# University of Tartu ICPC Team Notebook (2018-2019) November 22, 2018



## 1 Setup

```
set smartindent cindent
set ts=4 sw=4 expandtab
syntax enable
set clipboard=unnamedplus
# setxkbmap -option caps:escape
# valgrind --vgdb-error=0 ./a <inp &
# gdb a
# target remote | vgdb
```

## 2 crc.sh

```
#!/bin/env bash
for j in `seq 10 10 200`; do
  sed '/^\s*$/d' $1 | head -$j | tr -d '[[:space:]]' | cksum | cut -f1 -d ' ' | tail -c 4 #whitespaces don't matter.
done #there shouldn't be any COMMENTS.
#copy lines being checked to separate file.
# $ ./crc.sh tmp.cpp
```

## 3 gcc ordered set

```
#include <bits/stdc++.h>
typedef long long ll;
using namespace std;
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
  tree_order_statistics_node_update>;
int main() {                                                    #558
  ordered_set<int> cur;
  cur.insert(1);
  cur.insert(3);
  cout << cur.order_of_key(2)
       << endl; // the number of elements in the set less than 2
  cout << *cur.find_by_order(0)
       << endl; // the 0-th smallest number in the set(0-based)
  cout << *cur.find_by_order(1)
       << endl; // the 1-th smallest number in the set(0-based)
}                                                               %574
```

## 4 2D geometry

Define $\textbf{orient}(A,B,C) = \overline{AB} \times \overline{AC}$. CCW iff $> 0$.
Define $\textbf{perp}((a,b)) = (-b,a)$. The vectors are orthogonal.
For line $ax + by = c$ def $\overline{v} = (-b,a)$.
Line through $P$ and $Q$ has $\overline{v} = \overline{PQ}$ and $c = \overline{v} \times P$.
$\textbf{side}_l(P) = \overline{v_l} \times P - c_l$ sign determines which side $P$ is on from $l$.
$\textbf{dist}_l(P) = \textbf{side}_l(P)/\|v_l\|$ squared is integer.
Sorting points along a line: comparator is $\overline{v} \cdot A < \overline{v} \cdot B$.
Translating line by $\overline{t}$: new line has $c' = c + \overline{v} \times \overline{t}$.
Line intersection: is $(c_l \overline{v}_m - c_m \overline{v}_l)/(\overline{v}_l \times \overline{v}_m)$.
Project $P$ onto $l$: is $P - \textbf{perp}(v)\,\textbf{side}_l(P)/\|v\|^2$.
Angle bisectors: $\overline{v} = \overline{v}_l/\|\overline{v}_l\| + \overline{v}_m/\|\overline{v}_m\|$
$c = c_l/\|\overline{v}_l\| + c_m/\|\overline{v}_m\|$.
$P$ is on segment $AB$ iff $\textbf{orient}(A,B,P) = 0$ and $\overline{PA} \cdot \overline{PB} \leqslant 0$.
Proper intersection of $AB$ and $CD$ exists iff $\textbf{orient}(C,D,A)$ and $\textbf{orient}(C,D,B)$ have opp. signs and $\textbf{orient}(A,B,C)$ and $\textbf{orient}(A,B,D)$ have opp. signs. Coordinates:

$$\frac{A\,\textbf{orient}(C,D,B) - B\,\textbf{orient}(C,D,A)}{\textbf{orient}(C,D,B) - \textbf{orient}(C,D,A)}.$$

Circumcircle center:

```
pt circumCenter(pt a, pt b, pt c) {
  b = b-a, c = c-a; // consider coordinates
      relative to A
  assert(cross(b,c) != 0); // no circumcircle if
       A,B,C aligned
  return a + perp(b*sq(c) - c*sq(b))/cross(b,c)
      /2;
```

Circle-line intersect:

```
int circleLine(pt o, double r, line l, pair<pt,
    pt> &out) {
  double h2 = r*r - l.sqDist(o);
  if (h2 >= 0) { // the line touches the circle
    pt p = l.proj(o); // point P
    pt h = l.v*sqrt(h2)/abs(l.v); // vector
        paral to l, of len h
    out = {p-h, p+h};
  }
  return 1 + sgn(h2);
```

Circle-circle intersect:

```
int circleCircle(pt o1, double r1, pt o2, double
     r2, pair<pt,pt> &out) {
  pt d=o2-o1; double d2=sq(d);
  if (d2 == 0) {assert(r1 != r2); return 0;} //
      concentric circles
  double pd = (d2 + r1*r1 - r2*r2)/2; // = |O_1P
      | * d
  double h2 = r1*r1 - pd*pd/d2; // = h^2
  if (h2 >= 0) {
    pt p = o1 + d*pd/d2, h = perp(d)*sqrt(h2/d2)
       ;
    out = {p-h, p+h};}
  return 1 + sgn(h2);
```

Tangent lines:

```
int tangents(pt o1, double r1, pt o2, double r2,
     bool inner, vector<pair<pt,pt>> &out) {
  if (inner) r2 = -r2;
  pt d = o2-o1;
  double dr = r1-r2, d2 = sq(d), h2 = d2-dr*dr;
  if (d2 == 0 || h2 < 0) {assert(h2 != 0);
      return 0;}
  for (double sign : {-1,1}) {
    pt v = (d*dr + perp(d)*sqrt(h2)*sign)/d2;
    out.push_back({o1 + v*r1, o2 + v*r2});}
  return 1 + (h2 > 0);
```

## 5 3D geometry

$\textbf{orient}(P,Q,R,S) = (\overline{PQ} \times \overline{PR}) \cdot \overline{PS}$.
$S$ above $PQR$ iff $> 0$.
For plane $ax + by + cz = d$ def $\overline{n} = (a,b,c)$.
Line with normal $\overline{n}$ through point $P$ has $d = \overline{n} \cdot P$.
$\textbf{side}_\Pi(P) = \overline{n} \cdot P - d$ sign determines side from $\Pi$.
$\textbf{dist}_\Pi(P) = \textbf{side}_\Pi(P)/\|\overline{n}\|$.
Translating plane by $\overline{t}$ makes $d' = d + \overline{n} \cdot \overline{t}$.
Plane-plane intersection of has direction $\overline{n}_1 \times \overline{n}_2$ and goes through $((d_1\overline{n}_2 - d_2\overline{n}_1) \times \overline{d})/\|\overline{d}\|^2$.
Line-line distance:

```
double dist(line3d l1, line3d l2) {
  p3 n = l1.d*l2.d;
  if (n == zero) // parallel
    return l1.dist(l2.o);
  return abs((l2.o-l1.o)|n)/abs(n);
```

Spherical to Cartesian:
$(r\cos\varphi\cos\lambda, r\cos\varphi\sin\lambda, r\sin\varphi)$.
Sphere-line intersection:

```
int sphereLine(p3 o, double r, line3d l, pair<p3
    ,p3> &out) {
  double h2 = r*r - l.sqDist(o);
  if (h2 < 0) return 0; // the line doesn't
      touch the sphere
  p3 p = l.proj(o); // point P
  p3 h = l.d*sqrt(h2)/abs(l.d); // vector
      parallel to l, of length h
  out = {p-h, p+h};
  return 1 + (h2 > 0);
```

Great-circle distance between points $A$ and $B$ is $r\angle AOB$.
Spherical segment intersection:

```
bool properInter(p3 a, p3 b, p3 c, p3 d, p3 &out
    ) {
  p3 ab = a*b, cd = c*d; // normals of planes
      OAB and OCD
  int oa = sgn(cd|a),
    ob = sgn(cd|b),
    oc = sgn(ab|c),
    od = sgn(ab|d);
  out = ab*cd*od; // four multiplications =>
      careful with overflow !
  return (oa != ob && oc != od && oa != oc);
}
bool onSphSegment(p3 a, p3 b, p3 p) {
  p3 n = a*b;
  if (n == zero)
    return a*p == zero && (a|p) > 0;
  return (n|p) == 0 && (n|a*p) >= 0 && (n|b*p)
      <= 0;
}
struct directionSet : vector<p3> {
  using vector::vector; // import constructors
  void insert(p3 p) {
    for (p3 q : *this) if (p*q == zero) return;
    push_back(p);
  }
};
directionSet intersSph(p3 a, p3 b, p3 c, p3 d) {
  assert(validSegment(a, b) && validSegment(c, d
      ));
  p3 out;
  if (properInter(a, b, c, d, out)) return {out
      };
  directionSet s;
  if (onSphSegment(c, d, a)) s.insert(a);
  if (onSphSegment(c, d, b)) s.insert(b);
  if (onSphSegment(a, b, c)) s.insert(c);
  if (onSphSegment(a, b, d)) s.insert(d);
  return s;
}
```

Angle between spherical segments $AB$ and $AC$ is angle between $A \times B$ and $A \times C$.
Oriented angle: subtract from $2\pi$ if mixed product is negative.
Area of a spherical polygon:

$$r^2[\text{sum of interior angles} - (n-2)\pi].$$

## 6 Numerical integration with Simpson's rule

```
// computing power = how many times function integrate gets called
template <typename T>
double simps(T f, double a, double b) {
  return (f(a) + 4 * f((a + b) / 2) + f(b)) * (b - a) / 6;
}
template <typename T>
double integrate(T f, double a, double b, double computing_power) {
  double m = (a + b) / 2;
  double l = simps(f, a, m), r = simps(f, m, b), tot = simps(f, a, b);
  if (computing_power < 1) return tot;
  return integrate(f, a, m, computing_power / 2) +                #567
         integrate(f, m, b, computing_power / 2);
}                                                                 %360
```

