University of Tartu ICPC Team Notebook (2018-2019)

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Maxflow Complexity

$\mathcal{O}(V^2E)$ – Dinic

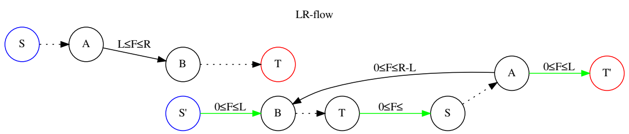
$\Theta(VE\log U)$ – Capacity scaling

$\Theta(\mathbf{flow}E)$ – Small flow

$\Theta(\min\{V^{\frac{2}{3}}, E^{\frac{1}{2}}\}E)$ – Unitary capacities

$\Theta(\sqrt{V}E)$ – Each vertex other than S,T has only a single incoming unitary edge or outgoing one (bipartite matching)

$\Theta(\mathbf{flow}E\log V)$ – Min-cost-max flow



Min Rotation of string

```
int a=0, N=s.size();
s += s;
ran(b,0,N){
  ran(i,0,N) {
    if (a+i == b || s[a+i] < s[b+i]) {
      b += max(0, i-1);
      break;
    }
    if (s[a+i] > s[b+i]) {
      a = b;
      break;
    }
  }
}
return a;
```

Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty)$$

Symmetric Submodular Functions; Queyrannes's algorithm

SSF: such function $f : V \to R$ that satisfies $f(A) = f(V/A)$ and for all $x \in V, X \subseteq Y \subseteq V$ it holds that $f(X+x) - f(X) \leq f(Y+x) - f(Y)$.
Hereditary family: such set $I \subseteq 2^V$ so that $X \subset Y \wedge Y \in I \Rightarrow X \in I$.
Loop: such $v \in V$ so that $v \notin I$.

```
def minimize():
  s = merge_all_loops()
  while size >= 3:
    t, u = find_pp()
    {u} is a possible minimizer
    tu = merge(t, u)
    if tu not in I:
      s = merge(tu, s)
  for x in V:
    {x} is a possible minimizer
def find_pp():
  W = {s} # s as in minimizer()
  todo = V/W
  ord = []
  while len(todo) > 0:
    x = min(todo, key=lambda x: f(W+{x}) -
    ↪  f({x}))
    W += {x}
    todo -= {x}
    ord.append(x)
  return ord[-1], ord[-2]

def enum_all_minimal_minimizers(X):
  # X is a inclusionwise minimal minimizer
  s = merge(s, X)
  yield X
  for {v} in I:
    if f({v}) == f(X):
      yield X
      s = merge(v, s)
  while size(V) >= 3:
    t, u = find_pp()
    tu = merge(t, u)
    if tu not in I:
      s = merge(tu, s)
    elif f({tu}) = f(X):
      yield tu
      s = merge(tu, s)
```

Subset convolution $\mathcal{O}(N \log N)$

$c[z] = \sum_{z=x\oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.

```
void FST(vi& a, bool inv) { //size(a) == 2^x
  for (int n =(int)a.size(), step = 1; step <
  ↪  n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step)
      ran(j, i, i + step) {
        int &u = a[j], &v = a[j + step];
        tie(u, v) =
          inv ? pii(v - u, u) : pii(v, u + v);
          ↪  // AND
          inv ? pii(v, u - v) : pii(u + v, u);
          ↪  // OR
          pii(u + v, u - v);
          ↪  // XOR
      }
  }
  if (inv) trav(x, a) x /= a.size(); // XOR
  ↪  only
}
vi conv(vi a, vi b) {
  FST(a, 0);
  FST(b, 0);
```

```
  ran(i, 0, (int)a.size()) a[i] *= b[i];
  FST(a, 1);
  return a;
}
```

Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2),\ \ b = k \cdot (2mn),\ \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\varphi(\varphi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n} \sum_{k|n} f(k)\varphi(n/k).$$

Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ \ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^n c(n,k)x^k = x(x+1)\dots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$

Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod p$$

Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2} C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

2D geometry

Define $\mathbf{orient}(A,B,C) = \overline{AB} \times \overline{AC}$. CCW iff > 0.
Define $\mathbf{perp}((a,b)) = (-b,a)$. The vectors are orthogonal.
For line $ax+by=c$ def $\overline{v} = (-b,a)$.
Line through P and Q has $\overline{v} = \overline{PQ}$ and $c = \overline{v} \times P$.
$\mathbf{side}_l(P) = \overline{v_l} \times P - c_l$ sign determines which side P is on from l.
$\mathbf{dist}_l(P) = \mathbf{side}_l(P)/\|v_l\|$ squared is integer.
Sorting points along a line: comparator is $\overline{v} \cdot A < \overline{v} \cdot B$.
Translating line by $\overline{t}$: new line has $c' = c + \overline{v} \times \overline{t}$.
Line intersection: is $(c_l \overline{v}_m - c_m \overline{v}_l)/(\overline{v}_l \times \overline{v}_m)$.
Project P onto l: is $P - \mathbf{perp}(v)\,\mathbf{side}_l(P)/\|v\|^2$.
Angle bisectors: $\overline{v} = \overline{v}_l/\|\overline{v}_l\| + \overline{v}_m/\|\overline{v}_m\|$
$c = c_l/\|\overline{v}_l\| + c_m/\|\overline{v}_m\|$.
P is on segment AB iff $\mathbf{orient}(A,B,P) = 0$ and $\overline{PA} \cdot \overline{PB} \leqslant 0$.
Proper intersection of AB and CD exists iff $\mathbf{orient}(C,D,A)$ and $\mathbf{orient}(C,D,B)$ have opp. signs and $\mathbf{orient}(A,B,C)$ and $\mathbf{orient}(A,B,D)$ have opp. signs. Coordinates:

$$\frac{A\,\mathbf{orient}(C,D,B) - B\,\mathbf{orient}(C,D,A)}{\mathbf{orient}(C,D,B) - \mathbf{orient}(C,D,A)}.$$

Circumcircle center:

```
pt circumCenter(pt a, pt b, pt c) {
  b = b-a, c = c-a; // consider coordinates relative to A
  assert(cross(b,c) != 0); // no circumcircle if A,B,C aligned
  return a + perp(b*sq(c) - c*sq(b))/cross(b,c)/2;
```

Circle-line intersect:

```
int circleLine(pt o, double r, line l, pair<pt, pt> &out) {
  double h2 = r*r - l.sqDist(o);
  if (h2 >= 0) { // the line touches the circle
    pt p = l.proj(o); // point P
    pt h = l.v*sqrt(h2)/abs(l.v); // vector paral to l, of len h
    out = {p-h, p+h};
  }
  return 1 + sgn(h2);
```

Circle-circle intersect:

```
int circleCircle(pt o1, double r1, pt o2, double r2, pair<pt,pt> &out) {
  pt d=o2-o1; double d2=sq(d);
  if (d2 == 0) {assert(r1 != r2); return 0;} // concentric circles
  double pd = (d2 + r1*r1 - r2*r2)/2; // = |O_1P| * d
  double h2 = r1*r1 - pd*pd/d2; // = h^2
  if (h2 >= 0) {
    pt p = o1 + d*pd/d2, h = perp(d)*sqrt(h2/d2);
    out = {p-h, p+h};}
  return 1 + sgn(h2);
```

Tangent lines:

```
int tangents(pt o1, double r1, pt o2, double r2, bool inner, vector<pair<pt,pt>> &out) {
  if (inner) r2 = -r2;
  pt d = o2-o1;
  double dr = r1-r2, d2 = sq(d), h2 = d2-dr*dr;
  if (d2 == 0 || h2 < 0) {assert(h2 != 0); return 0;}
  for (double sign : {-1,1}) {
    pt v = (d*dr + perp(d)*sqrt(h2)*sign)/d2;
    out.push_back({o1 + v*r1, o2 + v*r2});}
  return 1 + (h2 > 0);
```

3D geometry

$\mathbf{orient}(P,Q,R,S) = (\overline{PQ} \times \overline{PR}) \cdot \overline{PS}$.
S above PQR iff > 0.
For plane $ax+by+cz=d$ def $\overline{n} = (a,b,c)$.
Line with normal $\overline{n}$ through point P has $d = \overline{n} \cdot P$.
$\mathbf{side}_\Pi(P) = \overline{n} \cdot P - d$ sign determines side from Π.
$\mathbf{dist}_\Pi(P) = \mathbf{side}_\Pi(P)/\|\overline{n}\|$.
Translating plane by $\overline{t}$ makes $d' = d + \overline{n} \cdot \overline{t}$.
Plane-plane intersection of has direction $\overline{n}_1 \times \overline{n}_2$ and goes through $((d_1\overline{n}_2 - d_2\overline{n}_1) \times \overline{d})/\|\overline{d}\|^2$.
Line-line distance:

```
double dist(line3d l1, line3d l2) {
  p3 n = l1.d*l2.d;
  if (n == zero) // parallel
    return l1.dist(l2.o);
  return abs((l2.o-l1.o)|n)/abs(n);
```

Spherical to Cartesian:
$(r\cos\varphi\cos\lambda, r\cos\varphi\sin\lambda, r\sin\varphi)$.
Sphere-line intersection:

```
int sphereLine(p3 o, double r, line3d l, pair<p3,p3> &out) {
  double h2 = r*r - l.sqDist(o);
  if (h2 < 0) return 0; // the line doesn't touch the sphere
  p3 p = l.proj(o); // point P
  p3 h = l.d*sqrt(h2)/abs(l.d); // vector parallel to l, of length h
  out = {p-h, p+h};
  return 1 + (h2 > 0);
```

Great-circle distance between points A and B is $r\angle AOB$.
Spherical segment intersection:

```
bool properInter(p3 a, p3 b, p3 c, p3 d, p3 &out) {
  p3 ab = a*b, cd = c*d; // normals of planes OAB and OCD
  int oa = sgn(cd|a),
    ob = sgn(cd|b),
    oc = sgn(ab|c),
    od = sgn(ab|d);
  out = ab*cd*od; // four multiplications => careful with overflow !
  return (oa != ob && oc != od && oa != oc);
}
bool onSphSegment(p3 a, p3 b, p3 p) {
  p3 n = a*b;
  if (n == zero)
    return a*p == zero && (a|p) > 0;
  return (n|p) == 0 && (n|a*p) >= 0 && (n|b*p) <= 0;
}
struct directionSet : vector<p3> {
  using vector::vector; // import constructors
  void insert(p3 p) {
    for (p3 q : *this) if (p*q == zero) return;
    push_back(p);
  }
};
directionSet intersSph(p3 a, p3 b, p3 c, p3 d) {
  assert(validSegment(a, b) && validSegment(c, d));
  p3 out;
  if (properInter(a, b, c, d, out)) return {out};
  directionSet s;
  if (onSphSegment(c, d, a)) s.insert(a);
  if (onSphSegment(c, d, b)) s.insert(b);
  if (onSphSegment(a, b, c)) s.insert(c);
  if (onSphSegment(a, b, d)) s.insert(d);
  return s;
}
```

Angle between spherical segments AB and AC is angle between $A \times B$ and $A \times C$.
Oriented angle: subtract from 2π if mixed product is negative.
Area of a spherical polygon:

$$r^2[\text{sum of interior angles} - (n-2)\pi].$$

Triangle centers

```
const double min_delta = 1e-13;
const double coord_max = 1e6;
typedef complex<double> point;
point A, B, C; // vertixes of the triangle
bool collinear() {                                        0823
  double min_diff =
    min(abs(A - B), min(abs(A - C), abs(B - C)));
  if (min_diff < coord_max * min_delta) return true;
  point sp = (B - A) / (C - A);                           0033
  double ang = M_PI / 2 - abs(abs(arg(sp)) - M_PI / 2);
  return ang < min_delta;
  // positive angle with the real line
}                                                        %3048
point circum_center() {
  if (collinear()) return point(NAN, NAN);
  // squared lengths of sides
  double a2 = norm(B - C);
  double b2 = norm(A - C);
  double c2 = norm(A - B);                                6715
  // barycentric coordinates of the circumcenter
  // sin(2 * alpha) works also
  double c_A = a2 * (b2 + c2 - a2);
  double c_B = b2 * (a2 + c2 - b2);
  double c_C = c2 * (a2 + b2 - c2);
  double sum = c_A + c_B + c_C;
  c_A /= sum;                                             9407
  c_B /= sum;
  c_C /= sum;
  return c_A * A + c_B * B + c_C * C; // cartesian
}                                                        %6958
point centroid() { // center of mass
  return (A + B + C) / 3.0;
}
point ortho_center() { // euler line
  point O = circum_center();
  return O + 3.0 * (centroid() - O);                      6051
};
point nine_point_circle_center() { // euler line
  point O = circum_center();
  return O + 1.5 * (centroid() - O);
};                                                       %8193
point in_center() {
  if (collinear()) return point(NAN, NAN);
  double a = abs(B - C); // side lenghts
  double b = abs(A - C);
  double c = abs(A - B);                                  5954
  // trilinear coordinates are (1,1,1)
  double sum = a + b + c;
  a /= sum;
  b /= sum;
  c /= sum;                    // barycentric
  return a * A + b * B + c * C; // cartesian
```

```
}                                                        %9596
```

