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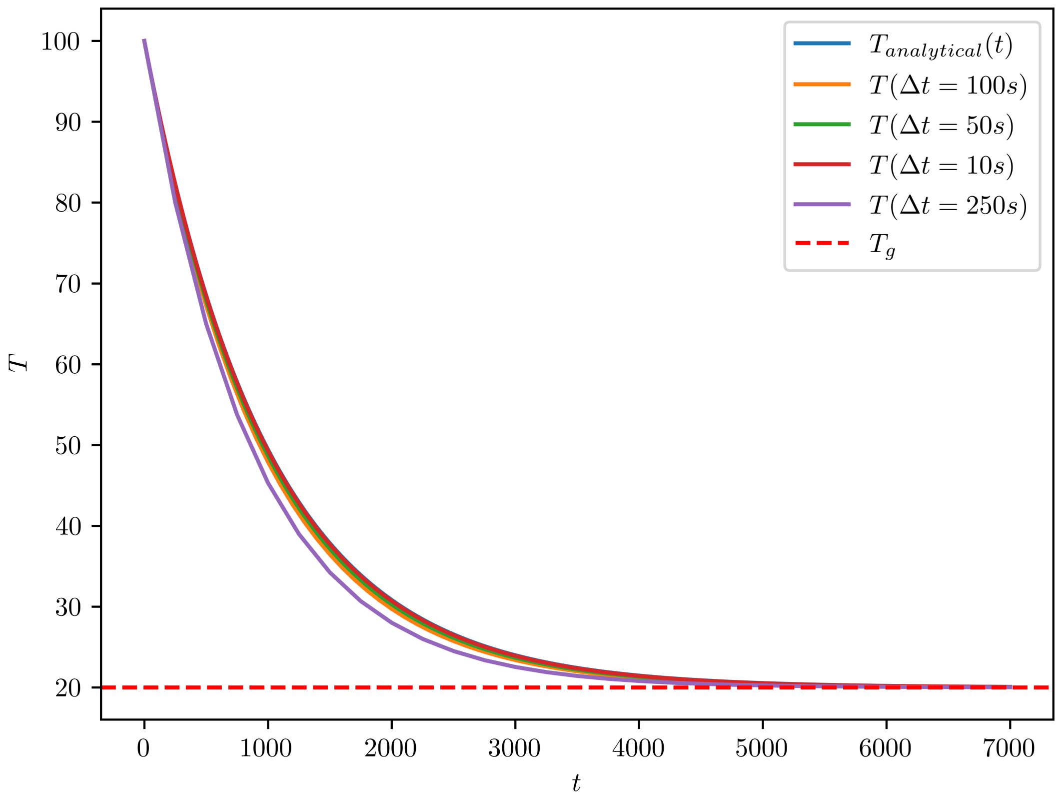
Kurss "Zinātniskā programmēšana fiziķiem"

