

University of Tartu ICPC Team Notebook (2018-2019)

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1 2D geometry

Define $\text{orient}(A, B, C) = \overline{AB} \times \overline{AC}$. CCW iff > 0 .
Define $\text{perp}((a, b)) = (-b, a)$. The vectors are orthogonal.

For line $ax + by = c$ def $\bar{v} = (-b, a)$.

Line through P and Q has $\bar{v} = \overline{PQ}$ and $c = \bar{v} \times P$.

$\text{side}_l(P) = \bar{v}_l \times P - c_l$ sign determines which side P is on from l .

$\text{dist}_l(P) = \text{side}_l(P) / \|\bar{v}_l\|$ squared is integer.

Sorting points along a line: comparator is $\bar{v} \cdot A < \bar{v} \cdot B$.

Translating line by \bar{t} : new line has $c' = c + \bar{v} \times \bar{t}$.

Line intersection: is $(c_l \bar{v}_m - c_m \bar{v}_l) / (\bar{v}_l \times \bar{v}_m)$.

Project P onto l : is $P - \text{perp}(v) \text{side}_l(P) / \|\bar{v}\|^2$.

Angle bisectors: $\bar{v} = \bar{v}_l / \|\bar{v}_l\| + \bar{v}_m / \|\bar{v}_m\|$

$c = c_l / \|\bar{v}_l\| + c_m / \|\bar{v}_m\|$.

P is on segment AB iff $\text{orient}(A, B, P) = 0$ and $\overline{PA} \cdot \overline{PB} \leq 0$.

Proper intersection of AB and CD exists iff $\text{orient}(C, D, A)$ and $\text{orient}(C, D, B)$ have opp. signs and $\text{orient}(A, B, C)$ and $\text{orient}(A, B, D)$ have opp. signs. Coordinates:

$$\frac{A \text{orient}(C, D, B) - B \text{orient}(C, D, A)}{\text{orient}(C, D, B) - \text{orient}(C, D, A)}.$$

Circumcircle center:

```
pt circumCenter(pt a, pt b, pt c) {
    b = b-a, c = c-a; // consider coordinates
    relative to A
    assert(cross(b,c) != 0); // no circumcircle
    if A,B,C aligned
    return a + perp(b*sq(c) - c*sq(b))/cross(b,c)
    /2;
```

Circle-line intersect:

```
int circleLine(pt o, double r, line l, pair<pt,
pt> &out) {
    double h2 = r*r - l.sqDist(o);
    if (h2 >= 0) { // the line touches the circle
        pt p = l.proj(o); // point P
        pt h = l.v*sqrt(h2)/abs(l.v); // vector
        paral to l, of len h
        out = {p-h, p+h};
    }
    return 1 + sgn(h2);
```

Circle-circle intersect:

```
int circleCircle(pt o1, double r1, pt o2,
double r2, pair<pt,pt> &out) {
    pt d=o2-o1; double d2=sq(d);
```

```
if (d2 == 0) {assert(r1 != r2); return 0;} //
concentric circles
double pd = (d2 + r1*r1 - r2*r2)/2; // = |
0_1P| * d
double h2 = r1*r1 - pd*pd/d2; // = h^2
if (h2 >= 0) {
    pt p = o1 + d*pd/d2, h = perp(d)*sqrt(h2/d2
);
    out = {p-h, p+h};}
return 1 + sgn(h2);
```

Tangent lines:

```
int tangents(pt o1, double r1, pt o2, double r2
, bool inner, vector<pair<pt,pt>> &out) {
    if (inner) r2 = -r2;
    pt d = o2-o1;
    double dr = r1-r2, d2 = sq(d), h2 = d2-dr*dr;
    if (d2 == 0 || h2 < 0) {assert(h2 != 0);
return 0;}
for (double sign : {-1,1}) {
    pt v = (d*dr + perp(d)*sqrt(h2)*sign)/d2;
    out.push_back({o1 + v*r1, o2 + v*r2});}
return 1 + (h2 > 0);
```

2 3D geometry

$\text{orient}(P, Q, R, S) = (\overline{PQ} \times \overline{PR}) \cdot \overline{PS}$.

S above PQR iff > 0 .

For plane $ax + by + cz = d$ def $\bar{n} = (a, b, c)$.

Line with normal \bar{n} through point P has $d = \bar{n} \cdot P$.

$\text{side}_\Pi(P) = \bar{n} \cdot P - d$ sign determines side from Π .

$\text{dist}_\Pi(P) = \text{side}_\Pi(P) / \|\bar{n}\|$.

Translating plane by \bar{t} makes $d' = d + \bar{n} \cdot \bar{t}$.

Plane-plane intersection of has direction $\bar{n}_1 \times \bar{n}_2$ and

goes through $((d_1 \bar{n}_2 - d_2 \bar{n}_1) \times \bar{d}) / \|\bar{d}\|^2$.

Line-line distance:

```
double dist(line3d l1, line3d l2) {
    p3 n = l1.d*l2.d;
    if (n == zero) // parallel
        return l1.dist(l2.o);
    return abs((l2.o-l1.o)|n)/abs(n);
```

Spherical to Cartesian:

$(r \cos \varphi \cos \lambda, r \cos \varphi \sin \lambda, r \sin \varphi)$.

Sphere-line intersection:

```
int sphereLine(p3 o, double r, line3d l, pair<
p3,p3> &out) {
    double h2 = r*r - l.sqDist(o);
    if (h2 < 0) return 0; // the line doesn't
touch the sphere
    p3 p = l.proj(o); // point P
    p3 h = l.d*sqrt(h2)/abs(l.d); // vector
    parallel to l, of length h
    out = {p-h, p+h};
    return 1 + (h2 > 0);
```

Great-circle distance between points A and B is $r \angle AOB$.

Spherical segment intersection:

```
bool properInter(p3 a, p3 b, p3 c, p3 d, p3 &
out) {
    p3 ab = a*b, cd = c*d; // normals of planes
OAB and OCD
    int oa = sgn(cd|a),
    ob = sgn(cd|b),
    oc = sgn(ab|c),
    od = sgn(ab|d);
    out = ab*cd*od; // four multiplications =>
careful with overflow !
    return (oa != ob && oc != od && oa != oc);
}
bool onSphSegment(p3 a, p3 b, p3 p) {
    p3 n = a*b;
    if (n == zero)
        return a*p == zero && (a|p) > 0;
    return (n|p) == 0 && (n|a*p) >= 0 && (n|b*p)
<= 0;
}
struct directionSet : vector<p3> {
    using vector::vector; // import constructors
    void insert(p3 p) {
        for (p3 q : *this) if (p*q == zero) return;
        push_back(p);
    }
};
directionSet intersSph(p3 a, p3 b, p3 c, p3 d)
{
    assert(validSegment(a, b) && validSegment(c,
d));
    p3 out;
    if (properInter(a, b, c, d, out)) return {out
};
    directionSet s;
    if (onSphSegment(c, d, a)) s.insert(a);
    if (onSphSegment(c, d, b)) s.insert(b);
    if (onSphSegment(a, b, c)) s.insert(c);
    if (onSphSegment(a, b, d)) s.insert(d);
    return s;
}
```

Angle between spherical segments AB and AC is angle between $A \times B$ and $A \times C$.

Oriented angle: subtract from 2π if mixed product is negative.

Area of a spherical polygon:

$$r^2[\text{sum of interior angles} - (n - 2)\pi].$$

3 Maxflow Complexity

$\mathcal{O}(V^2 E)$ – Dinic

$\Theta(VE \log U)$ – Capacity scaling

$\Theta(\text{flow} E)$ – Small flow

$\Theta(\min\{V^{\frac{2}{3}}, E^{\frac{1}{2}}\}E)$ – Unitary capacities

$\Theta(\sqrt{V}E)$ – Each vertex other than S,T has only
a single incoming unitary edge or outgoing one
(bipartite matching)

$\Theta(\text{flow} E \log V)$ – Min-cost-max flow

4 Min Rotation of string

```
int a=0, N=s.size();
s += s;
ran(b,0,N){
  ran(i,0,N) {
    if (a+i == b || s[a+i] < s[b+i]) {
      b += max(0, i-1);
      break;
    }
    if (s[a+i] > s[b+i]) {
      a = b;
      break;
    }
  }
}
return a;
```

5 Symmetric Submodular Functions; Queyrannes's algorithm

SSF: such function $f : V \rightarrow R$ that satisfies $f(A) = f(V/A)$ and for all $x \in V, X \subseteq Y \subseteq V$ it holds that $f(X + x) - f(X) \leq f(Y + x) - f(Y)$.

Hereditary family: such set $I \subseteq 2^V$ so that $X \subset Y \wedge Y \in I \Rightarrow X \in I$.

Loop: such $v \in V$ so that $v \notin I$.

```

1 def minimize():
2     s = merge_all_loops()
3     while size >= 3:
4         t, u = find_pp()
5         {u} is a possible minimizer
6         tu = merge(t, u)
7         if tu not in I:
8             s = merge(tu, s)
9     for x in V:
10        {x} is a possible minimizer
11 def find_pp():
12     W = {s} # s as in minimizer()
13     todo = V/W
14     ord = []
15     while len(todo) > 0:
16         x = min(todo, key=lambda x: f(W+{x}) - f({x}))
17         W += {x}
18         todo -= {x}
19         ord.append(x)
20     return ord[-1], ord[-2]
21
22 def enum_all_minimal_minimizers(X):
23     # X is a inclusionwise minimal minimizer
24     s = merge(s, X)
25     yield X
26     for {v} in I:
27         if f({v}) == f(X):
28             yield X
29             s = merge(v, s)
30     while size(V) >= 3:
31         t, u = find_pp()
32         tu = merge(t, u)
33         if tu not in I:
34             s = merge(tu, s)
35         elif f({tu}) = f(X):
36             yield tu
37             s = merge(tu, s)

