Gender as a Spherical Topological Space

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Abstract

We model gender as a system of socially realized categories that are defined by aggregating observers' individual perceptions of a subject's gender. In our Axis Model, we introduce the concept that gender lies on a woman–man line. Infinitely many gender categories can be created by midpoint-splitting when existing categories are descriptively insufficient. The Sphere Model generalizes this concept to a spherical space. In the Sphere Model, categories drift and adopt new specifications. The framework shows that under minimal assumptions and through natural social forces, gender categories proliferate and reconfigure over time. The theory was developed by examining gender, but applies to any consensus-driven system that discretizes continuous variation.

1 Introduction

Gender theory employs varied language to describe gender categories. As a result, debates often stall on definitions, and disagreement is as much about terminology as about substance. Without a structural framework, gender conversations risk becoming circular, fragmented, and confused. Rather than providing a prescriptive model of what gender should or could be, we focus on a descriptive approach that aggregates perceptions, ingesting individuals' perceptions of gender without making judgments about the validity of those perceptions.

To facilitate clarity, we model gender as a topological space. We describe how these categories emerge and drift over time. The approach captures both the proliferation of new genders and the shifting of existing ones.

Contributions

1. **Axis Model:** A scalar model on $[0, 1]$ that formalizes midpoint splitting, showing that infinitely many gender categories can arise.

2. **Sphere Model:** A spherical model on S^2 that generalizes the axis, introducing universes of midpoints, collapse to single points, and collective drift.

3. **Extensions:** The framework generalizes beyond gender to other social categories with certain features.

1.1   Assumptions and design choices

We use a model of gender that renders gender categories maximally descriptively useful. Genders are socially defined categories that are used to facilitate clarity when communicating about certain collections of behaviors and expectations. Each assumption reflects observed properties of gender.

- **Externalism:** Gender is socially realized, defined by aggregated observer judgments. This captures the relational nature of gender and places minimal importance on self-identification. The model avoids claims about the validity of gender self-identification or any relationship or lack of relationship between sex and gender[1], instead orienting evaluation to gender as it exists in society. It is possible for an observer to heavily weigh a person's self-identified gender or their sex in their evaluation, but no such commitments are privileged in the model.

- **Gender cue continuity:** Observers examine gender cues, which are continuous scores that influence their perception of a person's gender. The universe of gender cues is infinite, including physical indicators, social behaviors, and associations.

- **Observer heterogeneity:** Each observer has a distinct scoring function based on their own system for categorizing people into genders.

- **Equal influence baseline:** Consensus for gender categorization is the average of all observers. Individual observers are not formally privileged by the model, but influential observers have the ability to create new gender cues for others or influence others' scoring functions.

- **Descriptive usefulness:** Gender categories are assumed to tend toward maximizing distinctiveness between categories. This is based on the assumption that social categories are meant to be descriptively useful.

- **Emergence:** When existing categories are not sufficient to usefully describe significant subsets of people, a new category emerges.

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[1]Some examples and commentary acknowledge gender stereotypes that are rooted in a relationship between sex and gender, but the model does not require these examples.

2 Axis model

We model gender as a socially realized quantity, externally defined through observers. This framework consists of observable traits, observer-specific scoring functions, consensus values, fuzzy classification, and a midpoint-splitting process. The Axis Model situates gender on a woman–man line segment.

2.1 Cue space: observable traits

Let d denote the number of cues relevant to gender perception. Each person is represented by a *cue vector*

$$c = (c_1, c_2, \ldots, c_d) \in C = [0,1]^d,$$

where $c_j = 0$ is maximally woman-coded, $c_j = 1$ maximally man-coded, and intermediate values represent gradations.

2.2 Observers and scoring functions

Let $\mathcal{O}$ be the set of observers. Each observer $o \in \mathcal{O}$ evaluates an individual's cues $c \in [0,1]^d$ through a personal scoring function

$$s_o : [0,1]^d \to [0,1],$$

which assigns a continuous gender score. Here $s_o(c) = 0$ corresponds to the ideal "woman," $s_o(c) = 1$ to the ideal "man," and intermediate values indicate degrees of perceived ambiguity.[2]

Because gender perception depends on combinations of traits, s_o is generally nonlinear: the same cue can carry different gender implications depending on the presence or absence of others (for instance, long hair may be interpreted differently when paired with a deep voice). Formally, we may view

$$s_o(c) = f_o(c_1, \ldots, c_d),$$

where f_o is continuous but otherwise unconstrained, allowing for complex interactions among cues.