Seg-Seg intersection, halfplane intersection area

```
struct Seg {
  Vec a, b;
  Vec d() { return b - a; }
};
Vec intersection(Seg l, Seg r) {
  Vec dl = l.d(), dr = r.d();
  if (cross(dl, dr) == 0) return {nanl(""), nanl("")};
  double h = cross(dr, l.a - r.a) / len(dr);
  double dh = cross(dr, dl) / len(dr);                    3280
  return l.a + dl * (h / -dh);
}
// Returns the area bounded by halfplanes
double calc_area(const vector<Seg>& lines) {
  double lb = -HUGE_VAL, ub = HUGE_VAL;
  vector<Seg> slines[2];                                  9425
  for (auto line : lines) {
    if (line.b.y == line.a.y) {
      if (line.a.x < line.b.x) {
        lb = max(lb, line.a.y);
      } else {
        ub = min(ub, line.a.y);                           6288
      }
    } else if (line.a.y < line.b.y) {
      slines[1].push_back(line);
    } else {
      slines[0].push_back({line.b, line.a});
    }
  }
  ran(i, 0, 2) {                                          4028
    sort(slines[i].begin(), slines[i].end(),
      [&](Seg l, Seg r) {
        if (cross(l.d(), r.d()) == 0)
          return normal(l.d()) * l.a >                    6523
                 normal(r.d()) * r.a;
        return (1 - 2 * i) * cross(l.d(), r.d()) < 0;
      });
  }
  // Now find the application area of the lines and clean
  // up redundant ones
  vector<double> ap_s[2];
  ran(side, 0, 2) {
    vector<double>& apply = ap_s[side];                   5622
    vector<Seg> clines;
    for (auto line : slines[side]) {
      while (clines.size() > 0) {
        Seg other = clines.back();
        if (cross(line.d(), other.d()) != 0) {
          double start = intersection(line, other).y;
          if (start > apply.back()) break;                5287
        }
        clines.pop_back();
        apply.pop_back();
      }
      if (clines.size() == 0) {
        apply.push_back(-HUGE_VAL);
      } else {
        apply.push_back(                                  9835
          intersection(line, clines.back()).y);
      }
      clines.push_back(line);
    }
    slines[side] = clines;
  }
  ap_s[0].push_back(HUGE_VALL);                           8994
  ap_s[1].push_back(HUGE_VALL);
  double result = 0;
  {
    double lb = -HUGE_VALL, ub;
    for (int i = 0, j = 0; i < (int)slines[0].size() &&
                           j < (int)slines[1].size();
         lb = ub) {                                       4531
      ub = min(ap_s[0][i + 1], ap_s[1][j + 1]);
      double alb = lb, aub = ub;
      Seg l[2] = {slines[0][i], slines[1][j]};
      if (cross(l[1].d(), l[0].d()) > 0) {                5904
        alb = max(alb, intersection(l[0], l[1]).y);
      } else if (cross(l[1].d(), l[0].d()) < 0) {
        aub = min(aub, intersection(l[0], l[1]).y);
      }                                                   9883
      alb = max(alb, lb);
      aub = min(aub, ub);
      aub = max(aub, alb);
      ran(k, 0, 2) {
        double x1 = l[0].a.x + (alb - l[0].a.y) /
                               l[0].d().y * l[0].d().x;
        double x2 = l[0].a.x + (aub - l[0].a.y) /
                               l[0].d().y * l[0].d().x;
        result +=                                         8864
          (-1 + 2 * k) * (aub - alb) * (x1 + x2) / 2;
      }
      if (ap_s[0][i + 1] < ap_s[1][j + 1]) {
        i++;
      } else {
        j++;
      }
    }
  }
  return result;
}                                                        %3672
```

Convex polygon algorithms

```
typedef pair<int, int> Vec;
typedef pair<Vec, Vec> Seg;
typedef vector<Seg>::iterator SegIt;
#define F first
#define S second                                          2608
#define MP(x, y) make_pair(x, y)
Vec sub(const Vec &v1, const Vec &v2) {
  return MP(v1.F - v2.F, v1.S - v2.S);
}
ll dot(const Vec &v1, const Vec &v2) {                    0940
  return (ll)v1.F * v2.F + (ll)v1.S * v2.S;
}
```

```
ll cross(const Vec &v1, const Vec &v2) {
  return (ll)v1.F * v2.S - (ll)v2.F * v1.S;
}                                                     2634
ll dist_sq(const Vec &p1, const Vec &p2) {
  return (ll)(p2.F - p1.F) * (p2.S - p1.F) +
         (ll)(p2.S - p1.S) * (p2.S - p1.S);
}                                                     %4155
struct Point;
multiset<Point>::iterator end_node;
struct Point {
  Vec p;
  typename multiset<Point>::iterator get_it() const {
    // gcc rb_tree dependent                          1510
    tuple<void *> tmp = {(void *)this - 32};
    return *(multiset<Point>::iterator *)(&tmp);
  }
  bool operator<(const Point &rhs) const {            6790
    return (p.F < rhs.p.F); // sort by x
  }
  bool operator<(const Vec &q) const {
    auto nxt = next(get_it());     // convex hull trick
    if (nxt == end_node) return 0; // nxt == end()
    return q.S * dot(p, {q.F, 1}) <                   2658
           q.S * dot(nxt->p, {q.F, 1});
  }
};
template <int part> // 1 = upper, -1 = lower
struct HullDynamic : public multiset<Point, less<> > {
  bool bad(iterator y) {                              7293
    if (y == begin()) return 0;
    auto x = prev(y);
    auto z = next(y);
    if (z == end())
      return y->p.F == x->p.F && y->p.S <= x->p.S;
    return part *                                     5485
           cross(sub(y->p, x->p), sub(y->p, z->p)) <=
         0;
  }
  void insert_point(int m, int b) { // O(log(N))
    auto y = insert({{m, b}});
    if (bad(y)) {                                     0258
      erase(y);
      return;
    }
    while (next(y) != end() && bad(next(y)))
      erase(next(y));
    while (y != begin() && bad(prev(y))) erase(prev(y));
  }                                                   9920
  ll eval(
    int x) { // O(log(N)) upper maximize dot({x, 1}, v)
    end_node =
      end(); //       lower minimize dot({x, 1}, v)
    auto it = lower_bound((Vec){x, part});
    return (ll)it->p.F * x + it->p.S;
  }
};                                                    %8709
struct Hull {
  vector<Seg> hull;
  SegIt up_beg;
  template <typename It>
  void extend(It beg, It end) { // O(n)
    vector<Vec> r;
    for (auto it = beg; it != end; ++it) {            9981
      if (r.empty() || *it != r.back()) {
        while (r.size() >= 2) {
          int n = r.size();
          Vec v1 = {r[n - 1].F - r[n - 2].F,
            r[n - 1].S - r[n - 2].S};                 1365
          Vec v2 = {
            it->F - r[n - 2].F, it->S - r[n - 2].S};
          if (cross(v1, v2) > 0) break;
          r.pop_back();
        }
        r.push_back(*it);
      }
    }
    ran(i, 0, (int)r.size() - 1)                      8095
      hull.emplace_back(r[i], r[i + 1]);
  }
  Hull(vector<Vec> &vert) { // atleast 2 distinct points
    sort(vert.begin(), vert.end()); // O(n log(n))
    extend(vert.begin(), vert.end());                 6560
    int diff = hull.size();
    extend(vert.rbegin(), vert.rend());
    up_beg = hull.begin() + diff;
  }                                                   %0939
  bool contains(Vec p) { // O(log(n))
    if (p < hull.front().F || p > up_beg->F)
      return false;
    {
      auto it_low = lower_bound(hull.begin(), up_beg,
        MP(MP(p.F, (int)-2e9), MP(0, 0)));            1542
      if (it_low != hull.begin()) --it_low;
      Vec a = {it_low->S.F - it_low->F.F,
        it_low->S.S - it_low->F.S};
      Vec b = {p.F - it_low->F.F, p.S - it_low->F.S};
      if (cross(a, b) <                               1144
          0) // < 0 is inclusive, <=0 is exclusive
        return false;
    }
    {
      auto it_up = lower_bound(hull.rbegin(),
        hull.rbegin() + (hull.end() - up_beg),
        MP(MP(p.F, (int)2e9), MP(0, 0)));             9423
      if (it_up - hull.rbegin() == hull.end() - up_beg)
        --it_up;
      Vec a = {it_up->F.F - it_up->S.F,
        it_up->F.S - it_up->S.S};                     0193
      Vec b = {p.F - it_up->S.F, p.S - it_up->S.S};
      if (cross(a, b) >
          0) // > 0 is inclusive, >=0 is exclusive
        return false;
    }
    return true;
  }                                                   %3267
  // The function can have only one local min and max
  // and may be constant only at min and max.
  template <typename T>
  SegIt max(function<T(Seg &)> f) { // O(log(n))
    auto l = hull.begin();
    auto r = hull.end();
    SegIt b = hull.end();                             8566
    T b_v;
    while (r - l > 2) {
      auto m = l + (r - l) / 2;
      T l_v = f(*l);
      T l_n_v = f(*(l + 1));
      T m_v = f(*m);
      T m_n_v = f(*(m + 1));
      if (b == hull.end() || l_v > b_v) {             3580
        b = l; // If max is at l we may remove it from
               // the range.
        b_v = l_v;
      }
      if (l_n_v > l_v) {
        if (m_v < l_v) {
          r = m;
        } else {
          if (m_n_v > m_v) {
            l = m + 1;
          } else {
            r = m + 1;
          }
        }
      } else {
        if (m_v < l_v) {                              7715
          l = m + 1;
        } else {
          if (m_n_v > m_v) {
            l = m + 1;
          } else {
            r = m + 1;
          }
        }
      }
    }
    T l_v = f(*l);
    if (b == hull.end() || l_v > b_v) {
      b = l;
      b_v = l_v;
    }
    if (r - l > 1) {                                  2147
      T l_n_v = f(*(l + 1));
      if (b == hull.end() || l_n_v > b_v) {
        b = l + 1;
        b_v = l_n_v;
      }
    }
    return b;
  }                                                   %5939
  SegIt closest(
```

```
  Vec p) { // p can't be internal(can be on border),
           // hull must have atleast 3 points
  Seg &ref_p = hull.front(); // O(log(n))
  return max(function<double(Seg &)>(
    [&p, &ref_p](Seg &seg) { // accuracy of used type
                             // should be coord^-2
      if (p == seg.F) return 10 - M_PI;                   0134
      Vec v1 = {seg.S.F - seg.F.F, seg.S.S - seg.F.S};
      Vec v2 = {p.F - seg.F.F, p.S - seg.F.S};
      ll c_p = cross(v1, v2);
      if (c_p > 0) { // order the backside by angle
        Vec v1 = {ref_p.F.F - p.F, ref_p.F.S - p.S};
        Vec v2 = {seg.F.F - p.F, seg.F.S - p.S};
        ll d_p = dot(v1, v2);                             5063
        ll c_p = cross(v2, v1);
        return atan2(c_p, d_p) / 2;
      }
      ll d_p = dot(v1, v2);
      double res = atan2(d_p, c_p);
      if (d_p <= 0 && res > 0) res = -M_PI;
      if (res > 0) {                                      5050
        res += 20;
      } else {
        res = 10 - res;
      }
      return res;
    }));
}                                                        %5632
template <int DIRECTION> // 1 or -1
Vec tan_point(
  Vec p) { // can't be internal or on border
  //-1 iff CCW rotation of ray from p to res takes it
  // away from
  // polygon?
  Seg &ref_p = hull.front(); // O(log(n))
  auto best_seg = max(function<double(Seg &)>(
    [&p, &ref_p](Seg &seg) { // accuracy of used type
                             // should be coord^-2
      Vec v1 = {ref_p.F.F - p.F, ref_p.F.S - p.S};
      Vec v2 = {seg.F.F - p.F, seg.F.S - p.S};
      ll d_p = dot(v1, v2);                               0212
      ll c_p = DIRECTION * cross(v2, v1);
      return atan2(c_p, d_p); // order by signed angle
    }));
  return best_seg->F;
}                                                        %5890
SegIt max_in_dir(
  Vec v) { // first is the ans. O(log(n))
  return max(function<ll(Seg &)>(
    [&v](Seg &seg) { return dot(v, seg.F); }));
}                                                        %5805
pair<SegIt, SegIt> intersections(Seg l) { // O(log(n))
  int x = l.S.F - l.F.F;
  int y = l.S.S - l.F.S;
  Vec dir = {-y, x};
  auto it_max = max_in_dir(dir);                          4740
  auto it_min = max_in_dir(MP(y, -x));
```

```
  ll opt_val = dot(dir, l.F);
  if (dot(dir, it_max->F) < opt_val ||
      dot(dir, it_min->F) > opt_val)                      8921
    return MP(hull.end(), hull.end());
  SegIt it_r1, it_r2;
  function<bool(const Seg &, const Seg &)> inc_c(
    [&dir](const Seg &lft, const Seg &rgt) {
      return dot(dir, lft.F) < dot(dir, rgt.F);
    });                                                   1828
  function<bool(const Seg &, const Seg &)> dec_c(
    [&dir](const Seg &lft, const Seg &rgt) {
      return dot(dir, lft.F) > dot(dir, rgt.F);
    });                                                   1765
  if (it_min <= it_max) {
    it_r1 =
      upper_bound(it_min, it_max + 1, l, inc_c) - 1;
    if (dot(dir, hull.front().F) >= opt_val) {
      it_r2 = upper_bound(                                8531
               hull.begin(), it_min + 1, l, dec_c) -
             1;
    } else {
      it_r2 =
        upper_bound(it_max, hull.end(), l, dec_c) - 1;
    }
  } else {
    it_r1 =
      upper_bound(it_max, it_min + 1, l, dec_c) - 1;
    if (dot(dir, hull.front().F) <= opt_val) {
      it_r2 = upper_bound(                                1538
               hull.begin(), it_max + 1, l, inc_c) -
             1;
    } else {
      it_r2 =
        upper_bound(it_min, hull.end(), l, inc_c) - 1;
    }
  }
  return MP(it_r1, it_r2);                                2632
}                                                        %2632
Seg diameter() { // O(n)
  Seg res;
  ll dia_sq = 0;
  auto it1 = hull.begin();
  auto it2 = up_beg;
  Vec v1 = {hull.back().S.F - hull.back().F.F,
    hull.back().S.S - hull.back().F.S};                   2168
  while (it2 != hull.begin()) {
    Vec v2 = {(it2 - 1)->S.F - (it2 - 1)->F.F,
      (it2 - 1)->S.S - (it2 - 1)->F.S};
    if (cross(v1, v2) > 0) break;                         4807
    --it2;
  }
  while (
    it2 != hull.end()) { // check all antipodal pairs
    if (dist_sq(it1->F, it2->F) > dia_sq) {
      res = {it1->F, it2->F};
      dia_sq = dist_sq(res.F, res.S);                     7128
    }
```

```
    Vec v1 = {
      it1->S.F - it1->F.F, it1->S.S - it1->F.S};
    Vec v2 = {
      it2->S.F - it2->F.F, it2->S.S - it2->F.S};
    if (cross(v1, v2) == 0) {                             9381
      if (dist_sq(it1->S, it2->F) > dia_sq) {
        res = {it1->S, it2->F};
        dia_sq = dist_sq(res.F, res.S);
      }
      if (dist_sq(it1->F, it2->S) > dia_sq) {
        res = {it1->F, it2->S};                           8171
        dia_sq = dist_sq(res.F, res.S);
      } // report cross pairs at parallel lines.
      ++it1;
      ++it2;
    } else if (cross(v1, v2) < 0) {
      ++it1;
    } else {
      ++it2;
    }
  }
  return res;
}
};                                                       %1111
```