Laboratorijas darbs

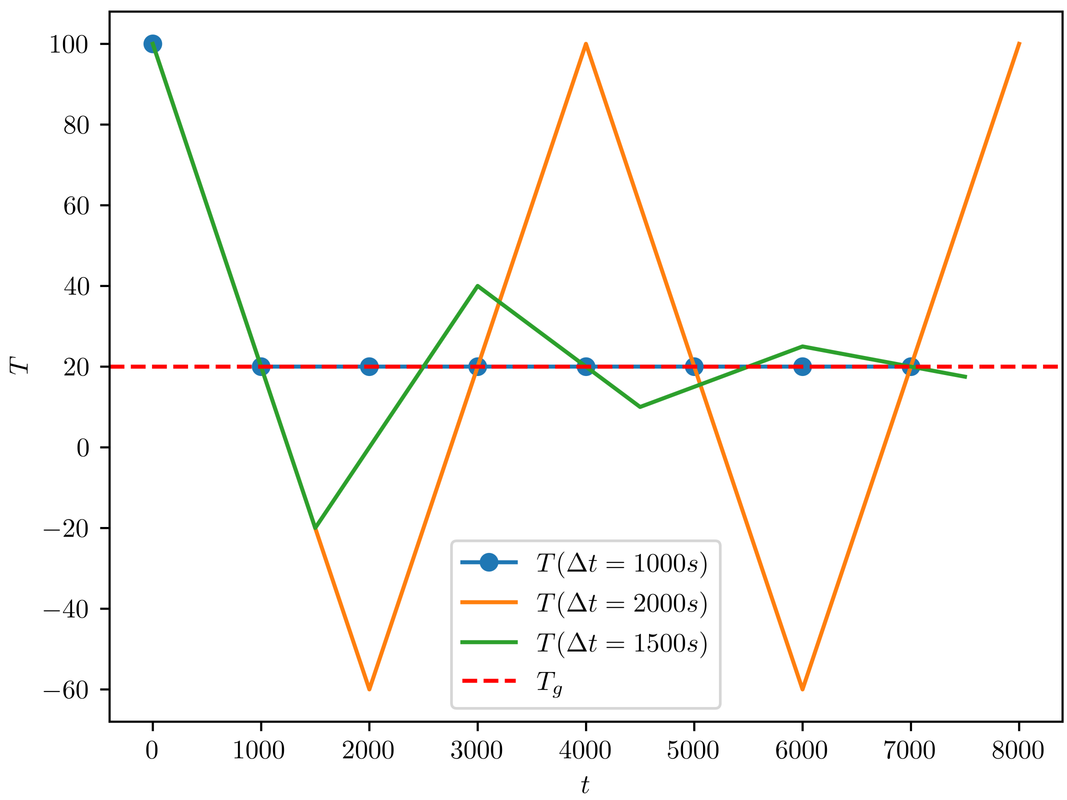
"Siltumvadīšana"

Uzsildīta objekta atdzišana

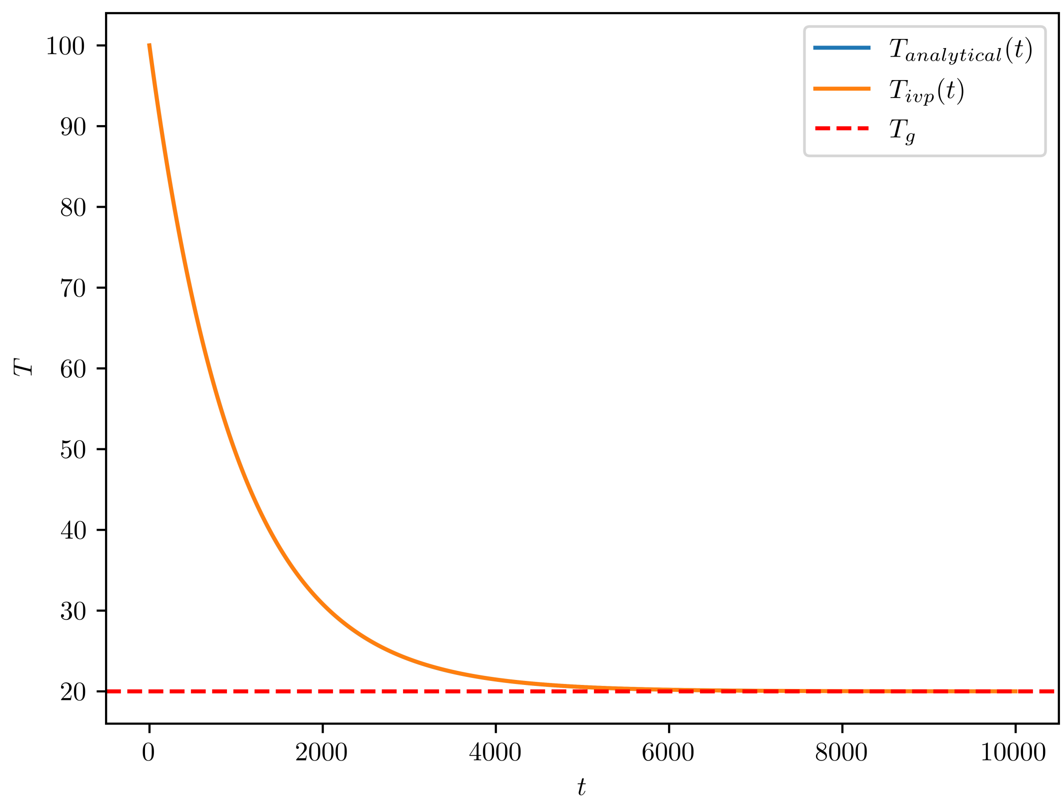
1. Attēls siltumvadīšanas vienādojuma atrisinājumam pie dažādām parametra Δt vērtībām un analītiskais atrisinājums.