## 7 Triangle centers

```
const double min_delta = 1e-13;
const double coord_max = 1e6;
typedef complex<double> point;
point A, B, C; // vertixes of the triangle
bool collinear() {
  double min_diff = min(abs(A - B), min(abs(A - C), abs(B - C)));
  if (min_diff < coord_max * min_delta) return true;
  point sp = (B - A) / (C - A);
  double ang =
    M_PI / 2 -                                                    #638
    abs(abs(arg(sp)) - M_PI / 2); // positive angle with the real line
  return ang < min_delta;
}                                                                 %446
point circum_center() {
  if (collinear()) return point(NAN, NAN);
  // squared lengths of sides
  double a2, b2, c2;
  a2 = norm(B - C);
  b2 = norm(A - C);
  c2 = norm(A - B);
  // barycentric coordinates of the circumcenter
  double c_A, c_B, c_C;
  c_A = a2 * (b2 + c2 - a2); // sin(2 * alpha) may be used as well
  c_B = b2 * (a2 + c2 - b2);
  c_C = c2 * (a2 + b2 - c2);                                      #403
  double sum = c_A + c_B + c_C;
  c_A /= sum;
  c_B /= sum;
  c_C /= sum;
  // cartesian coordinates of the circumcenter
  return c_A * A + c_B * B + c_C * C;
}                                                                 %742
point centroid() { // center of mass
  return (A + B + C) / 3.0;
}
point ortho_center() { // euler line
  point O = circum_center();
  return O + 3.0 * (centroid() - O);
};
point nine_point_circle_center() { // euler line
  point O = circum_center();
  return O + 1.5 * (centroid() - O);                              #193
};                                                                %031
point in_center() {
  if (collinear()) return point(NAN, NAN);
  double a, b, c; // side lengths
  a = abs(B - C);
  b = abs(A - C);
  c = abs(A - B);
  // trilinear coordinates are (1,1,1)
  // barycentric coordinates
  double c_A = a, c_B = b, c_C = c;
  double sum = c_A + c_B + c_C;
  c_A /= sum;
  c_B /= sum;                                                     #812
  c_C /= sum;
  // cartesian coordinates of the incenter
  return c_A * A + c_B * B + c_C * C;
}                                                                 %980
```

## 8 Seg-Seg intersection, halfplane intersection area

```
struct Seg {
  Vec a, b;
  Vec d() { return b - a; }
};
Vec intersection(Seg l, Seg r) {
  Vec dl = l.d(), dr = r.d();
  if (cross(dl, dr) == 0) return {nanl(""), nanl("")};
  double h = cross(dr, l.a - r.a) / len(dr);
  double dh = cross(dr, dl) / len(dr);
  return l.a + dl * (h / -dh);                                    #893
}
// Returns the area bounded by halfplanes
double calc_area(vector<Seg> lines) {
  double lb = -HUGE_VAL, ub = HUGE_VAL;
  vector<Seg> linesBySide[2];
  for (auto line : lines) {
    if (line.b.y == line.a.y) {
      if (line.a.x < line.b.x) {
        lb = max(lb, line.a.y);
      } else {
        ub = min(ub, line.a.y);                                   #029
      }
    } else if (line.a.y < line.b.y) {
      linesBySide[1].push_back(line);
    } else {
      linesBySide[0].push_back({line.b, line.a});
    }
```

```
  }
  sort(
    linesBySide[0].begin(), linesBySide[0].end(), [](Seg l, Seg r) {
      if (cross(l.d(), r.d()) == 0)                                    #123
        return normal(l.d()) * l.a > normal(r.d()) * r.a;
      return cross(l.d(), r.d()) < 0;
    });
  sort(
    linesBySide[1].begin(), linesBySide[1].end(), [](Seg l, Seg r) {
      if (cross(l.d(), r.d()) == 0)
        return normal(l.d()) * l.a < normal(r.d()) * r.a;
      return cross(l.d(), r.d()) > 0;
    });
  // Now find the application area of the lines and clean up redundant
  // ones
  vector<double> applyStart[2];                                         #597
  for (int side = 0; side < 2; side++) {
    vector<double> &apply = applyStart[side];
    vector<Seg> curLines;
    for (auto line : linesBySide[side]) {
      while (curLines.size() > 0) {
        Seg other = curLines.back();
        if (cross(line.d(), other.d()) != 0) {
          double start = intersection(line, other).y;
          if (start > apply.back()) break;
        }                                                               #503
        curLines.pop_back();
        apply.pop_back();
      }
      if (curLines.size() == 0) {
        apply.push_back(-HUGE_VAL);
      } else {
        apply.push_back(intersection(line, curLines.back()).y);
      }
      curLines.push_back(line);
    }                                                                   #047
    linesBySide[side] = curLines;
  }
  applyStart[0].push_back(HUGE_VALL);
  applyStart[1].push_back(HUGE_VALL);
  double result = 0;
  {
    double lb = -HUGE_VALL, ub;
    for (int i = 0, j = 0; i < (int)linesBySide[0].size() &&
                           j < (int)linesBySide[1].size();
          lb = ub) {                                                    #251
      ub = min(applyStart[0][i + 1], applyStart[1][j + 1]);
      double alb = lb, aub = ub;
      Seg l0 = linesBySide[0][i], l1 = linesBySide[1][j];
      if (cross(l1.d(), l0.d()) > 0) {
        alb = max(alb, intersection(l0, l1).y);
      } else if (cross(l1.d(), l0.d()) < 0) {
        aub = min(aub, intersection(l0, l1).y);
      }
      alb = max(alb, lb);
      aub = min(aub, ub);                                               #839
      aub = max(aub, alb);
      {
        double x1 = l0.a.x + (alb - l0.a.y) / l0.d().y * l0.d().x;
        double x2 = l0.a.x + (aub - l0.a.y) / l0.d().y * l0.d().x;
        result -= (aub - alb) * (x1 + x2) / 2;
      }
      {
        double x1 = l1.a.x + (alb - l1.a.y) / l1.d().y * l1.d().x;
        double x2 = l1.a.x + (aub - l1.a.y) / l1.d().y * l1.d().x;
        result += (aub - alb) * (x1 + x2) / 2;                          #717
      }
      if (applyStart[0][i + 1] < applyStart[1][j + 1]) {
        i++;
      } else {
        j++;
      }
    }
  }
  return result;
}                                                                       %103
```