Delaunay triangulation O(nlogn)

```
const int max_co = (1 << 28) - 5;
struct Vec {
  int x, y;
  bool operator==(const Vec &oth) {
    return x == oth.x && y == oth.y;
  }
  bool operator!=(const Vec &oth) {                       2500
    return !operator==(oth);
  }
  Vec operator-(const Vec &oth) {
    return {x - oth.x, y - oth.y};
  }
};
ll cross(Vec a, Vec b) {
  return (ll)a.x * b.y - (ll)a.y * b.x;                   8725
}
ll dot(Vec a, Vec b) {
  return (ll)a.x * b.x + (ll)a.y * b.y;
}
struct Edge {
  Vec tar;
  Edge *nxt;
  Edge *inv = NULL;
  Edge *rep = NULL;                                       4335
  bool vis = false;
};
struct Seg {
  Vec a, b;
  bool operator==(const Seg &oth) {
    return a == oth.a && b == oth.b;
  }
  bool operator!=(const Seg &oth) {                       8977
```

```
    return !operator==(oth);
  }
};
ll orient(Vec a, Vec b, Vec c) {
  return (ll)a.x * (b.y - c.y) + (ll)b.x * (c.y - a.y) +
         (ll)c.x * (a.y - b.y);                         3775
}                                                       %3775
bool in_c_circle(Vec *arr, Vec d) {
  if (cross(arr[1] - arr[0], arr[2] - arr[0]) == 0)
    return true; // degenerate
  ll m[3][3];
  ran(i, 0, 3) {                                        4264
    m[i][0] = arr[i].x - d.x;
    m[i][1] = arr[i].y - d.y;
    m[i][2] = m[i][0] * m[i][0];
    m[i][2] += m[i][1] * m[i][1];
  }
  __int128 res = 0;                                     3889
  res +=
    (__int128)(m[0][0] * m[1][1] - m[0][1] * m[1][0]) *
    m[2][2];
  res +=
    (__int128)(m[1][0] * m[2][1] - m[1][1] * m[2][0]) *
    m[0][2];                                            6577
  res -=
    (__int128)(m[0][0] * m[2][1] - m[0][1] * m[2][0]) *
    m[1][2];
  return res > 0;
}                                                       %1845
Edge *add_triangle(Edge *a, Edge *b, Edge *c) {
  Edge *old[] = {a, b, c};
  Edge *tmp = new Edge[3];
  ran(i, 0, 3) {
    old[i]->rep = tmp + i;                              8219
    tmp[i] = {
      old[i]->tar, tmp + (i + 1) % 3, old[i]->inv};
    if (tmp[i].inv) tmp[i].inv->inv = tmp + i;
  }
  return tmp;
}
Edge *add_point(                                        0087
  Vec p, Edge *cur) { // returns outgoing edge
  Edge *triangle[] = {cur, cur->nxt, cur->nxt->nxt};
  ran(i, 0, 3) {
    if (orient(triangle[i]->tar,
          triangle[(i + 1) % 3]->tar, p) < 0)
      return NULL;                                      0233
  }
  ran(i, 0, 3) {
    if (triangle[i]->rep) {
      Edge *res = add_point(p, triangle[i]->rep);
      if (res)
        return res; // unless we are on last layer we
                    // must exit here
    }
  }
  Edge p_as_e{p};                                       5490
  Edge tmp{cur->tar};
  tmp.inv = add_triangle(&p_as_e, &tmp, cur = cur->nxt);
  Edge *res = tmp.inv->nxt;
  tmp.tar = cur->tar;                                   4005
  tmp.inv = add_triangle(&p_as_e, &tmp, cur = cur->nxt);
  tmp.tar = cur->tar;
  res->inv = add_triangle(&p_as_e, &tmp, cur = cur->nxt);
  res->inv->inv = res;                                  3259
  return res;
}
Edge *delaunay(vector<Vec> &points) {
  random_shuffle(points.begin(), points.end());
  Vec arr[] = {{4 * max_co, 4 * max_co},                5591
    {-4 * max_co, max_co}, {max_co, -4 * max_co}};
  Edge *res = new Edge[3];
  ran(i, 0, 3) res[i] = {arr[i], res + (i + 1) % 3};
  for (Vec &cur : points) {                             4575
    Edge *loc = add_point(cur, res);
    Edge *out = loc;
    arr[0] = cur;
    while (true) {
      arr[1] = out->tar;
      arr[2] = out->nxt->tar;                           1771
      Edge *e = out->nxt->inv;
      if (e && in_c_circle(arr, e->nxt->tar)) {
        Edge tmp{cur};
        tmp.inv = add_triangle(&tmp, out, e->nxt);
        tmp.tar = e->nxt->tar;                          9851
        tmp.inv->inv =
          add_triangle(&tmp, e->nxt->nxt, out->nxt->nxt);
        out = tmp.inv->nxt;
        continue;
      }
      out = out->nxt->nxt->inv;                         0151
      if (out->tar == loc->tar) break;
    }
  }
  return res;
}                                                       %9625
void extract_triangles(
  Edge *cur, vector<vector<Seg> > &res) {
  if (!cur->vis) {
    bool inc = true;
    Edge *it = cur;
    do {
      it->vis = true;                                   9190
      if (it->rep) {
        extract_triangles(it->rep, res);
        inc = false;
      }
      it = it->nxt;
    } while (it != cur);
    if (inc) {
      Edge *triangle[3] = {cur, cur->nxt, cur->nxt->nxt};
      res.resize(res.size() + 1);                       6207
      vector<Seg> &tar = res.back();
      ran(i, 0, 3) {
        if ((abs(triangle[i]->tar.x) < max_co &&
              abs(triangle[(i + 1) % 3]->tar.x) <
                max_co))                                9617
          tar.push_back({triangle[i]->tar,
            triangle[(i + 1) % 3]->tar});
      }
      if (tar.empty()) res.pop_back();
    }
  }
}                                                       %8602
alias g++='g++ -g -Wall -Wshadow -Wconversion \  #.bashrc
  -fsanitize=undefined,address -DCDEBUG'         #.bashrc
alias a='setxkbmap us -option'                   #.bashrc
alias m='setxkbmap us -option caps:escape'       #.bashrc
alias ma='setxkbmap us -variant dvp \            #.bashrc
  -option caps:escape'                           #.bashrc
gsettings set org.compiz.core: \                 #settings
  /org/compiz/profiles/Default/plugins/core/ hsize 4
gsettings set org.gnome.desktop.wm.preferences \
  focus-mode 'sloppy'                            #settings
set si cin                                       #.vimrc
set ts=4 sw=4 noet                               #.vimrc
set cb=unnamedplus                               #.vimrc
(global-set-key (kbd "C-x <next>") 'other-window) #.emacs
(global-set-key (kbd "C-x <prior>") \            #.emacs
  'previous-multiframe-window)                   #.emacs
(global-set-key (kbd "C-M-z") 'ansi-term)        #.emacs
                                                 #.emacs
(global-linum-mode 1)                            #.emacs
(column-number-mode 1)                           #.emacs
(show-paren-mode 1)                              #.emacs
(setq-default indent-tabs-mode nil)              #.emacs
valgrind --vgdb-error=0 ./a <inp &               #valgrind
gdb a                                            #valgrind
target remote | vgdb                             #valgrind
```

crc.sh

```
#!/bin/envbash
for j in `seq $2 1 $3`; do #whitespaces don't matter.
  sed '/^\s*$/d' $1 | head -$j | tr -d '[[:space:]]' \
    | cksum | cut -f1 -d ' ' | tail -c 5
done #there shouldn't be any COMMENTS.
#copy lines being checked to separate file.
# $ ./crc.sh tmp.cpp 999 999
# $ ./crc.sh tmp.cpp 1 333 | grep XXXX
```

gcc ordered set, hashtable

```
#define DEBUG(...) cerr << __VA_ARGS__ << endl;
#ifndef CDEBUG
#undef DEBUG
#define DEBUG(...) ((void)0);
#define NDEBUG                                          4737
#endif
#define ran(i, a, b) for (auto i = (a); i < (b); i++)
#include <bits/stdc++.h>
typedef long long ll;
typedef long double ld;                                 8529
using namespace std;                                    %8529
#pragma GCC optimize("Ofast") // better vectorization
```

```
#pragma GCC target("avx,avx2")
// double vectorized performance
#include <bits/extc++.h>
using namespace __gnu_pbds;                                2960
template <typename T, typename U>
using hashmap = gp_hash_table<T, U>;
// dumb, 3x faster than stl
template <typename T>
using ordered_set = tree<T, null_type, less<T>,
  rb_tree_tag, tree_order_statistics_node_update>;
int main() {
  ordered_set<int> cur;
  cur.insert(1);
  cur.insert(3);
  cout << cur.order_of_key(2) << endl;
  // the number of elements in the set less than 2
  cout << *cur.find_by_order(1) << endl;
  // the 1-st smallest number in the set(0-based)
  ordered_set<int> oth;
  oth.insert(5); // to join: cur < oth
  cur.join(oth); // cur = {1, 3, 5}, oth = {}
  cur.split(1, oth); // cur = {1}, oth = {3, 5}
  hashmap<int, int> h({}, {}, {}, {}, {1 << 16});
}
```

PRNGs and Hash functions

```
mt19937 gen;
uint64_t rand64() {
  return gen() ^ ((uint64_t)gen() << 32);
}                                                          %5668
uint64_t rand64() {
  static uint64_t x = 1; //x != 0
  x ^= x >> 12;
  x ^= x << 25;
  x ^= x >> 27;
  return x * 0x2545f4914f6cdd1d; // can remove mult
}                                                          %6873
uint64_t mix(uint64_t x){ // deadbeef -> y allowed
variable uint64_t mem[2] = { x, 0xdeadbeeffeebdaedull };
        asm volatile (
                "pxor %%xmm0, %%xmm0;"                     4939
                "movdqa (%0), %%xmm1;"
                "aesenc %%xmm0, %%xmm1;"
                "movdqa %%xmm1, (%0);"
                :
                : "r" (&mem[0])
                : "memory"
        );
        return mem[0]; // use both slots for 128 bit
}                                                          %7419
uint64_t mix64(uint64_t x) { //x != 0
  x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
  x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
  x = x ^ (x >> 31);                                       7317
  return x;
}                                                          %8619
uint64_t unmix64(uint64_t x) {
  x = (x ^ (x >> 31) ^ (x >> 62)) * 0x319642b2d24d8ec3;
  x = (x ^ (x >> 27) ^ (x >> 54)) * 0x96de1b173f119089;
  x = x ^ (x >> 30) ^ (x >> 60);                           5864
  return x;
}                                                          %4224
uint64_t combine64(uint64_t x, uint64_t y) {
  if (y < x) swap(x, y); // remove for ord
  return mix64(mix64(x) + y);
}                                                          %1550
```

Memorypool

```
const int BLOCK = 8;
const int MEM_SIZE = 1 << 26;
char glob_buf[MEM_SIZE];
int glob_idx;
vector<bool> glob_used;
void init_mem() {                                          9423
  glob_used.resize(MEM_SIZE / BLOCK);
  glob_used[0] = true;
}
template <typename T>
struct Ptr {
  unsigned idx;
  explicit Ptr(T *tar) { idx = (char *)tar - glob_buf; }
  Ptr() { idx = 0; }                                       1466
  template <typename... Args>
  void construct(Args... args) {
    new (glob_buf + idx) T(args...);
  }
  T *operator->() {
    assert(idx);                                           1975
    return (T *)(glob_buf + idx);
  }
  T &operator*() { return *operator->(); }
  bool operator==(const Ptr &oth) const {
    return idx == oth.idx;                                 7762
  }
  operator unsigned() const { return idx; }
  Ptr &operator+=(int diff) {
    idx += diff * sizeof(T);
    return *this;
  }
  Ptr operator+(int diff) {                                4370
    Ptr res;
    res.idx = idx;
    return res += diff;
  }
  T &operator[](int diff) { return *operator+(diff); }
};
template <typename T, typename... Args>                    1487
Ptr<T> alloc(int n, Args... args) {
  unsigned len = 0; // TLE if running low on mem
  while (len < sizeof(T) * n) {
    if (!glob_idx) glob_idx = MEM_SIZE / BLOCK;
    if (glob_used[--glob_idx]) {                           9661
      len = 0;
    } else {
      len += BLOCK;
    }
  }
  ran(i, 0, ((int)sizeof(T) * n + BLOCK - 1) / BLOCK)
    glob_used[glob_idx + i] = true;
  Ptr<T> res;                                              8678
  if (n) res.idx = BLOCK * glob_idx;
  ran(i, 0, n)(res + i).construct(args...);
  return res;
}
template <typename T>
void dealloc(Ptr<T> ptr, int n) {                          2141
  ran(i, 0, ((int)sizeof(T) * n + BLOCK - 1) / BLOCK)
    glob_used[ptr.idx / BLOCK + i] = false;
}
template <typename T>
struct hash<Ptr<T> > {                                     3854
  std::size_t operator()(const Ptr<T> &cur) const {
    return cur.idx;
  }
};                                                         %3700
```

Radixsort 50M 64 bit integers as single array in 1 sec

```
template <typename T>
void rsort(T *a, T *b, int size, int d = sizeof(T) - 1) {
  int b_s[256]{};
  ran(i, 0, size) { ++b_s[(a[i] >> (d * 8)) & 255]; }
  // ++b_s[*((uchar *)(a + i) + d)];                       1489
  T *mem[257];
  mem[0] = b;
  T **l_b = mem + 1;
  l_b[0] = b;
  ran(i, 0, 255) { l_b[i + 1] = l_b[i] + b_s[i]; }
  for (T *it = a; it != a + size; ++it) {                  6813
    T id = ((*it) >> (d * 8)) & 255;
    *(l_b[id]++) = *it;
  }
  l_b = mem;
  if (d) {
    T *l_a[256];
    l_a[0] = a;
    ran(i, 0, 255) l_a[i + 1] = l_a[i] + b_s[i];
    ran(i, 0, 256) {                                       4380
      if (l_b[i + 1] - l_b[i] < 100) {
        sort(l_b[i], l_b[i + 1]);
        if (d & 1) copy(l_b[i], l_b[i + 1], l_a[i]);
      } else {
        rsort(l_b[i], l_a[i], b_s[i], d - 1);
      }                                                    7759
    }
  }
}                                                          %3895
const int nmax = 5e7;
ll arr[nmax], tmp[nmax];
int main() {
  for (int i = 0; i < nmax; ++i)
    arr[i] = ((ll)rand() << 32) | rand();
  rsort(arr, tmp, nmax);
  assert(is_sorted(arr, arr + nmax));
```

```
}
```