2. Attēls siltumvadīšanas vienādojuma atrisinājumam, kurā novērojamas skaitliskas nestabilitātes.



3. Attēls siltumvadīšanas vienādojuma atrisinājumam ar `solve_ivp` un analītiskais atrisinājums.



4. **PAMATOJUMS:** Izmantotais kods.

```
# initial conditions
y0 = [100, 20, 0.001]
T_0 = 100
T_g = 20
coefficient = 1/1000
# solve ivp solution # 3rd sort of point of the protocol

def dTdt(t, T, T_g=T_g, k=coefficient):
    return -k*(T - T_g)

# solve
sol = solve_ivp(dTdt, [0, 10000], [T_0], t_eval=np.linspace(0, 10000, 100000))

# plot
fig, ax = plt.subplots()
ax.plot(sol.t, sol.y[0], label=r'$T(t)$')
ax.set_xlabel(r'$t$')
ax.set_ylabel(r'$T$')
ax.axhline(y=20, color='r', linestyle='--', label=r'$T_g$')
ax.legend()
plt.show()

# eulers method # 1st and 2nd point of the protocol

dt = 1000
dt1 = 750
dt2 = 500
dt3 = 250
dt4 = 100
dt5 = 50
dt6 = 10
dt7 = 2000
dt8 = 1500
t = np.arange(0, 7000+dt, dt)
t1 = np.arange(0, 7000+dt1, dt1)
t2 = np.arange(0, 7000+dt2, dt2)
t3 = np.arange(0, 7000+dt3, dt3)
t4 = np.arange(0, 7000+dt4, dt4)
t5 = np.arange(0, 7000+dt5, dt5)
t6 = np.arange(0, 7000+dt6, dt6)
t7 = np.arange(0, 7000+dt7, dt7)
t8 = np.arange(0, 7000+dt8, dt8)
T_0 = 100

T_dt2 = np.zeros(len(t2))
T_dt2[0] = T_0
```

```
T_dt1 = np.zeros(len(t1))
T_dt1[0] = T_0

T_dt = np.zeros(len(t))
T_dt[0] = T_0

T_dt3 = np.zeros(len(t3))
T_dt3[0] = T_0

T_dt4 = np.zeros(len(t4))
T_dt4[0] = T_0

T_dt5 = np.zeros(len(t5))
T_dt5[0] = T_0

T_dt6 = np.zeros(len(t6))
T_dt6[0] = T_0

T_7d7 = np.zeros(len(t7))
T_7d7[0] = T_0

T_8d8 = np.zeros(len(t8))
T_8d8[0] = T_0

for j in range(0, len(t)-1):
    T_dt[j+1] = T_dt[j] + dt*dTdt(t[j], T_dt[j])

for i in range(0, len(t2)-1):
    T_dt2[i+1] = T_dt2[i] + dt2*dTdt(t2[i], T_dt2[i])

for k in range(0, len(t1)-1):
    T_dt1[k+1] = T_dt1[k] + dt1*dTdt(t1[k], T_dt1[k])

for l in range(0, len(t3)-1):
    T_dt3[l+1] = T_dt3[l] + dt3*dTdt(t3[l], T_dt3[l])

for m in range(0, len(t4)-1):
    T_dt4[m+1] = T_dt4[m] + dt4*dTdt(t4[m], T_dt4[m])

for n in range(0, len(t5)-1):
    T_dt5[n+1] = T_dt5[n] + dt5*dTdt(t5[n], T_dt5[n])

for o in range(0, len(t6)-1):
    T_dt6[o+1] = T_dt6[o] + dt6*dTdt(t6[o], T_dt6[o])

for p in range(0, len(t7)-1):
    T_7d7[p+1] = T_7d7[p] + dt7*dTdt(t7[p], T_7d7[p])
```