## 9 Convex polygon algorithms

```
typedef pair<int, int> Vec;
typedef pair<Vec, Vec> Seg;
typedef vector<Seg>::iterator SegIt;
#define F first
#define S second
#define MP(x, y) make_pair(x,y)
ll dot(const Vec &v1, const Vec &v2) {
  return (ll)v1.F * v2.F + (ll)v1.S * v2.S;
}
ll cross(const Vec &v1, const Vec &v2) {                                #914
  return (ll)v1.F * v2.S - (ll)v2.F * v1.S;
}
ll dist_sq(const Vec &p1, const Vec &p2) {
  return (ll)(p2.F - p1.F) * (p2.F - p1.F) +
         (ll)(p2.S - p1.S) * (p2.S - p1.S);
}                                                                       %623
struct Hull {
  vector<Seg> hull;
  SegIt upper_begin;
  template <typename It>
  void extend_hull(It begin, It end) { // O(n)
    vector<Vec> res;
    for (auto it = begin; it != end; ++it) {
      if (res.empty() || *it != res.back()) {
        while (res.size() >= 2) {
```

```
        Vec v1 = {res[res.size() - 1].F - res[res.size() - 2].F,
        ↪  #854
          res[res.size() - 1].S - res[res.size() - 2].S};
        Vec v2 = {it->F - res[res.size() - 2].F,
          it->S - res[res.size() - 2].S};
        if (cross(v1, v2) > 0) break;
        res.pop_back();
      }
      res.push_back(*it);
    }
  }
  for (int i = 0; i < res.size() - 1; ++i)                    #114
    hull.emplace_back(res[i], res[i + 1]);
}
Hull(vector<Vec> &vert) {          // atleast 2 distinct points
  sort(vert.begin(), vert.end()); // O(n log(n))
  extend_hull(vert.begin(), vert.end());
  int diff = hull.size();
  extend_hull(vert.rbegin(), vert.rend());
  upper_begin = hull.begin() + diff;
}                                                              %039
bool contains(Vec p) { // O(log(n))
  if (p < hull.front().F || p > upper_begin->F) return false;
  {
    auto it_low = lower_bound(
      hull.begin(), upper_begin, MP(MP(p.F, (int)-2e9), MP(0, 0)));
    if (it_low != hull.begin()) --it_low;
    Vec v1 = {it_low->S.F - it_low->F.F, it_low->S.S - it_low->F.S};
    Vec v2 = {p.F - it_low->F.F, p.S - it_low->F.S};
    if (cross(v1, v2) < 0) // < 0 is inclusive, <=0 is exclusive
      return false;                                            #287
  }
  {
    auto it_up = lower_bound(hull.rbegin(),
      hull.rbegin() + (hull.end() - upper_begin),
      MP(MP(p.F, (int)2e9), MP(0, 0)));
    if (it_up - hull.rbegin() == hull.end() - upper_begin) --it_up;
    Vec v1 = {it_up->F.F - it_up->S.F, it_up->F.S - it_up->S.S};
    Vec v2 = {p.F - it_up->S.F, p.S - it_up->S.S};
    if (cross(v1, v2) > 0) // > 0 is inclusive, >=0 is exclusive
      return false;                                            #906
  }
  return true;
}                                                              %673
// The function can have only one local min and max
// and may be constant only at min and max.
template <typename T>
SegIt max(function<T(const Seg &)> f) { // O(log(n))
  auto l = hull.begin();
  auto r = hull.end();
  SegIt best = hull.end();
  T best_val;
  while (r - l > 2) {
    auto mid = l + (r - l) / 2;
    T l_val = f(*l);
    T l_nxt_val = f(*(l + 1));                                 #485
    T mid_val = f(*mid);
    T mid_nxt_val = f(*(mid + 1));
    if (best == hull.end() ||
        l_val > best_val) { // If max is at l we may remove it from
                            // the range.
      best = l;
      best_val = l_val;
    }
    if (l_nxt_val > l_val) {
      if (mid_val < l_val) {
        r = mid;                                               #397
      } else {
        if (mid_nxt_val > mid_val) {
          l = mid + 1;
        } else {
          r = mid + 1;
        }
      }
    } else {
      if (mid_val < l_val) {
        l = mid + 1;                                           #634
      } else {
        if (mid_nxt_val > mid_val) {
          l = mid + 1;
        } else {
          r = mid + 1;
        }
      }
    }
  }
  T l_val = f(*l);                                             #470
  if (best == hull.end() || l_val > best_val) {
    best = l;
    best_val = l_val;
  }
  if (r - l > 1) {
    T l_nxt_val = f(*(l + 1));
    if (best == hull.end() || l_nxt_val > best_val) {
      best = l + 1;
      best_val = l_nxt_val;
    }                                                          #814
  }
  return best;
}                                                              %053
SegIt closest(Vec p) { // p can't be internal(can be on border),
                       // hull must have atleast 3 points
```

```
  const Seg &ref_p = hull.front(); // O(log(n))
  return max(function<double(const Seg &)>(
    [&p, &ref_p](
      const Seg &seg) { // accuracy of used type should be coord^2
      if (p == seg.F) return 10 - M_PI;
      Vec v1 = {seg.S.F - seg.F.F, seg.S.S - seg.F.S};
      Vec v2 = {p.F - seg.F.F, p.S - seg.F.S};
      ll cross_prod = cross(v1, v2);
      if (cross_prod > 0) { // order the backside by angle       #083
        Vec v1 = {ref_p.F.F - p.F, ref_p.F.S - p.S};
        Vec v2 = {seg.F.F - p.F, seg.F.S - p.S};
        ll dot_prod = dot(v1, v2);
        ll cross_prod = cross(v2, v1);
        return atan2(cross_prod, dot_prod) / 2;
      }
      ll dot_prod = dot(v1, v2);
      double res = atan2(dot_prod, cross_prod);
      if (dot_prod <= 0 && res > 0) res = -M_PI;
      if (res > 0) {                                            #195
        res += 20;
      } else {
        res = 10 - res;
      }
      return res;
    }));
}                                                               %368
template <int DIRECTION> // 1 or -1
Vec tan_point(Vec p) {   // can't be internal or on border
  //-1 iff CCW rotation of ray from p to res takes it away from
  //polygon?
  const Seg &ref_p = hull.front(); // O(log(n))
  auto best_seg = max(function<double(const Seg &)>(
    [&p, &ref_p](
      const Seg &seg) { // accuracy of used type should be coord^2
      Vec v1 = {ref_p.F.F - p.F, ref_p.F.S - p.S};
      Vec v2 = {seg.F.F - p.F, seg.F.S - p.S};
      ll dot_prod = dot(v1, v2);
      ll cross_prod = DIRECTION * cross(v2, v1);                #867
      return atan2(cross_prod, dot_prod); // order by signed angle
    }));
  return best_seg->F;
}                                                               %101
SegIt max_in_dir(Vec v) { // first is the ans. O(log(n))
  return max(function<ll(const Seg &)>(
    [&v](const Seg &seg) { return dot(v, seg.F); }));
}                                                               %861
pair<SegIt, SegIt> intersections(Seg line) { // O(log(n))
  int x = line.S.F - line.F.F;
  int y = line.S.S - line.F.S;
  Vec dir = {-y, x};
  auto it_max = max_in_dir(dir);
  auto it_min = max_in_dir(MP(y, -x));
  ll opt_val = dot(dir, line.F);
  if (dot(dir, it_max->F) < opt_val ||
      dot(dir, it_min->F) > opt_val)
    return MP(hull.end(), hull.end());                          #292
  SegIt it_r1, it_r2;
  function<bool(const Seg &, const Seg &)> inc_comp(
    [&dir](const Seg &lft, const Seg &rgt) {
      return dot(dir, lft.F) < dot(dir, rgt.F);
    });
  function<bool(const Seg &, const Seg &)> dec_comp(
    [&dir](const Seg &lft, const Seg &rgt) {
      return dot(dir, lft.F) > dot(dir, rgt.F);
    });
  if (it_min <= it_max) {                                       #402
    it_r1 = upper_bound(it_min, it_max + 1, line, inc_comp) - 1;
    if (dot(dir, hull.front().F) >= opt_val) {
      it_r2 =
        upper_bound(hull.begin(), it_min + 1, line, dec_comp) - 1;
    } else {
      it_r2 = upper_bound(it_max, hull.end(), line, dec_comp) - 1;
    }
  } else {
    it_r1 = upper_bound(it_max, it_min + 1, line, dec_comp) - 1;
    if (dot(dir, hull.front().F) <= opt_val) {                  #421
      it_r2 =
        upper_bound(hull.begin(), it_max + 1, line, inc_comp) - 1;
    } else {
      it_r2 = upper_bound(it_min, hull.end(), line, inc_comp) - 1;
    }
  }
  return MP(it_r1, it_r2);
}                                                               %567
Seg diameter() { // O(n)
  Seg res;
  ll dia_sq = 0;
  auto it1 = hull.begin();
  auto it2 = upper_begin;
  Vec v1 = {hull.back().S.F - hull.back().F.F,
    hull.back().S.S - hull.back().F.S};
  while (it2 != hull.begin()) {
    Vec v2 = {(it2 - 1)->S.F - (it2 - 1)->F.F,
      (it2 - 1)->S.S - (it2 - 1)->F.S};                         #386
    ll decider = cross(v1, v2);
    if (decider > 0) break;
    --it2;
  }
  while (it2 != hull.end()) { // check all antipodal pairs
    if (dist_sq(it1->F, it2->F) > dia_sq) {
      res = {it1->F, it2->F};
      dia_sq = dist_sq(res.F, res.S);
```

```
    }
    Vec v1 = {it1->S.F - it1->F.F, it1->S.S - it1->F.S};          #607
    Vec v2 = {it2->S.F - it2->F.F, it2->S.S - it2->F.S};
    ll decider = cross(v1, v2);
    if (decider == 0) { // report cross pairs at parallel lines.
      if (dist_sq(it1->S, it2->F) > dia_sq) {
        res = {it1->S, it2->F};
        dia_sq = dist_sq(res.F, res.S);
      }
      if (dist_sq(it1->F, it2->S) > dia_sq) {
        res = {it1->F, it2->S};
        dia_sq = dist_sq(res.F, res.S);                          #980
      }
      ++it1;
      ++it2;
    } else if (decider < 0) {
      ++it1;
    } else {
      ++it2;
    }
  }
  return res;                                                    #686
 }
};                                                               %781
```

## 10 Aho Corasick $\mathcal{O}(|\text{alpha}| \sum \text{len})$

```
const int alpha_size = 26;
struct node {
  node *nxt[alpha_size]; // May use other structures to move in trie
  node *suffix;
  node() { memset(nxt, 0, alpha_size * sizeof(node *)); }
  int cnt = 0;
};
node *aho_corasick(vector<vector<char> > &dict) {
  node *root = new node;
  root->suffix = 0;                                              #911
  vector<pair<vector<char> *, node *> > cur_state;
  for (vector<char> &s : dict) cur_state.emplace_back(&s, root);
  for (int i = 0; !cur_state.empty(); ++i) {
    vector<pair<vector<char> *, node *> > nxt_state;
    for (auto &cur : cur_state) {
      node *nxt = cur.second->nxt[(*cur.first)[i]];
      if (nxt) {
        cur.second = nxt;
      } else {
        nxt = new node;                                          #003
        cur.second->nxt[(*cur.first)[i]] = nxt;
        node *suf = cur.second->suffix;
        cur.second = nxt;
        nxt->suffix = root; // set correct suffix link
        while (suf) {
          if (suf->nxt[(*cur.first)[i]]) {
            nxt->suffix = suf->nxt[(*cur.first)[i]];
            break;
          }
          suf = suf->suffix;                                     #378
        }
      }
      if (cur.first->size() > i + 1) nxt_state.push_back(cur);
    }
    cur_state = nxt_state;
  }
  return root;
}                                                                %064
// auxilary functions for searhing and counting
node *walk(node *cur,
  char c) { // longest prefix in dict that is suffix of walked string.
  while (true) {
    if (cur->nxt[c]) return cur->nxt[c];
    if (!cur->suffix) return cur;
    cur = cur->suffix;
  }
}                                                                %127
void cnt_matches(node *root, vector<char> &match_in) {
  node *cur = root;
  for (char c : match_in) {
    cur = walk(cur, c);
    ++cur->cnt;
  }
}                                                                %286
void add_cnt(node *root) { // After counting matches propagete ONCE to
                           // suffixes for final counts
  vector<node *> to_visit = {root};
  for (int i = 0; i < to_visit.size(); ++i) {
    node *cur = to_visit[i];
    for (int j = 0; j < alpha_size; ++j) {
      if (cur->nxt[j]) to_visit.push_back(cur->nxt[j]);
    }
  }
  for (int i = to_visit.size() - 1; i > 0; --i)
    to_visit[i]->suffix->cnt += to_visit[i]->cnt;                #354
}                                                                %313
int main() { //
 ↪  http://codeforces.com/group/s3etJR5zZK/contest/212916/problem/4
  int n, len;
  scanf("%d %d", &len, &n);
  vector<char> a(len + 1);
  scanf("%s", a.data());
  a.pop_back();
  for (char &c : a) c -= 'a';
  vector<vector<char> > dict(n);
  for (int i = 0; i < n; ++i) {
    scanf("%d", &len);
```

```
    dict[i].resize(len + 1);
    scanf("%s", dict[i].data());
    dict[i].pop_back();
    for (char &c : dict[i]) c -= 'a';
  }
  node *root = aho_corasick(dict);
  cnt_matches(root, a);
  add_cnt(root);
  for (int i = 0; i < n; ++i) {
    node *cur = root;
    for (char c : dict[i]) cur = walk(cur, c);
    printf("%d\n", cur->cnt);
  }
}
```