FFT 10-15M length/sec

```
// integer c = a*b is accurate if c_i < 2^49
#pragma GCC optimize ("Ofast") //10% performance
#include <complex.h>
extern "C" __complex__ double __muldc3(
    double a, double b, double c, double d){
  return a*c-b*d+I*(a*d+b*c); // 40% performance
}                                                   6910
#include <bits/stdc++.h>
typedef complex<double> Comp;
void fft_rec(Comp *arr, Comp *root_pow, int len) {
  if (len != 1) {                                   6092
    fft_rec(arr,       root_pow, len >> 1);
    fft_rec(arr + len, root_pow, len >> 1);
  }
  root_pow += len;
  ran(i, 0, len){
    tie(arr[i], arr[i + len]) = pair<Comp, Comp> {
        arr[i] + root_pow[i] * arr[i + len],
        arr[i] - root_pow[i] * arr[i + len] };
  }                                                 5001
}
void fft(vector<Comp> &arr, int ord, bool invert) {
  assert(arr.size() == 1 << ord);
  static vector<Comp> root_pow(1);                  4070
  static int inc_pow = 1;
  static bool is_inv = false;
  if (inc_pow <= ord) {
    int idx = root_pow.size();
    root_pow.resize(1 << ord);                      0927
    for (; inc_pow <= ord; ++inc_pow) {
      for (int idx_p = 0; idx_p < 1 << (ord - 1);
          idx_p += 1 << (ord - inc_pow), ++idx) {
        root_pow[idx] = Comp {                      0935
          cos(-idx_p * M_PI / (1 << (ord - 1))),
          sin(-idx_p * M_PI / (1 << (ord - 1))) };
        if (is_inv) root_pow[idx] = conj(root_pow[idx]);
      }                                             5222
    }
  }
  if (invert != is_inv) {
    is_inv = invert;
    for (Comp &cur : root_pow) cur = conj(cur);
  }
  int j = 0;
  ran(i, 1, (1<<ord)){
    int m = 1 << (ord - 1);                         4150
    bool cont = true;
    while (cont) {
      cont = j & m;
      j ^= m;
      m >>= 1;
    }
    if (i < j) swap(arr[i], arr[j]);
  }
  fft_rec(arr.data(), root_pow.data(), 1 << (ord - 1));
  if (invert)                                       2958
    ran(i, 0, 1<<ord) arr[i] /= (1 << ord);
}                                                  %1362
void mult_poly_mod(vector<int> &a, vector<int> &b,
  vector<int> &c) { // c += a*b
  static vector<Comp> arr[4];
  // correct upto 0.5-2M elements(mod ~= 1e9)
  if (c.size() < 400) {                             5517
    ran(i, 0, (int)a.size())
      ran(j, 0, min((int)b.size(), (int)c.size()-i))
        c[i + j] = ((ll)a[i] * b[j] + c[i + j]) % mod;
  } else {                                          0158
    int ord = 32 - __builtin_clz((int)c.size()-1);
    if ((int)arr[0].size() != 1 << ord){
      ran(i, 0, 4) arr[i].resize(1 << ord);
    }                                               7344
    ran(i, 0, 4)
      fill(arr[i].begin(), arr[i].end(), Comp{});
    for (int &cur : a) if (cur < 0) cur += mod;
    for (int &cur : b) if (cur < 0) cur += mod;
    const int shift = 15;                           7118
    const int mask = (1 << shift) - 1;
    ran(i, 0, (int)min(a.size(), c.size())){
      arr[0][i] += a[i] & mask;
      arr[1][i] += a[i] >> shift;                   3548
    }
    ran(i, 0, (int)min(b.size(), c.size())){
      arr[2][i] += Comp{0, (b[i] & mask)};
      arr[3][i] += Comp{0, (b[i] >> shift)};
    }                                               2484
    ran(i, 0, 2) fft(arr[i], ord, false);
    ran(i, 0, 2){
      ran(j, 0, 2){
        int tar = 2 + (i + j) / 2;
        Comp mult = {0, -0.25};
        if (i ^ j) mult = {0.25, 0};                6876
        ran(k, 0, 1<<ord){
          int rev_k = ((1 << ord) - k) % (1 << ord);
          Comp ca = arr[i][k] + conj(arr[i][rev_k]);
          Comp cb = arr[j][k] - conj(arr[j][rev_k]);
          arr[tar][k] = arr[tar][k] + mult * ca * cb;
        }                                           8649
      }
    }
    ran(i, 2, 4){
      fft(arr[i], ord, true);
      ran(k, 0, (int)c.size()){
        c[k] = (c[k] + (((ll)(arr[i][k].real()+0.5)%mod)
                << (shift * (2 * (i-2) + 0)))) % mod;
        c[k] = (c[k] + (((ll)(arr[i][k].imag()+0.5)%mod)
                << (shift * (2 * (i-2) + 1)))) % mod;
      }                                             6208
    }
  }
}                                                  %4758
```

Fast mod mult, Rabbin Miller prime check, Pollard rho factorization O(p^0.5)

```
struct ModArithm {
  ull n;
  ld rec;
  ModArithm(ull _n) : n(_n) { // n in [2, 1<<63)
    rec = 1.0L / n;
  }
  // a, b in [0, min(2*n, 1<<63))
  ull multf(ull a, ull b) {
    ull mult = (ld)a * b * rec + 0.5L;              1609
    ll res = a * b - mult * n;
    if (res < 0) res += n;
    return res; // in [0, n-1)
  }
  ull sqp1(ull a) { return multf(a, a) + 1; }
};                                                 %7403
ull pow_mod(ull a, ull n, ModArithm &arithm) {
  ull res = 1;
  for (ull i = 1; i <= n; i <<= 1) {
    if (n & i) res = arithm.multf(res, a);          5920
    a = arithm.multf(a, a);
  }
  return res;
}                                                  %1380
vector<char> small_primes = {
  2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
bool is_prime(ull n) { // n <= 1<<63, 1M rand/s
  ModArithm arithm(n);
  if (n == 2 || n == 3) return true;                8104
  if (!(n & 1) || n == 1) return false;
  int s = __builtin_ctzll(n - 1);
  ull d = (n - 1) >> s;
  for (ull a : small_primes) {
    if (a >= n) break;                              8234
    a = pow_mod(a, d, arithm);
    if (a == 1 || a == n - 1) continue;
    ran(r, 1, s) {
      a = arithm.multf(a, a);
      if (a == 1) return false;
      if (a == n - 1) break;                        5793
    }
    if (a != n - 1) return false;
  }
  return true;
}                                                  %4147
ll pollard_rho(ll n) {
  ModArithm arithm(n);
  int cum_cnt = 64 - __builtin_clzll(n);
  cum_cnt *= cum_cnt / 5 + 1;
  while (true) {                                    2164
    ll lv = rand() % n;
    ll v = arithm.sqp1(lv);
    int idx = 1;
    int tar = 1;
    while (true) {
      ll cur = 1;
      ll v_cur = v;
      int j_stop = min(cum_cnt, tar - idx);
```

```
      for (int j = 0; j < j_stop; ++j) {                          4557
        cur = arithm.multf(cur, abs(v_cur - lv));
        v_cur = arithm.sqp1(v_cur);
        ++idx;
      }
      if (!cur) {
        for (int j = 0; j < cum_cnt; ++j) {
          ll g = __gcd(abs(v - lv), n);                           8079
          if (g == 1) {
            v = arithm.sqp1(v);
          } else if (g == n) {
            break;
          } else {
            return g;
          }
        }
        break;
      } else {
        ll g = __gcd(cur, n);
        if (g != 1) return g;                                     9567
      }
      v = v_cur;
      idx += j_stop;
      if (idx == tar) {
        lv = v;
        tar *= 2;
        v = arithm.sqp1(v);
        ++idx;
      }
    }
  }
}                                                                %4128
map<ll, int> prime_factor(
      ll n, map<ll, int> *res = NULL) {
  // n <= 1<<62, ~1000/s (<500/s on CF)
  if (!res) {
    map<ll, int> res_act;
    for (int p : small_primes) {                                  3770
      while (!(n % p)) {
        ++res_act[p];
        n /= p;
      }
    }
    if (n != 1) prime_factor(n, &res_act);
    return res_act;
  }
  if (is_prime(n)) {
    ++(*res)[n];                                                  1963
  } else {
    ll factor = pollard_rho(n);
    prime_factor(factor, res);
    prime_factor(n / factor, res);
  }
  return map<ll, int>();                                          1140
} // Usage: fact = prime_factor(n);                              %1140
```

Berlekamp-Massey O(LN)

```
template <typename T, T P>
struct intmod {
  intmod() {}
  constexpr intmod(T t) : x((t + P) % P) {}
  T value() const { return x; }                                   4043
  bool operator!=(const intmod<T, P> i) { return x != i.x; }
  bool operator==(const intmod<T, P> i) { return x == i.x; }
  intmod<T, P> &operator+=(const intmod<T, P> i) {
    x = (x + i.x) % P;                                            3220
    return *this;
  }
  intmod<T, P> &operator-=(const intmod<T, P> i) {
    x = (x + P - i.x) % P;
    return *this;
  }
  intmod<T, P> &operator*=(const intmod<T, P> i) {
    x = ((ll)x * i.x) % P;                                        6117
    return *this;
  }
  intmod<T, P> &operator/=(const intmod<T, P> i) {
    x = ((ll)x * i.inverse().x) % P;
    return *this;
  }
  intmod<T, P> operator+(const intmod<T, P> i) const {
    auto j = *this;                                               5239
    return j += i;
  }
  intmod<T, P> operator-(const intmod<T, P> i) const {
    auto j = *this;
    return j -= i;
  }
  intmod<T, P> operator*(const intmod<T, P> i) const {
    auto j = *this;                                               5405
    return j *= i;
  }
  intmod<T, P> operator/(const intmod<T, P> i) const {
    auto j = *this;
    return j /= i;
  }
  intmod<T, P> operator-() const {                                8184
    intmod<T, P> n;
    n.x = (P - x) % P;
    return n;
  }
  intmod<T, P> inverse() const {
    if (x == 0) return 0;
    T a = x, b = P;
    T aa = 1, ab = 0;
    T ba = 0, bb = 1;                                             6637
    while (a) {
      T q = b / a;
      T r = b % a;
      ba -= aa * q;
      bb -= ab * q;
      swap(ba, aa);
      swap(bb, ab);
      b = a;
      a = r;
    }
    intmod<T, P> ix = intmod<T, P>(aa) + intmod<T, P>(ba);
    assert(ix * x == unity);                                      1934
    return ix;
  }
  static const intmod<T, P> zero;
  static const intmod<T, P> unity;
 private:
  T x;
};
template <typename T, T P>                                        8968
constexpr intmod<T, P> intmod<T, P>::zero = 0;
template <typename T, T P>
constexpr intmod<T, P> intmod<T, P>::unity = 1;
using rem = intmod<char, 2>;                                     %8052
template <typename K>
static vector<K> berlekamp_massey(vector<K> ss) {
  vector<K> ts(ss.size());
  vector<K> cs(ss.size());                                        3987
  cs[0] = K::unity;
  fill(cs.begin() + 1, cs.end(), K::zero);
  vector<K> bs = cs;
  int l = 0, m = 1;
  K b = K::unity;
  for (int k = 0; k < (int)ss.size(); k++) {
    K d = ss[k];                                                  8023
    assert(l <= k);
    for (int i = 1; i <= l; i++) d += cs[i] * ss[k - i];
    if (d == K::zero) {
      m++;
    } else if (2 * l <= k) {
      K w = d / b;
      ts = cs;
      for (int i = 0; i < (int)cs.size() - m; i++)
        cs[i + m] -= w * bs[i];                                   9661
      l = k + 1 - l;
      swap(bs, ts);
      b = d;
      m = 1;
    } else {
      K w = d / b;
      for (int i = 0; i < (int)cs.size() - m; i++)
        cs[i + m] -= w * bs[i];
      m++;
    }
  }
  cs.resize(l + 1);                                               8403
  while (cs.back() == K::zero) cs.pop_back();
  return cs;
}                                                                %8013
```