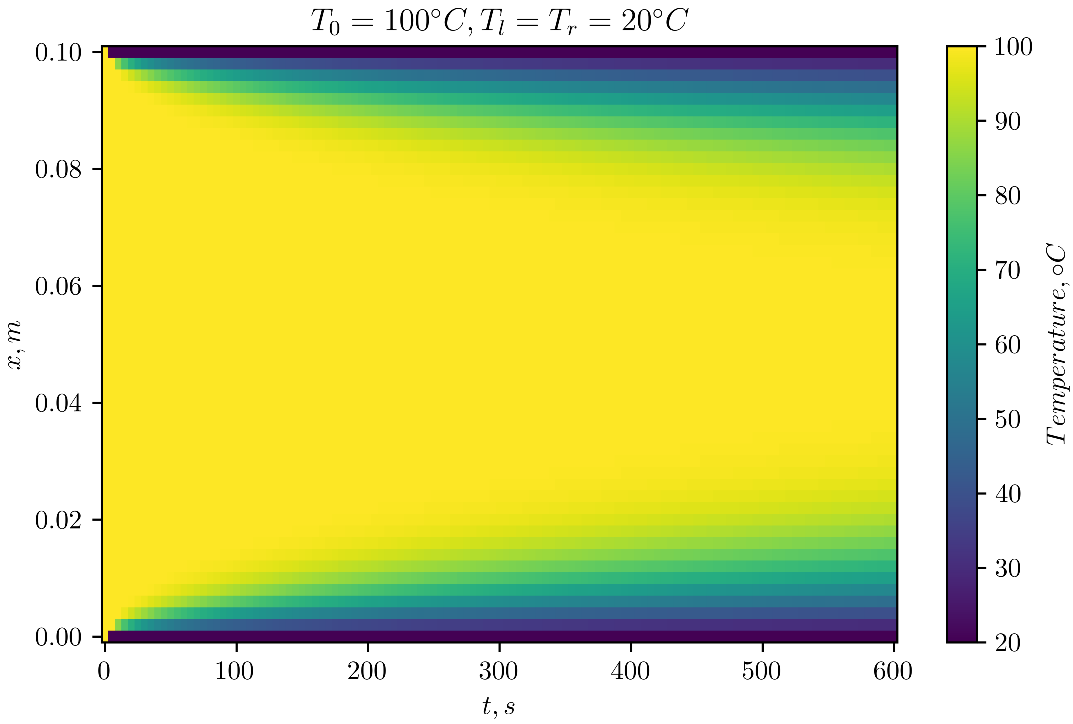
```
for q in range(0, len(t8)-1):
    T_8d8[q+1] = T_8d8[q] + dt8*dTdt(t8[q], T_8d8[q])

t_ana = np.linspace(0, 7000, 100000)
T_anal = (T_0 - T_g) * np.exp(-coefficient * t_ana) + T_g

fig, ax = plt.subplots()
# ax.plot(t_ana, T_anal, label=r'$T_{analytical}(t)$')
# ax.plot(t4, T_dt4, label=r'$T(\Delta{t=100}s)$')
# ax.plot(t5, T_dt5, label=r'$T(\Delta{t=50}s)$')
# ax.plot(t6, T_dt6, label=r'$T(\Delta{t=10}s)$')
ax.plot(t, T_dt, label=r'$T(\Delta{t=1000}s)$', marker = 'o')
# ax.plot(t3, T_dt3, label=r'$T(\Delta{t=250}s)$')
# ax.plot(t2, T_dt2, label=r'$T(\Delta{t=500}s)$')
# ax.plot(t1, T_dt1, label=r'$T(\Delta{t=750}s)$')
ax.plot(t7, T_7d7, label=r'$T(\Delta{t=2000}s)$')
ax.plot(t8, T_8d8, label=r'$T(\Delta{t=1500}s)$',)
ax.set_xlabel(r'$t$')
ax.set_ylabel(r'$T$')
ax.axhline(y=20, color='r', linestyle='--', label=r'$T_g$')
ax.legend()
plt.savefig(os.path.join(path, '1.2.png'), dpi=1000, bbox_inches='tight')
plt.show()
# analytical solution - shit,
currently ### to be fixed # 3rd point of the protocol # not shit anymore

t_ana = np.linspace(0, 7000, 10000)
T_anal = (T_0 - T_g) * np.exp(-coefficient * t_ana) + T_g

fig, ax = plt.subplots()
ax.plot(t_ana, T_anal, label=r'$T_{analytical}(t)$')
# ax.plot(t2, T_dt2, label=r'$T(t)_{broken}$') # he just like me :(
ax.plot(sol.t, sol.y[0], label=r'$T_{ivp}(t)$')
ax.set_xlabel(r'$t$')
ax.set_ylabel(r'$T$')
ax.axhline(y=20, color='r', linestyle='--', label=r'$T_g$')
ax.legend()
plt.savefig(os.path.join(path, '1.3.png'), dpi=1000, bbox_inches='tight')
plt.show()
```