## 11 Suffix automaton and tree $\mathcal{O}((n+q)\log(|\textbf{alpha}|))$

```
class AutoNode {
 private:
  map<char, AutoNode *>
    nxt_char; // Map is faster than hashtable and unsorted arrays
 public:
  int len; // Length of longest suffix in equivalence class.
  AutoNode *suf;
  bool has_nxt(char c) const { return nxt_char.count(c); }
  AutoNode *nxt(char c) {
    if (!has_nxt(c)) return NULL;                                #308
    return nxt_char[c];
  }
  void set_nxt(char c, AutoNode *node) { nxt_char[c] = node; }
  AutoNode *split(int new_len, char c) {
    AutoNode *new_n = new AutoNode;
    new_n->nxt_char = nxt_char;
    new_n->len = new_len;
    new_n->suf = suf;
    suf = new_n;
    return new_n;                                                #890
  }                                                              %677
  // Extra functions for matching and counting
  AutoNode *lower_depth(
    int depth) { // move to longest suffix of current with a maximum
                 // length of depth.
    if (suf->len >= depth) return suf->lower_depth(depth);
    return this;
  }
  AutoNode *walk(char c, int depth,
    int &match_len) { // move to longest suffix of walked path that is
                      // a substring
    match_len = min(match_len,
      len); // includes depth limit(needed for finding matches)
    if (has_nxt(c)) { // as suffixes are in classes match_len must
     ↪  be                                                        #091
                      // tracked externally
      ++match_len;
      return nxt(c)->lower_depth(depth);
    }
    if (suf) return suf->walk(c, depth, match_len);
    return this;
  }                                                              %955
  int paths_to_end = 0;
  void set_as_end() { // All suffixes of current node are marked as
                      // ending nodes.
    paths_to_end += 1;
    if (suf) suf->set_as_end();
  }
  bool vis = false;
  void calc_paths_to_end() { // Call ONCE from ROOT. For each node
                             // calculates number of ways to reach an
                             // end node.
    if (!vis) { // paths_to_end is ocurence count for any strings in
                // current suffix equivalence class.
      vis = true;
      for (auto cur : nxt_char) {                                #035
        cur.second->calc_paths_to_end();
        paths_to_end += cur.second->paths_to_end;
      }
    }
  }                                                              %996
  // Transform into suffix tree of reverse string
  map<char, AutoNode *> tree_links;
  int end_dist = 1 << 30;
  int calc_end_dist() {
    if (end_dist == 1 << 30) {
      if (nxt_char.empty()) end_dist = 0;
      for (auto cur : nxt_char)
        end_dist = min(end_dist, 1 + cur.second->calc_end_dist());
    }
    return end_dist;
  }                                                              #188
  bool vis_t = false;
  void build_suffix_tree(string &s) { // Call ONCE from ROOT.
    if (!vis_t) {
      vis_t = true;
      if (suf)
        suf->tree_links[s[s.size() - end_dist - suf->len - 1]] = this;
      for (auto cur : nxt_char) cur.second->build_suffix_tree(s);
    }
  }
};                                                               #748
struct SufAutomaton {
  AutoNode *last;
  AutoNode *root;
  void extend(char new_c) {
    AutoNode *new_end = new AutoNode;
```

```
    new_end->len = last->len + 1;
    AutoNode *suf_w_nxt = last;
    while (suf_w_nxt && !suf_w_nxt->has_nxt(new_c)) {
      suf_w_nxt->set_nxt(new_c, new_end);
      suf_w_nxt = suf_w_nxt->suf;                                    #705
    }
    if (!suf_w_nxt) {
      new_end->suf = root;
    } else {
      AutoNode *max_sbstr = suf_w_nxt->nxt(new_c);
      if (suf_w_nxt->len + 1 == max_sbstr->len) {
        new_end->suf = max_sbstr;
      } else {
        AutoNode *eq_sbstr =
          max_sbstr->split(suf_w_nxt->len + 1, new_c);               #169
        new_end->suf = eq_sbstr;
        AutoNode *w_edge_to_eq_sbstr = suf_w_nxt;
        while (w_edge_to_eq_sbstr != 0 &&
               w_edge_to_eq_sbstr->nxt(new_c) == max_sbstr) {
          w_edge_to_eq_sbstr->set_nxt(new_c, eq_sbstr);
          w_edge_to_eq_sbstr = w_edge_to_eq_sbstr->suf;
        }
      }
    }
    last = new_end;                                                  #356
  }                                                                  %628
  SufAutomaton(string &s) {
    root = new AutoNode;
    root->len = 0;
    root->suf = NULL;
    last = root;
    for (char c : s) extend(c);
    root->calc_end_dist(); // To build suffix tree use reversed string
    root->build_suffix_tree(s);
  }
};                                                                   %034
```

## 12 Dinic

```
struct MaxFlow {
  typedef long long ll;
  const ll INF = 1e18;
  struct Edge {
    int u, v;
    ll c, rc;
    shared_ptr<ll> flow;
    Edge(int _u, int _v, ll _c, ll _rc = 0)
      : u(_u), v(_v), c(_c), rc(_rc) {}
  };                                                                 #787
  struct FlowTracker {
    shared_ptr<ll> flow;
    ll cap, rcap;
    bool dir;
    FlowTracker(ll _cap, ll _rcap, shared_ptr<ll> _flow, int _dir)
      : cap(_cap), rcap(_rcap), flow(_flow), dir(_dir) {}
    ll rem() const {
      if (dir == 0) {
        return cap - *flow;
      } else {                                                       #844
        return rcap + *flow;
      }
    }
    void add_flow(ll f) {
      if (dir == 0)
        *flow += f;
      else
        *flow -= f;
      assert(*flow <= cap);
      assert(-*flow <= rcap);                                        #287
    }
    operator ll() const { return rem(); }
    void operator-=(ll x) { add_flow(x); }
    void operator+=(ll x) { add_flow(-x); }
  };
  int source, sink;
  vector<vector<int> > adj;
  vector<vector<FlowTracker> > cap;
  vector<Edge> edges;
  MaxFlow(int _source, int _sink) : source(_source), sink(_sink) {
  ↪ #080
    assert(source != sink);
  }
  int add_edge(int u, int v, ll c, ll rc = 0) {
    edges.push_back(Edge(u, v, c, rc));
    return edges.size() - 1;
  }
  vector<int> now, lvl;
  void prep() {
    int max_id = max(source, sink);
    for (auto edge : edges) max_id = max(max_id, max(edge.u, edge.v));
    ↪ #638
    adj.resize(max_id + 1);
    cap.resize(max_id + 1);
    now.resize(max_id + 1);
    lvl.resize(max_id + 1);
    for (auto &edge : edges) {
      auto flow = make_shared<ll>(0);
      adj[edge.u].push_back(edge.v);
      cap[edge.u].push_back(FlowTracker(edge.c, edge.rc, flow, 0));
      if (edge.u != edge.v) {
        adj[edge.v].push_back(edge.u);                               #789
        cap[edge.v].push_back(FlowTracker(edge.c, edge.rc, flow, 1));
      }
      assert(cap[edge.u].back() == edge.c);
```

```
      edge.flow = flow;
    }
  }
  bool dinic_bfs() {
    fill(now.begin(), now.end(), 0);
    fill(lvl.begin(), lvl.end(), 0);
    lvl[source] = 1;                                                    #448
    vector<int> bfs(1, source);
    for (int i = 0; i < bfs.size(); ++i) {
      int u = bfs[i];
      for (int j = 0; j < adj[u].size(); ++j) {
        int v = adj[u][j];
        if (cap[u][j] > 0 && lvl[v] == 0) {
          lvl[v] = lvl[u] + 1;
          bfs.push_back(v);
        }
      }                                                                 #722
    }
    return lvl[sink] > 0;
  }
  ll dinic_dfs(int u, ll flow) {
    if (u == sink) return flow;
    while (now[u] < adj[u].size()) {
      int v = adj[u][now[u]];
      if (lvl[v] == lvl[u] + 1 && cap[u][now[u]] != 0) {
        ll res = dinic_dfs(v, min(flow, (ll)cap[u][now[u]]));
        if (res > 0) {                                                  #459
          cap[u][now[u]] -= res;
          return res;
        }
      }
      ++now[u];
    }
    return 0;
  }
  ll calc_max_flow() {
    prep();                                                             #054
    ll ans = 0;
    while (dinic_bfs()) {
      ll cur = 0;
      do {
        cur = dinic_dfs(source, INF);
        ans += cur;
      } while (cur > 0);
    }
    return ans;
  }                                                                     #346
  ll flow_on_edge(int edge_index) {
    assert(edge_index < edges.size());
    return *edges[edge_index].flow;
  }
};                                                                      %583
int main() {
  int n, m;
  cin >> n >> m;
  auto mf = MaxFlow(
    1, n); // arguments source and sink, memory usage O(largest node
           // index + input size), sink doesn't need to be last index
  int edge_index;
  for (int i = 0; i < m; ++i) {
    int a, b, c;
    cin >> a >> b >> c;
    // mf.add_edge(a,b,c); // for directed edges
    edge_index = mf.add_edge(
      a, b, c, c); // store edge index if care about flow value
  }
  cout << mf.calc_max_flow() << '\n';
  // cout << mf.flow_on_edge(edge_index) << endl; // return flow on
  // this edge
}
```

## 13 Min Cost Max Flow with succesive dijkstra $\mathcal{O}(\text{flow} \cdot n^2)$

```
const int nmax = 1055;
const ll inf = 1e14;
int t, n, v; // 0 is source, v-1 sink
ll rem_flow[nmax][nmax];
// set [x][y] for directed capacity from x to y.
ll cost[nmax][nmax]; // set [x][y] for directed cost from x to y. SET
                     // TO inf IF NOT USED
ll min_dist[nmax];
int prev_node[nmax];
ll node_flow[nmax];
bool visited[nmax];                                                     %576
ll tot_cost, tot_flow; // output
void min_cost_max_flow() {
  tot_cost = 0; // Does not work with negative cycles.
  tot_flow = 0;
  ll sink_pot = 0;
  min_dist[0] = 0;                                                      %927
  for (int i = 1; i <= v; ++i) { // incase of no negative edges
                                 // Bellman-Ford can be removed.
    min_dist[i] = inf;
  }
  for (int i = 0; i < v - 1; ++i) {
    for (int j = 0; j < v; ++j) {
      for (int k = 0; k < v; ++k) {
        if (rem_flow[j][k] > 0 &&
            min_dist[j] + cost[j][k] < min_dist[k])
          min_dist[k] = min_dist[j] + cost[j][k];
      }                                                                 #040
    }
  }
  for (int i = 0; i < v; ++i) { // Apply potentials to edge costs.
```

```
  for (int j = 0; j < v; ++j) {
    if (cost[i][j] != inf) {
      cost[i][j] += min_dist[i];
      cost[i][j] -= min_dist[j];
    }
  }
}                                                                     #630
sink_pot += min_dist[v - 1]; // Bellman-Ford end.                    %849
while (true) {
  for (int i = 0; i <= v; ++i) { // node after sink is used as start
                                 // value for Dijkstra.
    min_dist[i] = inf;
    visited[i] = false;
  }
  min_dist[0] = 0;
  node_flow[0] = inf;
  int min_node;
  while (true) { // Use Dijkstra to calculate potentials
    int min_node = v;                                                 #782
    for (int i = 0; i < v; ++i) {
      if ((!visited[i]) && min_dist[i] < min_dist[min_node])
        min_node = i;
    }
    if (min_node == v) break visited[min_node] = true;
    for (int i = 0; i < v; ++i) {
      if ((!visited[i]) &&
          min_dist[min_node] + cost[min_node][i] < min_dist[i]) {
        min_dist[i] = min_dist[min_node] + cost[min_node][i];
        prev_node[i] = min_node;                                      #881
        node_flow[i] =
          min(node_flow[min_node], rem_flow[min_node][i]);
      }
    }
  }
  if (min_dist[v - 1] == inf)
    break for (int i = 0; i < v;
               ++i) { // Apply potentials to edge costs.
      for (int j = 0; j < v;
           ++j) { // Found path from source to sink becomes 0
           ↪  cost.                                                   #664
        if (cost[i][j] != inf) {
          cost[i][j] += min_dist[i];
          cost[i][j] -= min_dist[j];
        }
      }
    }
  sink_pot += min_dist[v - 1];
  tot_flow += node_flow[v - 1];
  tot_cost += sink_pot * node_flow[v - 1];
  int cur = v - 1;                                                    #946
  while (cur != 0) {
    // Backtrack along found path that now has 0 cost.
      rem_flow[prev_node[cur]][cur] -= node_flow[v - 1];
      rem_flow[cur][prev_node[cur]] += node_flow[v - 1];
      cost[cur][prev_node[cur]] = 0;
      if (rem_flow[prev_node[cur]][cur] == 0)
        cost[prev_node[cur]][cur] = inf;
      cur = prev_node[cur];
    }
  }
}                                                                     %803
int main() { // http://www.spoj.com/problems/GREED/
  cin >> t;
  for (int i = 0; i < t; ++i) {
    cin >> n;
    for (int j = 0; j < nmax; ++j) {
      for (int k = 0; k < nmax; ++k) {
        cost[j][k] = inf;
        rem_flow[j][k] = 0;
      }
    }
    for (int j = 1; j <= n; ++j) {
      cost[j][2 * n + 1] = 0;
      rem_flow[j][2 * n + 1] = 1;
    }
    for (int j = 1; j <= n; ++j) {
      int card;
      cin >> card;
      ++rem_flow[0][card];
      cost[0][card] = 0;
    }
    int ex_c;
    cin >> ex_c;
    for (int j = 0; j < ex_c; ++j) {
      int a, b;
      cin >> a >> b;
      if (b < a) swap(a, b);
      cost[a][b] = 1;
      rem_flow[a][b] = nmax;
      cost[b][n + b] = 0;
      rem_flow[b][n + b] = nmax;
      cost[n + b][a] = 1;
      rem_flow[n + b][a] = nmax;
    }
    v = 2 * n + 2;
    min_cost_max_flow();
    cout << tot_cost << '\n';
  }
}
```