Linear algebra

```
bitset<10> add(bitset<10> p, bitset<10> q) {
  return p ^ q;
}
bitset<10> mult(bitset<10> v, bool k) {
  if (k) {
    return v;
```

```
  } else {                                                8750
    return bitset<10>(0);
  }
}
bitset<10> normalize(bitset<10> v, int idx) { return v; }
bitset<10> neg(bitset<10> v) { return v; }                5844
template <typename T>
vector<T> add(vector<T> p, vector<T> q) {
  ran(i, 0, (int)p.size()) p[i] += q[i];
  return p;
}
template <typename T>                                     2733
vector<T> mult(vector<T> p, T k) {
  ran(i, 0, (int)p.size()) p[i] *= k;
  return p;
}
template <typename T>
vector<T> normalize(vector<T> v, int idx) {
  return mult(v, (T)1 / v[idx]);                          1772
}
template <typename T>
vector<T> neg(vector<T> p) {
  return mult(p, (T)-1);
}                                                         %1000
/* V is the class implementing a vector, T is the type
 * within. examples: <bitset<10>, bool>; <vector<double>,
 * double> etc. V must have an "add" operation defined */
template <typename V, typename T>
pair<vector<V>, pair<vector<int>, vector<int>>>
diagonalize(vector<V> matrix, int width) {                9747
  /* width is the number of columns we consider for
   * diagonalizing. all columns after that can be used
   * for things after equal sign etc */
  int cur_row = 0;
  vector<int> crap_columns;
  vector<int> diag_columns;
  ran(i, 0, width) {
    int row_id = -1;
    T best_val = 0; /* may want to replace with epsilon
                       if working over reals */
    ran(j, cur_row, (int)matrix.size()) {                 0164
      if (abs(matrix[j][i]) > abs(best_val)) {
        row_id = j;
        best_val = matrix[j][i];
      }
    }
    if (row_id == -1) {
      crap_columns.push_back(i);                          5068
    } else {
      diag_columns.push_back(i);
      swap(matrix[cur_row], matrix[row_id]);
      matrix[cur_row] = normalize(matrix[cur_row], i);
      ran(j, cur_row + 1, j < (int)matrix.size()) {
        if (matrix[j][i] != 0) {                          6006
          matrix[j] = add(neg(normalize(matrix[j], i)),
            matrix[cur_row]);
        }
      }
      cur_row++;
    }
  }
  for (int i = (int)diag_columns.size() - 1; i >= 0;
      --i) {                                              1210
    for (int j = i - 1; j >= 0; --j) {
      matrix[j] = add(matrix[j],
        neg(
          mult(matrix[i], matrix[j][diag_columns[i]])));
    }
  }
  return {matrix, {diag_columns, crap_columns}};
}                                                         %9471
template <typename V, typename T>
int matrix_rank(vector<V> matrix, int width) {
  return diagonalize<V, T>(matrix, width)                 5622
    .second.first.size();
}                                                         %6175
template <typename V, typename T>
vector<T> one_solution(
  vector<V> matrix, int width, vector<T> y) {
  /* finds one solution to the system Ax = y.
   * each row in matrix must have width at least width
   * + 1. aborts if there is no solution (you can check
   * whether solution exists using matrix_rank) */
  assert(matrix.size() == y.size());                      8765
  ran(i, 0, (int)matrix.size()) matrix[i][width] = y[i];
  pair<vector<V>, pair<vector<int>, vector<int>>> prr =
    diagonalize<V, T>(matrix, width);                     5091
  vector<V> diag = prr.first;
  vector<int> diag_cols = prr.second.first;
  vector<T> ans(width, 0);
  ran(i, 0, (int)matrix.size()) {                         6769
    if (i < (int)diag_cols.size()) {
      ans[diag_cols[i]] = diag[i][width];
    } else {
      assert(diag[i][width] == T(0));
      /* replace with epsilon if working over reals */
    }
  }
  return ans;                                             4744
}                                                         %4744
template <typename V, typename T>
vector<vector<T>> homog_basis(
  vector<V> matrix, int width) {
  /* finds the basis of the nullspace of matrix */
  pair<vector<V>, pair<vector<int>, vector<int>>> prr =
    diagonalize<V, T>(matrix, width);                     7752
  vector<V> diag = prr.first;
  vector<int> diag_cols = prr.second.first;
  vector<int> crap_cols = prr.second.second;
  if (diag_cols.size() != matrix.size())                  8673
    return vector<vector<T>>(0);
  vector<vector<T>> ans;
  for (int u : crap_cols) {
    vector<T> row(width, 0);
    row[u] = 1;
    ran(i, 0, (int)diag.size()) row[i] =                  1447
      -diag[diag_cols[i]][u];
    ans.push_back(row);
  }
  return ans;
}                                                         %1837
```

Polynomial roots and O(n^2) interpolation

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for (int i = (int)a.size(); i--;) (val *= x) += a[i];
    return val;                                           3663
  }
  void diff() {
    ran(i, 1, (int)a.size()) a[i - 1] = i * a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c;                               5829
    a.back() = 0;
    for (int i = (int)a.size() - 1; i--;)
      c = a[i], a[i] = a[i + 1] * x0 + b, b = c;
    a.pop_back();
  }
};                                                        %2226
/* Description: Finds the real roots to a polynomial.
 * Usage: poly_roots({{2,-3,1}},-1e9,1e9) // solve
 * x^2-3x+2 = 0 Time: O(n^2 \log(1/\epsilon)) */
vector<double> poly_roots(
  Poly p, double xmin, double xmax) {
  if (sz(p.a) == 2) return {-p.a[0] / p.a[1]};
  vector<double> ret;                                     4231
  Poly der = p;
  der.diff();
  auto dr = poly_roots(der, xmin, xmax);
  dr.push_back(xmin - 1);
  dr.push_back(xmax + 1);
  sort(dr.begin(), dr.end());                             1733
  ran(i, 0, (int)dr.size() - 1) {
    double l = dr[i], h = dr[i + 1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      ran(it, 0, 60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);                 1929
        if ((f <= 0) ^ sign) {
          l = m;
        } else {
          h = m;
        }
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
}                                                         %2596
```

```
/* Description: Given $n$ points (x[i], y[i]), computes
 * an n-1-degree polynomial $p$ that passes through them:
 * $p(x) = a[0]*x^0 + ... + a[n-1]*x^{n-1}$. For
 * numerical precision, pick $x[k] = c*\cos(k/(n-1)*\pi),
 * k=0 \dots n-1$. Time: O(n^2) */
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  ran(k, 0, n - 1) ran(i, k + 1, n) y[i] =
    (y[i] - y[k]) / (x[i] - x[k]);                    5711
  double last = 0;
  temp[0] = 1;
  ran(k, 0, n) {
    ran(i, 0, n) {
      res[i] += y[k] * temp[i];
      swap(last, temp[i]);
      temp[i] -= last * x[k];                         8691
    }
  }
  return res;
}                                                    %2093
```

Simplex algorithm

```
/* Description: Solves a general linear maximization
 * problem: maximize $c^T x$ subject to $Ax \le b$, $x
 * \ge 0$. Returns -inf if there is no solution, inf if
 * there are arbitrarily good solutions, or the maximum
 * value of $c^T x$ otherwise. The input vector is set to
 * an optimal $x$ (or in the unbounded case, an arbitrary
 * solution fulfilling the constraints). Numerical
 * stability is not guaranteed. For better performance,
 * define variables such that $x = 0$ is viable. Usage:
 * vvd A = {{1,-1}, {-1,1}, {-1,-2}};
 * vd b = {1,1,-4}, c = {-1,-1}, x;
 * T val = LPSolver(A, b, c).solve(x);
 * Time: O(NM * \#pivots), where a pivot may be e.g. an
 * edge relaxation. O(2^n) in the general case. Status:
 * seems to work? */
typedef double
  T; // long double, Rational, double + mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;
const T eps = 1e-8, inf = 1 / .0;
#define MP make_pair
#define ltj(X) \                                      3913
  if (s == -1 || MP(X[j], N[j]) < MP(X[s], N[s])) s = j
struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;
  LPSolver(const vvd& A, const vd& b, const vd& c)
    : m(sz(b)),                                       2418
      n(sz(c)),
      N(n + 1),
      B(m),
      D(m + 2, vd(n + 2)) {
    ran(i, 0, m) ran(j, 0, n) D[i][j] = A[i][j];
    ran(i, 0, m) {
      B[i] = n + i;
      D[i][n] = -1;                                   2216
      D[i][n + 1] = b[i];
    }
    ran(j, 0, n) {
      N[j] = j;
      D[m][j] = -c[j];
    }
    N[n] = -1;
    D[m + 1][n] = 1;
  }
  void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];               1099
    ran(i, 0, m + 2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s] * inv;
      ran(j, 0, n + 2) b[j] -= a[j] * inv2;
      b[s] = a[s] * inv2;                             8058
    }
    ran(j, 0, n + 2) if (j != s) D[r][j] *= inv;
    ran(i, 0, m + 2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }
  bool simplex(int phase) {                           0674
    int x = m + phase - 1;
    for (;;) {
      int s = -1;
      ran(j, 0, n + 1) if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;                                     0227
      ran(i, 0, m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || MP(D[i][n + 1] / D[i][s], B[i]) <
                         MP(D[r][n + 1] / D[r][s], B[r]))
          r = i;                                      0657
      }
      if (r == -1) return false;
      pivot(r, s);
    }
  }
  T solve(vd& x) {
    int r = 0;
    ran(i, 1, m) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -eps) {                         2015
      pivot(r, n);
      if (!simplex(2) || D[m + 1][n + 1] < -eps)
        return -inf;
      ran(i, 0, m) if (B[i] == -1) {
        int s = 0;
        ran(j, 1, n + 1) ltj(D[i]);                   5590
        pivot(i, s);
      }
    }
    bool ok = simplex(1);
    x = vd(n);
    ran(i, 0, m) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return ok ? D[m][n + 1] : inf;                    6872
  }
};                                                   %3986
```

Dinic

```
struct MaxFlow {
  const static ll INF = 1e18;
  int source, sink;
  vector<int> start, now, lvl, adj, rcap, cap_loc, bfs;
  vector<int> cap, orig_cap;                          2347
  ll sink_pot = 0;
  vector<bool> visited;
  vector<ll> cost;
  priority_queue<pair<ll, int>, vector<pair<ll, int> >,
    greater<pair<ll, int> > >                         7035
    dist_que;
  void add_flow(int idx, ll flow, bool cont = true) {
    cap[idx] -= flow;
    if (cont) add_flow(rcap[idx], -flow, false);
  }                                                   7491
  MaxFlow(                                            2758
    const vector<tuple<int, int, ll, ll, ll > > &edges) {
    for (auto &cur : edges) { //from, to, cap, rcap, cost
      start.resize(max(max(get<0>(cur), get<1>(cur)) + 2,
             (int)start.size()));                8990 6901
      ++start[get<0>(cur) + 1];
      ++start[get<1>(cur) + 1];
    }
    ran(i, 1, (int)start.size()) start[i] += start[i-1];
    now = start;                                 5067 7619
    adj.resize(start.back());
    cap.resize(start.back());
    rcap.resize(start.back());
    cost.resize(start.back());                        8183
    for (auto &cur : edges) {
      int u, v;                                       2600
      ll c, rc, c_cost;
      tie(u, v, c, rc, c_cost) = cur;
      assert(u != v);
      adj[now[u]] = v;
      adj[now[v]] = u;                                9009
      rcap[now[u]] = now[v];
      rcap[now[v]] = now[u];                          3424
      cap_loc.push_back(now[u]);
      cost[now[u]] = c_cost;
      cost[now[v]] = -c_cost;                         1155
      cap[now[u]++] = c;
      cap[now[v]++] = rc;
      orig_cap.push_back(c);
    }
  }
  bool dinic_bfs(int min_cap) {                       1782
    lvl.clear();
    lvl.resize(start.size());                         1409
    bfs.clear();
    bfs.resize(1, source);
    now = start;
    lvl[source] = 1;
    ran(i, 0, (int)bfs.size()) {                      7194
```

```
    int u = bfs[i];
    while (now[u] < start[u + 1]) {                    4157
      int v = adj[now[u]];
      if (cost[now[u]] == 0 &&
        cap[now[u]] >= min_cap && lvl[v] == 0) {
        lvl[v] = lvl[u] + 1;                           0292
        if(v==sink) return true;                       9445
        bfs.push_back(v);
      }
      ++now[u];
    }
  }
  return false;
}
ll dinic_dfs(int u, ll flow, int min_cap) {
  if (u == sink) return flow;                     0286 0667
  if (lvl[u] == lvl[sink]) return 0;
  ll res = 0;
  while (now[u] < start[u + 1]) {
    int v = adj[now[u]];
    if (lvl[v] == lvl[u] + 1 && cost[now[u]] == 0 &&
        cap[now[u]] >= min_cap) {                 3898 5811
      ll cur = dinic_dfs(v, min(flow, (ll)cap[now[u]]),
          min_cap);
      if (cur) {
        add_flow(now[u], cur);
        flow -= cur;
        res += cur;                               8689 6305
        if(!flow) break;
      }
    }
    ++now[u];
  }
  return res;
}
bool recalc_dist(bool check_imp = false) {
  now = start;
  visited.clear();                                     2399
  visited.resize(start.size());
  dist_que.emplace(0, source);
  bool imp = false;
  while (!dist_que.empty()) {
    int u;                                             1146
    ll dist;
    tie(dist, u) = dist_que.top();                     7076
    dist_que.pop();
    if (!visited[u]) {
      visited[u] = true;
      if (check_imp && dist != 0) imp = true;
      if (u == sink) sink_pot += dist;                 1736
      while (now[u] < start[u + 1]) {
        int v = adj[now[u]];                           1746
        if (!visited[v] && cap[now[u]])
          dist_que.emplace(dist + cost[now[u]], v);
        cost[now[u]] += dist;                          6628
        cost[rcap[now[u]++]] -= dist;                  3870
      }
    }
  }
  if (check_imp) return imp;
  return visited[sink];
}
// return whether there is a negative cycle
bool recalc_dist_bellman_ford() {                      6214
  int i = 0;
  for (; i < (int)start.size() - 1 &&
      recalc_dist(true); ++i) {}
  return i == (int)start.size() - 1;
}
pair<ll, ll> calc_flow(int _source, int _sink) {
  source = _source;                                    7113
  sink = _sink;
  assert(max(source, sink) < start.size() - 1);
  ll tot_flow = 0;                                0704 0619
  ll tot_cost = 0;
  if (recalc_dist_bellman_ford()) {                    8769
    assert(false);
  } else {
    while (recalc_dist()){
      ll flow = 0;
      for(int min_cap = 1<<30; min_cap; min_cap >>= 1){
        while (dinic_bfs(min_cap)) {                   9291
          now = start;                                 6673
          ll cur;
          while (cur = dinic_dfs(source, INF, min_cap))
            flow += cur;                               8615
        }
      }
      tot_flow += flow;
      tot_cost += sink_pot * flow;                     8204
    }
  }
  return {tot_flow, tot_cost};                         9075
}                                          %6688%9207%9075
ll flow_on_edge(int idx) {
  assert(idx < cap.size());
  return orig_cap[idx] - cap[cap_loc[idx]];
}
};                                                    %2070
const int nmax = 1055;
int main() {
  int t;
  scanf("%d", &t);
  for (int i = 0; i < t; ++i) {
    vector<tuple<int, int, ll, ll, ll> > edges;
    int n;
    scanf("%d", &n);
    for (int j = 1; j <= n; ++j) {
      edges.emplace_back(j, 2 * n + 1, 1, 0, 0);
    }
    for (int j = 1; j <= n; ++j) {
      int card;
      scanf("%d", &card);
      edges.emplace_back(0, card, 1, 0, 0);
    }
    int ex_c;
    scanf("%d", &ex_c);
    for (int j = 0; j < ex_c; ++j) {
      int a, b;
      scanf("%d %d", &a, &b);
      if (b < a) swap(a, b);
      edges.emplace_back(a, b, nmax, 0, 1);
      edges.emplace_back(b, n + b, nmax, 0, 0);
      edges.emplace_back(n + b, a, nmax, 0, 1);
    }
    int v = 2 * n + 2;
    MaxFlow mf(edges);
    printf("%d\n", (int)mf.calc_flow(0, v - 1).second);
    //cout << mf.flow_on_edge(edge_index) << endl;
  }
}
```

Min Cost Max Flow with Cycle Cancelling O(Cnm)