Siltumvadīšanas vienādojums

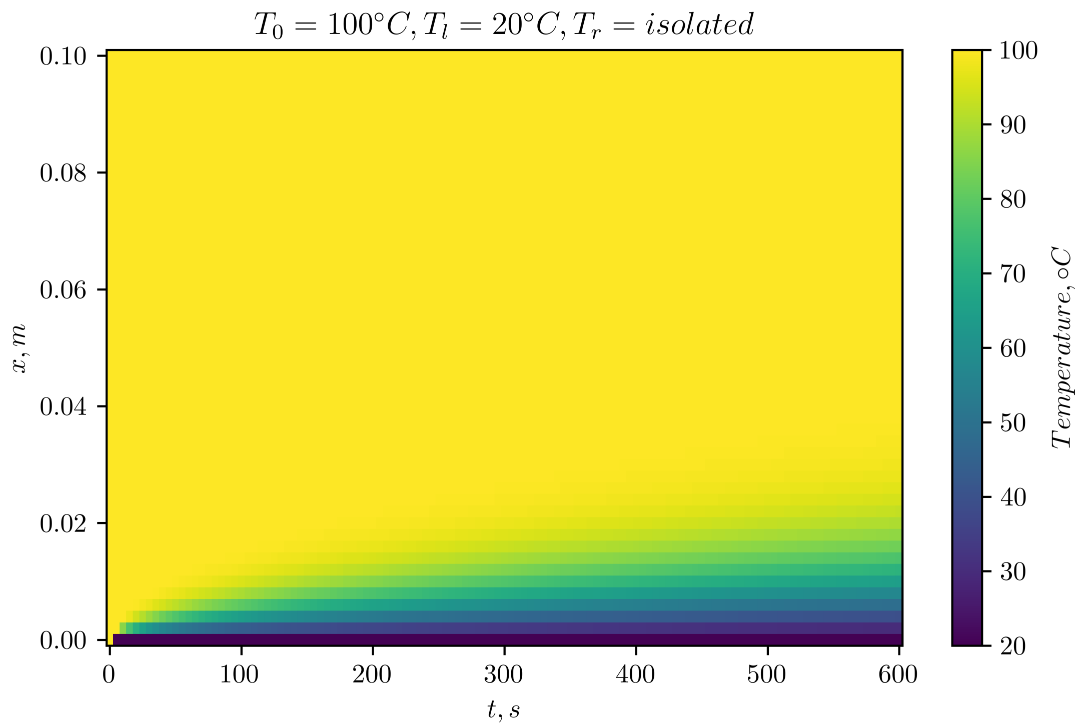
1. Izmantotās parametru vērtības un krūzītes izmēri.

```
rho = 1000.0          # kg/m^3, density of water
cp = 4186.0           # J/(kg·K), specific heat capacity of water
k = 0.5918            # W/(m·K), thermal conductivity of water
```