```

```

alias g++='g++ -g -Wall -Wshadow -Wconversion \ #.bashrc
-fsanitize=undefined,address -DCDEBUG' #.bashrc
alias a='setxkbmap us -option' #.bashrc
alias m='setxkbmap us -option caps:escape' #.bashrc
alias ma='setxkbmap us -variant dvp \ #.bashrc
-option caps:escape' #.bashrc
gsettings set org.compiz.core: \ #settings
/org/compiz/profiles/Default/plugins/core/ hsize 4
gsettings set org.gnome.desktop.wm.preferences \
focus-mode 'sloppy' #settings
set si cin #.vimrc
set ts=4 sw=4 noet #.vimrc
set cb=unnamedplus #.vimrc
(global-set-key (kbd "C-x <next>") 'other-window) #.emacs
(global-set-key (kbd "C-x <prior>") \ #.emacs
'previous-multiframe-window)
(global-set-key (kbd "C-M-z") 'ansi-term)

(global-linum-mode 1)
(column-number-mode 1)
(show-paren-mode 1)
(setq-default indent-tabs-mode nil)
valgrind --vgdb-error=0 ./a <inp &
gdb a
target remote | vgdb

crc.sh

#!/bin/envbash
for j in `seq $2 1 $3`; do #whitespaces don't matter.
sed '/^s*/d' $1 | head -$j | tr -d '[:space:]' \
| cksum | cut -f1 -d ' ' | tail -c 5
done #there shouldn't be any COMMENTS.
#copy lines being checked to separate file.
# $ ./crc.sh tmp.cpp 999 999
# $ ./crc.sh tmp.cpp 1 333 | grep XXXX

gcc ordered set, hashtable

#define DEBUG(...) cerr << __VA_ARGS__ << endl;
#ifdef CDEBUG
#undef DEBUG
#define DEBUG(...) ((void)0);
#define NDEBUD
#endif
#define ran(i, a, b) for (auto i = (a); i < (b); i++)
#include <bits/stdc++.h>
typedef long long ll;
typedef long double ld;
using namespace std;
#pragma GCC optimize("Ofast") // better vectorization
#pragma GCC target("avx,avx2")
// double vectorized performance
#include <bits/extc++.h>
using namespace __gnu_pbds;
template <typename T, typename U>
using hashmap = gp_hash_table<T, U>

// dumb, 3x faster than stl
template <typename T>
using ordered_set = tree<T, null_type, less<T>,
rb_tree_tag, tree_order_statistics_node_update>;

int main() {
ordered_set<int> cur;
cur.insert(1);
cur.insert(3);
cout << cur.order_of_key(2) << endl;
// the number of elements in the set less than 2
cout << *cur.find_by_order(1) << endl;
// the 1-st smallest number in the set(0-based)
ordered_set<int> oth;
oth.insert(5); // to join: cur < oth
cur.join(oth); // cur = {1, 3, 5}, oth = {}
cur.split(1, oth); // cur = {1}, oth = {3, 5}
hashmap<int, int> h({}, {}, {}, {}, {1 << 16});

PRNGs and Hash functions

mt19937 gen;
uint64_t rand64() {
return gen() ^ ((uint64_t)gen() << 32);
}
uint64_t rand64() {
static uint64_t x = 1; //x != 0
x ^= x >> 12;
x ^= x << 25;
x ^= x >> 27;
return x * 0x2545f4914f6cdd1d; // can remove mult
}
uint64_t mix(uint64_t x){ // deadbeef -> y allowed
variable uint64_t mem[2] = { x, 0xdeadbeeffeebdaedull };
asm volatile (
"pxor %%xmm0, %%xmm0;"
"movdqa (%0), %%xmm1;"
"aesenc %%xmm0, %%xmm1;"
"movdqa %%xmm1, (%0);"
:
: "r" (&mem[0])
: "memory"
);
return mem[0]; // use both slots for 128 bit
}
uint64_t mix64(uint64_t x) { //x != 0
x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
x = x ^ (x >> 31);
return x;
}
uint64_t unmix64(uint64_t x) {
x = (x ^ (x >> 31) ^ (x >> 62)) * 0x319642b2d24d8ec3;
x = (x ^ (x >> 27) ^ (x >> 54)) * 0x96de1b173f119089;
x = x ^ (x >> 30) ^ (x >> 60);
return x;
}
uint64_t combine64(uint64_t x, uint64_t y) {
if (y < x) swap(x, y); // remove for ord
}

return mix64(mix64(x) + y);
}

Triangle centers
const double min_delta = 1e-13;
const double coord_max = 1e6;
typedef complex<double> point;
point A, B, C; // vertices of the triangle
bool collinear() {
double min_diff =
min(abs(A - B), min(abs(A - C), abs(B - C)));
if (min_diff < coord_max * min_delta) return true;
point sp = (B - A) / (C - A);
double ang = M_PI / 2 - abs(abs(arg(sp)) - M_PI / 2);
return ang < min_delta;
// positive angle with the real line
}
point circum_center() {
if (collinear()) return point(NAN, NAN);
// squared lengths of sides
double a2 = norm(B - C);
double b2 = norm(A - C);
double c2 = norm(A - B);
// barycentric coordinates of the circumcenter
// sin(2 * alpha) works also
double c_A = a2 * (b2 + c2 - a2);
double c_B = b2 * (a2 + c2 - b2);
double c_C = c2 * (a2 + b2 - c2);
double sum = c_A + c_B + c_C;
c_A /= sum;
c_B /= sum;
c_C /= sum;
return c_A * A + c_B * B + c_C * C; // cartesian
}
point centroid() { // center of mass
return (A + B + C) / 3.0;
}
point ortho_center() { // euler line
point O = circum_center();
return O + 3.0 * (centroid() - O);
}
point nine_point_circle_center() { // euler line
point O = circum_center();
return O + 1.5 * (centroid() - O);
}
point in_center() {
if (collinear()) return point(NAN, NAN);
double a = abs(B - C); // side lengths
double b = abs(A - C);
double c = abs(A - B);
// trilinear coordinates are (1,1,1)
double sum = a + b + c;
a /= sum;
b /= sum;
c /= sum;
return a * A + b * B + c * C; // barycentric
// cartesian
}

Seg-Seg intersection, halfplane intersection area

```

```

struct Seg {
    Vec a, b;
    Vec d() { return b - a; }
};
Vec intersection(Seg l, Seg r) {
    Vec dl = l.d(), dr = r.d();
    if (cross(dl, dr) == 0) return {nanl(""), nanl("")};
    double h = cross(dr, l.a - r.a) / len(dr);
    double dh = cross(dr, dl) / len(dr);
    return l.a + dl * (h / -dh);
}
// Returns the area bounded by halfplanes
double calc_area(const vector<Seg>& lines) {
    double lb = -HUGE_VAL, ub = HUGE_VAL;
    vector<Seg> slines[2];
    for (auto line : lines) {
        if (line.b.y == line.a.y) {
            if (line.a.x < line.b.x) {
                lb = max(lb, line.a.x);
            } else {
                ub = min(ub, line.a.x);
            }
        } else if (line.a.y < line.b.y) {
            slines[1].push_back(line);
        } else {
            slines[0].push_back({line.b, line.a});
        }
    }
    ran(i, 0, 2) {
        sort(slines[i].begin(), slines[i].end(),
            [&](Seg l, Seg r) {
                if (cross(l.d(), r.d()) == 0)
                    return normal(l.d()) * l.a >
                        normal(r.d()) * r.a;
                return (1 - 2 * i) * cross(l.d(), r.d()) < 0;
            });
    }
    // Now find the application area of the lines and clean
    // up redundant ones
    vector<double> ap_s[2];
    ran(side, 0, 2) {
        vector<double>& apply = ap_s[side];
        vector<Seg> clines;
        for (auto line : slines[side]) {
            while (clines.size() > 0) {
                Seg other = clines.back();
                if (cross(line.d(), other.d()) != 0) {
                    double start = intersection(line, other).y;
                    if (start > apply.back()) break;
                }
                clines.pop_back();
                apply.pop_back();
            }
            if (clines.size() == 0) {
                apply.push_back(-HUGE_VAL);
            } else {
                apply.push_back(
                    intersection(line, clines.back()).y);
                clines.push_back(line);
            }
            slines[side] = clines;
        }
        ap_s[0].push_back(HUGE_VAL);
        ap_s[1].push_back(HUGE_VAL);
        double result = 0;
        {
            double lb = -HUGE_VAL, ub;
            for (int i = 0, j = 0; i < (int)slines[0].size() &&
                j < (int)slines[1].size();
                lb = ub) {
                ub = min(ap_s[0][i + 1], ap_s[1][j + 1]);
                double alb = lb, aub = ub;
                Seg l[2] = {slines[0][i], slines[1][j]};
                if (cross(l[1].d(), l[0].d()) > 0) {
                    alb = max(alb, intersection(l[0], l[1]).y);
                } else if (cross(l[1].d(), l[0].d()) < 0) {
                    aub = min(aub, intersection(l[0], l[1]).y);
                }
                alb = max(alb, lb);
                aub = min(aub, ub);
                aub = max(aub, alb);
                ran(k, 0, 2) {
                    double x1 = l[0].a.x + (alb - l[0].a.y) /
                        l[0].d().y * l[0].d().x;
                    double x2 = l[0].a.x + (aub - l[0].a.y) /
                        l[0].d().y * l[0].d().x;
                    result +=
                        (-1 + 2 * k) * (aub - alb) * (x1 + x2) / 2;
                }
                if (ap_s[0][i + 1] < ap_s[1][j + 1]) {
                    i++;
                } else {
                    j++;
                }
            }
        }
        return result;
    }
}

Convex polygon algorithms
typedef pair<int, int> Vec;
typedef pair<Vec, Vec> Seg;
typedef vector<Seg>::iterator SegIt;
#define F first
#define S second
#define MP(x, y) make_pair(x, y)
Vec sub(const Vec &v1, const Vec &v2) {
    return MP(v1.F - v2.F, v1.S - v2.S);
}
ll dot(const Vec &v1, const Vec &v2) {
    return (ll)v1.F * v2.F + (ll)v1.S * v2.S;
}
ll cross(const Vec &v1, const Vec &v2) {
    return (ll)v1.F * v2.S - (ll)v2.F * v1.S;
}

}
ll dist_sq(const Vec &p1, const Vec &p2) {
    return (ll)(p2.F - p1.F) * (p2.F - p1.F) +
        (ll)(p2.S - p1.S) * (p2.S - p1.S);
}
struct Point;
multiset<Point>::iterator end_node;
struct Point {
    Vec p;
    typename multiset<Point>::iterator get_it() const {
        // gcc rb_tree dependent
        tuple<void*> tmp = {(void*)this - 32};
        return *(multiset<Point>::iterator*)(&tmp);
    }
    bool operator<(const Point &rhs) const {
        return (p.F < rhs.p.F); // sort by x
    }
    bool operator<(const Vec &q) const {
        auto nxt = next(get_it()); // convex hull trick
        if (nxt == end_node) return 0; // nxt == end()
        return q.S * dot(p, {q.F, 1}) <
            q.S * dot(nxt->p, {q.F, 1});
    }
};
template <int part> // 1 = upper, -1 = lower
struct HullDynamic : public multiset<Point, less<>> {
    bool bad(iterator y) {
        if (y == begin()) return 0;
        auto x = prev(y);
        auto z = next(y);
        if (z == end())
            return y->p.F == x->p.F && y->p.S <= x->p.S;
        return part *
            cross(sub(y->p, x->p), sub(y->p, z->p)) <=
                0;
    }
    void insert_point(int m, int b) { // O(log(N))
        auto y = insert({m, b});
        if (bad(y)) {
            erase(y);
            return;
        }
        while (next(y) != end() && bad(next(y)))
            erase(next(y));
        while (y != begin() && bad(prev(y))) erase(prev(y));
    }
    ll eval(
        int x) { // O(log(N)) upper maximize dot({x, 1}, v)
        end_node =
            end(); // lower minimize dot({x, 1}, v)
        auto it = lower_bound((Vec){x, part});
        return (ll)it->p.F * x + it->p.S;
    }
};
struct Hull {
    vector<Seg> hull;
    SegIt up_beg;
}