```
struct Network {
  struct Node;
  struct Edge {
    Node *u, *v;
    int f, c, cost;
    Node* from(Node* pos) {
      if (pos == u) return v;
      return u;                                        9547
    }
    int getCap(Node* pos) {
      if (pos == u) return c - f;
      return f;
    }
    int addFlow(Node* pos, int toAdd) {
      if (pos == u) {
        f += toAdd;                                    1565
        return toAdd * cost;
      } else {
        f -= toAdd;
        return -toAdd * cost;
      }
    }
  };
  struct Node {
    vector<Edge*> conn;
    int index;
  };
  deque<Node> nodes;                                   4534
  deque<Edge> edges;
  Node* addNode() {
    nodes.push_back(Node());
    nodes.back().index = nodes.size() - 1;
    return &nodes.back();                              3762
  }
  Edge* addEdge(
    Node* u, Node* v, int f, int c, int cost) {
    edges.push_back({u, v, f, c, cost});
    u->conn.push_back(&edges.back());                  5814
    v->conn.push_back(&edges.back());
```

```
    return &edges.back();
  }
  // Assumes all needed flow has already been added
  int minCostMaxFlow() {
    int n = nodes.size();
    int result = 0;                                     7253
    struct State {
      int p;
      Edge* used;
    };
    while (1) {
      vector<vector<State> > state(
        1, vector<State>(n, {0, 0}));
      for (int lev = 0; lev < n; lev++) {               3158
        state.push_back(state[lev]);
        for (int i = 0; i < n; i++) {
          if (lev == 0 ||
              state[lev][i].p < state[lev - 1][i].p) {
            for (Edge* edge : nodes[i].conn) {
              if (edge->getCap(&nodes[i]) > 0) {
                int np =                                7871
                  state[lev][i].p + (edge->u == &nodes[i]
                                      ? edge->cost
                                      : -edge->cost);
                int ni = edge->from(&nodes[i])->index;
                if (np < state[lev + 1][ni].p) {
                  state[lev + 1][ni].p = np;
                  state[lev + 1][ni].used = edge;
                }                                       8767
              }
            }
          }
        }
      }
      // Now look at the last level
      bool valid = false;
      for (int i = 0; i < n; i++)
        if (state[n - 1][i].p > state[n][i].p) {
          valid = true;
          vector<Edge*> path;                           4080
          int cap = 1000000000;
          Node* cur = &nodes[i];
          int clev = n;
          vector<bool> explr(n, false);
          while (!explr[cur->index]) {                  5898
            explr[cur->index] = true;
            State cstate = state[clev][cur->index];
            cur = cstate.used->from(cur);
            path.push_back(cstate.used);                7277
          }
          reverse(path.begin(), path.end());
          {
            int i = 0;
            Node* cur2 = cur;
            do {
              cur2 = path[i]->from(cur2);
              i++;
            } while (cur2 != cur);                      9838
            path.resize(i);
          }
          for (auto edge : path) {
            cap = min(cap, edge->getCap(cur));
            cur = edge->from(cur);
          }
          for (auto edge : path) {                      6010
            result += edge->addFlow(cur, cap);
            cur = edge->from(cur);
          }
        }
      if (!valid) break;
    }
    return result;
  }
};                                                     %4029
pair<int, vi> GetMinCut(vector<vi>& weights) {
  int N = sz(weights);
  vi used(N), cut, best_cut;
  int best_weight = -1;                                 9745
  for (int phase = N - 1; phase >= 0; phase--) {
    vi w = weights[0], added = used;
    int prev, k = 0;
    rep(i, 0, phase) {
      prev = k;
      k = -1;                                           4363
      rep(j, 1, N)
        if (!added[j] && (k == -1 || w[j] > w[k])) k = j;
      if (i == phase - 1) {
        rep(j, 0, N) weights[prev][j] += weights[k][j];
        rep(j, 0, N) weights[j][prev] = weights[prev][j];
        used[k] = true;                                 3181
        cut.push_back(k);
        if (best_weight == -1 || w[k] < best_weight) {
          best_cut = cut;
          best_weight = w[k];
        }
      } else {
        rep(j, 0, N) w[j] += weights[k][j];
        added[k] = true;                                2381
      }
    }
  }
  return {best_weight, best_cut};
}                                                      %8654
```

Aho Corasick O(|alpha|*sum(len))

```
const int alpha_size = 26;
struct Node {
  Node *nxt[alpha_size]; // May use other structures to
                         // move in trie
  Node *suffix;
  Node() { memset(nxt, 0, alpha_size * sizeof(Node *)); }
  int cnt = 0;                                          1006
};
Node *aho_corasick(vector<vector<char> > &dict) {
  Node *root = new Node;
  root->suffix = 0;
  vector<pair<vector<char> *, Node *> > state;
  for (vector<char> &s : dict)                          2117
    state.emplace_back(&s, root);
  for (int i = 0; !state.empty(); ++i) {
    vector<pair<vector<char> *, Node *> > nstate;
    for (auto &cur : state) {                           9306
      Node *nxt = cur.second->nxt[(*cur.first)[i]];
      if (nxt) {
        cur.second = nxt;
      } else {
        nxt = new Node;
        cur.second->nxt[(*cur.first)[i]] = nxt;
        Node *suf = cur.second->suffix;                 5283
        cur.second = nxt;
        nxt->suffix = root; // set correct suffix link
        while (suf) {
          if (suf->nxt[(*cur.first)[i]]) {
            nxt->suffix = suf->nxt[(*cur.first)[i]];
            break;                                      3580
          }
          suf = suf->suffix;
        }
      }
      if (cur.first->size() > i + 1)
        nstate.push_back(cur);
    }
    state = nstate;
  }
  return root;
}                                                      %8654
// auxilary functions for searhing and counting
Node *walk(Node *cur,
  char c) { // longest prefix in dict that is suffix of
            // walked string.
  while (true) {
    if (cur->nxt[c]) return cur->nxt[c];
    if (!cur->suffix) return cur;
    cur = cur->suffix;                                  3162
  }
}                                                      %1423
void cnt_matches(Node *root, vector<char> &match_in) {
  Node *cur = root;
  for (char c : match_in) {
    cur = walk(cur, c);
    ++cur->cnt;                                         0015
  }
}                                                      %3991
void add_cnt(
  Node *root) { // After counting matches propagete ONCE
                // to suffixes for final counts
  vector<Node *> to_visit = {root};
  ran(i, 0, to_visit.size()) {
    Node *cur = to_visit[i];
    ran(j, 0, alpha_size) {                             0662
      if (cur->nxt[j]) to_visit.push_back(cur->nxt[j]);
    }
```

```
  }
  for (int i = to_visit.size() - 1; i > 0; --i)
    to_visit[i]->suffix->cnt += to_visit[i]->cnt;
}                                                    %7950
int main() {
  int n, len;
  scanf("%d %d", &len, &n);
  vector<char> a(len + 1);
  scanf("%s", a.data());
  a.pop_back();
  for (char &c : a) c -= 'a';
  vector<vector<char> > dict(n);
  ran(i, 0, n) {
    scanf("%d", &len);
    dict[i].resize(len + 1);
    scanf("%s", dict[i].data());
    dict[i].pop_back();
    for (char &c : dict[i]) c -= 'a';
  }
  Node *root = aho_corasick(dict);
  cnt_matches(root, a);
  add_cnt(root);
  ran(i, 0, n) {
    Node *cur = root;
    for (char c : dict[i]) cur = walk(cur, c);
    printf("%d\n", cur->cnt);
  }
}
```

Suffix automaton and tree O((n+q)log(|alpha|)) - 10+M length/s

```
struct Node;
typedef Ptr<Node> P;
struct Node {
  int act = 0;
  Ptr<P> out;
  int len; // Length of longest suffix in equivalence
  P suf;   // class.
  char size = 0;
  char cap = 0;
  Node(int _len) : len(_len) {};                     5245
  Node(int &_act, Ptr<P> &_out, int &_len, P &_suf,
       int _size, int _cap) : act(_act), len(_len),
       suf(_suf), size(_size), cap(_cap) {           2571
    out = alloc<P>(cap);
    ran(i, 0, size)
      out[i]  = _out[i];
  }
  int has_nxt(char c) {
    return act & (1<<(c-'a'));
  }
  P nxt(char c) {                                    5014
    return
      out[__builtin_popcount(act & ((1<<(c-'a'))-1))];
  }
  void set_nxt(char c, P nxt) {
    int idx = __builtin_popcount(act & ((1<<(c-'a'))-1));
    if(has_nxt(c)){                                  8690
      out[idx] = nxt;
    } else{
      if(size == cap){
        cap *= 2;
        if(!size)
          cap = 2;
        Ptr<P> nout = alloc<P>(cap);
        ran(i, 0, idx)
          nout[i] = out[i];                          8164
        ran(i, idx, size)
          nout[i+1] = out[i];
        dealloc(out, size);
        out = nout;
      } else {
        for(int i=size; i>idx; --i)
          out[i] = out[i-1];                         4046
      }
      act |= (1<<(c-'a'));
      out[idx] = nxt;
      ++size;
    }
  }
  P split(int new_len) {
    return suf = alloc<Node>(1, act, out, new_len,
        suf, size, cap);                             8441
  }                                                 %8441
  // Extra functions for matching and counting
  P lower(int depth) {
    // move to longest suf of current with a maximum
    // length of depth.
    if (suf->len >= depth) return suf->lower(depth);
    return (P)this;
  }
  P walk(char c, int depth, int &match_len) {
    // move to longest suffix of walked path that is a
    // substring                                     3466
    match_len = min(match_len, len);
    // includes depth limit(needed for finding matches)
    if (has_nxt(c)) { // as suffixes are in classes,
             // match_len must be tracked externally
      ++match_len;
      return nxt(c)->lower(depth);
    }
    if (suf) return suf->walk(c, depth, match_len);
    return (P)this;                                  4494
  }                                                 %4494
  bool vis = false;
  bool vis_t = false;
  int paths_to_end = 0;
  void set_as_end() { // All suffixes of current node are
    paths_to_end += 1;// marked as ending nodes.
    if (suf) suf->set_as_end();                      0905
  }
  void calc_paths() {
    /* Call ONCE from ROOT. For each node  calculates
     * number of ways to reach an end node. paths_to_end
     * is ocurence count for any strings in current
     * suffix equivalence class. */
    if (!vis) {
      vis = true;
      ran(i, 0, size){
        out[i]->calc_paths();
        paths_to_end += out[i]->paths_to_end;
      }                                              0806
    }
  }
  // Transform into suffix tree of reverse string
  P tree_links[26];
  int end_d_v = 1 << 30;
  int end_d() {
    if (end_d_v == 1 << 30) {
      ran(i, 0, size){                               2648
        end_d_v = min(end_d_v, 1 + out[i]->end_d());
      }                                              6170
      if (end_d_v == 1 << 30)
        end_d_v = 0;
    }
    return end_d_v;
  }
  void build_suffix_tree(                            6985
    string &s) { // Call ONCE from ROOT.
    if (!vis_t) {
      vis_t = true;
      if (suf)                                       4268
        suf->tree_links[s[(int)s.size() - end_d() -
                          suf->len - 1]-'a'] = (P)this;
      ran(i, 0, size){                               2958
        out[i]->build_suffix_tree(s);                1410
      }
    }
  }
};                                       %7187%1877%5307
struct SufAuto {
  P last;
  P root;
  void extend(char new_c) {
    P nlast = alloc<Node>(1, last->len + 1);
    P swn = last;
    while (swn && !swn->has_nxt(new_c)) {            7641
      swn->set_nxt(new_c, nlast);
      swn = swn->suf;
    }
    if (!swn) {
      nlast->suf = root;
    } else {
      P max_sbstr = swn->nxt(new_c);
      if (swn->len + 1 == max_sbstr->len) {
        nlast->suf = max_sbstr;                      1138
      } else { // remove for minimal DFA that matches
               // suffixes and crap
        P eq_sbstr = max_sbstr->split(swn->len + 1);
        nlast->suf = eq_sbstr;
        P x = swn; // x = with_edge_to_eq_sbstr
        while (x != 0 && x->nxt(new_c) == max_sbstr) {
```

```
          x->set_nxt(new_c, eq_sbstr);   1131
          x = x->suf;
        }
      }
    }
    last = nlast;
  }
  SufAuto(string &s) {
    last = root = alloc<Node>(1, 0);
    for (char c : s) extend(c);
    // To build suffix tree use reversed string
    root->build_suffix_tree(s);   1055
  }
};   %5553%5020
```

DMST O(E log V)