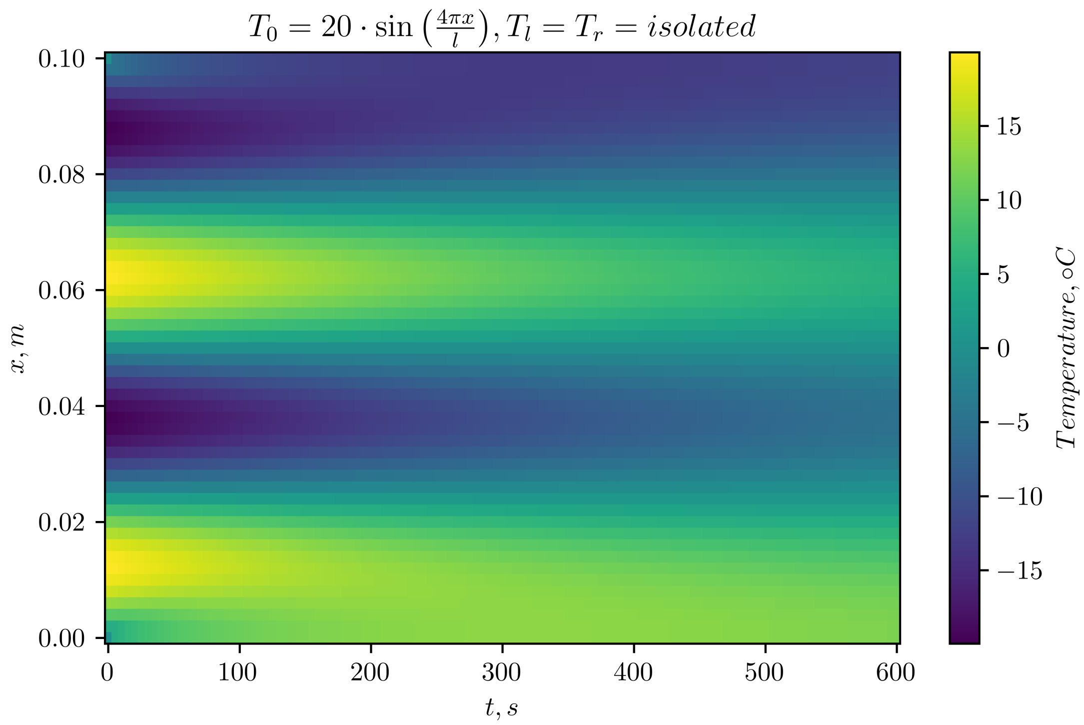
2. Atrisināt siltumvadīšanas vienādojumu stienī, kur sākuma temperatūra ir 100°C, bet temperatūra uz stieņa galiem ir 20°C.
   Attēls problēmas atrisinājumam.