```

```

template <typename It>
void extend(It beg, It end) { // O(n)
    vector<Vec> r;
    for (auto it = beg; it != end; ++it) {
        if (r.empty() || *it != r.back()) {
            while (r.size() >= 2) {
                int n = r.size();
                Vec v1 = {r[n - 1].F - r[n - 2].F,
                    r[n - 1].S - r[n - 2].S};
                Vec v2 = {
                    it->F - r[n - 2].F, it->S - r[n - 2].S};
                if (cross(v1, v2) > 0) break;
                r.pop_back();
            }
            r.push_back(*it);
        }
    }
    ran(i, 0, (int)r.size() - 1)
    hull.emplace_back(r[i], r[i + 1]);
}

Hull(vector<Vec> &vert) { // at least 2 distinct points
    sort(vert.begin(), vert.end()); // O(n log(n))
    extend(vert.begin(), vert.end());
    int diff = hull.size();
    extend(vert.rbegin(), vert.rend());
    up_beg = hull.begin() + diff;
}

bool contains(Vec p) { // O(log(n))
    if (p < hull.front().F || p > up_beg->F)
        return false;
    {
        auto it_low = lower_bound(hull.begin(), up_beg,
            MP(MP(p.F, (int)-2e9), MP(0, 0)));
        if (it_low != hull.begin()) --it_low;
        Vec a = {it_low->S.F - it_low->F.F,
            it_low->S.S - it_low->F.S};
        Vec b = {p.F - it_low->F.F, p.S - it_low->F.S};
        if (cross(a, b) <
            0) // < 0 is inclusive, <= 0 is exclusive
            return false;
    }
    {
        auto it_up = lower_bound(hull.rbegin(),
            hull.rbegin() + (hull.end() - up_beg),
            MP(MP(p.F, (int)2e9), MP(0, 0)));
        if (it_up - hull.rbegin() == hull.end() - up_beg)
            --it_up;
        Vec a = {it_up->F.F - it_up->S.F,
            it_up->F.S - it_up->S.S};
        Vec b = {p.F - it_up->S.F, p.S - it_up->S.S};
        if (cross(a, b) >
            0) // > 0 is inclusive, >= 0 is exclusive
            return false;
    }
    return true;
}
// The function can have only one local min and max

```

```

// and may be constant only at min and max.
template <typename T>
SegIt max(function<T(Seg &)> f) { // O(log(n))
    auto l = hull.begin();
    auto r = hull.end();
    SegIt b = hull.end();
    T b_v;
    while (r - l > 2) {
        auto m = l + (r - l) / 2;
        T l_v = f(*l);
        T l_n_v = f(*(l + 1));
        T m_v = f(*m);
        T m_n_v = f(*(m + 1));
        if (b == hull.end() || l_v > b_v) {
            b = l; // If max is at l we may remove it from
                // the range.
            b_v = l_v;
        }
        if (l_n_v > l_v) {
            if (m_v < l_v) {
                r = m;
            } else {
                if (m_n_v > m_v) {
                    l = m + 1;
                } else {
                    r = m + 1;
                }
            }
        } else {
            if (m_v < l_v) {
                l = m + 1;
            } else {
                if (m_n_v > m_v) {
                    l = m + 1;
                } else {
                    r = m + 1;
                }
            }
        }
    }
    T l_v = f(*l);
    if (b == hull.end() || l_v > b_v) {
        b = l;
        b_v = l_v;
    }
    if (r - l > 1) {
        T l_n_v = f(*(l + 1));
        if (b == hull.end() || l_n_v > b_v) {
            b = l + 1;
            b_v = l_n_v;
        }
    }
    return b;
}

SegIt closest(
    Vec p) { // p can't be internal(can be on border),
    // hull must have at least 3 points

```

```

Seg &ref_p = hull.front(); // O(log(n))
return max(function<double>(Seg &)>{
    [&p, &ref_p](Seg &seg) { // accuracy of used type
        // should be coord^2
        if (p == seg.F) return 10 - M_PI;
        Vec v1 = {seg.S.F - seg.F.F, seg.S.S - seg.F.S};
        Vec v2 = {p.F - seg.F.F, p.S - seg.F.S};
        ll c_p = cross(v1, v2);
        if (c_p > 0) { // order the backside by angle
            Vec v1 = {ref_p.F.F - p.F, ref_p.F.S - p.S};
            Vec v2 = {seg.F.F - p.F, seg.F.S - p.S};
            ll d_p = dot(v1, v2);
            ll c_p = cross(v2, v1);
            return atan2(c_p, d_p) / 2;
        }
        ll d_p = dot(v1, v2);
        double res = atan2(d_p, c_p);
        if (d_p <= 0 && res > 0) res = -M_PI;
        if (res > 0) {
            res += 20;
        } else {
            res = 10 - res;
        }
    });
}

template <int DIRECTION> // 1 or -1
Vec tan_point(
    Vec p) { // can't be internal or on border
    // -1 iff CCW rotation of ray from p to res takes it
    // away from
    // polygon?
    Seg &ref_p = hull.front(); // O(log(n))
    auto best_seg = max(function<double>(Seg &)>{
        [&p, &ref_p](Seg &seg) { // accuracy of used type
            // should be coord^2
            Vec v1 = {ref_p.F.F - p.F, ref_p.F.S - p.S};
            Vec v2 = {seg.F.F - p.F, seg.F.S - p.S};
            ll d_p = dot(v1, v2);
            ll c_p = DIRECTION * cross(v2, v1);
            return atan2(c_p, d_p); // order by signed angle
        });
    return best_seg->F;
}

SegIt max_in_dir(
    Vec v) { // first is the ans. O(log(n))
    return max(function<ll>(Seg &)>{
        [&v](Seg &seg) { return dot(v, seg.F); });
    });
}

pair<SegIt, SegIt> intersections(Seg l) { // O(log(n))
    int x = l.S.F - l.F.F;
    int y = l.S.S - l.F.S;
    Vec dir = {-y, x};
    auto it_max = max_in_dir(dir);
    auto it_min = max_in_dir(MP(y, -x));
    ll opt_val = dot(dir, l.F);
    if (dot(dir, it_max->F) < opt_val ||

```

```

    dot(dir, it_min->F) > opt_val)
    return MP(hull.end(), hull.end());
SegIt it_r1, it_r2;
function<bool(const Seg &, const Seg &)> inc_c(
    [dir](const Seg &lft, const Seg &rgt) {
        return dot(dir, lft.F) < dot(dir, rgt.F);
    });
function<bool(const Seg &, const Seg &)> dec_c(
    [dir](const Seg &lft, const Seg &rgt) {
        return dot(dir, lft.F) > dot(dir, rgt.F);
    });
if (it_min <= it_max) {
    it_r1 =
        upper_bound(it_min, it_max + 1, 1, inc_c) - 1;
    if (dot(dir, hull.front().F) >= opt_val) {
        it_r2 = upper_bound(
            hull.begin(), it_min + 1, 1, dec_c) -
            1;
    } else {
        it_r2 =
            upper_bound(it_max, hull.end(), 1, dec_c) - 1;
    }
} else {
    it_r1 =
        upper_bound(it_max, it_min + 1, 1, dec_c) - 1;
    if (dot(dir, hull.front().F) <= opt_val) {
        it_r2 = upper_bound(
            hull.begin(), it_max + 1, 1, inc_c) -
            1;
    } else {
        it_r2 =
            upper_bound(it_min, hull.end(), 1, inc_c) - 1;
    }
}
return MP(it_r1, it_r2);
}
Seg diameter() { // O(n)
    Seg res;
    ll dia_sq = 0;
    auto it1 = hull.begin();
    auto it2 = up_beg;
    Vec v1 = {hull.back().S.F - hull.back().F.F,
        hull.back().S.S - hull.back().F.S};
    while (it2 != hull.begin()) {
        Vec v2 = {(it2 - 1)->S.F - (it2 - 1)->F.F,
            (it2 - 1)->S.S - (it2 - 1)->F.S};
        if (cross(v1, v2) > 0) break;
        --it2;
    }
    while (
        it2 != hull.end()) { // check all antipodal pairs
        if (dist_sq(it1->F, it2->F) > dia_sq) {
            res = {it1->F, it2->F};
            dia_sq = dist_sq(res.F, res.S);
        }
        Vec v1 = {
            it1->S.F - it1->F.F, it1->S.S - it1->F.S};
            Vec v2 = {
                it2->S.F - it2->F.F, it2->S.S - it2->F.S};
            if (cross(v1, v2) == 0) {
                if (dist_sq(it1->S, it2->F) > dia_sq) {
                    res = {it1->S, it2->F};
                    dia_sq = dist_sq(res.F, res.S);
                }
                if (dist_sq(it1->F, it2->S) > dia_sq) {
                    res = {it1->F, it2->S};
                    dia_sq = dist_sq(res.F, res.S);
                } // report cross pairs at parallel lines.
                ++it1;
                ++it2;
            } else if (cross(v1, v2) < 0) {
                ++it1;
            } else {
                ++it2;
            }
        }
        return res;
    }
}

Delaunay triangulation O(nlogn)
const int max_co = (1 << 28) - 5;
struct Vec {
    int x, y;
    bool operator==(const Vec &oth) {
        return x == oth.x && y == oth.y;
    }
    bool operator!=(const Vec &oth) {
        return !operator==(oth);
    }
    Vec operator-(const Vec &oth) {
        return {x - oth.x, y - oth.y};
    }
}

ll cross(Vec a, Vec b) {
    return (ll)a.x * b.y - (ll)a.y * b.x;
}

ll dot(Vec a, Vec b) {
    return (ll)a.x * b.x + (ll)a.y * b.y;
}

struct Edge {
    Vec tar;
    Edge *nxt;
    Edge *inv = NULL;
    Edge *rep = NULL;
    bool vis = false;
};

struct Seg {
    Vec a, b;
    bool operator==(const Seg &oth) {
        return a == oth.a && b == oth.b;
    }
    bool operator!=(const Seg &oth) {
        return !operator==(oth);
    }
}

};
ll orient(Vec a, Vec b, Vec c) {
    return (ll)a.x * (b.y - c.y) + (ll)b.x * (c.y - a.y) +
        (ll)c.x * (a.y - b.y);
}

bool in_c_circle(Vec *arr, Vec d) {
    if (cross(arr[1] - arr[0], arr[2] - arr[0]) == 0)
        return true; // degenerate
    ll m[3][3];
    ran(i, 0, 3) {
        m[i][0] = arr[i].x - d.x;
        m[i][1] = arr[i].y - d.y;
        m[i][2] = m[i][0] * m[i][0];
        m[i][2] += m[i][1] * m[i][1];
    }
    __int128 res = 0;
    res +=
        (__int128)(m[0][0] * m[1][1] - m[0][1] * m[1][0]) *
        m[2][2];
    res +=
        (__int128)(m[1][0] * m[2][1] - m[1][1] * m[2][0]) *
        m[0][2];
    res -=
        (__int128)(m[0][0] * m[2][1] - m[0][1] * m[2][0]) *
        m[1][2];
    return res > 0;
}

Edge *add_triangle(Edge *a, Edge *b, Edge *c) {
    Edge *old[] = {a, b, c};
    Edge *tmp = new Edge[3];
    ran(i, 0, 3) {
        old[i]->rep = tmp + i;
        tmp[i] = {
            old[i]->tar, tmp + (i + 1) % 3, old[i]->inv;
            if (tmp[i].inv) tmp[i].inv->inv = tmp + i;
        }
        return tmp;
    }

Edge *add_point(
    Vec p, Edge *cur) { // returns outgoing edge
    Edge *triangle[] = {cur, cur->nxt, cur->nxt->nxt};
    ran(i, 0, 3) {
        if (orient(triangle[i]->tar,
            triangle[(i + 1) % 3]->tar, p) < 0)
            return NULL;
    }
    ran(i, 0, 3) {
        if (triangle[i]->rep) {
            Edge *res = add_point(p, triangle[i]->rep);
            if (res)
                return res; // unless we are on last layer we
                // must exit here
        }
    }
    Edge p_as_e[p];
    Edge tmp[cur->tar];
    tmp.inv = add_triangle(&p_as_e, &tmp, cur = cur->nxt);
}

