[i] /= (1 << ord);
67 }
68 void mult_poly_mod(vector< int > &a, vector< int > &b, vector< int >
69     ↪ &c) { // c += a*b
70     static vector< Complex > arr[4]; // correct upto 0.5-2M elements(mod
71     ↪ ~ = 1e9)
72     if (c.size() < 400) {
73         for (int i = 0; i < a.size(); ++i)
74             for (int j = 0; j < b.size() && i + j < c.size(); ++j)
75                 c[i + j] = ((ll)a[i] * b[j] + c[i + j]) % mod;
76     } else {
77         int fft_ord = 32 - __builtin_clz(c.size());
78         if (arr[0].size() != 1 << fft_ord)
79             for (int i = 0; i < 4; ++i) arr[i].resize(1 << fft_ord);
80         for (int i = 0; i < 4; ++i) fill(arr[i].begin(), arr[i].end(),
81             ↪ Complex{});
82         for (int &cur : a)
83             if (cur < 0) cur += mod;
84         for (int &cur : b)
85             if (cur < 0) cur += mod;
86         const int shift = 15;
87         const int mask = (1 << shift) - 1;
88         for (int i = 0; i < min(a.size(), c.size()); ++i) {
89             arr[0][i].a = a[i] & mask;
90             arr[1][i].a = a[i] >> shift;

```

#710

#750

#844

%380

#809

```

87 }
88 for (int i = 0; i < min(b.size(), c.size()); ++i) {
89     arr[0][i].b = b[i] & mask;
90     arr[1][i].b = b[i] >> shift;
91 }
92 for (int i = 0; i < 2; ++i) fft(arr[i], fft_ord, false);
93 for (int i = 0; i < 2; ++i) {
94     for (int j = 0; j < 2; ++j) {
95         int tar = 2 + (i + j)/2;
96         Complex mult = {0, -0.25};
97         if(i~j)
98             mult = {0.25, 0};
99         for (int k = 0; k < (1 << fft_ord); ++k){
100             int rev_k = ((1 << fft_ord)-k)%(1 << fft_ord);
101             Complex ca = arr[i][k] + conj(arr[i][rev_k]);
102             Complex cb = arr[j][k] - conj(arr[j][rev_k]);
103             arr[tar][k] = arr[tar][k] + mult*ca*cb;
104         }
105     }
106 }
107 for (int i = 2; i < 4; ++i) {
108     fft(arr[i], fft_ord, true);
109     for (int k = 0; k < (int)c.size(); ++k){
110         c[k] = (c[k] + (((ll)(arr[i][k].a + 0.5) % mod) << (shift *
111             ↪ 2*(i - 2)))) % mod;
112         c[k] = (c[k] + (((ll)(arr[i][k].b + 0.5) % mod) << (shift *
113             ↪ (2*(i - 2)+1)))) % mod;
114     }
115 }

```

#066

#623

%231

## 22 Fast mod mult, Rabbin Miller prime check, Pollard rho factorization $\mathcal{O}(\sqrt{p})$

```

1 struct ModArithm {
2     ull n;
3     ld rec;
4     ModArithm(ull _n) : n(_n) { // n in [2, 1<<63)
5         rec = 1.0L/n;
6     }
7     ull multf(ull a, ull b) { // a, b in [0, min(2*n, 1<<63))
8         ull mult = (ld)a*b*rec+0.5L;
9         ll res = a*b-mult*n;
10        if(res < 0) res += n;
11        return res; // in [0, n-1)
12    }
13    ull sqp1(ull a) { return multf(a, a) + 1; }
14 };
15 ull pow_mod(ull a, ull n, ModArithm &arithm) {
16     ull res = 1;
17     for (ull i = 1; i <= n; i <= 1) {
18         if (n & i) res = arithm.multf(res, a);
19         a = arithm.multf(a, a);

```

#780

%493

```

20 }
21 return res;
22 }
23 vector< char > small_primes = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,
    ↪ 37};
24 bool is_prime(ull n) { // n <= 1<<63, 1M rand/s
25     ModArithm arithm(n);
26     if (n == 2 || n == 3) return true;
27     if (!(n & 1) || n == 1) return false;
28     ull s = __builtin_ctz(n - 1);
29     ull d = (n - 1) >> s;
30     for (ull a : small_primes) {
31         if (a >= n) break;
32         a = pow_mod(a, d, arithm);
33         if (a == 1 || a == n - 1) continue;
34         for (ull r = 1; r < s; ++r) {
35             a = arithm.multf(a, a);
36             if (a == 1) return false;
37             if (a == n - 1) break;
38         }
39         if (a != n - 1) return false;
40     }
41     return true;
42 }
43 ll pollard_rho(ll n) {
44     ModArithm arithm(n);
45     int cum_cnt = 64 - __builtin_clz(n);
46     cum_cnt *= cum_cnt / 5 + 1;
47     while (true) {
48         ll lv = rand() % n;
49         ll v = arithm.sqp1(lv);
50         int idx = 1;
51         int tar = 1;
52         while (true) {
53             ll cur = 1;
54             ll v_cur = v;
55             int j_stop = min(cum_cnt, tar-idx);
56             for (int j = 0; j < j_stop; ++j) {
57                 cur = arithm.multf(cur, abs(v_cur - lv));
58                 v_cur = arithm.sqp1(v_cur);
59                 ++idx;
60             }
61             if (!cur) {
62                 for (int j = 0; j < cum_cnt; ++j) {
63                     ll g = __gcd(abs(v-lv), n);
64                     if (g == 1) {
65                         v = arithm.sqp1(v);
66                     } else if (g == n) {
67                         break;
68                     } else {
69                         return g;
70                     }
71                 }

```

#356

%975

#290

#912