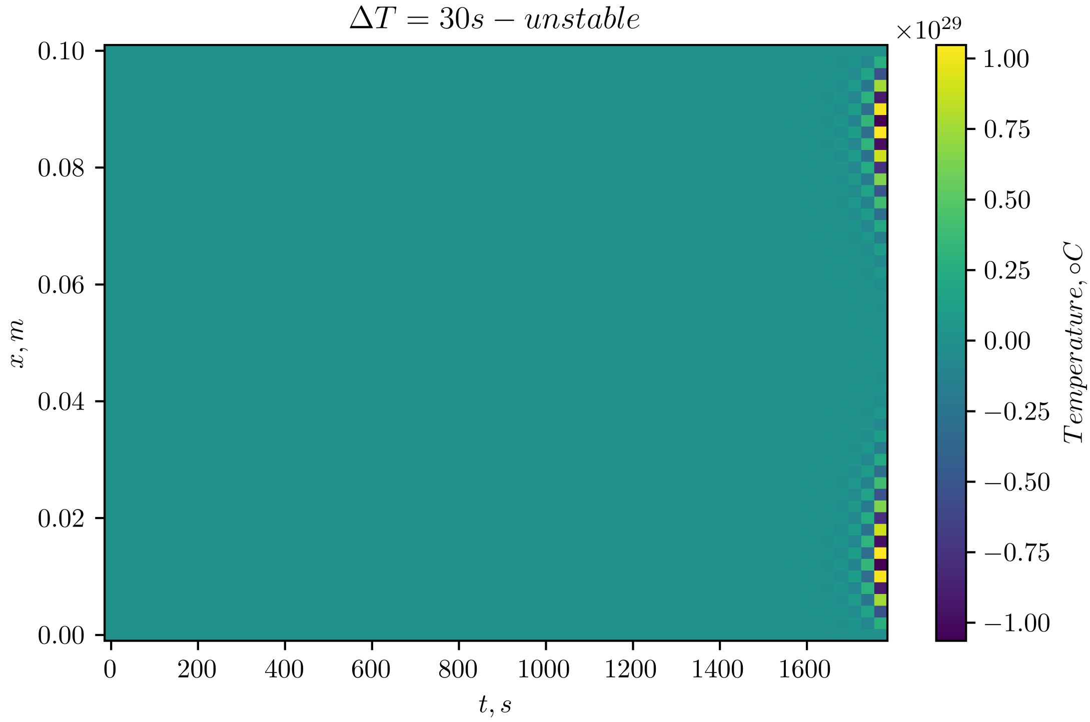
3. Atrisināt siltumvadīšanas vienādojumu stienī, kur sākuma temperatūra ir 100°C, bet temperatūra uz viena stieņa gala ir 20°C, bet otrs gals ir izolēts.
   Attēls problēmas atrisinājumam.



4. Atrisināt siltumvadīšanas vienādojumu stienī, kur sākuma temperatūra ir $20 \cdot \sin\left(\frac{4\pi x}{l}\right)$, bet abi stieņa gali ir izolēti.
   Attēls problēmas atrisinājumam.



5. Grafiks atrisinājumam ar skaitliskām nestabilitātēm.



6. **PAMATOJUMS:** Izmantotais kods siltumvadīšanas vienādojuma atrisināšanai.

```
alpha = k / (rho * cp)
L = 0.10
N = 50
dx = L / N

# stability criterion for explicit FTCS: dt <= dx^2/(2*alpha)
dt_stable = 5.0
r = alpha * dt_stable / dx**2
(should be <= 0.5 for stability)

t_total = 600.0
simulated time (s)
steps = int(t_total / dt_stable) + 1
times = np.linspace(0, t_total, steps)

x = np.linspace(0, L, N + 1)

def lifeisasimulation(initial_T, bc_left, bc_right, dt, steps):
    """
    Simulate 1-D heat conduction with explicit FTCS.
    bc_left, bc_right:
        - 'fixed': value (float) for Dirichlet
        - 'insulated': Neumann zero-flux
    """
    T = initial_T.copy()
    T_record = np.empty((steps, N + 1))
```

```
    T_record[0] = T

    r_local = alpha * dt / dx**2

    for n in range(1, steps):
        T_new = T.copy()

        # interior points
        T_new[1:-1] = T[1:-1] + r_local * (T[2:] - 2.0 * T[1:-1] + T[:-2])

        # left boundary # lefty hehehe (:<
        if bc_left[0] == 'fixed':
            T_new[0] = bc_left[1]
        elif bc_left[0] == 'insulated':
            T_new[0] = T_new[1]    # T/x = 0 → T0 = T1

        # right boundary # righty hahaha >:)
        if bc_right[0] == 'fixed':
            T_new[-1] = bc_right[1]
        elif bc_right[0] == 'insulated':
            T_new[-1] = T_new[-2]

        T = T_new
        T_record[n] = T

    return T_record

# 2nd point of the protocol
initial_T_case2 = np.full(N + 1, 100.0)
T_case2 = lifeisasimulation(initial_T_case2, ('fixed', 20.0),
('fixed', 20.0), dt_stable, steps)

# 3rd point of the protocol
initial_T_case3 = np.full(N + 1, 100.0)
T_case3 = lifeisasimulation(initial_T_case3, ('fixed', 20.0),
('insulated', None), dt_stable, steps)

# my sanity is long gone, 4th point of the protocol
initial_T_case4 = 20.0 * np.sin(4 * np.pi * x / L)
T_case4 = lifeisasimulation(initial_T_case4, ('insulated',
None), ('insulated', None), dt_stable, steps)

# what is blud waffling about, 6th point of the protocol
dt_unstable = 30.0
steps_unstable = 60
T_unstable = lifeisasimulation(initial_T_case2, ('fixed',
20.0), ('fixed', 20.0), dt_unstable, steps_unstable)
```