```



```

Edge *res = tmp.inv->nxt;
tmp.tar = cur->tar;
tmp.inv = add_triangle(&p_as_e, &tmp, cur = cur->nxt);
tmp.tar = cur->tar;
res->inv = add_triangle(&p_as_e, &tmp, cur = cur->nxt);
res->inv->inv = res;
return res;
}
Edge *delanay(vector<Vec> &points) {
    random_shuffle(points.begin(), points.end());
    Vec arr[] = {{4 * max_co, 4 * max_co},
        {-4 * max_co, max_co}, {max_co, -4 * max_co}};
    Edge *res = new Edge[3];
    ran(i, 0, 3) res[i] = {arr[i], res + (i + 1) % 3};
    for (Vec &cur : points) {
        Edge *loc = add_point(cur, res);
        Edge *out = loc;
        arr[0] = cur;
        while (true) {
            arr[1] = out->tar;
            arr[2] = out->nxt->tar;
            Edge *e = out->nxt->inv;
            if (e && in_c_circle(arr, e->nxt->tar)) {
                Edge tmp{cur};
                tmp.inv = add_triangle(&tmp, out, e->nxt);
                tmp.tar = e->nxt->tar;
                tmp.inv->inv =
                    add_triangle(&tmp, e->nxt->nxt, out->nxt->nxt);
                out = tmp.inv->nxt;
                continue;
            }
            out = out->nxt->nxt->inv;
            if (out->tar == loc->tar) break;
        }
    }
    return res;
}
void extract_triangles(
    Edge *cur, vector<vector<Seg> > &res) {
    if (!cur->vis) {
        bool inc = true;
        Edge *it = cur;
        do {
            it->vis = true;
            if (it->rep) {
                extract_triangles(it->rep, res);
                inc = false;
            }
            it = it->nxt;
        } while (it != cur);
    }
    if (inc) {
        Edge *triangle[3] = {cur, cur->nxt, cur->nxt->nxt};
        res.resize(res.size() + 1);
        vector<Seg> &tar = res.back();
        ran(i, 0, 3) {
            if ((abs(triangle[i]->tar.x) < max_co &&
                abs(triangle[(i + 1) % 3]->tar.x) <
                    max_co))
                tar.push_back({triangle[i]->tar,
                    triangle[(i + 1) % 3]->tar});
            if (tar.empty()) res.pop_back();
        }
    }
}

Aho Corasick O(|alpha|*sum(len))
const int alpha_size = 26;
struct Node {
    Node *nxt[alpha_size]; // May use other structures to
                          // move in trie
    Node *suffix;
    Node() { memset(nxt, 0, alpha_size * sizeof(Node *)); }
    int cnt = 0;
};
Node *aho_corasick(vector<vector<char> > &dict) {
    Node *root = new Node;
    root->suffix = 0;
    vector<pair<vector<char> *, Node *> > state;
    for (vector<char> &s : dict)
        state.emplace_back(&s, root);
    for (int i = 0; !state.empty(); ++i) {
        vector<pair<vector<char> *, Node *> > nstate;
        for (auto &cur : state) {
            Node *nxt = cur.second->nxt[(cur.first)[i]];
            if (nxt) {
                cur.second = nxt;
            } else {
                nxt = new Node;
                cur.second->nxt[(cur.first)[i]] = nxt;
                Node *suf = cur.second->suffix;
                cur.second = nxt;
                nxt->suffix = root; // set correct suffix link
                while (suf) {
                    if (suf->nxt[(cur.first)[i]]) {
                        nxt->suffix = suf->nxt[(cur.first)[i]];
                        break;
                    }
                    suf = suf->suffix;
                }
            }
            if (cur.first->size() > i + 1)
                nstate.push_back(cur);
        }
        state = nstate;
    }
    return root;
}
// auxiliary functions for searhing and counting
Node *walk(Node *cur,
    char c) { // longest prefix in dict that is suffix of
              // walked string.
    while (true) {
        if (cur->nxt[c]) return cur->nxt[c];
        if (!cur->suffix) return cur;
    }
}

cur = cur->suffix;
}
void cnt_matches(Node *root, vector<char> &match_in) {
    Node *cur = root;
    for (char c : match_in) {
        cur = walk(cur, c);
        ++cur->cnt;
    }
}
void add_cnt(
    Node *root) { // After counting matches propagete ONCE
                  // to suffixes for final counts
    vector<Node *> to_visit = {root};
    ran(i, 0, to_visit.size()) {
        Node *cur = to_visit[i];
        ran(j, 0, alpha_size) {
            if (cur->nxt[j]) to_visit.push_back(cur->nxt[j]);
        }
    }
    for (int i = to_visit.size() - 1; i > 0; --i)
        to_visit[i]->suffix->cnt += to_visit[i]->cnt;
}
int main() {
    int n, len;
    scanf("%d %d", &n, &len);
    vector<char> a(len + 1);
    scanf("%s", a.data());
    a.pop_back();
    for (char &c : a) c -= 'a';
    vector<vector<char> > dict(n);
    ran(i, 0, n) {
        scanf("%d", &len);
        dict[i].resize(len + 1);
        scanf("%s", dict[i].data());
        dict[i].pop_back();
        for (char &c : dict[i]) c -= 'a';
    }
    Node *root = aho_corasick(dict);
    cnt_matches(root, a);
    add_cnt(root);
    ran(i, 0, n) {
        Node *cur = root;
        for (char c : dict[i]) cur = walk(cur, c);
        printf("%d\n", cur->cnt);
    }
}
Suffix automaton and tree O((n+q)log(|alpha|))
struct Node {
    map<char, Node *> nxt_char;
    // Map is faster than hashtable and unsorted arrays
    int len; // Length of longest suffix in equivalence
              // class.
    Node *suf;
    bool has_nxt(char c) const {
        return nxt_char.count(c);
    }
}

```

```

Node *nxt(char c) {
    if (!has_nxt(c)) return NULL;
    return nxt_char[c];
}
void set_nxt(char c, Node *node) {
    nxt_char[c] = node;
}
Node *split(int new_len, char c) {
    Node *new_n = new Node;
    new_n->nxt_char = nxt_char;
    new_n->len = new_len;
    new_n->suf = suf;
    return new_n;
}
// Extra functions for matching and counting
Node *lower(int depth) {
    // move to longest suf of current with a maximum
    // length of depth.
    if (suf->len >= depth) return suf->lower(depth);
    return this;
}
Node *walk(char c, int depth, int &match_len) {
    // move to longest suffix of walked path that is a
    // substring
    match_len = min(match_len, len);
    // includes depth limit (needed for finding matches)
    if (has_nxt(c)) { // as suffixes are in classes match_len
        // must be tracked externally
        ++match_len;
        return nxt(c)->lower(depth);
    }
    if (suf) return suf->walk(c, depth, match_len);
    return this;
}
int paths_to_end = 0;
void set_as_end() { // All suffixes of current node are
    // marked as ending nodes.
    paths_to_end += 1;
    if (suf) suf->set_as_end();
}
bool vis = false;
void calc_paths() {
    /* Call ONCE from ROOT. For each node calculates
    * number of ways to reach an end node. paths_to_end
    * is occurrence count for any strings in current
    * suffix equivalence class. */
    if (!vis) {
        vis = true;
        for (auto cur : nxt_char) {
            cur.second->calc_paths();
            paths_to_end += cur.second->paths_to_end;
        }
    }
}
// Transform into suffix tree of reverse string

map<char, Node *> tree_links;
int end_dist = 1 << 30;
int calc_end_dist() {
    if (end_dist == 1 << 30) {
        if (nxt_char.empty()) end_dist = 0;
        for (auto cur : nxt_char)
            end_dist =
                min(end_dist, 1 + cur.second->calc_end_dist());
    }
    return end_dist;
}
bool vis_t = false;
void build_suffix_tree(
    string &s) { // Call ONCE from ROOT.
    if (!vis_t) {
        vis_t = true;
        if (suf)
            suf->tree_links[s[s.size() - end_dist -
                suf->len - 1]] = this;
        for (auto cur : nxt_char)
            cur.second->build_suffix_tree(s);
    }
}
struct SufAuto {
    Node *last;
    Node *root;
    void extend(char new_c) {
        Node *nlast = new Node;
        nlast->len = last->len + 1;
        Node *sw_n = last;
        while (sw_n && !sw_n->has_nxt(new_c)) {
            sw_n->set_nxt(new_c, nlast);
            sw_n = sw_n->suf;
        }
        if (!sw_n) {
            nlast->suf = root;
        }
        else {
            Node *max_sbstr = sw_n->nxt(new_c);
            if (sw_n->len + 1 == max_sbstr->len) {
                nlast->suf = max_sbstr;
            }
            else { // remove for minimal DFA that matches
                // suffixes and crap
                Node *eq_sbstr =
                    max_sbstr->split(sw_n->len + 1, new_c);
                nlast->suf = eq_sbstr;
                Node *x = sw_n; // x = with_edge_to_eq_sbstr
                while (x != 0 && x->nxt(new_c) == max_sbstr) {
                    x->set_nxt(new_c, eq_sbstr);
                    x = x->suf;
                }
            }
        }
        last = nlast;
    }
}
SufAuto(string &s) {
    root = new Node;
    root->len = 0;
    root->suf = NULL;
    last = root;
    for (char c : s) extend(c);
    root->calc_end_dist(); // To build suffix tree use
    // reversed string
    root->build_suffix_tree(s);
}
Dinic
struct MaxFlow {
    const static ll INF = 1e18;
    int source, sink;
    vector<int> start, now, lvl, adj, rcap, cap_loc, bfs;
    vector<int> cap, orig_cap;
    ll sink_pot = 0;
    vector<bool> visited;
    vector<ll> cost;
    priority_queue<pair<ll, int>, vector<pair<ll, int> >,
        greater<pair<ll, int> > >
        dist_que;
    void add_flow(int idx, ll flow, bool cont = true) {
        cap[idx] -= flow;
        if (cont) add_flow(rcap[idx], -flow, false);
    }
    MaxFlow(
        const vector<tuple<int, int, ll, ll, ll> > &edges) {
        for (auto &cur : edges) { //from, to, cap, rcap, cost
            start.resize(max(max(get<0>(cur), get<1>(cur)) + 2,
                (int)start.size()));
            ++start[get<0>(cur) + 1];
            ++start[get<1>(cur) + 1];
        }
        ran(i, 1, (int)start.size()) start[i] += start[i-1];
        now = start;
        adj.resize(start.back());
        cap.resize(start.back());
        rcap.resize(start.back());
        cost.resize(start.back());
        for (auto &cur : edges) {
            int u, v;
            ll c, rc, c_cost;
            tie(u, v, c, rc, c_cost) = cur;
            assert(u != v);
            adj[now[u]] = v;
            adj[now[v]] = u;
            rcap[now[u]] = now[v];
            rcap[now[v]] = now[u];
            cap_loc.push_back(now[u]);
            cost[now[u]] = c_cost;
            cost[now[v]] = -c_cost;
            cap[now[u]++] = c;
            cap[now[v]++] = rc;
            orig_cap.push_back(c);
        }
    }
    bool dinic_bfs(int min_cap) {