An overly simple illustrative case is a weighted average:

$$s_o(c) = \frac{\sum_{j=1}^{d} w_{oj}\, c_j}{\sum_{j=1}^{d} w_{oj}}, \qquad w_{oj} \geq 0,$$

where the weights w_{oj} express how strongly observer o considers each cue in their gender assignment.

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[2]Note that reduction to a one-dimensional space simplifies the model to facilitate clarity, but in the Sphere Model, the cue function may reduce to a two-dimensional space to preserve information.

2.3 Socially realized gender value

The consensus score is the average of all observer scores

$$\phi(c) = \frac{1}{|\mathcal{O}|} \sum_{o \in \mathcal{O}} s_o(c).$$

Thus $\phi(c) \in [0, 1]$ is the socially realized gender score.

2.4 Gender categories as labels

At time t, society recognizes a finite set of gender categories,

$$K_t \subseteq [0, 1].$$

Initially $K_0 = \{0, 1\}$, where 0 and 1 represent the categories "woman" and "man." As social perception encounters individuals whose consensus scores fall between existing labels, new categories emerge at midpoints, such as

$$K_1 = \{0, \tfrac{1}{2}, 1\},$$

where $\frac{1}{2}$ corresponds to the category "nonbinary."

Each individual's socially realized gender score $\phi(c)$ is then assigned to the closest recognized category:

$$A_t(c) = \arg \min_{\ell \in K_t} |\phi(c) - \ell|.$$

This captures how continuous variation in gender presentation is discretized by society into a finite set of recognized labels.

2.5 Midpoint-splitting rule

Over time, regions of the gender spectrum can become densely populated with individuals whose gender scores $\phi(c)$ fall between existing category labels and are thus not accurately represented by existing labels. When the distance between many individuals and their nearest labels becomes too large, the existing classification system loses descriptive usefulness.

Let $a < b$ be adjacent labels in K_t. For any individual with score $x = \phi(c)$ between them, define the distance to the nearest label as

$$d(x; a, b) = \min\{\, x - a, \; b - x \,\}.$$

We use $\tau > 0$ as a tolerance threshold. It represents the maximum acceptable ambiguity a social group is willing to tolerate before introducing a new label. Intuitively, τ measures how far an

individual can be from the nearest recognized categories before observers begin to feel that a new name is useful.

If many individuals fall within the interval (a, b) and their distances satisfy

$$d(x; a, b) > \tau,$$

then the existing categories are insufficient. A new category is introduced at the midpoint

$$m = \frac{a + b}{2},$$

and the label set is updated to

$$K_{t+1} = K_t \cup \{m\}.$$

This rule models how new genders emerge socially. They don't appear arbitrarily, but as the observer set's adaptive response to classification failure. The parameter τ can vary culturally: smaller τ corresponds to a society that demands finer categorical distinctions. For example, niche online communities fixated on hyper-specific genders have small τ values. A social group committed to a binary understanding of gender would have a high τ value and would not find it necessary to introduce new categories.

2.6   Limitations

The Axis Model assumes category meanings are fixed (what it means to be "woman-coded," "man-coded," etc.). In practice, these shift. Gender classification is not simple enough to be accurately represented by a two-dimensional model. The Axis Model is thus a base case.

3   Sphere model

The Axis Model is a special case of the Sphere Model, restricted to a single great circle. The spherical formulation removes privileged endpoints and allows categories to reconfigure continuously. The core mechanism of social gender assignment—aggregation of observers' cue-based judgments—remains unchanged. However, because the sphere introduces a two-dimensional topology, we can preserve information by reducing the d-dimensional cue matrix to a two-dimensional representation rather than to a single scalar value. This preserves the same intuition as in the Axis Model while accommodating the richer geometry of the sphere.