Aprēķinu apakšprogrammas noformēšana

1. Izmantotais kods.

```
class Heat1D:
"""
Explicit FTCS solver for 1-D heat-conduction equation.
Results returned as pandas.DataFrame (rows = time, columns = x).
"""
def __init__(self, L, N, dt, t_total, rho, cp, k):  # length,
cells, time step, total time, density, coefficient, conductivity #
self.L, self.N = L, N
self.dx = L / N
self.dt = dt
self.t_total = t_total
self.steps = int(np.ceil(t_total / dt)) + 1

self.rho, self.cp, self.k = rho, cp, k
self.alpha = k / (rho * cp)
self.r = self.alpha * dt / self.dx**2

if self.r > 0.5:
    raise ValueError(f"Unstable: r = {self.r:.3f} > 0.5 "
                     "(reduce dt or refine grid)")

# coordinates and time vector
self.x = np.linspace(0.0, L, N + 1)
self.t = np.linspace(0.0, t_total, self.steps)

# placeholders for IC/BC (initial condition, boundary condition)
self.T0 = np.zeros_like(self.x)
self.bc_left = ('fixed', 0.0)      # default Dirichlet 0 °C
self.bc_right = ('fixed', 0.0)

# ---------- setters ----------------------------------------------------
def set_initial(self, T0):
"""T0: scalar, 1-D array of size N+1, or callable f(x)."""
if np.isscalar(T0):
    self.T0[:] = T0
elif callable(T0):
    self.T0[:] = T0(self.x)
else:
    T0 = np.asarray(T0, dtype=float)
    if T0.shape != self.x.shape:
        raise ValueError("T0 length must be N+1")
    self.T0[:] = T0

def set_boundary(self, left, right):
"""
```

```
left, right : tuple ('fixed', value) or ('insulated', None)
"""
self.bc_left = left
self.bc_right = right

# ---------- core solver --------------------------------------------
def solve(self):
T = self.T0.copy()
out = np.empty((self.steps, self.N + 1), dtype=float)
out[0] = T

for n in range(1, self.steps):
    T_new = T.copy()
    # interior nodes
    T_new[1:-1] = (T[1:-1] +
                   self.r * (T[2:] - 2.0*T[1:-1] + T[:-2]))

    # left BC
    if self.bc_left[0] == 'fixed':
        T_new[0] = self.bc_left[1]
    elif self.bc_left[0] == 'insulated':
        T_new[0] = T_new[1]

    # right BC
    if self.bc_right[0] == 'fixed':
        T_new[-1] = self.bc_right[1]
    elif self.bc_right[0] == 'insulated':
        T_new[-1] = T_new[-2]

    T = T_new
    out[n] = T

# build DataFrame: index=time, columns=x coordinate
df = pd.DataFrame(out, index=self.t, columns=self.x)
df.index.name = "time_s"
df.columns.name = "x_m"
return df
```

Izmantotā literatūra

[1] Bernard Knaepen and Yelyzaveta Velizhanina. One dimensional heat equation: Implicit methods, 2022. Part of the Jupyter Book "Solving Partial Differential Equations – MOOC".

[2] Bernard Knaepen and Yelyzaveta Velizhanina. Numerical methods for partial differential equations (solving pde mooc), 2025. GitHub repository, MIT licence.

[3] John C. Tannehill, Dale A. Anderson, and Richard H. Pletcher. *Computational Fluid Mechanics and Heat Transfer*. Taylor & Francis, Washington, D.C., 2 edition, 1997.

[4] Wikipedia contributors. Ftcs scheme, 2025.