```

```

lvl.clear();
lvl.resize(start.size());
bfs.clear();
bfs.resize(1, source);
now = start;
lvl[source] = 1;
ran(i, 0, (int)bfs.size()) {
    int u = bfs[i];
    while (now[u] < start[u + 1]) {
        int v = adj[now[u]];
        if (cost[now[u]] == 0 &&
            cap[now[u]] >= min_cap && lvl[v] == 0) {
            lvl[v] = lvl[u] + 1;
            if (v == sink) return true;
            bfs.push_back(v);
        }
        ++now[u];
    }
}
return false;
}
ll dinic_dfs(int u, ll flow, int min_cap) {
    if (u == sink) return flow;
    if (lvl[u] == lvl[sink]) return 0;
    ll res = 0;
    while (now[u] < start[u + 1]) {
        int v = adj[now[u]];
        if (lvl[v] == lvl[u] + 1 && cost[now[u]] == 0 &&
            cap[now[u]] >= min_cap) {
            ll cur = dinic_dfs(v, min(flow, (ll)cap[now[u]]),
                min_cap);
            if (cur) {
                add_flow(now[u], cur);
                flow -= cur;
                res += cur;
                if (!flow) break;
            }
        }
        ++now[u];
    }
    return res;
}
bool recalc_dist(bool check_imp = false) {
    now = start;
    visited.clear();
    visited.resize(start.size());
    dist_que.emplace(0, source);
    bool imp = false;
    while (!dist_que.empty()) {
        int u;
        ll dist;
        tie(dist, u) = dist_que.top();
        dist_que.pop();
        if (!visited[u]) {
            visited[u] = true;
            if (check_imp && dist != 0) imp = true;
            if (u == sink) sink_pot += dist;
            while (now[u] < start[u + 1]) {
                int v = adj[now[u]];
                if (!visited[v] && cap[now[u]])
                    dist_que.emplace(dist + cost[now[u]], v);
                cost[now[u]] += dist;
                cost[rcap[now[u]++]] -= dist;
            }
        }
        if (check_imp) return imp;
        return visited[sink];
    }
    // return whether there is a negative cycle
    bool recalc_dist_bellman_ford() {
        int i = 0;
        for (; i < (int)start.size() - 1 &&
            recalc_dist(true); ++i) {}
        return i == (int)start.size() - 1;
    }
    pair<ll, ll> calc_flow(int _source, int _sink) {
        source = _source;
        sink = _sink;
        assert(max(source, sink) < start.size() - 1);
        ll tot_flow = 0;
        ll tot_cost = 0;
        if (recalc_dist_bellman_ford()) {
            assert(false);
        } else {
            while (recalc_dist()) {
                ll flow = 0;
                for (int min_cap = 1 << 30; min_cap; min_cap >= 1) {
                    while (dinic_bfs(min_cap)) {
                        now = start;
                        ll cur;
                        while (cur = dinic_dfs(source, INF, min_cap))
                            flow += cur;
                    }
                    tot_flow += flow;
                    tot_cost += sink_pot * flow;
                }
                return {tot_flow, tot_cost};
            }
        }
        ll flow_on_edge(int idx) {
            assert(idx < cap.size());
            return orig_cap[idx] - cap[cap_loc[idx]];
        }
    };
    const int nmax = 1055;
    int main() {
        int t;
        scanf("%d", &t);
        for (int i = 0; i < t; ++i) {
            vector<tuple<int, int, ll, ll, ll>> edges;
            int n;
            scanf("%d", &n);
            for (int j = 1; j <= n; ++j) {
                edges.emplace_back(j, 2 * n + 1, 1, 0, 0);
            }
            for (int j = 1; j <= n; ++j) {
                int card;
                scanf("%d", &card);
                edges.emplace_back(0, card, 1, 0, 0);
            }
            int ex_c;
            scanf("%d", &ex_c);
            for (int j = 0; j < ex_c; ++j) {
                int a, b;
                scanf("%d %d", &a, &b);
                if (b < a) swap(a, b);
                edges.emplace_back(a, b, nmax, 0, 1);
                edges.emplace_back(b, n + b, nmax, 0, 0);
                edges.emplace_back(n + b, a, nmax, 0, 1);
            }
            int v = 2 * n + 2;
            MaxFlow mf(edges);
            printf("%d\n", (int)mf.calc_flow(0, v - 1).second);
            //cout << mf.flow_on_edge(edge_index) << endl;
        }
    }
}

```

Min Cost Max Flow with Cycle Cancelling O(Cnm)

```

struct Network {
    struct Node;
    struct Edge {
        Node *u, *v;
        int f, c, cost;
    };
    Node* from(Node* pos) {
        if (pos == u) return v;
        return u;
    }
    int getCap(Node* pos) {
        if (pos == u) return c - f;
        return f;
    }
    int addFlow(Node* pos, int toAdd) {
        if (pos == u) {
            f += toAdd;
            return toAdd * cost;
        } else {
            f -= toAdd;
            return -toAdd * cost;
        }
    }
};
struct Node {
    vector<Edge*> conn;
    int index;
};
deque<Node> nodes;
deque<Edge> edges;
Node* addNode() {
    nodes.push_back(Node());
    nodes.back().index = nodes.size() - 1;
}

```



```

    return super;
}
};
Node *croot;
vector<Node> graph;
vector<Edge> edges;
DMST(int n, vector<EdgeDesc> &desc, int r) {
    // Self loops and multiple edges are okay.
    graph.resize(n);
    croot = &graph[r];
    for (auto &cur : desc)
        // Edges are reversed internally
        edges.push_back(
            Edge(&graph[cur.to], &graph[cur.from], cur.w));
    for (int i = 0; i < desc.size(); ++i)
        graph[desc[i].to].con.emplace_back(
            desc[i].w, &edges[i]);
    for (int i = 0; i < n; ++i)
        make_heap(
            graph[i].con.begin(), graph[i].con.end(), comp);
}
bool find() {
    root = croot;
    no_dmst = false;
    for (auto &cur : graph) {
        cur.con_to_root();
        to_proc.clear();
        if (no_dmst) return false;
    }
    return true;
}
ll weight() {
    ll res = 0;
    for (auto &cur : edges) {
        if (cur.inc) res += cur.w;
    }
    return res;
}
};
void DMST::Circle::clean(int idx) {
    if (!vis) {
        vis = true;
        for (int i = 0; i < cont.size(); ++i) {
            if (i != idx) {
                cont[i]->inc = true;
                cont[i]->from->clean();
            }
        }
    }
}
const greater<pair<ll, DMST::Edge *> > DMST::comp;
vector<DMST::Circle> DMST::to_proc;
bool DMST::no_dmst;
DMST::Node *DMST::root;
Bridges 0(n)
struct vert;
struct edge {

```

```

    bool exists = true;
    vert *dest;
    edge *rev;
    edge(vert *_dest) : dest(_dest) { rev = NULL; }
    vert &operator*() { return *dest; }
    vert *operator->() { return dest; }
    bool is_bridge();
};
struct vert {
    deque<edge> con;
    int val = 0;
    int seen;
    int dfs(int upd, edge *ban) { // handles multiple edges
        if (!val) {
            val = upd;
            seen = val;
            for (edge &nxt : con) {
                if (nxt.exists && (&nxt) != ban)
                    seen = min(seen, nxt->dfs(upd + 1, nxt.rev));
            }
            return seen;
        }
    }
    void remove_adj_bridges() {
        for (edge &nxt : con) {
            if (nxt.is_bridge()) nxt.exists = false;
        }
    }
    int cnt_adj_bridges() {
        int res = 0;
        for (edge &nxt : con) res += nxt.is_bridge();
        return res;
    }
};
bool edge::is_bridge() {
    return exists && (dest->seen > rev->dest->val ||
        dest->val < rev->dest->seen);
}
vert graph[nmax];
int main() { // Mechanics Practice BRIDGES
    int n, m;
    cin >> n >> m;
    for (int i = 0; i < m; ++i) {
        int u, v;
        scanf("%d %d", &u, &v);
        graph[u].con.emplace_back(graph + v);
        graph[v].con.emplace_back(graph + u);
        graph[u].con.back().rev = &graph[v].con.back();
        graph[v].con.back().rev = &graph[u].con.back();
    }
    graph[1].dfs(1, NULL);
    int res = 0;
    for (int i = 1; i <= n; ++i)
        res += graph[i].cnt_adj_bridges();
    cout << res / 2 << endl;
}