We represent gender as points on the unit sphere $S^2 \subset \mathbb{R}^3$. A sphere is chosen to represent the gender space due primarily to three qualities:

1. *Boundarylessness:* No edges, unlike $[0, 1]$.

2. *Rotational invariance:* SO(3) acts transitively.

3. *Geodesics:* The woman–man binary is a geodesic arc; $\theta(x, y) = \arccos(x \cdot y)$ defines distance.

3.1 Midpoint universes collapse to points

Supposed antipodes u, v represent "woman" and "man." The set of points equidistant from both is the equator

$$E = \{x \in S^2 : u \cdot x = v \cdot x = 0\}.$$

When there are only two categories, every point in E maximizes the geodesic distance from the existing ones. The space of potential new categories is therefore infinite—there is no distinguished position for a third point. The need for a category ideal forces the introduction of a third point somewhere on E.

3.2 Collective drift

Points $u_1, \ldots, u_k \in S^2$ drift to maximize their minimal pairwise separation:

$$u_1, \ldots, u_k \quad \approx \quad \arg \max_{x_1, \ldots, x_k \in S^2} \min_{i \neq j} \theta(x_i, x_j),$$

where $\theta(x_i, x_j)$ is the geodesic distance between points.

3.3 Instantiation

(A) Sphere with poles (man, woman) and equator E; nonbinary is a midpoint on E.

(B) The equator E collapses to a single category point: nonbinary.

(C) Collective drift: man, woman, and nonbinary spread to maximize minimal pairwise distance.
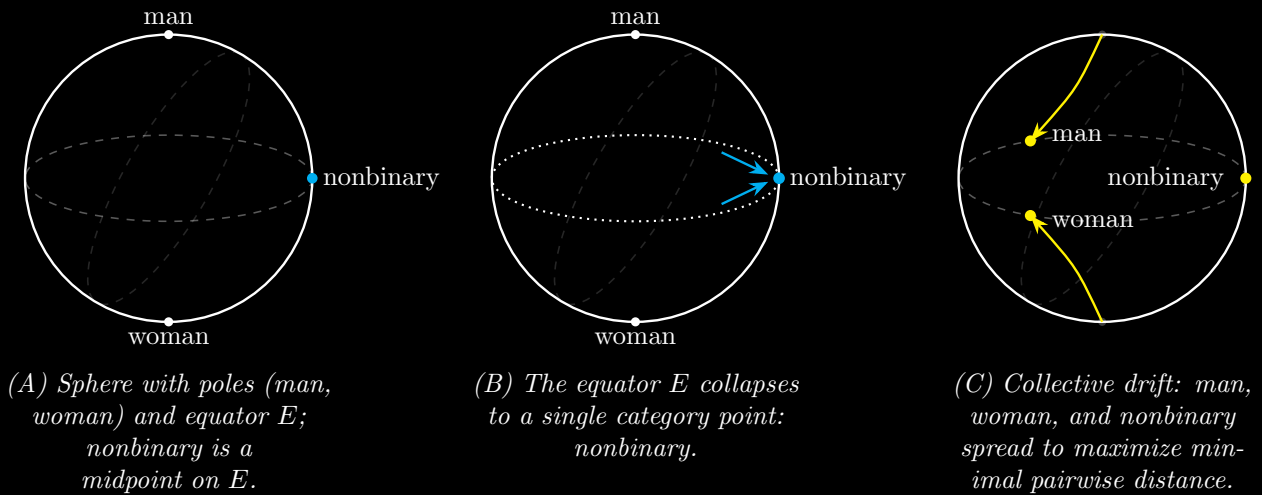
Figure 1: Emergence and drift on a sphere. (A) Poles (man, woman) with equator E and midpoint nonbinary. (B) The equator collapses to the nonbinary point. (C) All three labels co-move to maximize distinctiveness.

The Sphere Model is more easily understood when instantiated. Suppose there is a spherical space where the genders "woman" and "man" exist at the poles and "nonbinary" exists at the equator (see Figure 1A). Because "nonbinary" is initially defined in opposition to the categories "man" and "woman," it exists at all midpoints between the two (along the entire equator). The category lacks an ideal, as opposed to "man" and "woman," which have ideals represented by points.