```

72     break;
73 } else {
74     ll g = __gcd(cur, n);
75     if (g != 1) return g;
76 }
77 v = v_cur;
78 idx += j_stop;
79 if (idx == tar) {
80     lv = v;
81     tar *= 2;
82     v = arithm.sqp1(v);
83     ++idx;
84 }
85 }
86 }
87 }
88 map< ll, int > prime_factor(ll n, map< ll, int > *res = NULL) { // n
    ↪ <= 1<<61, ~1000/s (<500/s on CF)
89     if (!res) {
90         map< ll, int > res_act;
91         for (int p : small_primes) {
92             while (!(n % p)) {
93                 ++res_act[p];
94                 n /= p;
95             }
96         }
97         if (n != 1) prime_factor(n, &res_act);
98         return res_act;
99     }
100     if (is_prime(n)) {
101         ++(*res)[n];
102     } else {
103         ll factor = pollard_rho(n);
104         prime_factor(factor, res);
105         prime_factor(n / factor, res);
106     }
107     return map< ll, int >();
108 } //Usage: fact = prime_factor(n);

```

#208

#174

%542

#023

#140

%477





## Combinatorics Cheat Sheet

## Useful formulas

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$  — number of ways to choose  $k$  objects out of  $n$

$\binom{n+k-1}{k-1}$  — number of ways to choose  $k$  objects out of  $n$  with repetitions

$[n]$  — Stirling numbers of the first kind; number of permutations of  $n$  elements with  $k$  cycles

$$[n+1] = n[n] + [n-1]$$

$$(x)_n = x(x-1)\dots x-n+1 = \sum_{k=0}^n (-1)^{n-k} [n]_k x^k$$

$\left\{ \begin{smallmatrix} n \\ m \end{smallmatrix} \right\}$  — Stirling numbers of the second kind; number of partitions of set  $1, \dots, n$  into  $k$  disjoint subsets.

$$\left\{ \begin{smallmatrix} n+1 \\ m \end{smallmatrix} \right\} = k \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\} + \left\{ \begin{smallmatrix} n \\ k-1 \end{smallmatrix} \right\}$$

$$\sum_{k=0}^n \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\} (x)_k = x^n$$

$$C_n = \frac{1}{n+1} \binom{2n}{n} \text{ — Catalan numbers}$$

$$C(x) = \frac{1-\sqrt{1-4x}}{2x}$$

## Binomial transform

If  $a_n = \sum_{k=0}^n \binom{n}{k} b_k$ , then  $b_n = \sum_{k=0}^n (-1)^{n-k} \binom{n}{k} a_k$

$$\bullet a = (1, x, x^2, \dots), b = (1, (x+1), (x+1)^2, \dots)$$

$$\bullet a_i = i^k, b_i = \left\{ \begin{smallmatrix} n \\ i \end{smallmatrix} \right\} i!$$

## Burnside's lemma

Let  $G$  be a group of *action* on set  $X$  (Ex.: cyclic shifts of array, rotations and symmetries of  $n \times n$  matrix, ...)

Call two objects  $x$  and  $y$  *equivalent* if there is an action  $f$  that transforms  $x$  to  $y$ :  $f(x) = y$ .

The number of equivalence classes then can be calculated as follows:  $C = \frac{1}{|G|} \sum_{f \in G} |X^f|$ , where  $X^f$  is the set of *fixed points* of  $f$ :  $X^f = \{x | f(x) = x\}$

## Generating functions

Ordinary generating function (o.g.f.) for sequence  $a_0, a_1, \dots, a_n, \dots$  is  $A(x) = \sum_{n=0}^{\infty} a_n x^n$

Exponential generating function (e.g.f.) for sequence  $a_0, a_1, \dots, a_n, \dots$  is  $A(x) = \sum_{n=0}^{\infty} a_n \frac{x^n}{n!}$

$$B(x) = A'(x), b_{n-1} = n \cdot a_n$$

$$c_n = \sum_{k=0}^n a_k b_{n-k} \text{ (o.g.f. convolution)}$$

$$c_n = \sum_{k=0}^n \binom{n}{k} a_k b_{n-k} \text{ (e.g.f. convolution, compute with FFT using } \widetilde{a_n} = \frac{a_n}{n!} \text{)}$$

## General linear recurrences

If  $a_n = \sum_{k=1}^n b_k a_{n-k}$ , then  $A(x) = \frac{a_0}{1-B(x)}$ . We also can compute all  $a_n$  with Divide-and-Conquer algorithm in  $O(n \log^2 n)$ .

Inverse polynomial modulo  $x^l$ 

Given  $A(x)$ , find  $B(x)$  such that  $A(x)B(x) = 1 + x^l \cdot Q(x)$  for some  $Q(x)$

$$1. \text{ Start with } B_0(x) = \frac{1}{a_0}$$

$$2. \text{ Double the length of } B(x): B_{k+1}(x) = (-B_k(x)^2 A(x) + 2B_k(x)) \bmod x^{2^{k+1}}$$

## Fast subset convolution

Given array  $a_i$  of size  $2^k$ , calculate  $b_i = \sum_{j \& i = i} b_j$

```
for b = 0..k-1
  for i = 0..2^k-1
    if (i & (1 << b)) != 0:
      a[i + (1 << b)] += a[i]
```

## Hadamard transform

Treat array  $a$  of size  $2^k$  as  $k$ -dimensional array of size  $2 \times 2 \times \dots \times 2$ , calculate FFT of that array:

```
for b = 0..k-1
  for i = 0..2^k-1
    if (i & (1 << b)) != 0:
      u = a[i], v = a[i + (1 << b)]
      a[i] = u + v
      a[i + (1 << b)] = u - v
```