## 14 Min Cost Max Flow with Cycle Cancelling $\mathcal{O}(\textbf{flow} \cdot nm)$

```
struct Network {
  struct Node;
```

```
struct Edge {
  Node *u, *v;
  int f, c, cost;
  Node* from(Node* pos) {
    if (pos == u) return v;
    return u;
  }
  int getCap(Node* pos) {                                    #145
    if (pos == u) return c - f;
    return f;
  }
  int addFlow(Node* pos, int toAdd) {
    if (pos == u) {
      f += toAdd;
      return toAdd * cost;
    } else {
      f -= toAdd;
      return -toAdd * cost;                                  #987
    }
  }
};
struct Node {
  vector<Edge*> conn;
  int index;
};
deque<Node> nodes;
deque<Edge> edges;
Node* addNode() {                                            #057
  nodes.push_back(Node());
  nodes.back().index = nodes.size() - 1;
  return &nodes.back();
}
Edge* addEdge(Node* u, Node* v, int f, int c, int cost) {
  edges.push_back({u, v, f, c, cost});
  u->conn.push_back(&edges.back());
  v->conn.push_back(&edges.back());
  return &edges.back();
}                                                            #692
// Assumes all needed flow has already been added
int minCostMaxFlow() {
  int n = nodes.size();
  int result = 0;
  struct State {
    int p;
    Edge* used;
  };
  while (1) {
    vector<vector<State> > state(1, vector<State>(n, {0, 0}));
    for (int lev = 0; lev < n; lev++) {                      #158
      state.push_back(state[lev]);
      for (int i = 0; i < n; i++) {
        if (lev == 0 || state[lev][i].p < state[lev - 1][i].p) {
          for (Edge* edge : nodes[i].conn) {
            if (edge->getCap(&nodes[i]) > 0) {
              int np =
                state[lev][i].p +
                (edge->u == &nodes[i] ? edge->cost : -edge->cost);
              int ni = edge->from(&nodes[i])->index;
              if (np < state[lev + 1][ni].p) {               #281
                state[lev + 1][ni].p = np;
                state[lev + 1][ni].used = edge;
              }
            }
          }
        }
      }
    }
    // Now look at the last level
    bool valid = false;
    for (int i = 0; i < n; i++)                              #283
      if (state[n - 1][i].p > state[n][i].p) {
        valid = true;
        vector<Edge*> path;
        int cap = 1000000000;
        Node* cur = &nodes[i];
        int clev = n;
        vector<bool> explr(n, false);
        while (!explr[cur->index]) {
          explr[cur->index] = true;
          State cstate = state[clev][cur->index];            #954
          cur = cstate.used->from(cur);
          path.push_back(cstate.used);
        }
        reverse(path.begin(), path.end());
        {
          int i = 0;
          Node* cur2 = cur;
          do {
            cur2 = path[i]->from(cur2);
            i++;                                             #990
          } while (cur2 != cur);
          path.resize(i);
        }
        for (auto edge : path) {
          cap = min(cap, edge->getCap(cur));
          cur = edge->from(cur);
        }
        for (auto edge : path) {
          result += edge->addFlow(cur, cap);
          cur = edge->from(cur);                             #599
        }
      }
    if (!valid) break;
```

```
    }
    return result;
  }
}; %900
```

## 15 DMST $\mathcal{O}(E \log V)$

```
struct EdgeDesc {
  int from, to, w;
};
struct DMST {
  struct Node;
  struct Edge {
    Node *from;
    Node *tar;
    int w;
    bool inc; #186
  };
  struct Circle {
    bool vis = false;
    vector<Edge *> contents;
    void clean(int idx);
  };
  const static greater<pair<ll, Edge *> >
    comp; // Can use inline static since C++17
  static vector<Circle> to_process;
  static bool no_dmst; #478
  static Node *root;
  struct Node {
    Node *par = NULL;
    vector<pair<int, int> > out_cands; // Circ, edge idx
    vector<pair<ll, Edge *> > con;
    bool in_use = false;
    ll w = 0; // extra to add to edges in con
    Node *anc() {
      if (!par) return this;
      while (par->par) par = par->par; #721
      return par;
    }
    void clean() {
      if (!no_dmst) {
        in_use = false;
        for (auto &cur : out_cands)
          to_process[cur.first].clean(cur.second);
      }
    }
    Node *con_to_root() { #488
      if (anc() == root) return root;
      in_use = true;
      Node *super = this; // Will become root or the first Node
                          // encountered in a loop.
      while (super == this) {
        while (
          !con.empty() && con.front().second->tar->anc() == anc()) {
          pop_heap(con.begin(), con.end(), comp);
          con.pop_back();
        }
        if (con.empty()) { #506
          no_dmst = true;
          return root;
        }
        pop_heap(con.begin(), con.end(), comp);
        auto nxt = con.back();
        con.pop_back();
        w = -nxt.first;
        if (nxt.second->tar
              ->in_use) { // anc() wouldn't change anything
          super = nxt.second->tar->anc(); #174
          to_process.resize(to_process.size() + 1);
        } else {
          super = nxt.second->tar->con_to_root();
        }
        if (super != root) {
          to_process.back().contents.push_back(nxt.second);
          out_cands.emplace_back(to_process.size() - 1,
            to_process.back().contents.size() - 1);
        } else { // Clean circles
          nxt.second->inc = true; #848
          nxt.second->from->clean();
        }
      }
      if (super != root) { // we are some loops non first Node.
        if (con.size() > super->con.size()) {
          swap(con,
            super->con); // Largest con in loop should not be copied.
          swap(w, super->w);
        }
        for (auto cur : con) { #064
          super->con.emplace_back(
            cur.first - super->w + w, cur.second);
          push_heap(super->con.begin(), super->con.end(), comp);
        }
      }
      par = super; // root or anc() of first Node encountered in a
                   // loop
      return super;
    }
  };
  Node *cur_root; #995
  vector<Node> graph;
  vector<Edge> edges;
  DMST(int n, vector<EdgeDesc> &desc,
    int r) { // Self loops and multiple edges are okay.
    graph.resize(n);
```

```
    cur_root = &graph[r];
    for (auto &cur : desc) // Edges are reversed internally
      edges.push_back(Edge{&graph[cur.to], &graph[cur.from], cur.w});
    for (int i = 0; i < desc.size(); ++i)
      graph[desc[i].to].con.emplace_back(desc[i].w, &edges[i]);   #895
    for (int i = 0; i < n; ++i)
      make_heap(graph[i].con.begin(), graph[i].con.end(), comp);
  }
  bool find() {
    root = cur_root;
    no_dmst = false;
    for (auto &cur : graph) {
      cur.con_to_root();
      to_process.clear();
      if (no_dmst) return false;                                 #405
    }
    return true;
  }                                                              %732
  ll weight() {
    ll res = 0;
    for (auto &cur : edges) {
      if (cur.inc) res += cur.w;
    }
    return res;
  }                                                              %477
};
void DMST::Circle::clean(int idx) {
  if (!vis) {
    vis = true;
    for (int i = 0; i < contents.size(); ++i) {
      if (i != idx) {
        contents[i]->inc = true;
        contents[i]->from->clean();
      }
    }                                                            #711
  }
}
const greater<pair<ll, DMST::Edge *> > DMST::comp;
vector<DMST::Circle> DMST::to_process;
bool DMST::no_dmst;
DMST::Node *DMST::root;                                          %771
```