```

2-Sat 0(n) and SCC 0(n)

```

struct Graph {
    int n;
    vector<vector<int> > con;
    Graph(int nsize) {
        n = nsize;
        con.resize(n);
    }
    void add_edge(int u, int v) { con[u].push_back(v); }
    void top_dfs(int pos, vector<int> &result,
        vector<bool> &explr, vector<vector<int> > &revcon) {
        if (explr[pos]) return;
        explr[pos] = true;
        for (auto next : revcon[pos])
            top_dfs(next, result, explr, revcon);
        result.push_back(pos);
    }
    vector<int> topsort() {
        vector<vector<int> > revcon(n);
        ran(u, 0, n) {
            for (auto v : con[u]) revcon[v].push_back(u);
        }
        vector<int> result;
        vector<bool> explr(n, false);
        ran(i, 0, n) top_dfs(i, result, explr, revcon);
        reverse(result.begin(), result.end());
        return result;
    }
    void dfs(
        int pos, vector<int> &result, vector<bool> &explr) {
        if (explr[pos]) return;
        explr[pos] = true;
        for (auto next : con[pos]) dfs(next, result, explr);
        result.push_back(pos);
    }
    vector<vector<int> > scc() {
        vector<int> order = topsort();
        reverse(order.begin(), order.end());
        vector<bool> explr(n, false);
        vector<vector<int> > res;
        for (auto it = order.rbegin(); it != order.rend();
            ++it) {
            vector<int> comp;
            top_dfs(*it, comp, explr, con);
            sort(comp.begin(), comp.end());
            res.push_back(comp);
        }
        sort(res.begin(), res.end());
        return res;
    }
};
int main() {
    int n, m;
    cin >> n >> m;
    Graph g(2 * m);
    ran(i, 0, n) {
        int a, sa, b, sb;
        cin >> a >> sa >> b >> sb;
    }
}

```

```

a--, b--;
g.add_edge(2 * a + 1 - sa, 2 * b + sb);
g.add_edge(2 * b + 1 - sb, 2 * a + sa);
}
vector<int> state(2 * m, 0);
{
    vector<int> order = g.topsort();
    vector<bool> explr(2 * m, false);
    for (auto u : order) {
        vector<int> traversed;
        g.dfs(u, traversed, explr);
        if (traversed.size() > 0 &&
            !state[traversed[0] ^ 1]) {
            for (auto c : traversed) state[c] = 1;
        }
    }
}
ran(i, 0, m) {
    if (state[2 * i] == state[2 * i + 1]) {
        cout << "IMPOSSIBLE\n";
        return 0;
    }
}
ran(i, 0, m) cout << state[2 * i + 1] << '\n';
return 0;
}

Generic persistent compressed lazy segment tree
struct Seg {
    ll sum = 0;
    void recalc(const Seg &lhs_seg, int lhs_len,
               const Seg &rhs_seg, int rhs_len) {
        sum = lhs_seg.sum + rhs_seg.sum;
    }
} __attribute__((packed));
struct Lazy {
    ll add;
    ll assign_val; // LLONG_MIN if no assign;
    void init() {
        add = 0;
        assign_val = LLONG_MIN;
    }
    Lazy() { init(); }
    void split(Lazy &lhs_lazy, Lazy &rhs_lazy, int len) {
        lhs_lazy = *this;
        rhs_lazy = *this;
        init();
    }
    void merge(Lazy &oth, int len) {
        if (oth.assign_val != LLONG_MIN) {
            add = 0;
            assign_val = oth.assign_val;
        }
        add += oth.add;
    }
    void apply_to_seg(Seg &cur, int len) const {
        if (assign_val != LLONG_MIN) {
            cur.sum = len * assign_val;
        }
        cur.sum += len * add;
    }
} __attribute__((packed));
struct Node { // Following code should not need to be
              // modified
    int ver;
    bool is_lazy = false;
    Seg seg;
    Lazy lazy;
    Node *lc = NULL, *rc = NULL;
    void init() {
        if (!lc) {
            lc = new Node(ver);
            rc = new Node(ver);
        }
    }
    Node *upd(
        int L, int R, int l, int r, Lazy &val, int tar_ver) {
        if (ver != tar_ver) {
            Node *rep = new Node(*this);
            rep->ver = tar_ver;
            return rep->upd(L, R, l, r, val, tar_ver);
        }
        if (L >= l && R <= r) {
            val.apply_to_seg(seg, R - L);
            lazy.merge(val, R - L);
            is_lazy = true;
        } else {
            init();
            int M = (L + R) / 2;
            if (is_lazy) {
                Lazy l_val, r_val;
                lazy.split(l_val, r_val, R - L);
                lc = lc->upd(L, M, l, M, l_val, ver);
                rc = rc->upd(M, R, M, R, r_val, ver);
                is_lazy = false;
            }
            Lazy l_val, r_val;
            val.split(l_val, r_val, R - L);
            if (l < M) lc = lc->upd(L, M, l, r, l_val, ver);
            if (M < r) rc = rc->upd(M, R, l, r, r_val, ver);
            seg.recalc(lc->seg, M - L, rc->seg, R - M);
        }
        return this;
    }
    void get(int L, int R, int l, int r, Seg *lft_res,
             Seg *rht_res, bool last_ver) {
        if (L >= l && R <= r) {
            *lft_res = seg;
            *rht_res = seg;
            return;
        }
        if (is_lazy) {
            init();
            int M = (L + R) / 2;
            if (is_lazy) {
                Lazy l_val, r_val;
                lazy.split(l_val, r_val, R - L);
                lc = lc->upd(L, M, l, M, l_val, ver);
                rc = rc->upd(M, R, M, R, r_val, ver);
                is_lazy = false;
            }
            Lazy l_val, r_val;
            val.split(l_val, r_val, R - L);
            if (l < M) lc = lc->upd(L, M, l, r, l_val, ver);
            if (M < r) rc = rc->upd(M, R, l, r, r_val, ver);
            seg.recalc(lc->seg, M - L, rc->seg, R - M);
        }
        return this;
    }
}

lc = lc->upd(L, M, l, M, l_val, ver + last_ver);
lc->ver = ver;
rc = rc->upd(M, R, M, R, r_val, ver + last_ver);
rc->ver = ver;
is_lazy = false;
}
if (l < M)
    lc->get(L, M, l, r, lft_res, tmp, last_ver);
if (M < r)
    rc->get(M, R, l, r, lft_res, tmp, last_ver);
}
} __attribute__((packed));
struct SegTree { // indexes start from 0, ranges are
                 // [beg, end)
    vector<Node *> roots; // versions start from 0
    int len;
    SegTree(int len) : len(len) {
        roots.push_back(new Node{0});
    }
    int upd(
        int l, int r, Lazy &val, bool new_ver = false) {
        Node *cur_root = roots.back()->upd(
            0, len, l, r, val, roots.size() - !new_ver);
        if (cur_root != roots.back())
            roots.push_back(cur_root);
        return roots.size() - 1;
    }
    Seg get(int l, int r, int ver = -1) {
        if (ver == -1) ver = roots.size() - 1;
        Seg seg1, seg2;
        Seg *pres = &seg1, *ptmp = &seg2;
        roots[ver]->get(
            0, len, l, r, pres, ptmp, roots.size() - 1);
        return *pres;
    }
};

int main() {
    int n, m; // solves Mechanics Practice LAZY
    cin >> n >> m;
    SegTree seg_tree(1 << 17);
    for (int i = 0; i < n; ++i) {
        Lazy tmp;
        scanf("%lld", &tmp.assign_val);
        seg_tree.upd(i, i + 1, tmp);
    }
    for (int i = 0; i < m; ++i) {
        int o;
        int l, r;
        scanf("%d %d %d", &o, &l, &r);
        --l;
        if (o == 1) {
            Lazy tmp;
            scanf("%lld", &tmp.add);
            seg_tree.upd(l, r, tmp);
        } else if (o == 2) {
            Lazy tmp;

```



```

scanf("%lld", &tmp.assign_val);
seg_tree.upd(1, r, tmp);
} else {
    Seg res = seg_tree.get(1, r);
    printf("%lld\n", res.sum);
}
}

Templated HLD O(M(n) log n) per query
class dummy {
public:
    dummy() {}
    dummy(int, int) {}
    void set(int, int) {}
    int query(int left, int right) {
        cout << this << ' ' << left << ' ' << right << endl;
    }
};
/* T should be the type of the data stored in each
 * vertex; DS should be the underlying data structure
 * that is used to perform the group operation. It should
 * have the following methods:
 * * DS () - empty constructor
 * * DS (int size, T initial) - constructs the structure
 * with the given size, initially filled with initial.
 * * void set (int index, T value) - set the value at
 * index 'index' to 'value'
 * * T query (int left, int right) - return the "sum" of
 * elements between left and right, inclusive.
 */
template <typename T, class DS>
class HLD {
    int vertexc;
    vector<int> *adj;
    vector<int> subtree_size;
    DS structure;
    DS aux;
    void build_sizes(int vertex, int parent) {
        subtree_size[vertex] = 1;
        for (int child : adj[vertex]) {
            if (child != parent) {
                build_sizes(child, vertex);
                subtree_size[vertex] += subtree_size[child];
            }
        }
    }
    int cur;
    vector<int> ord;
    vector<int> chain_root;
    vector<int> par;
    void build_hld(
        int vertex, int parent, int chain_source) {
        cur++;
        ord[vertex] = cur;
        chain_root[vertex] = chain_source;
        par[vertex] = parent;
        if (adj[vertex].size() > 1 ||
            (vertex == 1 && adj[vertex].size() == 1)) {
            int big_child, big_size = -1;
            for (int child : adj[vertex]) {
                if ((child != parent) &&
                    (subtree_size[child] > big_size)) {
                    big_child = child;
                    big_size = subtree_size[child];
                }
            }
            build_hld(big_child, vertex, chain_source);
            for (int child : adj[vertex]) {
                if ((child != parent) && (child != big_child))
                    build_hld(child, vertex, child);
            }
        }
    }
public:
    HLD(int _vertexc) {
        vertexc = _vertexc;
        adj = new vector<int>[vertexc + 5];
    }
    void add_edge(int u, int v) {
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    void build(T initial) {
        subtree_size = vector<int>(vertexc + 5);
        ord = vector<int>(vertexc + 5);
        chain_root = vector<int>(vertexc + 5);
        par = vector<int>(vertexc + 5);
        cur = 0;
        build_sizes(1, -1);
        build_hld(1, -1, 1);
        structure = DS(vertexc + 5, initial);
        aux = DS(50, initial);
    }
    void set(int vertex, int value) {
        structure.set(ord[vertex], value);
    }
    T query_path(int u,
        int v) { /* returns the "sum" of the path u->v */
        int cur_id = 0;
        while (chain_root[u] != chain_root[v]) {
            if (ord[u] > ord[v]) {
                cur_id++;
                aux.set(cur_id,
                    structure.query(ord[chain_root[u]], ord[u]));
                u = par[chain_root[u]];
            } else {
                cur_id++;
                aux.set(cur_id,
                    structure.query(ord[chain_root[v]], ord[v]));
                v = par[chain_root[v]];
            }
        }
        cur_id++;
        aux.set(cur_id, structure.query(min(ord[u], ord[v]),
            max(ord[u], ord[v])));
        return aux.query(1, cur_id);
    }
    void print() {
        for (int i = 1; i <= vertexc; i++)
            cout << i << ' ' << ord[i] << ' ' << chain_root[i]
                << ' ' << par[i] << endl;
    }
};

int main() {
    int vertexc;
    cin >> vertexc;
    HLD<int, dummy> hld(vertexc);
    for (int i = 0; i < vertexc - 1; i++) {
        int u, v;
        cin >> u >> v;
        hld.add_edge(u, v);
    }
    hld.build(0);
    hld.print();
    int queryc;
    cin >> queryc;
    for (int i = 0; i < queryc; i++) {
        int u, v;
        cin >> u >> v;
        hld.query_path(u, v);
        cout << endl;
    }
}

Splay Tree + Link-Cut O(NlogN)
struct Tree *treev;
struct Tree {
    struct T {
        int i;
        constexpr T() : i(-1) {}
        T(int _i) : i(_i) {}
        operator int() const { return i; }
        explicit operator bool() const { return i != -1; }
        Tree *operator->() { return treev + i; }
    };
    T c[2], p;
    /* insert monoid here */
    T link;
    Tree() {
        /* init monoid here */
        link = -1;
    }
};
using T = Tree::T;
constexpr T NIL;
void update(T t) { /* recalculate the monoid here */
}
void propagate(T t) {
    assert(t);
    for (T c : t->c)
        if (c) c->link = t->link;
    /* lazily propagate updates here */
}