```
struct EdgeDesc {
  int from, to, w;
};
struct DMST {
  struct Node;
  struct Edge {
    Node *from;
    Node *tar;
    int w;
    bool inc;
  };
  struct Circle {   9251
    bool vis = false;
    vector<Edge *> cont;
    void clean(int idx);
  };
  const static greater<pair<ll, Edge *> > comp;
  static vector<Circle> to_proc;   8693
  static bool no_dmst;
  static Node *root; // Can use inline static since C++17
  struct Node {
    Node *par = NULL;
    vector<pair<int, int> > out_cands; // Circ, edge idx
    vector<pair<ll, Edge *> > con;   8937
    bool in_use = false;
    ll w = 0; // extra to add to edges in con
    Node *anc() {
      if (!par) return this;
      while (par->par) par = par->par;
      return par;
    }
    void clean() {   0300
      if (!no_dmst) {
        in_use = false;
        for (auto &cur : out_cands)
          to_proc[cur.first].clean(cur.second);
      }
    }
    Node *con_to_root() {   6779
      if (anc() == root) return root;
      in_use = true;
      Node *super = this;
      // Will become root or the first Node encountered
      // in a loop.
      while (super == this) {
        while (!con.empty() &&
               con.front().second->tar->anc() == anc()) {
          pop_heap(con.begin(), con.end(), comp);
          con.pop_back();   1880
        }
        if (con.empty()) {
          no_dmst = true;
          return root;
        }
        pop_heap(con.begin(), con.end(), comp);
        auto nxt = con.back();
        con.pop_back();   1711
        w = -nxt.first;
        if (nxt.second->tar->in_use) {
          super = nxt.second->tar->anc();
          to_proc.resize(to_proc.size() + 1);
        } else {   6356
          super = nxt.second->tar->con_to_root();
        }
        if (super != root) {
          to_proc.back().cont.push_back(nxt.second);
          out_cands.emplace_back(to_proc.size() - 1,
            to_proc.back().cont.size() - 1);
        } else { // Clean circles   7094
          nxt.second->inc = true;
          nxt.second->from->clean();
        }
      }
      if (super != root) {
        // we are some loops non first Node.
        if (con.size() > super->con.size()) {
          swap(con, super->con);   0198
          // Largest con in loop should not be copied.
          swap(w, super->w);
        }
        for (auto cur : con) {
          super->con.emplace_back(
            cur.first - super->w + w, cur.second);
          push_heap(   2509
            super->con.begin(), super->con.end(), comp);
        }
      }
      par = super; // root or anc() of first Node
                   // encountered in a loop
      return super;
    }
  };
  Node *croot;
  vector<Node> graph;
  vector<Edge> edges;   0270
  DMST(int n, vector<EdgeDesc> &desc, int r) {
    // Self loops and multiple edges are okay.
    graph.resize(n);
    croot = &graph[r];
    for (auto &cur : desc)
      // Edges are reversed internally
      edges.push_back(   2636
        Edge{&graph[cur.to], &graph[cur.from], cur.w});
    for (int i = 0; i < desc.size(); ++i)
      graph[desc[i].to].con.emplace_back(   5825
        desc[i].w, &edges[i]);
    for (int i = 0; i < n; ++i)
      make_heap(
        graph[i].con.begin(), graph[i].con.end(), comp);
  }
  bool find() {   0310
    root = croot;
    no_dmst = false;
    for (auto &cur : graph) {
      cur.con_to_root();
      to_proc.clear();
      if (no_dmst) return false;   3580
    }
    return true;
  }
  ll weight() {
    ll res = 0;
    for (auto &cur : edges) {
      if (cur.inc) res += cur.w;
    }
    return res;
  }
};
void DMST::Circle::clean(int idx) {   8969
  if (!vis) {
    vis = true;
    for (int i = 0; i < cont.size(); ++i) {
      if (i != idx) {   2000
        cont[i]->inc = true;
        cont[i]->from->clean();   5761
      }
    }
  }
}
const greater<pair<ll, DMST::Edge *> > DMST::comp;
vector<DMST::Circle> DMST::to_proc;   4425
bool DMST::no_dmst;   7169
DMST::Node *DMST::root;   %1911%7169
```

Dominator tree O(NlogN)

```
struct Tree {
  /* insert structure here */
  void set_root(int u) {
    cout << "root is " << u << endl;
  }
  void add_edge(int u, int v) {
    cout << u << "->" << v << endl;   9295
  }
};
struct Graph {
  vector<vector<int>> in_edges, out_edges;   4936
  vector<int> ord, dfs_idx, parent;
```

```
vector<int> sdom, idom;                                      3559 2843
vector<vector<int>> rsdom; /* inverse of sdom */
/* slightly modified version of dsu-s root[] */
vector<int> dsu;
vector<int> label;                                                8541
void dfs(int cur, int par, vector<int> &vis) {
  ord.push_back(cur);                                        7901 9919
  parent[cur] = par;
  dfs_idx[cur] = (int)ord.size() - 1;                             7813
  vis[cur] = 1;
  for (int nxt : out_edges[cur]) {
    in_edges[nxt].push_back(cur);                            9341 8446
    if (!vis[nxt])
      dfs(nxt, cur, vis);
  }
}
void add_edge(int u, int v) {                                     1297
  out_edges[u].push_back(v);
}
Graph(int n) {
  in_edges.resize(n, vector<int>(0));                             2678
  out_edges.resize(n, vector<int>(0));                            3692
  rsdom.resize(n, vector<int>(0));                                2528
  dfs_idx.resize(n, -1);
  parent.resize(n, -1);                                           5034
  ran(i, 0, n) {
    sdom.push_back(i);                                            3309
    idom.push_back(i);
    dsu.push_back(i);                                             6047
    label.push_back(i);
  }
}
int find(int u, int x = 0) {                                      1942
  if (u == dsu[u]) {
    if (x) {
      return -1;                                                  7922
    } else {
      return u;
    }
  }
  int v = find(dsu[u], x + 1);                                    5963
  if (v < 0) {
    return u;
  }
  if (dfs_idx[sdom[label[dsu[u]]]] <                              5992
      dfs_idx[sdom[label[u]]]) {                                  0401
        label[u] = label[dsu[u]];
  }
  dsu[u] = v;                                                     2576
  return x ? v : label[u];
}
void merge(int u, int v) { dsu[v] = u; }                          7954
Tree dom_tree(int src) {                                          7503
  vector<int> vis(idom.size(), 0);                                5374
  dfs(src, -1, vis);
  for (int i = (int)ord.size() - 1; i >= 0; --i) {
    int u = ord[i];                                               5015
    for (int v : in_edges[u]) {                                   3975
      int w = find(v);                                            4547
      if (dfs_idx[sdom[u]] > dfs_idx[sdom[w]]) {
        sdom[u] = sdom[w];
      }
    }
    if (i > 0) {
      rsdom[sdom[u]].push_back(u);                           0705 4218
    }
    for (int w : rsdom[u]) {                                      7099
      int v = find(w);
      if (sdom[v] == sdom[w]) {
        idom[w] = sdom[w];
      } else {
        idom[w] = v;
      }
    }
    if (i > 0) {
      merge(parent[u], u);                                   8824 8104
    }
  }
  Tree ans; /* if your constructor needs # of vertices,
             * use (int) idom.size() + 5 for example */
  ran(i, 1, (int)ord.size()) {                                    6999
    int u = ord[i];
    if (idom[u] != sdom[u]) {
      idom[u] = idom[idom[u]];
    }
    ans.add_edge(idom[u], u);                                5230 2307
  }
  ans.set_root(src);
  return ans;                                                     6690
 }
};                                                      %7388%1935%7257
```

Bridges O(n)

```
struct vert;
struct edge {
  bool exists = true;
  vert *dest;
  edge *rev;
  edge(vert *_dest) : dest(_dest) { rev = NULL; }
  vert &operator*() { return *dest; }                             8609
  vert *operator->() { return dest; }
  bool is_bridge();
};
struct vert {
  deque<edge> con;
  int val = 0;
  int seen;
  int dfs(int upd, edge *ban) { // handles multiple edges
    if (!val) {                                                   1288
      val = upd;
      seen = val;
      for (edge &nxt : con) {
        if (nxt.exists && (&nxt) != ban)
          seen = min(seen, nxt->dfs(upd + 1, nxt.rev));
      }                                                           8194
    }
    return seen;
  }
  void remove_adj_bridges() {
    for (edge &nxt : con) {
      if (nxt.is_bridge()) nxt.exists = false;
    }
  }
  int cnt_adj_bridges() {                                         0310
    int res = 0;
    for (edge &nxt : con) res += nxt.is_bridge();
    return res;
  }
};
bool edge::is_bridge() {                                          4882
  return exists && (dest->seen > rev->dest->val ||
                    dest->val < rev->dest->seen);
}                                                       %3548%8614%4558
vert graph[nmax];
int main() { // Mechanics Practice BRIDGES
  int n, m;
  cin >> n >> m;
  for (int i = 0; i < m; ++i) {
    int u, v;
    scanf("%d %d", &u, &v);
    graph[u].con.emplace_back(graph + v);
    graph[v].con.emplace_back(graph + u);
    graph[u].con.back().rev = &graph[v].con.back();
    graph[v].con.back().rev = &graph[u].con.back();
  }
  graph[1].dfs(1, NULL);
  int res = 0;
  for (int i = 1; i <= n; ++i)
    res += graph[i].cnt_adj_bridges();
  cout << res / 2 << endl;
}
```

2-Sat O(n) and SCC O(n)

```
struct Graph {
  int n;
  vector<vector<int> > con;
  Graph(int nsize) {
    n = nsize;
    con.resize(n);
  }
  void add_edge(int u, int v) { con[u].push_back(v); }
  void top_dfs(int pos, vector<int> &result,
    vector<bool> &explr, vector<vector<int> > &revcon) {
    if (explr[pos]) return;                                       1224
    explr[pos] = true;
    for (auto next : revcon[pos])
      top_dfs(next, result, explr, revcon);
    result.push_back(pos);
  }
  vector<int> topsort() {                                         7420
    vector<vector<int> > revcon(n);
    ran(u, 0, n) {
      for (auto v : con[u]) revcon[v].push_back(u);
```

```
    }
    vector<int> result;
    vector<bool> explr(n, false);                    2387
    ran(i, 0, n) top_dfs(i, result, explr, revcon);
    reverse(result.begin(), result.end());
    return result;
  }
  void dfs(                                          5654
    int pos, vector<int> &result, vector<bool> &explr) {
    if (explr[pos]) return;
    explr[pos] = true;
    for (auto next : con[pos]) dfs(next, result, explr);
    result.push_back(pos);                           0239
  }
  vector<vector<int> > scc() {
    vector<int> order = topsort();
    reverse(order.begin(), order.end());
    vector<bool> explr(n, false);                    6484
    vector<vector<int> > res;
    for (auto it = order.rbegin(); it != order.rend();
         ++it) {
      vector<int> comp;
      top_dfs(*it, comp, explr, con);                5220
      sort(comp.begin(), comp.end());
      res.push_back(comp);
    }
    sort(res.begin(), res.end());
    return res;
  }
};                                        %0503%6965%4511
int main() {
  int n, m;
  cin >> n >> m;
  Graph g(2 * m);
  ran(i, 0, n) {
    int a, sa, b, sb;
    cin >> a >> sa >> b >> sb;
    a--, b--;
    g.add_edge(2 * a + 1 - sa, 2 * b + sb);
    g.add_edge(2 * b + 1 - sb, 2 * a + sa);
  }
  vector<int> state(2 * m, 0);
  {
    vector<int> order = g.topsort();
    vector<bool> explr(2 * m, false);
    for (auto u : order) {
      vector<int> traversed;
      g.dfs(u, traversed, explr);
      if (traversed.size() > 0 &&
          !state[traversed[0] ^ 1]) {
        for (auto c : traversed) state[c] = 1;
      }
    }
  }
  ran(i, 0, m) {
    if (state[2 * i] == state[2 * i + 1]) {
      cout << "IMPOSSIBLE\n";
      return 0;
    }
  }
  ran(i, 0, m) cout << state[2 * i + 1] << '\n';
  return 0;
}
```

Templated multi dimensional BIT O(log(n)^d) per query

```
// Fully overloaded any dimensional BIT, use any type for
// coordinates, elements, return_value. Includes
// coordinate compression.
template <class E_T, class C_T, C_T n_inf, class R_T>
struct BIT {
  vector<C_T> pos;
  vector<E_T> elems;
  bool act = false;                                  9342
  BIT() { pos.push_back(n_inf); }
  void init() {
    if (act) {
      for (E_T &c_elem : elems) c_elem.init();
    } else {
      act = true;
      sort(pos.begin(), pos.end());                  6080
      pos.resize(
        unique(pos.begin(), pos.end()) - pos.begin());
      elems.resize(pos.size());
    }
  }
  template <typename... loc_form>                    5478
  void update(C_T cx, loc_form... args) {
    if (act) {
      int x = lower_bound(pos.begin(), pos.end(), cx) -
              pos.begin();
      for (; x < (int)pos.size(); x += x & -x)
        elems[x].update(args...);                    5773
    } else {
      pos.push_back(cx);
    }
  }
  template <typename... loc_form>
  R_T query(
    C_T cx, loc_form... args) { // sum in (-inf, cx)
    R_T res = 0;
    int x = lower_bound(pos.begin(), pos.end(), cx) -
            pos.begin() - 1;                         9513
    for (; x > 0; x -= x & -x)
      res += elems[x].query(args...);
    return res;
  }
};
template <typename I_T>
struct wrapped {
  I_T a = 0;                                         7930
  void update(I_T b) { a += b; }
  I_T query() { return a; }
  // Should never be called, needed for compilation
  void init() { DEBUG('i') }
  void update() { DEBUG('u') }
};                                                   %0186
int main() {
  // retun type should be same as type inside wrapped
  BIT<BIT<wrapped<ll>, int, INT_MIN, ll>, int, INT_MIN,
    ll>
    fenwick;
  int dim = 2;
  vector<tuple<int, int, ll> > to_insert;
  to_insert.emplace_back(1, 1, 1);
  // set up all pos that are to be used for update
  for (int i = 0; i < dim; ++i) {
    for (auto &cur : to_insert)
      fenwick.update(get<0>(cur), get<1>(cur));
    // May include value which won't be used
    fenwick.init();
  }
  // actual use
  for (auto &cur : to_insert)
    fenwick.update(
      get<0>(cur), get<1>(cur), get<2>(cur));
  cout << fenwick.query(2, 2) << '\n';
}
```

Treap O(log (n)) per query

```
mt19937 randgen;
struct Treap {
  struct Node {
    int key;
    int value;
    unsigned int priority;
    ll total;
    Node* lch;
    Node* rch;                                       0909
    Node(int new_key, int new_value) {
      key = new_key;
      value = new_value;
      priority = randgen();
      total = new_value;
      lch = 0;
      rch = 0;                                       8691
    }
    void update() {
      total = value;
      if (lch) total += lch->total;
      if (rch) total += rch->total;
    }
  };
  deque<Node> nodes;
  Node* root = 0;                                    9983
  pair<Node*, Node*> split(int key, Node* cur) {
    if (cur == 0) return {0, 0};
    pair<Node*, Node*> result;
    if (key <= cur->key) {                           7646
      auto ret = split(key, cur->lch);
      cur->lch = ret.second;
      result = {ret.first, cur};
    } else {
```