## 16 Bridges $\mathcal{O}(n)$

```
struct vert;
struct edge {
  bool exists = true;
  vert *dest;
  edge *rev;
  edge(vert *_dest) : dest(_dest) { rev = NULL; }
  vert &operator*() { return *dest; }
  vert *operator->() { return dest; }
  bool is_bridge();
};                                                               #116
struct vert {
  deque<edge> con;
  int val = 0;
  int seen;
  int dfs(int upd, edge *ban) { // handles multiple edges
    if (!val) {
      val = upd;
      seen = val;
      for (edge &nxt : con) {
        if (nxt.exists && (&nxt) != ban)                         #866
          seen = min(seen, nxt->dfs(upd + 1, nxt.rev));
      }
    }
    return seen;
  }                                                              %624
  void remove_adj_bridges() {
    for (edge &nxt : con) {
      if (nxt.is_bridge()) nxt.exists = false;
    }
  }                                                              %106
  int cnt_adj_bridges() {
    int res = 0;
    for (edge &nxt : con) res += nxt.is_bridge();
    return res;
  }                                                              %056
};
bool edge::is_bridge() {
  return exists &&
         (dest->seen > rev->dest->val || dest->val < rev->dest->seen);
}                                                                %223
vert graph[nmax];
int main() { // Mechanics Practice BRIDGES
  int n, m;
  cin >> n >> m;
  for (int i = 0; i < m; ++i) {
    int u, v;
    scanf("%d %d", &u, &v);
    graph[u].con.emplace_back(graph + v);
    graph[v].con.emplace_back(graph + u);
    graph[u].con.back().rev = &graph[v].con.back();
    graph[v].con.back().rev = &graph[u].con.back();
  }
  graph[1].dfs(1, NULL);
  int res = 0;
  for (int i = 1; i <= n; ++i) res += graph[i].cnt_adj_bridges();
  cout << res / 2 << endl;
}
```

## 17 2-Sat $\mathcal{O}(n)$ and SCC $\mathcal{O}(n)$

```
struct Graph {
  int n;
```

```
  vector<vector<int> > conn;
  Graph(int nsize) {
    n = nsize;
    conn.resize(n);
  }
  void add_edge(int u, int v) { conn[u].push_back(v); }
  void _topsort_dfs(int pos, vector<int> &result, vector<bool> &explr,
    vector<vector<int> > &revconn) {                                  #592
    if (explr[pos]) return;
    explr[pos] = true;
    for (auto next : revconn[pos])
      _topsort_dfs(next, result, explr, revconn);
    result.push_back(pos);
  }
  vector<int> topsort() {
    vector<vector<int> > revconn(n);
    for (int u = 0; u < n; u++) {
      for (auto v : conn[u]) revconn[v].push_back(u);                 #775
    }
    vector<int> result;
    vector<bool> explr(n, false);
    for (int i = 0; i < n; i++)
      _topsort_dfs(i, result, explr, revconn);
    reverse(result.begin(), result.end());
    return result;
  }
  void dfs(int pos, vector<int> &result, vector<bool> &explr) {
    if (explr[pos]) return;                                           #591
    explr[pos] = true;
    for (auto next : conn[pos]) dfs(next, result, explr);
    result.push_back(pos);
  }                                                                   %603
  vector<vector<int> > scc() {
    vector<int> order = topsort();
    reverse(order.begin(), order.end());
    vector<bool> explr(n, false);
    vector<vector<int> > results;
    for (auto it = order.rbegin(); it != order.rend(); ++it) {
      vector<int> component;
      _topsort_dfs(*it, component, explr, conn);
      sort(component.begin(), component.end());
      results.push_back(component);                                   #741
    }
    sort(results.begin(), results.end());
    return results;
  }
};                                                                    %983
// Solution for:
// http://codeforces.com/group/PjzGiggT71/contest/221700/problem/C
int main() {
  int n, m;
  cin >> n >> m;
  Graph g(2 * m);
  for (int i = 0; i < n; i++) {
    int a, sa, b, sb;
    cin >> a >> sa >> b >> sb;
    a--, b--;
    g.add_edge(2 * a + 1 - sa, 2 * b + sb);
    g.add_edge(2 * b + 1 - sb, 2 * a + sa);
  }
  vector<int> state(2 * m, 0);
  {
    vector<int> order = g.topsort();
    vector<bool> explr(2 * m, false);
    for (auto u : order) {
      vector<int> traversed;
      g.dfs(u, traversed, explr);
      if (traversed.size() > 0 && !state[traversed[0] ^ 1]) {
        for (auto c : traversed) state[c] = 1;
      }
    }
  }
  for (int i = 0; i < m; i++) {
    if (state[2 * i] == state[2 * i + 1]) {
      cout << "IMPOSSIBLE\n";
      return 0;
    }
  }
  for (int i = 0; i < m; i++) {
    cout << state[2 * i + 1] << '\n';
  }
  return 0;
}
```

## 18 Generic persistent compressed lazy segment tree

```
struct Seg {
  ll sum = 0;
  void recalc(const Seg &lhs_seg, int lhs_len, const Seg &rhs_seg,
    int rhs_len) {
    sum = lhs_seg.sum + rhs_seg.sum;
  }
} __attribute__((packed));
struct Lazy {
  ll add;
  ll assign_val; // LLONG_MIN if no assign;                           #529
  void init() {
    add = 0;
    assign_val = LLONG_MIN;
  }
  Lazy() { init(); }
  void split(Lazy &lhs_lazy, Lazy &rhs_lazy, int len) {
    lhs_lazy = *this;
    rhs_lazy = *this;
```

```cpp
    init();
  }                                                          #953
  void merge(Lazy &oth, int len) {
    if (oth.assign_val != LLONG_MIN) {
      add = 0;
      assign_val = oth.assign_val;
    }
    add += oth.add;
  }
  void apply_to_seg(Seg &cur, int len) const {
    if (assign_val != LLONG_MIN) {
      cur.sum = len * assign_val;                            #204
    }
    cur.sum += len * add;
  }
} __attribute__((packed));                                   %625
struct Node { // Following code should not need to be modified
  int ver;
  bool is_lazy = false;
  Seg seg;
  Lazy lazy;
  Node *lc = NULL, *rc = NULL;
  void init() {
    if (!lc) {
      lc = new Node{ver};
      rc = new Node{ver};                                    #313
    }
  }
  Node *upd(int L, int R, int l, int r, Lazy &val, int tar_ver) {
    if (ver != tar_ver) {
      Node *rep = new Node(*this);
      rep->ver = tar_ver;
      return rep->upd(L, R, l, r, val, tar_ver);
    }
    if (L >= l && R <= r) {
      val.apply_to_seg(seg, R - L);                          #138
      lazy.merge(val, R - L);
      is_lazy = true;
    } else {
      init();
      int M = (L + R) / 2;
      if (is_lazy) {
        Lazy l_val, r_val;
        lazy.split(l_val, r_val, R - L);
        lc = lc->upd(L, M, L, M, l_val, ver);
        rc = rc->upd(M, R, M, R, r_val, ver);                #104
        is_lazy = false;
      }
      Lazy l_val, r_val;
      val.split(l_val, r_val, R - L);
      if (l < M) lc = lc->upd(L, M, l, r, l_val, ver);
      if (M < r) rc = rc->upd(M, R, l, r, r_val, ver);
      seg.recalc(lc->seg, M - L, rc->seg, R - M);
    }
    return this;
  }                                                          #441
  void get(int L, int R, int l, int r, Seg *&lft_res, Seg *&tmp,
    bool last_ver) {
    if (L >= l && R <= r) {
      tmp->recalc(*lft_res, L - l, seg, R - L);
      swap(lft_res, tmp);
    } else {
      init();
      int M = (L + R) / 2;
      if (is_lazy) {
        Lazy l_val, r_val;                                   #803
        lazy.split(l_val, r_val, R - L);
        lc = lc->upd(L, M, L, M, l_val, ver + last_ver);
        lc->ver = ver;
        rc = rc->upd(M, R, M, R, r_val, ver + last_ver);
        rc->ver = ver;
        is_lazy = false;
      }
      if (l < M) lc->get(L, M, l, r, lft_res, tmp, last_ver);
      if (M < r) rc->get(M, R, l, r, lft_res, tmp, last_ver);
    }                                                        #770
  }
} __attribute__((packed));
struct SegTree {        // indexes start from 0, ranges are [beg, end)
  vector<Node *> roots; // versions start from 0
  int len;
  SegTree(int _len) : len(_len) { roots.push_back(new Node{0}); }
  int upd(int l, int r, Lazy &val, bool new_ver = false) {
    Node *cur_root =
      roots.back()->upd(0, len, l, r, val, roots.size() - !new_ver);
    if (cur_root != roots.back()) roots.push_back(cur_root);      #700
    return roots.size() - 1;
  }
  Seg get(int l, int r, int ver = -1) {
    if (ver == -1) ver = roots.size() - 1;
    Seg seg1, seg2;
    Seg *pres = &seg1, *ptmp = &seg2;
    roots[ver]->get(0, len, l, r, pres, ptmp, roots.size() - 1);
    return *pres;
  }
};                                #542                       %542
int main() {
  int n, m; // solves Mechanics Practice LAZY
  cin >> n >> m;
  SegTree seg_tree(1 << 17);
  for (int i = 0; i < n; ++i) {
    Lazy tmp;
    scanf("%lld", &tmp.assign_val);
```

```
    seg_tree.upd(i, i + 1, tmp);
  }
  for (int i = 0; i < m; ++i) {
    int o;
    int l, r;
    scanf("%d %d %d", &o, &l, &r);
    --l;
    if (o == 1) {
      Lazy tmp;
      scanf("%lld", &tmp.add);
      seg_tree.upd(l, r, tmp);
    } else if (o == 2) {
      Lazy tmp;
      scanf("%lld", &tmp.assign_val);
      seg_tree.upd(l, r, tmp);
    } else {
      Seg res = seg_tree.get(l, r);
      printf("%lld\n", res.sum);
    }
  }
}
```