```

```

}
void lazy_reverse(T t) { /* lazily reverse t here */
}
T splay(T n) {
    for (;;) {
        propagate(n);
        T p = n->p;
        if (p == NIL) break;
        propagate(p);
        ll px = p->c[1] == n;
        assert(p->c[px] == n);
        T g = p->p;
        if (g == NIL) { /* zig */
            p->c[px] = n->c[px ^ 1];
            p->c[px]->p = p;
            n->c[px ^ 1] = p;
            n->c[px ^ 1]->p = n;
            n->p = NIL;
            update(p);
            update(n);
            break;
        }
        propagate(g);
        ll gx = g->c[1] == p;
        assert(g->c[gx] == p);
        T gg = g->p;
        ll ggx = gg && gg->c[1] == g;
        if (gg) assert(gg->c[ggx] == g);
        if (gx == px) { /* zig zig */
            g->c[gx] = p->c[gx ^ 1];
            g->c[gx]->p = g;
            p->c[gx ^ 1] = g;
            p->c[gx ^ 1]->p = p;
            p->c[gx] = n->c[gx ^ 1];
            p->c[gx]->p = p;
            n->c[gx ^ 1] = p;
            n->c[gx ^ 1]->p = n;
        } else { /* zig zag */
            g->c[gx] = n->c[gx ^ 1];
            g->c[gx]->p = g;
            n->c[gx ^ 1] = g;
            n->c[gx ^ 1]->p = n;
            p->c[gx ^ 1] = n->c[gx];
            p->c[gx ^ 1]->p = p;
            n->c[gx] = p;
            n->c[gx]->p = n;
        }
        if (gg) gg->c[ggx] = n;
        n->p = gg;
        update(g);
        update(p);
        update(n);
        if (gg) update(gg);
    }
    return n;
}
T extreme(T t, int x) {
    while (t->c[x]) t = t->c[x];
    return t;
}
T set_child(T t, int x, T a) {
    T o = t->c[x];
    t->c[x] = a;
    update(t);
    o->p = NIL;
    a->p = t;
    return o;
}
/***** Link-Cut Tree: *****/
T expose(T t) {
    set_child(splay(t), 1, NIL);
    T leader = splay(extreme(t, 0));
    if (leader->link == NIL) return t;
    set_child(splay(leader), 0, expose(leader->link));
    return splay(t);
}
void link(T t, T p) {
    assert(t->link == NIL);
    t->link = p;
}
T cut(T t) {
    T p = t->link;
    if (p) expose(p);
    t->link = NIL;
    return p;
}
void make_root(T t) {
    expose(t);
    lazy_reverse(extreme(splay(t), 0));
}
Templated multi dimensional BIT  $O(\log(n)^d)$  per query
// Fully overloaded any dimensional BIT, use any type for coordinates, elements, return_value. Includes coordinate compression.
template <class E_T, class C_T, C_T n_inf, class R_T>
struct BIT {
    vector<C_T> pos;
    vector<E_T> elems;
    bool act = false;
    BIT() { pos.push_back(n_inf); }
    void init() {
        if (act) {
            for (E_T &c_elem : elems) c_elem.init();
        } else {
            act = true;
            sort(pos.begin(), pos.end());
            pos.resize(
                unique(pos.begin(), pos.end()) - pos.begin());
            elems.resize(pos.size());
        }
    }
};
template <typename... loc_form>
void update(C_T cx, loc_form... args) {
    if (act) {
        int x = lower_bound(pos.begin(), pos.end(), cx) - pos.begin();
        for (; x < (int)pos.size(); x += x & -x)
            elems[x].update(args...);
        else {
            pos.push_back(cx);
        }
    }
}
template <typename... loc_form>
R_T query(
    C_T cx, loc_form... args) { // sum in (-inf, cx)
    R_T res = 0;
    int x = lower_bound(pos.begin(), pos.end(), cx) - pos.begin() - 1;
    for (; x > 0; x -= x & -x)
        res += elems[x].query(args...);
    return res;
}
};
template <typename I_T>
struct wrapped {
    I_T a = 0;
    void update(I_T b) { a += b; }
    I_T query() { return a; }
    // Should never be called, needed for compilation
    void init() { DEBUG('i') }
    void update() { DEBUG('u') }
};
int main() {
    // return type should be same as type inside wrapped
    BIT<BIT<wrapped<ll>, int, INT_MIN, ll>, int, INT_MIN, ll>
        fenwick;
    int dim = 2;
    vector<tuple<int, int, ll>> to_insert;
    to_insert.emplace_back(1, 1, 1);
    // set up all pos that are to be used for update
    for (int i = 0; i < dim; ++i) {
        for (auto &cur : to_insert)
            fenwick.update(get<0>(cur), get<1>(cur));
        // May include value which won't be used
        fenwick.init();
    }
    // actual use
    for (auto &cur : to_insert)
        fenwick.update(
            get<0>(cur), get<1>(cur), get<2>(cur));
    cout << fenwick.query(2, 2) << '\n';
}
Treap  $O(\log(n))$  per query
mt19937 randgen;
struct Treap {
    struct Node {
        int key;
        int value;
        unsigned int priority;
    };
};

```



```

11 total;
Node* lch;
Node* rch;
Node(int new_key, int new_value) {
    key = new_key;
    value = new_value;
    priority = randgen();
    total = new_value;
    lch = 0;
    rch = 0;
}
void update() {
    total = value;
    if (lch) total += lch->total;
    if (rch) total += rch->total;
}
};
deque<Node> nodes;
Node* root = 0;
pair<Node*, Node*> split(int key, Node* cur) {
    if (cur == 0) return {0, 0};
    pair<Node*, Node*> result;
    if (key <= cur->key) {
        auto ret = split(key, cur->lch);
        cur->lch = ret.second;
        result = {ret.first, cur};
    } else {
        auto ret = split(key, cur->rch);
        cur->rch = ret.first;
        result = {cur, ret.second};
    }
    cur->update();
    return result;
}
Node* merge(Node* left, Node* right) {
    if (left == 0) return right;
    if (right == 0) return left;
    Node* top;
    if (left->priority < right->priority) {
        left->rch = merge(left->rch, right);
        top = left;
    } else {
        right->lch = merge(left, right->lch);
        top = right;
    }
    top->update();
    return top;
}
void insert(int key, int value) {
    nodes.push_back(Node(key, value));
    Node* cur = &nodes.back();
    pair<Node*, Node*> ret = split(key, root);
    cur = merge(ret.first, cur);
    cur = merge(cur, ret.second);
    root = cur;
}
void erase(int key) {
    Node *left, *mid, *right;
    tie(left, mid) = split(key, root);
    tie(mid, right) = split(key + 1, mid);
    root = merge(left, right);
}
11 sum_upto(int key, Node* cur) {
    if (cur == 0) return 0;
    if (key <= cur->key) {
        return sum_upto(key, cur->lch);
    } else {
        11 result = cur->value + sum_upto(key, cur->rch);
        if (cur->lch) result += cur->lch->total;
        return result;
    }
}
11 get(int l, int r) {
    return sum_upto(r + 1, root) - sum_upto(l, root);
}
};
int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(0);
    int m;
    Treap treap;
    cin >> m;
    for (int i = 0; i < m; i++) {
        int type;
        cin >> type;
        if (type == 1) {
            int x, y;
            cin >> x >> y;
            treap.insert(x, y);
        } else if (type == 2) {
            int x;
            cin >> x;
            treap.erase(x);
        } else {
            int l, r;
            cin >> l >> r;
            cout << treap.get(l, r) << endl;
        }
    }
    return 0;
}
Radixsort 50M 64 bit integers as single array in 1 sec
template <typename T>
void rsort(T *a, T *b, int size, int d = sizeof(T) - 1) {
    8465 int b_s[256]{};
    ran(i, 0, size) { ++b_s[(a[i] >> (d * 8)) & 255]; }
    // ++b_s[*((uchar *) (a + i) + d)];
    T *mem[257];
    1599 mem[0] = b;
    T *l_b = mem + 1;
    l_b[0] = b;
    ran(i, 0, 255) { l_b[i + 1] = l_b[i] + b_s[i]; }
    for (T *it = a; it != a + size; ++it) {
        T id = ((*it) >> (d * 8)) & 255;
        *(l_b[id]++) = *it;
    }
    l_b = mem;
    if (d) {
        T *l_a[256];
        1009 l_a[0] = a;
        ran(i, 0, 255) l_a[i + 1] = l_a[i] + b_s[i];
        ran(i, 0, 256) {
            if (l_b[i + 1] - l_b[i] < 100) {
                sort(l_b[i], l_b[i + 1]);
                if (d & 1) copy(l_b[i], l_b[i + 1], l_a[i]);
            } else {
                rsort(l_b[i], l_a[i], b_s[i], d - 1);
            }
        }
    }
}
const int nmax = 5e7;
11 arr[nmax], tmp[nmax];
int main() {
    for (int i = 0; i < nmax; ++i)
        arr[i] = ((11)rand() << 32) | rand();
    rsort(arr, tmp, nmax);
    assert(is_sorted(arr, arr + nmax));
}
FFT 10-15M length/sec
// integer c = a*b is accurate if c_i < 2^49
#pragma GCC optimize ("Ofast") //10% performance
#include <complex.h>
extern "C" __complex__ double __muldc3(
    double a, double b, double c, double d){
    return a*c-b*d+I*(a*d+b*c); // 40% performance
}
#include <bits/stdc++.h>
typedef complex<double> Comp;
void fft_rec(Comp *arr, Comp *root_pow, int len) {
    if (len != 1) {
        6092 fft_rec(arr, root_pow, len >> 1);
        fft_rec(arr + len, root_pow, len >> 1);
    }
    root_pow += len;
    ran(i, 0, len){
        tie(arr[i], arr[i + len]) = pair<Comp, Comp> {
            arr[i] + root_pow[i] * arr[i + len],
            arr[i] - root_pow[i] * arr[i + len] };
    }
}
void fft(vector<Comp> &arr, int ord, bool invert) {
    assert(arr.size() == 1 << ord);
    static vector<Comp> root_pow(1);
    static int inc_pow = 1;
    static bool is_inv = false;
    if (inc_pow <= ord) {
        int idx = root_pow.size();
        root_pow.resize(1 << ord);
        0927 for (; inc_pow <= ord; ++inc_pow) {