```
      auto ret = split(key, cur->rch);                                5818
      cur->rch = ret.first;
      result = {cur, ret.second};
    }
    cur->update();
    return result;
  }
  Node* merge(Node* left, Node* right) {                              1748
    if (left == 0) return right;
    if (right == 0) return left;
    Node* top;
    if (left->priority < right->priority) {
      left->rch = merge(left->rch, right);                            0054
      top = left;
    } else {
      right->lch = merge(left, right->lch);
      top = right;
    }
    top->update();
    return top;
  }
  void insert(int key, int value) {                                   8465
    nodes.push_back(Node(key, value));
    Node* cur = &nodes.back();
    pair<Node*, Node*> ret = split(key, root);
    cur = merge(ret.first, cur);                                      1599
    cur = merge(cur, ret.second);
    root = cur;
  }
  void erase(int key) {
    Node *left, *mid, *right;
    tie(left, mid) = split(key, root);                                1926
    tie(mid, right) = split(key + 1, mid);
    root = merge(left, right);
  }
  ll sum_upto(int key, Node* cur) {
    if (cur == 0) return 0;                                           1009
    if (key <= cur->key) {
      return sum_upto(key, cur->lch);
    } else {
      ll result = cur->value + sum_upto(key, cur->rch);
      if (cur->lch) result += cur->lch->total;
      return result;                                                  6764
    }
  }
  ll get(int l, int r) {
    return sum_upto(r + 1, root) - sum_upto(l, root);
  }
};                                                                   %1936
int main() {
  ios_base::sync_with_stdio(false);
  cin.tie(0);
  int m;
  Treap treap;
  cin >> m;
  for (int i = 0; i < m; i++) {
    int type;
    cin >> type;
    if (type == 1) {
      int x, y;
      cin >> x >> y;
      treap.insert(x, y);
    } else if (type == 2) {
      int x;
      cin >> x;
      treap.erase(x);
    } else {
      int l, r;
      cin >> l >> r;
      cout << treap.get(l, r) << endl;
    }
  }
  return 0;
}
```

Generic persistent compressed lazy segment tree

```
struct Seg {
  ll sum = 0;
  void recalc(const Seg &lhs_seg, int lhs_len,
    const Seg &rhs_seg, int rhs_len) {
    sum = lhs_seg.sum + rhs_seg.sum;                                  7684
  }
} __attribute__((packed));
struct Lazy {
  ll add;
  ll assign_val; // LLONG_MIN if no assign;
  void init() {
    add = 0;
    assign_val = LLONG_MIN;
  }
  Lazy() { init(); }                                                  2819
  void split(Lazy &lhs_lazy, Lazy &rhs_lazy, int len) {
    lhs_lazy = *this;
    rhs_lazy = *this;
    init();
  }
  void merge(Lazy &oth, int len) {                                    8470
    if (oth.assign_val != LLONG_MIN) {
      add = 0;
      assign_val = oth.assign_val;
    }
    add += oth.add;
  }
  void apply_to_seg(Seg &cur, int len) const {
    if (assign_val != LLONG_MIN) {                                    7562
      cur.sum = len * assign_val;
    }
    cur.sum += len * add;
  }
} __attribute__((packed));                                           %7609
struct Node { // Following code should not need to be
              // modified
  int ver;
  bool is_lazy = false;
  Seg seg;
  Lazy lazy;
  Node *lc = NULL, *rc = NULL;
  void init() {
    if (!lc) {
      lc = new Node{ver};                                             6808
      rc = new Node{ver};
    }
  }
  Node *upd(
    int L, int R, int l, int r, Lazy &val, int tar_ver) {
    if (ver != tar_ver) {
      Node *rep = new Node(*this);                                    8874
      rep->ver = tar_ver;
      return rep->upd(L, R, l, r, val, tar_ver);
    }
    if (L >= l && R <= r) {
      val.apply_to_seg(seg, R - L);
      lazy.merge(val, R - L);                                         0020
      is_lazy = true;
    } else {
      init();
      int M = (L + R) / 2;
      if (is_lazy) {
        Lazy l_val, r_val;
        lazy.split(l_val, r_val, R - L);
        lc = lc->upd(L, M, L, M, l_val, ver);
        rc = rc->upd(M, R, M, R, r_val, ver);
        is_lazy = false;                                              9948
      }
      Lazy l_val, r_val;
      val.split(l_val, r_val, R - L);
      if (l < M) lc = lc->upd(L, M, l, r, l_val, ver);
      if (M < r) rc = rc->upd(M, R, l, r, r_val, ver);
      seg.recalc(lc->seg, M - L, rc->seg, R - M);
    }                                                                 8245
    return this;
  }
  void get(int L, int R, int l, int r, Seg *&lft_res,
    Seg *&tmp, bool last_ver) {
    if (L >= l && R <= r) {
      tmp->recalc(*lft_res, L - l, seg, R - L);
      swap(lft_res, tmp);                                             8394
    } else {
      init();
      int M = (L + R) / 2;
      if (is_lazy) {
        Lazy l_val, r_val;
        lazy.split(l_val, r_val, R - L);
        lc = lc->upd(L, M, L, M, l_val, ver + last_ver);
        lc->ver = ver;                                                2185
        rc = rc->upd(M, R, M, R, r_val, ver + last_ver);
        rc->ver = ver;
        is_lazy = false;
      }
      if (l < M)
        lc->get(L, M, l, r, lft_res, tmp, last_ver);
      if (M < r)                                                      9755
```

```
      rc->get(M, R, l, r, lft_res, tmp, last_ver);
    }
  }
} __attribute__((packed));
struct SegTree { // indexes start from 0, ranges are
                 // [beg, end)
  vector<Node *> roots; // versions start from 0
  int len;                                              4873
  SegTree(int _len) : len(_len) {
    roots.push_back(new Node{0});
  }
  int upd(
    int l, int r, Lazy &val, bool new_ver = false) {
    Node *cur_root = roots.back()->upd(                 3935
      0, len, l, r, val, roots.size() - !new_ver);
    if (cur_root != roots.back())
      roots.push_back(cur_root);
    return roots.size() - 1;                            1461
  }
  Seg get(int l, int r, int ver = -1) {
    if (ver == -1) ver = roots.size() - 1;
    Seg seg1, seg2;
    Seg *pres = &seg1, *ptmp = &seg2;
    roots[ver]->get(                                    7894
      0, len, l, r, pres, ptmp, roots.size() - 1);
    return *pres;
  }
};                                                      %3905
int main() {
  int n, m; // solves Mechanics Practice LAZY
  cin >> n >> m;
  SegTree seg_tree(1 << 17);
  for (int i = 0; i < n; ++i) {
    Lazy tmp;
    scanf("%lld", &tmp.assign_val);
    seg_tree.upd(i, i + 1, tmp);
  }
  for (int i = 0; i < m; ++i) {
    int o;
    int l, r;
    scanf("%d %d %d", &o, &l, &r);
    --l;
    if (o == 1) {
      Lazy tmp;
      scanf("%lld", &tmp.add);
      seg_tree.upd(l, r, tmp);
    } else if (o == 2) {
      Lazy tmp;
      scanf("%lld", &tmp.assign_val);
      seg_tree.upd(l, r, tmp);
    } else {
      Seg res = seg_tree.get(l, r);
      printf("%lld\n", res.sum);
    }
  }
}
```

Templated HLD O(M(n) log n) per query

```
class dummy {
 public:
  dummy() {}
  dummy(int, int) {}
  void set(int, int) {}
  int query(int left, int right) {
    cout << this << ' ' << left << ' ' << right << endl;
  }                                                     6131
};                                                      %6131
/* T should be the type of the data stored in each
 * vertex; DS should be the underlying data structure
 * that is used to peform the group operation. It should
 * have the following methods:
 * * DS () - empty constructor
 * * DS (int size, T initial) - constructs the structure
 * with the given size, initially filled with initial.
 * * void set (int index, T value) - set the value at
 * index `index` to `value`
 * * T query (int left, int right) - return the "sum" of
 * elements between left and right, inclusive.
 */
template <typename T, class DS>
class HLD {
  int vertexc;
  vector<int> *adj;
  vector<int> subtree_size;
  DS structure;
  DS aux;                                               7513
  void build_sizes(int vertex, int parent) {
    subtree_size[vertex] = 1;
    for (int child : adj[vertex]) {
      if (child != parent) {                            7279
        build_sizes(child, vertex);
        subtree_size[vertex] += subtree_size[child];
      }
    }
  }
  int cur;
  vector<int> ord;
  vector<int> chain_root;                               3893
  vector<int> par;
  void build_hld(
    int vertex, int parent, int chain_source) {
    cur++;
    ord[vertex] = cur;
    chain_root[vertex] = chain_source;                  0708
    par[vertex] = parent;
    if (adj[vertex].size() > 1 ||
        (vertex == 1 && adj[vertex].size() == 1)) {
      int big_child, big_size = -1;                     7661
      for (int child : adj[vertex]) {
        if ((child != parent) &&
            (subtree_size[child] > big_size)) {
          big_child = child;
          big_size = subtree_size[child];               7393
        }
      }
      build_hld(big_child, vertex, chain_source);
      for (int child : adj[vertex]) {
        if ((child != parent) && (child != big_child))
          build_hld(child, vertex, child);              8363
      }
    }
  }
public:
  HLD(int _vertexc) {
    vertexc = _vertexc;
    adj = new vector<int>[vertexc + 5];
  }
  void add_edge(int u, int v) {
    adj[u].push_back(v);                                8422
    adj[v].push_back(u);
  }
  void build(T initial) {
    subtree_size = vector<int>(vertexc + 5);
    ord = vector<int>(vertexc + 5);                     6999
    chain_root = vector<int>(vertexc + 5);
    par = vector<int>(vertexc + 5);
    cur = 0;
    build_sizes(1, -1);
    build_hld(1, -1, 1);                                7373
    structure = DS(vertexc + 5, initial);
    aux = DS(50, initial);
  }
  void set(int vertex, int value) {
    structure.set(ord[vertex], value);                  8170
  }
  T query_path(int u,
    int v) { /* returns the "sum" of the path u->v */
    int cur_id = 0;
    while (chain_root[u] != chain_root[v]) {
      if (ord[u] > ord[v]) {
        cur_id++;
        aux.set(cur_id,                                 3518
          structure.query(ord[chain_root[u]], ord[u]));
        u = par[chain_root[u]];
      } else {
        cur_id++;
        aux.set(cur_id,
          structure.query(ord[chain_root[v]], ord[v]));
        v = par[chain_root[v]];                         3845
      }
    }
    cur_id++;
    aux.set(cur_id, structure.query(min(ord[u], ord[v]),
                      max(ord[u], ord[v])));
    return aux.query(1, cur_id);                        7150
  }                                                     %7150
  void print() {
    for (int i = 1; i <= vertexc; i++)
      cout << i << ' ' << ord[i] << ' ' << chain_root[i]
           << ' ' << par[i] << endl;
  }
};
```

```
int main() {
  int vertexc;
  cin >> vertexc;
  HLD<int, dummy> hld(vertexc);
  for (int i = 0; i < vertexc - 1; i++) {
    int u, v;
    cin >> u >> v;
    hld.add_edge(u, v);
  }
  hld.build(0);
  hld.print();
  int queryc;
  cin >> queryc;
  for (int i = 0; i < queryc; i++) {
    int u, v;
    cin >> u >> v;
    hld.query_path(u, v);
    cout << endl;
  }
}
```

Splay Tree + Link-Cut O(NlogN)

```
struct Tree *treev;
struct Tree {
  struct T {
    int i;
    constexpr T() : i(-1) {}
    T(int _i) : i(_i) {}
    operator int() const { return i; }                         5021
    explicit operator bool() const { return i != -1; }
    Tree *operator->() { return treev + i; }
  };
  T c[2], p;
  /* insert monoid here */
  T link;
  Tree() {
    /* init monoid here */
    link = -1;                                                 7237
  }
};
using T = Tree::T;                                             5875
constexpr T NIL;
void update(T t) { /* recalculate the monoid here */
}
void propagate(T t) {
  assert(t);
  for (T c : t->c)
    if (c) c->link = t->link;                                  8249
  /* lazily propagate updates here */
}
void lazy_reverse(T t) { /* lazily reverse t here */
}
T splay(T n) {
  for (;;) {
    propagate(n);
    T p = n->p;
    if (p == NIL) break;                                       5925
    propagate(p);
    ll px = p->c[1] == n;                                      2795
    assert(p->c[px] == n);                                     0722
    T g = p->p;
    if (g == NIL) { /* zig */
      p->c[px] = n->c[px ^ 1];
      p->c[px]->p = p;                                         2987
      n->c[px ^ 1] = p;
      n->c[px ^ 1]->p = n;                                     7572
      n->p = NIL;
      update(p);                                               4253
      update(n);
      break;
    }
    propagate(g);
    ll gx = g->c[1] == p;
    assert(g->c[gx] == p);                                     2867
    T gg = g->p;
    ll ggx = gg && gg->c[1] == g;                              8789
    if (gg) assert(gg->c[ggx] == g);                           8979
    if (gx == px) { /* zig zig */
      g->c[gx] = p->c[gx ^ 1];
      g->c[gx]->p = g;                                         7232
      p->c[gx ^ 1] = g;
      p->c[gx ^ 1]->p = p;                                     6588
      p->c[gx] = n->c[gx ^ 1];
      p->c[gx]->p = p;                                         8474
      n->c[gx ^ 1] = p;
      n->c[gx ^ 1]->p = n;
    } else { /* zig zag */
      g->c[gx] = n->c[gx ^ 1];                                 3340
      g->c[gx]->p = g;                                         6351
      n->c[gx ^ 1] = g;
      n->c[gx ^ 1]->p = n;
      p->c[gx ^ 1] = n->c[gx];                                 2638
      p->c[gx ^ 1]->p = p;
      n->c[gx] = p;
      n->c[gx]->p = n;                                         8629
    }
    if (gg) gg->c[ggx] = n;                                    9629
    n->p = gg;
    update(g);
    update(p);
    update(n);
    if (gg) update(gg);                                        3586
  }
  return n;
}
T extreme(T t, int x) {                                        6751
  while (t->c[x]) t = t->c[x];                                 2290
  return t;
}
T set_child(T t, int x, T a) {
  T o = t->c[x];
  t->c[x] = a;                                                 4115
  update(t);
  o->p = NIL;
  a->p = t;                                                    7177
  return o;
}                                                    %8008%4627%1017
/************* Link-Cut Tree: **************/
T expose(T t) {
  set_child(splay(t), 1, NIL);
  T leader = splay(extreme(t, 0));
  if (leader->link == NIL) return t;
  set_child(splay(leader), 0, expose(leader->link));
  return splay(t);
}
void link(T t, T p) {
  assert(t->link == NIL);
  t->link = p;
}
T cut(T t) {
  T p = t->link;
  if (p) expose(p);
  t->link = NIL;
  return p;
}
void make_root(T t) {
  expose(t);
  lazy_reverse(extreme(splay(t), 0));
}                                                        %7295%6269
```