## 19 Templated HLD $\mathcal{O}(M(n)\log n)$ per query

```
class dummy {
 public:
  dummy() {}
  dummy(int, int) {}
  void set(int, int) {}
  int query(int left, int right) {
    cout << this << ' ' << left << ' ' << right << endl;
  }
};                                                          %932
/* T should be the type of the data stored in each vertex;
 * DS should be the underlying data structure that is used to peform
 * the group operation. It should have the following methods:
 * * DS () - empty constructor
 * * DS (int size, T initial) - constructs the structure with the
 * given size, initially filled with initial.
 * * void set (int index, T value) - set the value at index `index` to
 * `value`
 * * T query (int left, int right) - return the "sum" of elements
 * between left and right, inclusive.
 */
template <typename T, class DS>
class HLD {
  int vertexc;
  vector<int> *adj;
  vector<int> subtree_size;
  DS structure;
  DS aux;
  void build_sizes(int vertex, int parent) {
    subtree_size[vertex] = 1;
    for (int child : adj[vertex]) {                         #037
      if (child != parent) {
        build_sizes(child, vertex);
        subtree_size[vertex] += subtree_size[child];
      }
    }
  }
  int cur;
  vector<int> ord;
  vector<int> chain_root;
  vector<int> par;                                          #593
  void build_hld(int vertex, int parent, int chain_source) {
    cur++;
    ord[vertex] = cur;
    chain_root[vertex] = chain_source;
    par[vertex] = parent;
    if (adj[vertex].size() > 1 ||
        (vertex == 1 && adj[vertex].size() == 1)) {
      int big_child, big_size = -1;
      for (int child : adj[vertex]) {
        if ((child != parent) && (subtree_size[child] > big_size)) {
          ↪ #042
          big_child = child;
          big_size = subtree_size[child];
        }
      }
      build_hld(big_child, vertex, chain_source);
      for (int child : adj[vertex]) {
        if ((child != parent) && (child != big_child))
          build_hld(child, vertex, child);
      }
    }                                                       #461
  }
 public:
  HLD(int _vertexc) {
    vertexc = _vertexc;
    adj = new vector<int>[vertexc + 5];
  }
  void add_edge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
  }                                                         #587
  void build(T initial) {
    subtree_size = vector<int>(vertexc + 5);
    ord = vector<int>(vertexc + 5);
    chain_root = vector<int>(vertexc + 5);
    par = vector<int>(vertexc + 5);
    cur = 0;
    build_sizes(1, -1);
    build_hld(1, -1, 1);
    structure = DS(vertexc + 5, initial);
```

```
    aux = DS(50, initial);                                          #638
  }
  void set(int vertex, int value) {
    structure.set(ord[vertex], value);
  }
  T query_path(
    int u, int v) { /* returns the "sum" of the path u->v */
    int cur_id = 0;
    while (chain_root[u] != chain_root[v]) {
      if (ord[u] > ord[v]) {
        cur_id++;                                                   #052
        aux.set(cur_id, structure.query(ord[chain_root[u]], ord[u]));
        u = par[chain_root[u]];
      } else {
        cur_id++;
        aux.set(cur_id, structure.query(ord[chain_root[v]], ord[v]));
        v = par[chain_root[v]];
      }
    }
    cur_id++;
    aux.set(cur_id,                                                 #041
      structure.query(min(ord[u], ord[v]), max(ord[u], ord[v])));
    return aux.query(1, cur_id);
  }                                                                 %905
  void print() {
    for (int i = 1; i <= vertexc; i++)
      cout << i << ' ' << ord[i] << ' ' << chain_root[i] << ' '
           << par[i] << endl;
  }
};
int main() {
  int vertexc;
  cin >> vertexc;
  HLD<int, dummy> hld(vertexc);
  for (int i = 0; i < vertexc - 1; i++) {
    int u, v;
    cin >> u >> v;
    hld.add_edge(u, v);
  }
  hld.build(0);
  hld.print();
  int queryc;
  cin >> queryc;
  for (int i = 0; i < queryc; i++) {
    int u, v;
    cin >> u >> v;
    hld.query_path(u, v);
    cout << endl;
  }
}
```

## 20 Templated multi dimensional BIT $\mathcal{O}(\log(n)^{\textbf{dim}})$ per query

```
// Fully overloaded any dimensional BIT, use any type for coordinates,
// elements, return_value. Includes coordinate compression.
template <typename elem_t, typename coord_t, coord_t n_inf,
  typename ret_t>
class BIT {
  vector<coord_t> positions;
  vector<elem_t> elems;
  bool initiated = false;
 public:
  BIT() { positions.push_back(n_inf); }
  void initiate() {
    if (initiated) {                                                #330
      for (elem_t &c_elem : elems) c_elem.initiate();
    } else {
      initiated = true;
      sort(positions.begin(), positions.end());
      positions.resize(unique(positions.begin(), positions.end()) -
                       positions.begin());
      elems.resize(positions.size());
    }
  }
  template <typename... loc_form>                                   #620
  void update(coord_t cord, loc_form... args) {
    if (initiated) {
      int pos =
        lower_bound(positions.begin(), positions.end(), cord) -
        positions.begin();
      for (; pos < positions.size(); pos += pos & -pos)
        elems[pos].update(args...);
    } else {
      positions.push_back(cord);
    }                                                               #542
  }
  template <typename... loc_form>
  ret_t query(coord_t cord,
    loc_form... args) { // sum in open interval (-inf, cord)
    ret_t res = 0;
    int pos = (lower_bound(positions.begin(), positions.end(), cord) -
               positions.begin()) -
              1;
    for (; pos > 0; pos -= pos & -pos)
      res += elems[pos].query(args...);                             #549
    return res;
  }
};
template <typename internal_type>
struct wrapped {
  internal_type a = 0;
  void update(internal_type b) { a += b; }
  internal_type query() { return a; }
```

```
  // Should never be called, needed for compilation
  void initiate() { cerr << 'i' << endl; }
  void update() { cerr << 'u' << endl; }                          #636
};                                                                %714
int main() {
  // retun type should be same as type inside wrapped
  BIT<BIT<wrapped<ll>, int, INT_MIN, ll>, int, INT_MIN, ll> fenwick;
  int dim = 2;
  vector<tuple<int, int, ll> > to_insert;
  to_insert.emplace_back(1, 1, 1);
  // set up all positions that are to be used for update
  for (int i = 0; i < dim; ++i) {
    for (auto &cur : to_insert)
      fenwick.update(get<0>(cur),
        get<1>(cur)); // May include value which won't be used
    fenwick.initiate();
  }
  // actual use
  for (auto &cur : to_insert)
    fenwick.update(get<0>(cur), get<1>(cur), get<2>(cur));
  cout << fenwick.query(2, 2) << '\n';
}
```

## 21 Treap $\mathcal{O}(\log n)$ per query

```
mt19937 randgen;
struct Treap {
  struct Node {
    int key;
    int value;
    unsigned int priority;
    long long total;
    Node* lch;
    Node* rch;
    Node(int new_key, int new_value) {                            #698
      key = new_key;
      value = new_value;
      priority = randgen();
      total = new_value;
      lch = 0;
      rch = 0;
    }
    void update() {
      total = value;
      if (lch) total += lch->total;                               #295
      if (rch) total += rch->total;
    }
  };
  deque<Node> nodes;
  Node* root = 0;
  pair<Node*, Node*> split(int key, Node* cur) {
    if (cur == 0) return {0, 0};
    pair<Node*, Node*> result;
    if (key <= cur->key) {
      auto ret = split(key, cur->lch);                            #233
      cur->lch = ret.second;
      result = {ret.first, cur};
    } else {
      auto ret = split(key, cur->rch);
      cur->rch = ret.first;
      result = {cur, ret.second};
    }
    cur->update();
    return result;
  }                                                               #230
  Node* merge(Node* left, Node* right) {
    if (left == 0) return right;
    if (right == 0) return left;
    Node* top;
    if (left->priority < right->priority) {
      left->rch = merge(left->rch, right);
      top = left;
    } else {
      right->lch = merge(left, right->lch);
      top = right;                                                #510
    }
    top->update();
    return top;
  }
  void insert(int key, int value) {
    nodes.push_back(Node(key, value));
    Node* cur = &nodes.back();
    pair<Node*, Node*> ret = split(key, root);
    cur = merge(ret.first, cur);
    cur = merge(cur, ret.second);                                 #760
    root = cur;
  }
  void erase(int key) {
    Node *left, *mid, *right;
    tie(left, mid) = split(key, root);
    tie(mid, right) = split(key + 1, mid);
    root = merge(left, right);
  }
  long long sum_upto(int key, Node* cur) {
    if (cur == 0) return 0;                                       #634
    if (key <= cur->key) {
      return sum_upto(key, cur->lch);
    } else {
      long long result = cur->value + sum_upto(key, cur->rch);
      if (cur->lch) result += cur->lch->total;
      return result;
    }
  }
  long long get(int l, int r) {
```

```
    return sum_upto(r + 1, root) - sum_upto(l, root);   #509
  }
};   %959
// Solution for:
// http://codeforces.com/group/U01GDa2Gwb/contest/219104/problem/TREAP
int main() {
  ios_base::sync_with_stdio(false);
  cin.tie(0);
  int m;
  Treap treap;
  cin >> m;
  for (int i = 0; i < m; i++) {
    int type;
    cin >> type;
    if (type == 1) {
      int x, y;
      cin >> x >> y;
      treap.insert(x, y);
    } else if (type == 2) {
      int x;
      cin >> x;
      treap.erase(x);
    } else {
      int l, r;
      cin >> l >> r;
      cout << treap.get(l, r) << endl;
    }
  }
  return 0;
}
```

## 22 Radixsort 50M 64 bit integers as single array in 1 sec

```
typedef unsigned char uchar;
template <typename T>
void msd_radixsort(
  T *start, T *sec_start, int arr_size, int d = sizeof(T) - 1) {
  const int msd_radix_lim = 100;
  const T mask = 255;
  int bucket_sizes[256]{};
  for (T *it = start; it != start + arr_size; ++it) {
    ++bucket_sizes[((*it) >> (d * 8)) & mask];
    //++bucket_sizes[*((uchar*)it + d)];
  }   #772
  T *locs_mem[257];
  locs_mem[0] = sec_start;
  T **locs = locs_mem + 1;
  locs[0] = sec_start;
  for (int j = 0; j < 255; ++j) {
    locs[j + 1] = locs[j] + bucket_sizes[j];
  }
  for (T *it = start; it != start + arr_size; ++it) {
    uchar bucket_id = ((*it) >> (d * 8)) & mask;
    *(locs[bucket_id]++) = *it;   #361
  }
  locs = locs_mem;
  if (d) {
    T *locs_old[256];
    locs_old[0] = start;
    for (int j = 0; j < 255; ++j) {
      locs_old[j + 1] = locs_old[j] + bucket_sizes[j];
    }
    for (int j = 0; j < 256; ++j) {
      if (locs[j + 1] - locs[j] < msd_radix_lim) {   #867
        std::sort(locs[j], locs[j + 1]);
        if (d & 1) {
          copy(locs[j], locs[j + 1], locs_old[j]);
        }
      } else {
        msd_radixsort(locs[j], locs_old[j], bucket_sizes[j], d - 1);
      }
    }
  }
}   %225
const int nmax = 5e7;
ll arr[nmax], tmp[nmax];
int main() {
  for (int i = 0; i < nmax; ++i) arr[i] = ((ll)rand() << 32) | rand();
  msd_radixsort(arr, tmp, nmax);
  assert(is_sorted(arr, arr + nmax));
}
```