As the term "nonbinary" is used to describe individuals, it is no longer defined only in opposition to the other genders, but begins to adopt an identity of its own. For example, while having a septum piercing may not have had strongly defined gender associations with "man" and "woman," enough "nonbinary" people may have one for the pattern to be identifiable to observers. As "nonbinary" becomes a category, it takes on such associations which causes it to develop an ideal and collapse to a point. This phenomenon is represented by Figure 1B.

When "nonbinary" takes on a category identity and collapses to a point, it is no longer the case that all the gender categories are as far away from each other as possible. This means they are not maximally descriptively useful. As a result, the points shift to maximize the distance between them (represented in Figure 1C). If these points did not shift, there would be an unnecessarily large gap in the gender space with no categories to define it.
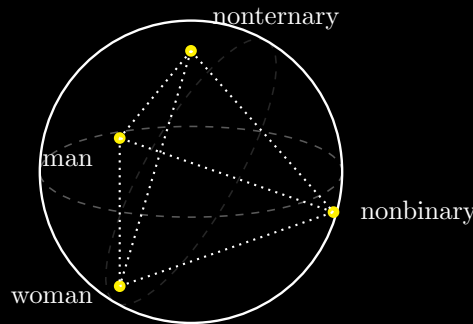
3.4   Implications

Figure 2: Four maximally separated categories on a sphere. Dotted lines indicate the tetrahedral edges, showing that all pairwise distances are equal.

The process of category emergence can, in theory, be repeated indefinitely. When individuals begin to cluster in regions of the space that are not close to the existing categories "man," "woman," and "nonbinary," a new category—"nonternary"—emerges. These categories will likely be re-named as they develop. When the "nonternary" gender appears, the categories must again maximally differentiate, as shown in Figure 2.

The emergence of a fourth category (and any subsequent ones) is theoretically simpler than the initial introduction of the third. The midpoint-splitting that produced "nonbinary" was ambiguous—its equatorial universe contained infinitely many potential positions. The addition of a

fourth point is discrete: the configuration of three existing categories determines only two antipodal locations that maximize separation.

Once category drift is introduced, the minimal distance between categories decreases as new ones are added. Consequently, the original categories "man" and "woman" become closer in relative terms over time.

Socially, this model implies that the recognition of the "nonbinary" category lowers the threshold for further category emergence. Each new distinction makes subsequent ones easier to imagine, while the difference between man-coded and woman-coded behavior diminishes.

4 Generalization to social categories

The framework applies beyond gender. It is especially suited to category systems where classification is based on collective perception and where the goal is to transform continuous variation into discrete, socially meaningful categories.

This framework is therefore most applicable to domains where consensus-driven perception discretizes an underlying continuum. Examples of such domains include gender, political identity, musical genres, and occupational roles.

5 Conclusion

We have proposed a mathematical framework for modeling gender as a socially realized quantity, defined externally through the aggregation of observers' judgments. Beginning with the Axis Model, we showed how midpoint-splitting yields a potentially infinite proliferation of categories. In the Sphere Model, we argued that a spherical topology better captures the dynamics of category formation: sets of possible midpoints collapse into single points and categories drift collectively to maximize distinctiveness.

The benefit of this approach lies in its clarity and extensibility. Familiar claims about the instability and proliferation of gender categories receive a precise formal treatment. The same structure can be applied to other consensus-driven category systems. Continuous variation is discretized into labels through social consensus, fuzzy membership, midpoint emergence, and drift.

The framework is deliberately minimal and focused on explaining the core dynamics of category definition and emergence. Future work can make the model more concrete and descriptive by defining observer weights or aggregation patterns. The details of these enhancements could be informed by empirical study.

The model advances a simple claim: when categories are socially realized, consensus-based, and grounded in continuous variation, they can proliferate without bound and reconfigure over

time. Emergence and reconfiguration become easier over time because midpoint splitting becomes more straightforward and category drift requires less reorientation. The pursuit of descriptive usefulness for gender categories encourages the proliferation of new gender categories.