```

```

    for (int idx_p = 0; idx_p < 1 << (ord - 1);
        idx_p += 1 << (ord - inc_pow), ++idx) {
        root_pow[idx] = Comp {
            cos(-idx_p * M_PI / (1 << (ord - 1))),
            sin(-idx_p * M_PI / (1 << (ord - 1))) };
        if (is_inv) root_pow[idx] = conj(root_pow[idx]);
    }
}
if (invert != is_inv) {
    is_inv = invert;
    for (Comp &cur : root_pow) cur = conj(cur);
}
int j = 0;
ran(i, 1, (1<<ord)){
    int m = 1 << (ord - 1);
    bool cont = true;
    while (cont) {
        cont = j & m;
        j ^= m;
        m >>= 1;
    }
    if (i < j) swap(arr[i], arr[j]);
}
fft_rec(arr.data(), root_pow.data(), 1 << (ord - 1));
if (invert)
    ran(i, 0, 1<<ord) arr[i] /= (1 << ord);
}
void mult_poly_mod(vector<int> &a, vector<int> &b,
    vector<int> &c) { // c += a*b
    static vector<Comp> arr[4];
    // correct upto 0.5-2M elements(mod ~ 1e9)
    if (c.size() < 400) {
        ran(i, 0, (int)a.size())
            ran(j, 0, min((int)b.size(), (int)c.size()-i))
            c[i + j] = ((ll)a[i] * b[j] + c[i + j]) % mod;
    } else {
        int ord = 32 - __builtin_clz((int)c.size()-1);
        if ((int)arr[0].size() != 1 << ord){
            ran(i, 0, 4) arr[i].resize(1 << ord);
        }
        ran(i, 0, 4)
            fill(arr[i].begin(), arr[i].end(), Comp{});
        for (int &cur : a) if (cur < 0) cur += mod;
        for (int &cur : b) if (cur < 0) cur += mod;
        const int shift = 15;
        const int mask = (1 << shift) - 1;
        ran(i, 0, (int)min(a.size(), c.size())){
            arr[0][i] += a[i] & mask;
            arr[1][i] += a[i] >> shift;
        }
        ran(i, 0, (int)min(b.size(), c.size())){
            arr[0][i] += Comp{0, (b[i] & mask)};
            arr[1][i] += Comp{0, (b[i] >> shift)};
        }
        ran(i, 0, 2) fft(arr[i], ord, false);
        ran(i, 0, 2){
            ran(j, 0, 2){
                int tar = 2 + (i + j) / 2;
                Comp mult = {0, -0.25};
                if (i ^ j) mult = {0.25, 0};
                ran(k, 0, 1<<ord){
                    int rev_k = ((1 << ord) - k) % (1 << ord);
                    Comp ca = arr[i][k] + conj(arr[i][rev_k]);
                    Comp cb = arr[j][k] - conj(arr[j][rev_k]);
                    arr[tar][k] = arr[tar][k] + mult * ca * cb;
                }
            }
        }
        ran(i, 2, 4){
            fft(arr[i], ord, true);
            ran(k, 0, (int)c.size()){
                c[k] = (c[k] + (((ll)(arr[i][k].real()+0.5)%mod)
                    << (shift * (2 * (i-2) + 0)))) % mod;
                c[k] = (c[k] + (((ll)(arr[i][k].imag()+0.5)%mod)
                    << (shift * (2 * (i-2) + 1)))) % mod;
            }
        }
    }
}
Fast mod mult, Rabbin Miller prime check, Pollard
rho factorization O(p^0.5)
struct ModArithm {
    ull n;
    ld rec;
    ModArithm(ull _n) : n(_n) { // n in [2, 1<<63]
        rec = 1.0L / n;
    }
    // a, b in [0, min(2*n, 1<<63))
    ull multf(ull a, ull b) {
        ull mult = (ld)a * b * rec + 0.5L;
        ll res = a * b - mult * n;
        if (res < 0) res += n;
        return res; // in [0, n-1]
    }
    ull sqp1(ull a) { return multf(a, a) + 1; }
};
ull pow_mod(ull a, ull n, ModArithm &arithm) {
    ull res = 1;
    for (ull i = 1; i <= n; i <<= 1) {
        if (n & i) res = arithm.multf(res, a);
        a = arithm.multf(a, a);
    }
    return res;
}
vector<char> small_primes = {
    2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
bool is_prime(ull n) { // n <= 1<<63, 1M rand/s
    ModArithm arithm(n);
    if (n == 2 || n == 3) return true;
    if (!(n & 1) || n == 1) return false;
    int s = __builtin_ctzll(n - 1);
    ull d = (n - 1) >> s;
    for (ull a : small_primes) {
        if (a >= n) break;
        a = pow_mod(a, d, arithm);
        if (a == 1 || a == n - 1) continue;
        ran(r, 1, s) {
            a = arithm.multf(a, a);
            if (a == 1) return false;
            if (a == n - 1) break;
        }
        if (a != n - 1) return false;
    }
    return true;
}
ll pollard_rho(ll n) {
    ModArithm arithm(n);
    int cum_cnt = 64 - __builtin_clzll(n);
    cum_cnt *= cum_cnt / 5 + 1;
    while (true) {
        ll lv = rand() % n;
        ll v = arithm.sqp1(lv);
        int idx = 1;
        int tar = 1;
        while (true) {
            ll cur = 1;
            ll v_cur = v;
            int j_stop = min(cum_cnt, tar - idx);
            for (int j = 0; j < j_stop; ++j) {
                cur = arithm.multf(cur, abs(v_cur - lv));
                v_cur = arithm.sqp1(v_cur);
                ++idx;
            }
            if (!cur) {
                for (int j = 0; j < cum_cnt; ++j) {
                    ll g = __gcd(abs(v - lv), n);
                    if (g == 1) {
                        v = arithm.sqp1(v);
                    } else if (g == n) {
                        break;
                    } else {
                        return g;
                    }
                }
                break;
            } else {
                ll g = __gcd(cur, n);
                if (g != 1) return g;
            }
            v = v_cur;
            idx += j_stop;
            if (idx == tar) {
                lv = v;
                tar *= 2;
                v = arithm.sqp1(v);
                ++idx;
            }
        }
    }
}

```

%4128

```

map<ll, int> prime_factor(
    ll n, map<ll, int> *res = NULL) {
    // n <= 1<<62, ~1000/s (<500/s on CF)
    if (!res) {
        map<ll, int> res_act;
        for (int p : small_primes) {
            while (!(n % p)) {
                ++res_act[p];
                n /= p;
            }
        }
        if (n != 1) prime_factor(n, &res_act);
        return res_act;
    }
    if (is_prime(n)) {
        ++(*res)[n];
    } else {
        ll factor = pollard_rho(n);
        prime_factor(factor, res);
        prime_factor(n / factor, res);
    }
    return map<ll, int>();
} // Usage: fact = prime_factor(n);
    Berlekamp-Massey O(LN)
template <typename T, T P>
struct intmod {
    intmod() {}
    constexpr intmod(T t) : x((t + P) % P) {}
    T value() const { return x; }
    bool operator!=(const intmod<T, P> i) { return x != i.x; }
    bool operator==(const intmod<T, P> i) { return x == i.x; }
    intmod<T, P> &operator+=(const intmod<T, P> i) {
        x = (x + i.x) % P;
        return *this;
    }
    intmod<T, P> &operator-=(const intmod<T, P> i) {
        x = (x + P - i.x) % P;
        return *this;
    }
    intmod<T, P> &operator*=(const intmod<T, P> i) {
        x = ((long long)x * i.x) % P;
        return *this;
    }
    intmod<T, P> &operator/=(const intmod<T, P> i) {
        x = ((long long)x * i.inverse().x) % P;
        return *this;
    }
}

intmod<T, P> operator+(const intmod<T, P> i) const {
    auto j = *this;
    return j += i;
}
intmod<T, P> operator-(const intmod<T, P> i) const {
    auto j = *this;
    return j -= i;
}
intmod<T, P> operator*(const intmod<T, P> i) const {
    auto j = *this;
    return j *= i;
}
intmod<T, P> operator/(const intmod<T, P> i) const {
    auto j = *this;
    return j /= i;
}
intmod<T, P> operator-() const {
    intmod<T, P> n;
    n.x = (P - x) % P;
    return n;
}
intmod<T, P> inverse() const {
    if (x == 0) return 0;
    T a = x, b = P;
    T aa = 1, ab = 0;
    T ba = 0, bb = 1;
    while (a) {
        T q = b / a;
        T r = b % a;
        ba -= aa * q;
        bb -= ab * q;
        swap(ba, aa);
        swap(bb, ab);
        b = a;
        a = r;
    }
    intmod<T, P> ix = intmod<T, P>(aa) + intmod<T, P>(ba);
    assert(ix * x == unity);
    return ix;
}
static const intmod<T, P> zero;
static const intmod<T, P> unity;
private:
    T x;
};
template <typename T, T P>
constexpr intmod<T, P> intmod<T, P>::zero = 0;
    template <typename T, T P>
    constexpr intmod<T, P> intmod<T, P>::unity = 1;
    using rem = intmod<char, 2>;
    template <typename K>
    static vector<K> berlekamp_massey(vector<K> ss) {
        vector<K> ts(ss.size());
        vector<K> cs(ss.size());
        cs[0] = K::unity;
        fill(cs.begin() + 1, cs.end(), K::zero);
        vector<K> bs = cs;
        int l = 0, m = 1;
        K b = K::unity;
        for (int k = 0; k < (int)ss.size(); k++) {
            K d = ss[k];
            assert(l <= k);
            for (int i = 1; i <= l; i++) d += cs[i] * ss[k - i];
            if (d == K::zero) {
                m++;
            } else if (2 * l <= k) {
                K w = d / b;
                ts = cs;
                for (int i = 0; i < (int)cs.size() - m; i++)
                    cs[i + m] -= w * bs[i];
                l = k + 1 - l;
                swap(bs, ts);
                b = d;
                m = 1;
            } else {
                K w = d / b;
                for (int i = 0; i < (int)cs.size() - m; i++)
                    cs[i + m] -= w * bs[i];
                m++;
            }
        }
        cs.resize(l + 1);
        while (cs.back() == K::zero) cs.pop_back();
        return cs;
    }

```

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