## 23 FFT 5M length/sec

integer $c = a * b$ is accurate if $c_i < 2^{49}$

```
struct Complex {
  double a = 0, b = 0;
  Complex &operator/=(const int &oth) {
    a /= oth;
    b /= oth;
    return *this;
  }
};
Complex operator+(const Complex &lft, const Complex &rgt) {
  return Complex{lft.a + rgt.a, lft.b + rgt.b};   #384
}
Complex operator-(const Complex &lft, const Complex &rgt) {
  return Complex{lft.a - rgt.a, lft.b - rgt.b};
}
Complex operator*(const Complex &lft, const Complex &rgt) {
  return Complex{
    lft.a * rgt.a - lft.b * rgt.b, lft.a * rgt.b + lft.b * rgt.a};
}
Complex conj(const Complex &cur) { return Complex{cur.a, -cur.b}; }
void fft_rec(Complex *arr, Complex *root_pow, int len) {   #385
```

```
  if (len != 1) {
    fft_rec(arr, root_pow, len >> 1);
    fft_rec(arr + len, root_pow, len >> 1);
  }
  root_pow += len;
  for (int i = 0; i < len; ++i) {
    Complex tmp = arr[i] + root_pow[i] * arr[i + len];
    arr[i + len] = arr[i] - root_pow[i] * arr[i + len];
    arr[i] = tmp;
  }                                                          #249
}
void fft(vector<Complex> &arr, int ord, bool invert) {
  assert(arr.size() == 1 << ord);
  static vector<Complex> root_pow(1);
  static int inc_pow = 1;
  static bool is_inv = false;
  if (inc_pow <= ord) {
    int idx = root_pow.size();
    root_pow.resize(1 << ord);
    for (; inc_pow <= ord; ++inc_pow) {                      #517
      for (int idx_p = 0; idx_p < 1 << (ord - 1);
           idx_p += 1 << (ord - inc_pow), ++idx) {
        root_pow[idx] = Complex{cos(-idx_p * M_PI / (1 << (ord - 1))),
          sin(-idx_p * M_PI / (1 << (ord - 1)))};
        if (is_inv) root_pow[idx].b = -root_pow[idx].b;
      }
    }
  }
  if (invert != is_inv) {
    is_inv = invert;                                         #750
    for (Complex &cur : root_pow) cur.b = -cur.b;
  }
  for (int i = 1, j = 0; i < (1 << ord); ++i) {
    int m = 1 << (ord - 1);
    bool cont = true;
    while (cont) {
      cont = j & m;
      j ^= m;
      m >>= 1;
    }                                                        #844
    if (i < j) swap(arr[i], arr[j]);
  }
  fft_rec(arr.data(), root_pow.data(), 1 << (ord - 1));
  if (invert)
    for (int i = 0; i < (1 << ord); ++i) arr[i] /= (1 << ord);
}                                                            %380
void mult_poly_mod(
  vector<int> &a, vector<int> &b, vector<int> &c) { // c += a*b
  static vector<Complex>
    arr[4]; // correct upto 0.5-2M elements(mod ~= 1e9)
  if (c.size() < 400) {
    for (int i = 0; i < a.size(); ++i)
      for (int j = 0; j < b.size() && i + j < c.size(); ++j)
        c[i + j] = ((ll)a[i] * b[j] + c[i + j]) % mod;
  } else {
    int fft_ord = 32 - __builtin_clz(c.size());              #629
    if (arr[0].size() != 1 << fft_ord)
      for (int i = 0; i < 4; ++i) arr[i].resize(1 << fft_ord);
    for (int i = 0; i < 4; ++i)
      fill(arr[i].begin(), arr[i].end(), Complex{});
    for (int &cur : a)
      if (cur < 0) cur += mod;
    for (int &cur : b)
      if (cur < 0) cur += mod;
    const int shift = 15;
    const int mask = (1 << shift) - 1;                       #625
    for (int i = 0; i < min(a.size(), c.size()); ++i) {
      arr[0][i].a = a[i] & mask;
      arr[1][i].a = a[i] >> shift;
    }
    for (int i = 0; i < min(b.size(), c.size()); ++i) {
      arr[0][i].b = b[i] & mask;
      arr[1][i].b = b[i] >> shift;
    }
    for (int i = 0; i < 2; ++i) fft(arr[i], fft_ord, false);
    for (int i = 0; i < 2; ++i) {                            #644
      for (int j = 0; j < 2; ++j) {
        int tar = 2 + (i + j) / 2;
        Complex mult = {0, -0.25};
        if (i ^ j) mult = {0.25, 0};
        for (int k = 0; k < (1 << fft_ord); ++k) {
          int rev_k = ((1 << fft_ord) - k) % (1 << fft_ord);
          Complex ca = arr[i][k] + conj(arr[i][rev_k]);
          Complex cb = arr[j][k] - conj(arr[j][rev_k]);
          arr[tar][k] = arr[tar][k] + mult * ca * cb;
        }                                                    #471
      }
    }
    for (int i = 2; i < 4; ++i) {
      fft(arr[i], fft_ord, true);
      for (int k = 0; k < (int)c.size(); ++k) {
        c[k] = (c[k] + (((ll)(arr[i][k].a + 0.5) % mod)
                        << (shift * 2 * (i - 2)))) %
               mod;
        c[k] = (c[k] + (((ll)(arr[i][k].b + 0.5) % mod)
                        << (shift * (2 * (i - 2) + 1)))) %   #108
               mod;
      }
    }
  }
}                                                            %231
```

## 24 Fast mod mult, Rabbin Miller prime check, Pollard rho factorization $\mathcal{O}(\sqrt{p})$

```
struct ModArithm {
  ull n;
  ld rec;
  ModArithm(ull _n) : n(_n) { // n in [2, 1<<63)
    rec = 1.0L / n;
  }
  ull multf(ull a, ull b) { // a, b in [0, min(2*n, 1<<63))
    ull mult = (ld)a * b * rec + 0.5L;
    ll res = a * b - mult * n;
    if (res < 0) res += n;                                  #780
    return res; // in [0, n-1)
  }
  ull sqp1(ull a) { return multf(a, a) + 1; }
};                                                          %493
ull pow_mod(ull a, ull n, ModArithm &arithm) {
  ull res = 1;
  for (ull i = 1; i <= n; i <<= 1) {
    if (n & i) res = arithm.multf(res, a);
    a = arithm.multf(a, a);
  }
  return res;
}                                                           %144
vector<char> small_primes = {
  2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
bool is_prime(ull n) { // n <= 1<<63, 1M rand/s
  ModArithm arithm(n);
  if (n == 2 || n == 3) return true;
  if (!(n & 1) || n == 1) return false;
  ull s = __builtin_ctz(n - 1);
  ull d = (n - 1) >> s;
  for (ull a : small_primes) {
    if (a >= n) break;                                      #402
    a = pow_mod(a, d, arithm);
    if (a == 1 || a == n - 1) continue;
    for (ull r = 1; r < s; ++r) {
      a = arithm.multf(a, a);
      if (a == 1) return false;
      if (a == n - 1) break;
    }
    if (a != n - 1) return false;
  }
  return true;                                              #806
}                                                           %975
ll pollard_rho(ll n) {
  ModArithm arithm(n);
  int cum_cnt = 64 - __builtin_clz(n);
  cum_cnt *= cum_cnt / 5 + 1;
  while (true) {
    ll lv = rand() % n;
    ll v = arithm.sqp1(lv);
    int idx = 1;
    int tar = 1;
    while (true) {                                          #290
      ll cur = 1;
      ll v_cur = v;
      int j_stop = min(cum_cnt, tar - idx);
      for (int j = 0; j < j_stop; ++j) {
        cur = arithm.multf(cur, abs(v_cur - lv));
        v_cur = arithm.sqp1(v_cur);
        ++idx;
      }
      if (!cur) {
        for (int j = 0; j < cum_cnt; ++j) {                 #912
          ll g = __gcd(abs(v - lv), n);
          if (g == 1) {
            v = arithm.sqp1(v);
          } else if (g == n) {
            break;
          } else {
            return g;
          }
        }
        break;                                              #208
      } else {
        ll g = __gcd(cur, n);
        if (g != 1) return g;
      }
      v = v_cur;
      idx += j_stop;
      if (idx == tar) {
        lv = v;
        tar *= 2;
        v = arithm.sqp1(v);                                 #174
        ++idx;
      }
    }
  }
}                                                           %542
map<ll, int> prime_factor(ll n,
  map<ll, int> *res = NULL) { // n <= 1<<61, ~1000/s (<500/s on CF)
  if (!res) {
    map<ll, int> res_act;
    for (int p : small_primes) {
      while (!(n % p)) {
        ++res_act[p];
        n /= p;
      }
    }                                                       #612
    if (n != 1) prime_factor(n, &res_act);
    return res_act;
```

```
  }
  if (is_prime(n)) {
    ++(*res)[n];
  } else {
    ll factor = pollard_rho(n);
    prime_factor(factor, res);
    prime_factor(n / factor, res);
  }                                                  #350
  return map<ll, int>();
} // Usage: fact = prime_factor(n);                  %477
```

## 25 Symmetric Submodular Functions; Queyrannes's algorithm

**SSF**: such function $f : V \to R$ that satisfies $f(A) = f(V/A)$ and for all $x \in V, X \subseteq Y \subseteq V$ it holds that $f(X+x) - f(X) \leq f(Y+x) - f(Y)$. **Hereditary family**: such set $I \subseteq 2^V$ so that $X \subset Y \wedge Y \in I \Rightarrow X \in I$. **Loop**: such $v \in V$ so that $v \notin I$.

```
def minimize():
    s = merge_all_loops()
    while size >= 3:
        t, u = find_pp()
        {u} is a possible minimizer
        tu = merge(t, u)
        if tu not in I:
            s = merge(tu, s)
    for x in V:
        {x} is a possible minimizer
def find_pp():
    W = {s} # s as in minimizer()
    todo = V/W
    ord = []
    while len(todo) > 0:
        x = min(todo, key=lambda x: f(W+{x}) - f({x}))
        W += {x}
        todo -= {x}
        ord.append(x)
    return ord[-1], ord[-2]
def enum_all_minimal_minimizers(X):
  # X is a inclusionwise minimal minimizer
    s = merge(s, X)
    yield X
    for {v} in I:
        if f({v}) == f(X):
            yield X
            s = merge(v, s)
    while size(V) >= 3:
        t, u = find_pp()
        tu = merge(t, u)
        if tu not in I:
            s = merge(tu, s)
        elif f({tu}) = f(X):
            yield tu
            s = merge(tu, s)
```